Financial Risk Management in Electricity Markets

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Abstract

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This research studies a decision problem to allocate an electricity trading firm's budget to its trading strategies using a risk management framework. The considered problem consists of maximizing a firm's profit while controlling two risk measures: the variance of the portfolio and the conditional value at risk. The dependence structure of the returns associated with different trading strategies is modelled using vine copulas and it is assumed that the marginal distribution of the returns originates from the Johnson family of distributions. The studied problem is formulated as a stochastic integer quadratic program and solved it with a commercial optimization software. The proposed mathematical program is assessed on the firm's portfolio and the obtained results are discussed.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

After Tomas Edison invented the light bulb in 1879, he founded the first electrical power plant in the United States. The next step for him was to find and build an electrical distribution system as there was no infrastructure to deliver electricity at that time. Therefore, the very first model of electricity distribution was a "vertically-integrated " model where all the power plants, transmission, and distribution lines belonged to the one company which in term, delivered the electricity directly to the end customer.

Shortly afterwards, other electricity companies started working with the same model and they all had their unique customers. As a result, if a company encountered any problems in their plants or the transmission lines, it would result in a blackout for its customers.

The problem was solved when the businessmen lobbied and argued that the publicly-regulated monopolies would lower prices and make the power grid safer and more reliable. They eliminated the competition so that each utility company had the authority to operate within a specific geographical region.

The "vertically-integrated" markets, also known as "traditionally-regulated" markets still exist in different parts of the United States.

As time passed and people were more in need of electricity, the existing electricity market model was no longer adequate. As a result, a federal committee known as the Federal Energy Regulatory Commission (FERC) was formed to supervise and regulate the transmission and sale of electricity in the states and federal borders. Subsequently, the generator companies, transmission line owners, and utilities formed non-profit private organizations referred to as Independent System Operators (ISOs) to share the transmission responsibilities. They obtained the FERC's approval to regulate independently. The goal of the ISOs is to ensure reliability by controlling the power dispatch, transmission and distribution of electricity (Barron, 2019). Figure 1.1 shows the different ISOs and their operation region in North America.

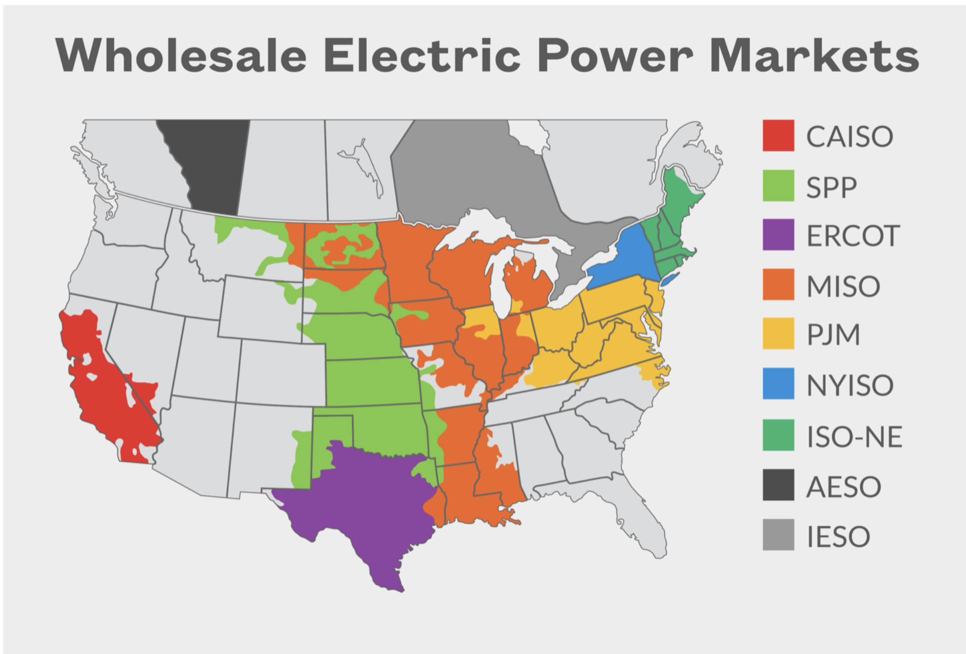


Figure 1.1: The ISOs in North America (Barron, 2019)

Nowadays each ISO has its own electricity market. Different individuals and companies can participate in different electricity markets. One of the most important aspects of the companies that participate in electricity markets is financial risk management. One of the definitions of financial risk is the possibility of losing money on an investment or business venture (Chen, 2020). This definition is also applied to electricity markets. The electricity market participants are always concerned about their profits and losses (P&L) similar to participants in other markets. However, the challenge of electricity is that it cannot be stored. This feature of electricity makes its market more volatile than other markets. The electricity market participants are more exposed to risk as volatility makes risk heavier. In a risk management framework, one of the classic problems is selecting a portfolio such that the net exposure to risk is minimized for a suitable level of profitability.

There are several securities and derivatives in electricity markets that market participants can use in order to avoid risk exposure. A few instances of these instruments are forward contracts, future contracts and different options among others. Forward contracts allow the holder to buy or sell a specific amount of electricity at a predetermined price and time in the future as stated in their contract. Future contracts are similar to the forward contracts except that electricity is not physically traded. Options also operate in a similar manner as in other markets.

An optimal selection of physical and financial approaches using these securities and derivatives can be made in order to create a portfolio with controlled risk. The physical trading approaches are techniques in which the electricity is actually traded such as forward contracts. The financial approaches are the ones that the trade is settled financially, but the electricity will not be traded physically. Using a diverse set of trading approaches can help market participants to hedge against the risk of a single approach.

Some studies provide solutions for the portfolio selection problem in electricity markets. Kaye et al. (1990) studies forward contracts as a risk management instrument for market participants. The authors use a simulation model to analyse the effectiveness of forward contracts in hedging against spot price fluctuations. Collins (2002) argue the effect of future contracts in electricity markets and how their special features can be used to avoid risk. Vehviläinen and Keppo (2003) formulate the portfolio selection problem as a static optimization problem. The proposed model can cover a wide range of instruments in electricity markets, unlike the classic Markowitz (1952) portfolio optimization model which is not easily adaptable in practice. Their optimization model maximizes the profit of an agent, while controlling the risk taken. A Monte Carlo simulation is used to transform the problem to a deterministic non-linear optimization problem. The risk measure used is value at risk, which is the worst possible loss at a given confidence level over a specific time horizon (Jorion, 1996). Liu and Wu (2007) address the problem of portfolio selection from the perspective of a generator company (Genco). The authors optimize a Genco's portfolio in a manner that the profit is maximized while considering the related risk factors. They use modern portfolio theory for their portfolio optimization approach. Modern portfolio theory, also called the

Markowitz portfolio theory, attempts to maximize the expected return of portfolio while simultaneously minimizing the risk (Mangram, 2013). In Liu and Wu's method, decision-makers' risk aversion, and the correlation of different revenues are considered. Furthermore, the risks associated with physical approaches, such as price risk and delivery risk (due to the transmission constraints) are reflected in their model. The portfolio optimization model is formulated as a quadratic programming problem which is then solved in two steps. In the first step, a single risk free asset and a fixed number of risky assets are optimally selected, afterwards, the budget is optimally allocated to the selected assets. Denton et al. (2003) also addresses the issue from a generator/producer perspective. They discuss the market price based commitment unit to evaluate the risk. Eichhorn et al. (2004) use a mixed-integer stochastic program to address the issue. They assume that the observed spot prices and the load data are the realizations of a specific bivariate random variable. Also, they model the joint distribution of the stochastic process by a time series model. In addition, they use a scenario generation method using Monte Carlo simulation from the time series model. The considered risk measure corresponds to the conditional value at risk (CVaR).

Several studies in the literature discuss portfolio optimizations problem from different perspectives and approaches while using different risk measures. In this thesis, a portfolio optimization problem of relevance to an undisclosed electricity trading firm is studied. The firm seeks to allocate a limited budget to four trading strategies. The objective is to maximize their profits, while simultaneously controlling two risk measures: the variance and CVaR of the portfolio.

The main contributions of this thesis are the following. First, a new portfolio optimization model is introduced to represent a complex real-world problem arising in the North-American electricity market. Second, the model is formulated as a stochastic integer quadratic program where the marginal distribution of the trading strategies' returns originates from a family of Johnson distributions. The means of the marginal distributions are used as coefficients in the objective function. In order to estimate the variance of the portfolio, the sample covariance matrix is used. To estimate the CVaR, several scenarios are generated from the joint distribution of the trading strategies' P&Ls. The joint distribution is modelled using vine copulas. The scenarios are different possible P&Ls for different trading strategies in the electricity trading firm's portfolio in a given planning horizon. The third and final contribution is to analyse the results of computational experiments performed using real data. The relative performance of the portfolios obtained with the proposed formulation with respect to the portfolios used by the firm is assessed.

The structure of this thesis is as follows. In Chapter 2, the preliminaries needed for this study, from the details about how one specific ISO operates to the mathematical tools utilized in this research, are provided. Chapter 3 describes in detail the formal definition of the considered problem and its formulation. In Chapter 4 the methods used to solve the problem are explained. Finally, Chapter 5 gives a conclusion of this study and provides some future research directions.

Chapter 2

Preliminaries

In this chapter, a succinct overview of the theoretical, methodological, and practical concepts used during this thesis is provided. The first section introduces the New York Independent System Operators (NYISO). All the other ISOs operate in a similar way. The following sections explain the mathematical tools used in this study in order to formulate the problem of maximizing the profit of the partnering electricity trading company while controlling the risk. The second section introduces the Johnson family of distributions. Afterwards, conditional value at risk is defined. Furthermore, copulas are introduced and in the final section, details about modern portfolio theory are provided.

2.1 New York Independent System Operator (NYISO)

The New York Independent System Operator or NYISO is responsible for managing the electricity market in the New York state. NYISO works with power generation companies, transmission owners, and other utilities to manage electricity through the New York power grid, and meet the customers' demand in order to sustain the reliability of the whole system.

NYISO consists of eleven pricing zones and four interfaces. The interfaces are for importing/exporting electricity from/to the other ISO. These different zones and interfaces are listed in Table 2.1 .

Multiple different entities participate in the electricity market. They include all organizations that participate in producing, transmitting, selling, and/or purchasing for

Zone A	West
Zone B	Genesee
Zone C	Central
Zone D	North
Zone E	Mohawk Valley
Zone F	Capital
Zone G	Hudson Valley
Zone H	Millwood
Zone I	Dunwoodie
Zone J	New York City
Zone K	Long Island
Interface	IESO
Interface	PJM
Interface	Hydro-Quebec
Interface	ISO-NE

Table 2.1: List of the pricing zones and interfaces in NYISO (NYISO, 2020b)

resale capacity, energy, and ancillary services in the wholesale market (NYISO, 2020a).

NYISO's main mission is to administer the power grid in New York State to maintain the reliability and safety of the system. Market participants can submit their bids and asks (also known as bids and offers) to NYISO on a day-ahead basis or on a real-time basis which is explained later in this study. Furthermore, NYISO receives all the bids and asks and uses an algorithm called security-constrained unit commitment to plan the day-ahead market. In the day-ahead planning process, several generators are selected to be dispatched at every hour of the next day. After the dispatch scheduling is done, the electricity prices are calculated. The system of pricing in NYISO referred to as locational based marginal pricing (LBMP). As Dupuis et al. (2017) defines

> "LBMP is essentially the cost to serve the next incremental megawatt (MW) of load at a specific location on the grid, and it is determined by the NYISO following bids and offers. Congestion and transmission losses lead to unequal LBMP at different locations."

The formula to calculate the LBMP is as follows (NYISO, 2020a)

$$LBMP = Marginal\ Cost\ of\ Energy - Congestion + Losses.$$

The marginal cost of energy is the price of electricity offered by a generator which is going to be dispatched if another megawatt (MW) is needed on the grid. When the most economical generator is not able to dispatch due to transmission constraints, dispatch will happen from another generator with a higher price. The difference between this higher price and the most economical price (initial price) is called congestion. Losses refer to the price of the wasted electrical energy during the transmission. All of these three components together determine the price of electricity at the pricing locations in NYISO.

The pricing locations in NYISO are zones. This system of pricing in NYISO is called zonal pricing, meaning that the price of the electricity is constant all over the zone. There is also another system of pricing referred to as nodal pricing where there are different nodes in each zone and each node has a different price.

2.1.1 Electricty Market in NYISO

The NYISO electricity market is a two settlement market. Firstly, there is a spot market, which is the real-time market which is going to be explained later in this section. Secondly, there is a forward market, which is the day-ahead market. The day-ahead energy market lets market participants commit to buy or sell wholesale electricity one day before the operating day, to help avoid price volatility. The day-ahead market is only a financial market.

NYISO receives all the bids and offers after the day-ahead market closes at 5 am the day before the operating day. Afterwards, it solves a co-optimization model simultaneously for every hour of the next operating day to clear supply offers, and demand bids for each hour of the operating day to yield day-ahead schedules. Furthermore, it efficiently allocates transmission capacity to day-ahead schedules by resolving transmission congestion. In addition, NYISO sets the prices for the day-ahead market and releases the information at 9:00 A.M., the day before the operating day.

When the operating day arrives, the market participants are able to trade electricity in real-time. The real-time market is a physical market where market participants buy and sell energy physically. Although the ISO has scheduled everything for the operating day, there are cases where the demand might change. For example, the weather conditions might differ from the forecast. In general, a drop or increase in temperature, unforeseen storms, etc.

are the reasons explaining the difference between the predicted demand and the real-time demand in electricity markets.

The electricity price in the day-ahead market is hourly while it changes every 5 minutes in the real-time market. Furthermore, the hourly price in the real-time market is calculated as the average of the 5 minutes prices.

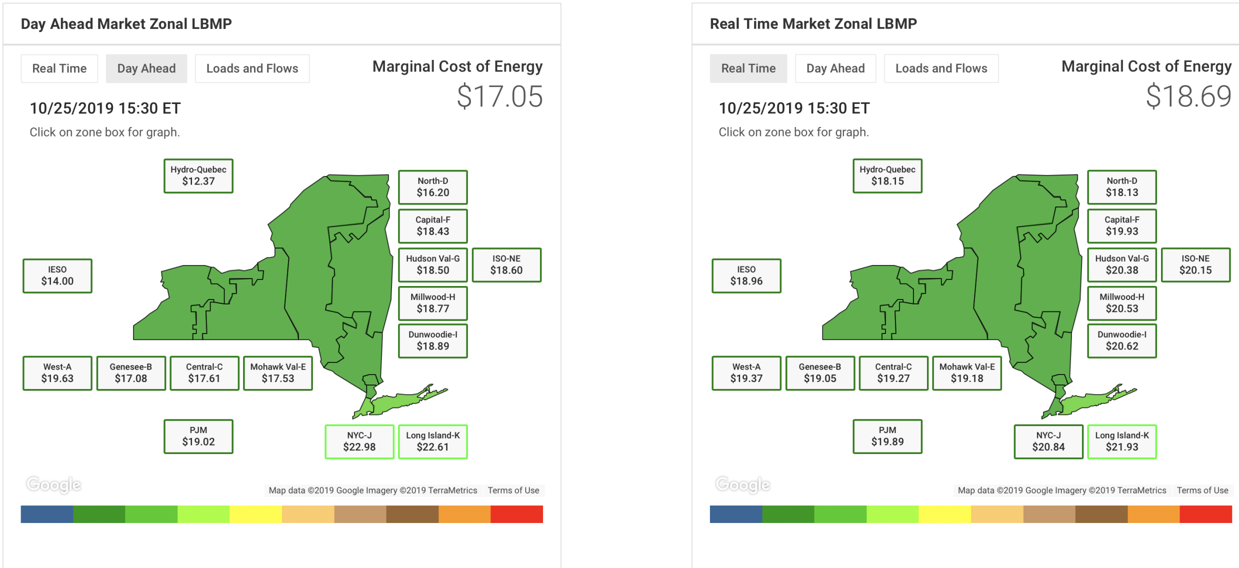
Generally, there are two major participants in the electricity market: physical traders and virtual traders. They can both submit bids and offers in the market, and the ISO considers both of them in the determination of prices. Of course, when it comes to scheduling the dispatch of the electricity, only physical demand and supply are considered. Moreover, virtual transactions only happen in the day-ahead market. Virtual traders can buy or sell a specific amount of electricity (determined by the trader) virtually in the day-ahead market at the day-ahead price, and the exact amount should be sold back or repurchased in the real-time market at the real-time price since electricity cannot be stored.

The ISO can distinguish between the physical and the financial (virtual) bids and offers. The allocation of the transmission capacity is only based on physical bids and offers. Also, when the demand deviates from what was scheduled the day before, throughout the operating day, the ISO commits unscheduled resources at least-cost to meet the energy requirements.

Usually, when demand is lower than the scheduled demand, the real-time prices are less than the day-ahead prices. Furthermore, when the demand is higher than what was scheduled, the real-time prices will be higher than the day-ahead prices since the more expensive generators get dispatched. However, this is not always the case; sometimes higher real-time prices are observed because of the outage of a generator or damage to electric transmission lines.

One of the main differences between the electricity markets and markets with other commodities is that electricity cannot be stored or is simply too expensive to store. Therefore, there needs to be a continuous balance between supply and demand in real-time. However, this usually does not happen in practice. This deviation from the day-ahead schedule causes a non-zero spread between the day-ahead and the real-time prices. The market is said to be efficient when the spread is zero.

Figure 2.1 shows the graphical representation of the day-ahead and real-time prices for



(a) Day-ahead prices

(b) Real-time prices

Figure 2.1: A graphical representation of the day-ahead and real-time prices in NYISO. (NYISO, 2020b)

the zones in NYISO. The day ahead prices are the ones that scheduled the day before.

In virtual bidding, the bids are settled hourly before the day-ahead market closes the day before the operating day. Virtual bidders can set a maximum and a minimum for the price such that if the price is not in that interval, they will not trade. Furthermore, they have specified the trade volume and have decided if they are going to buy or sell that energy at the day-ahead price or not.

2.2 Johnson SU distribution

The family of Johnson distributions was introduced by Johnson (1949) which contains four distributions: normal, lognormal, Johnson SB, and Johnson SU distribution. Johnson SB models the bounded distributions and Johnson SU models the unbounded distributions. One of the features of this family of distributions is that by using the elementary functions, they can be transformed into a normal distribution. Furthermore, this transformation is invertible.

Moreover, Johnson SU and Johnson SB distributions have four parameters, and this fact makes them cover a broad range of distribution shapes.

The Probability Distribution Function (pdf) of this family of Johnson distribution is

$$f(x) = \frac{\delta}{\lambda\sqrt{2\pi}} g'\left(\frac{x-\xi}{\lambda}\right) e^{-\frac{1}{2}\left(\gamma+\delta g(\frac{x-\xi}{\lambda})\right)^2}$$

where $g(y) = ln(\frac{y}{1-y})$, $g'(y) = \frac{1}{y(1-y)}$ for the SB family and $g(y) = ln(y + \sqrt{y^2+1})$, $g'(y) = \frac{1}{\sqrt{y^2+1}}$ for the SU family. For the SB family the support of x is $[\xi, \xi+\lambda]$ and for the SU family the support of x is $(-\infty, +\infty)$ (George and Ramachandran, 2011). This family of distributions are defined on $(-\infty, +\infty)$.

The distribution used in this study is the Johnson SU distribution. It has two shape parameters, γ and $\delta > 0$, a scale parameter $\lambda > 0$ and a location parameter ξ. The pdf of this distribution is as follows

$$f(x) = \frac{\delta}{\lambda\sqrt{2\pi(1+\left(\frac{x-\xi}{\lambda}\right)^2)}} e^{-\frac{1}{2}\left(\gamma+\delta sinh^{-1}\left(\frac{x-\xi}{\lambda}\right)^2\right)}.$$

The mean of the distribution is given by

$$\xi - \lambda \ \exp\frac{\delta^2}{2} sinh\left(\frac{\gamma}{\delta}\right).$$

The use of this distribution is explained in detail in the next chapter. Figure 2.2 shows the probability density function of the Johnson SU distribution with different values for γ and δ. For all the curves in this figure, $\theta = 0$ and $\sigma = 1$.

2.3 Conditional Value At Risk

Value at risk (VaR) and conditional value at risk (CVaR) are two popular risk measures. VaR is the maximum loss at a given confidence level in a determined time horizon while CVaR, also known as expected shortfall or mean excess loss, is, under some assumptions, the conditional expectation of the losses that exceed VaR.

Let $x = (x_1, \cdots, x_n)$ be a real vector containing the number of units allocated to all components of a portfolio and $f(x, y)$ is the associated random loss function to the portfolio

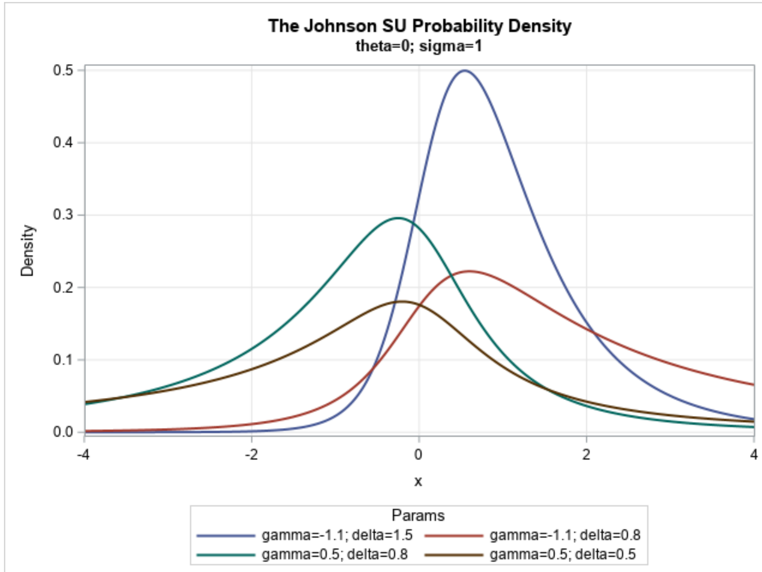


Figure 2.2: The probability density function of Johnson SU distribution with different values for γ and δ (Wicklin, 2020)

x, where y is the realization of the random events (such as the P&L of the electricity trading firm for different trading strategies). Assume that Y is absolutely continuous and $p(y)$ is the probability density function of the random vector Y.

The cumulative distribution function (cdf) of the loss associated with portfolio x can be defined by

$$\Psi(x, \gamma) := \int_{f(x,y)<\gamma} p(y)dy.$$

With this definition, the VaR_α of portfolio x is defined as follows

$$VaR_\alpha(x) := min\{\gamma \in \mathbb{R} : \Psi(x, \gamma) \geq \alpha\}.$$

As a result, the $CVaR_\alpha$ of portfolio x at given confidence level α is

$$CVaR_\alpha(x) := \frac{1}{1-\alpha} \int_{f(x,y)\geq VaR_\alpha(x)} f(x, y)p(y)dy.$$

Figure 2.3 shows a graphical representation of VaR and CVaR.

It can be shown that $CVaR_\alpha(x)$ is always greater than or equal to $VaR_\alpha(x)$. So, VaR_α is a lower bound for $CVaR_\alpha$.

In the case of having a discrete probability distribution for random events y, the expected

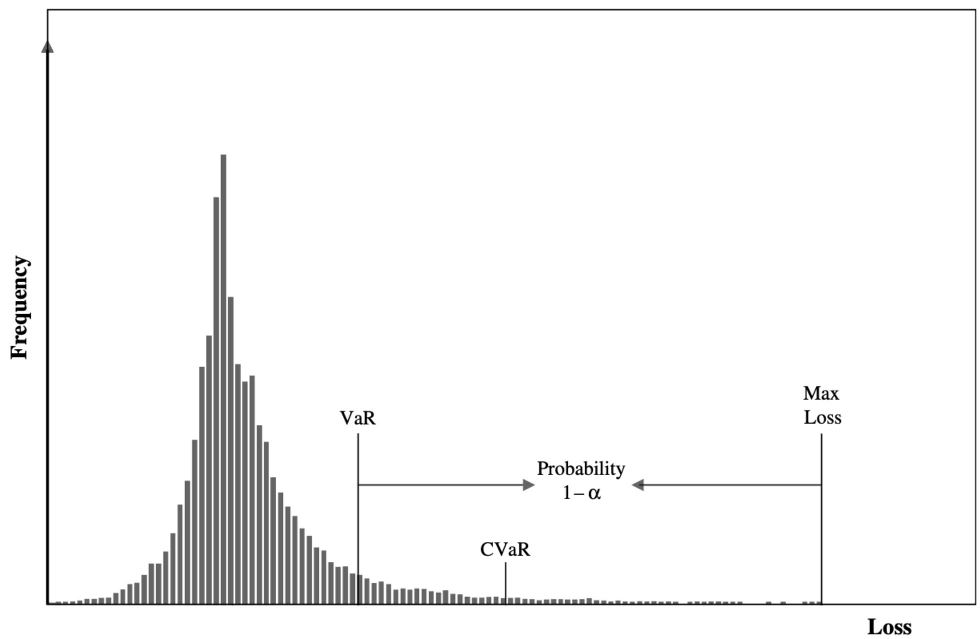


Figure 2.3: The graphical representation of VaR and CVaR (Sarykalin et al., 2008)

shortfall can be defined as

$$CVaR_{\alpha}(x) = \frac{1}{1-\alpha} \sum_{j:f(x,y_j)\geq VaR_{\alpha}(x)} p_j f(x, y_j)$$

Where p_j is the probability of the event y_j.

The reason for using CVaR instead of VaR as a risk measure is the undesirable features of VaR. The absence of subadditivity and convexity made the VaR a hard to use tool in mathematics and specially in optimization (Artzner et al., 1999). Furthermore, CVaR has the VaR function in its definition. As a result, an auxiliary function is proposed to be used instead of CVaR as the following

$$F_{\alpha}(x, \gamma) := \gamma + \frac{1}{1-\alpha} \int_{f(x,y)\geq\gamma} (f(x, y) - \gamma)p(y)dy$$

This function has the following properties (Cornuejols and Tütüncü, 2006):

1. $F_{\alpha}(x, \gamma)$ is a convex function of γ

2. $VaR_\alpha(x)$ is a minimizer over γ of $F_\alpha(x,\gamma)$

3. The minimum value over γ of the function $F_\alpha(x,\gamma)$ is $CVaR_\alpha(x)$

As a consequence, instead of minimizing the $CVaR_\alpha(x)$ over x, $F_\alpha(x,\gamma)$ can be optimized over x and γ with no need to calculate VaR explicitly.

Furthermore, an approximation of this function can be made with discretizing. Instead of a random event, there are a number of scenarios that are different possible realizations of the random event. For, $k = 1, \cdots, K$, let y_k be the scenarios (possible values) for the random variable y and p_k be the probability of scenario k. In this case, the above function can be approximated as

$$\tilde{F}_\alpha(x,\gamma) := \gamma + \frac{1}{1-\alpha}\sum_{k=1}^{K} p_k(f(x,y_k)-\gamma)^+.$$

2.4 Copulas

The material explained in this section is mostly from Schmidt (2007). To model the dependence structure between random variables, several tools in mathematics can be used. One of the tools which has been developed for a long time but has triggered extensive recent interest are copulas. Basically, a copula is a multivariate distribution function with uniform marginals. The first appearance of the name "copula" was in Sklar (1959). The root of the name "copula" is the Latin word *copular*, meaning to connect or to join. A d-dimensional copula $C : [0,1]^d :\rightarrow [0,1]$ is a cumulative distribution function with uniform marginals (Schmidt, 2007). Copulas are mostly famous for modelling the dependence structure and the marginals separately. The following proposition is the reason that copulas can do the magic.

In this proposition, $F^{\leftarrow}(y)$ is defined as $F^{\leftarrow}(y) := inf\{x : F(x) \geq y\}$, the generalized inverse of F.

Proposition 2.4.1. *If $U \sim U[0,1]$ and F is a cumulative distribution function, then*

$$P(F^{\leftarrow}(U) \leq x) = F(x).$$

On the contrary, if the real-valued random variable Y has a distribution function F and F is *continuous, then*

$$F(Y) \sim U[0,1]$$

.

As a result of this proposition, the copula does not depend on the marginal distributions of its random variables and it can be determined separately. Therefore, if the marginal distribution of one or more random variables changes but the dependence structure remains the same, the copula would be the same as well.

Sklar (1959) proved that all multivariate cumulative distributions can be written in terms of copulas in the Sklar representation theorem. Also, it indicates that the copula can be determined uniquely if the marginals are continuous.

Theorem 2.4.2. *Consider a d-dimensional cdf F with marginals $F_1, \ldots, F_d$. There exists a copula C, such that*

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d))$$

for all x_i in $[-\infty, +\infty]$, $i = 1, \ldots, d$. If F_i is continuous for all $i = 1, \ldots, d$ then C is unique; otherwise C is uniquely determined only on $Ran\, F_1 \times \ldots \times Ran\, F_d$, where $Ran\, F_i$ denotes the range of the cdf F_i.

On the other hand, consider a copula C and univariate cdfs $F_1, \ldots, F_d$. Then F as defined above is a multivariate cdf with marginals $F_1, ..., F_d$.

Moreover, if F is absolutely continuous and $F_1, \cdots, F_d$ are strictly increasing continuous, we have:

$$f(x_1, \cdots, x_d) = \Big[\prod_{k=1}^{d} f_k(x_k)\Big] \times c(F_1(x_1), \cdots, F_d(x_d))$$

where f and f_k represent respectively the joint density underlying distribution F and marginal densities of each of its components, and c is the associated copula.

Furthermore, Hoeffding and Frechet stated that a copula always has a certain upper bound and lower bound. The following theorem is what Hoeffding and Frechet came up with independently for the bounds of a copula.

Theorem 2.4.3. ***(Frechet-Hoeffding bounds)*** *Consider a copula $C(u) = C(u_1, \ldots, u_d)$.*

Then

$$max\Big\{\sum_{i=1}^{d} u_i + 1 - d, 0\Big\} \leq C(u) \leq min\{u_1, \ldots, u_d\}$$

.

In the next section, some of the most important families of copulas and the ones that are used in this research are introduced. Some of them are derived from other multivariate distributions such as multivariate normal distribution and some are defined explicitly.

2.4.1 Perfect Dependence and Independence

In the case of no dependence between the random variables $u_1, \ldots, u_d$, their copula is the *independence copula* defined as

$$\Pi(u) = \prod_{i=1}^{d} u_i.$$

Also, the copula that models the perfect positive dependence between random variables is the *comonotononicity copula* or *Frechet-Hoeffding upper bound* which is given by

$$M(u) = min\{u_1, \ldots, u_d\}.$$

In addition, there is *countermonotonicity copula* or *Frechet- Hoeffding lower bound* which can only be obtained in the two-dimensional case. The countermonotonicity copula is defined as

$$W(u_1, u_2) = max\{u_1, u_2 - 1, 0\}.$$

2.4.2 Gaussian Copula and T-Copula

There are some families of copulas that are derived from multivariate distributions. The first copula that is introduced in this section is the Gaussian copula, which is part of a larger family of copulas called the Ellipitical family of copulas. There is no closed-form expression for the Gaussian copula but using the Sklar's representation theorem, the two-dimensional Gaussian copula can be represented by

$$C_{\rho}^{Ga}(u_1, u_2) = \Phi_{\Sigma}\Big(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\Big).$$

In this representation, Σ is the correlation matrix with 1 on the diagonal and ρ otherwise. Φ is the standard normal distribution and Φ_Σ is the bivariate normal distribution with mean equal to zero and correlation matrix Σ. The above representation can also be written as

$$C_\rho^{Ga}(u_1, u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} exp\Big(- \frac{s_1^2 - 2\rho s_1 s_2 + s_2^2}{2(1-\rho^2)}\Big) ds_1 ds_2.$$

Here, for $\rho = 0$ the Gaussian copula becomes the independence copula, for $\rho = 1$, it becomes the comonotonicity copula and for $\rho = -1$, it becomes the countermonotonicity copula.

A t-copula (or student t-copula) represents the copula of a multivariate t-distribution whose marginal distributions in the latter are student-t. The representation of a d-dimensional t-copula is given by

$$C_{\nu,\Sigma}^t(u_1, u_2) = t_{\nu,\Sigma}\Big(t_\nu^{-1}(u_1), \ldots, t_\nu^{-1}(u_d)\Big)$$

where Σ is the correlation matrix, t_ν is the cumulative distribution of a univariate t-student distribution with ν degree of freedom and $t_{\nu,\Sigma}$ is multivariate cumulative distribution with correlation matrix Σ and ν degree of freedom.

Figure 2.4 demonstrates the density of a Gaussian copula and a t-copula.

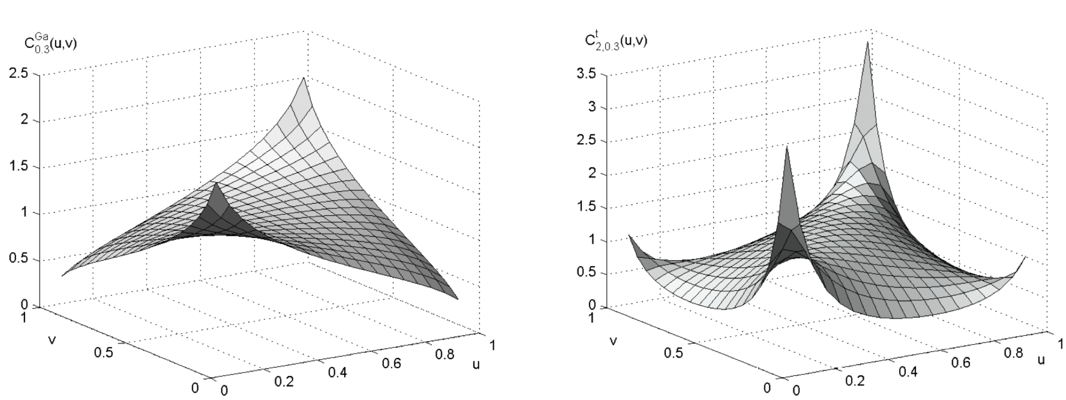


Figure 2.4: The graphical representation of a density associated to the Gaussian copula (left) and a t-copula (right). The correlation coefficient for both of the copulas is $\rho = 0.3$ and the degree of freedom for the t-copula equals to 2 (Schmidt, 2007)

2.4.3 Archimedean Copulas

This family of copulas are defined explicitly and they are not derived from any multivariate distributions. The Archimedean copulas have closed from for their density functions. In this section only two-dimensional Archimedean copulas are introduced, unless otherwise stated, for the sake of simplicity. The form of a two-dimensional Archimedean copula is given by

$$C_\theta(u_1, u_2) = \phi^{-1}[\phi(u_1; \theta) + \phi(u_2; \theta); \theta],\ (u_1, u_2) \in [0, 1]^2,\ \theta \in \Theta$$

where $\phi : [0, 1] \times \Theta \to \mathbb{R}_+$ is the generator function of the copula which is a strictly decreasing convex function with dependence parameter θ. ϕ^{-1} represents the inverse function of ϕ. Different generator functions leads to different copulas in this family.

The *Gumbel copula* Gumbel (1960) is defined as

$$C_\theta^{Gu}(u_1, u_2) = exp[-\Big((-ln(u_1)^\theta + -ln(u_2)^\theta)\Big)^{\frac{1}{\theta}}],\ \theta \in [1, \infty)$$

by using $\phi_{Gu}(u) = -ln(u)^\theta$ as the generator function. If $\theta = 1$ the independence copula can be obtained and when $\theta \to \infty$ the Gumbel copula becomes the comonotonicity copula.

If the generator function $\phi_{Cl}(u) = \frac{1}{\theta}(u^{-\theta} - 1)$ is used, the *Clayton copula* Clayton (1978) would be obtained. The closed form for the Clayton copula is

$$C_\theta^{Cl}(u_1, u_2) = \Big(max\{u_1^{-\theta} + u_2^{-\theta}, 0\}\Big)^{-\frac{1}{\theta}},\ \theta \in [-1, \infty)\backslash\{0\}.$$

Here, the results from setting $\theta = 0$ and $\theta \to \infty$ are the same as in the Gumbel copula. Furthermore, for $\theta = -1$ the Clayton copula becomes the countermonotonicity copula.

The generator function $\phi_{Fr}(u_1, u_2) = ln(e^{-\theta} - 1) - ln(e^{-\theta u} - 1)$ gives the *Frank copula* Frank (1979) defined as follows

$$C_\theta^{Fr}(u) = -\frac{1}{\theta} ln\Big(1 + \frac{(e^{-\theta}u_1 - 1)(e^{-\theta}u_2 - 1)}{e^{-\theta} - 1}\Big),\ \theta \in \mathbb{R}\backslash\{0\}.$$

Frank copula reaches both the upper and lower Frechet bounds as $\theta \to \infty$ and $\theta \to -\infty$, respectively. Also, it becomes the independence copula as $\theta \to 0$.

Joe copula Joe (1993) is another copula in the Archimedean family of copulas. The generator function for joe copulas is $\phi_{Jo}(u) = -ln[1-(1-u)^{\theta}]$. Joe copula has the following copula form

$$C_{\theta}^{Jo}(u_1, u_2) = 1 - [(1-u_1)^{\theta} + (1-u_2)^{\theta} - (1-u_1)^{\theta}(1-u_2)^{\theta}]^{\frac{1}{\theta}},\ \theta \in [1, \infty).$$

Joe copula becomes the comonotonicity copula when $\theta \to \infty$.

Figure 2.5 is the graphical representation of the copulas which are introduced above.

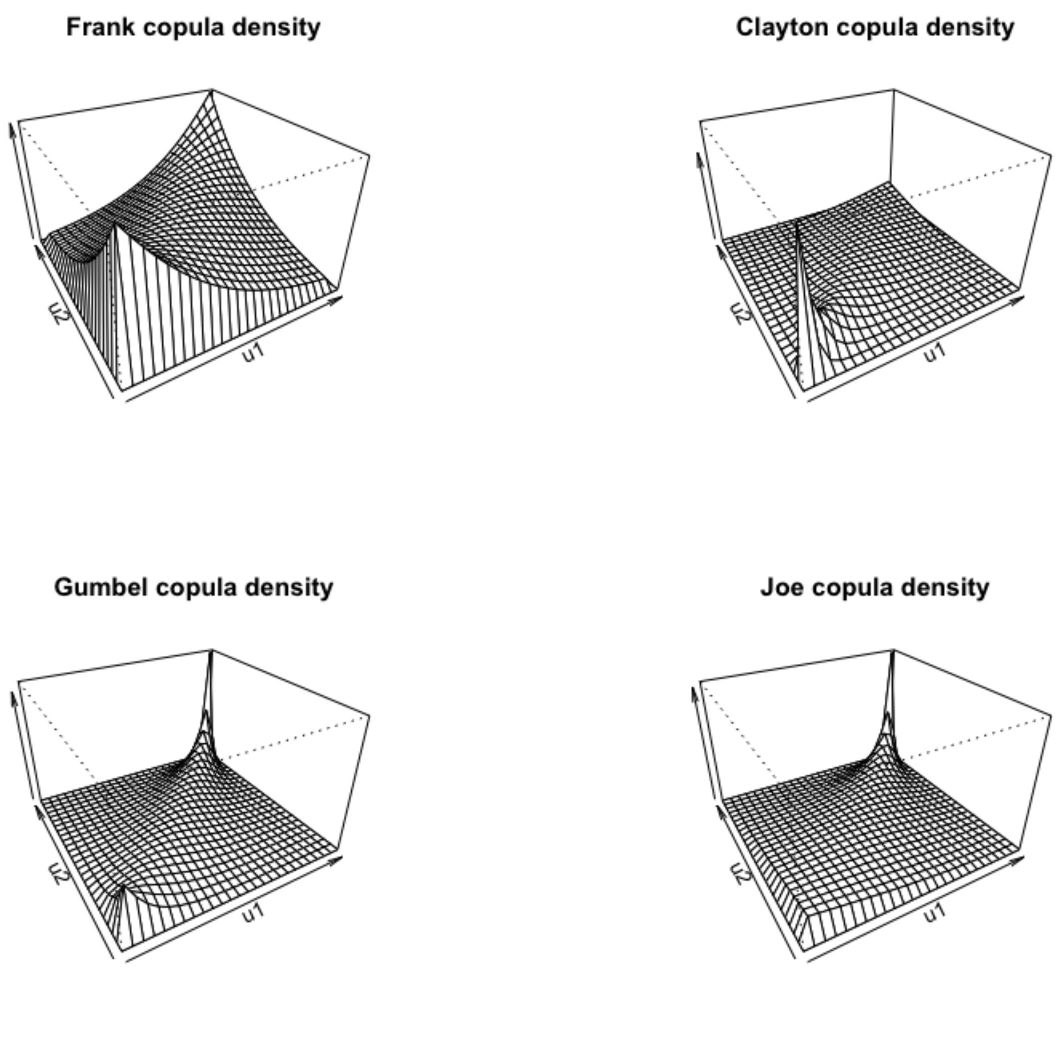


Figure 2.5: Densities of Frank copula (upper left), Clayton copula (upper right), Gumbel copula (lower left) and Joe Copula (lower right). For all the copulas $\theta = 3$

Some important families of copulas have been introduced in two dimensions. Fitting a two-dimensional copula to data is usually an easy task but modelling the dependence in higher dimensions is a challenge. The dependence structure of a random vector in practice is usually not as symmetric in tails as for the Gaussian copula and t-copula. Furthermore, the Archimedean copulas which usually have one or two parameters apply a strong dependence structure which might not be representative of empirical observations in practice. Vine copulas overcome this issue by using bivariate copulas as building blocks of a higher dimensional copula. In the next section, the vine copulas are explained in detail.

2.5 Vine Copulas

Pair copula constructions (PCCs) were introduced by Aas et al. (2009). Vines are graphical representations of PCCs. For illustrative purposes, a three dimensional PCC is going to be introduced which will make explanations for higher dimension cases more straightforward.

Let $X = (X_1, X_2, X_3)' \sim F$ and suppose that all required densities exist. It holds that

$$f(x_1, x_2, x_3) = f_1(x_1)f(x_2|x_1)f(x_3|x_1, x_2).$$

The following result can be obtained by using the Sklar's theorem

$$\begin{aligned} f(x_2|x_1) =& \frac{f(x_1, x_2)}{f_1(x_1)} = \frac{c_{1,2}(F_1(x_1), F_2(x_2))f_1(x_1)f_2(x_2)}{f_1(x_1)} \\ =& c_{1,2}(F_1(x_1), F_2(x_2))f_2(x_2) \end{aligned}$$

and

$$\begin{aligned} f(x_3|x_1, x_2) =& \frac{f(x_2, x_3|x_1)}{f(x_2|x_1)} = \frac{c_{2,3|1}(F(x_2|x_1), F(x_3|x_1))f(x_2|x_1)f(x_3|x_1)}{f(x_2|x_1)} \\ =& c_{2,3|1}(F(x_2|x_1), F(x_3|x_1))f(x_3|x_1) \\ =& c_{2,3|1}(F(x_2|x_1), F(x_3|x_1))c_{1,3}(F_1(x_1), F_3(x_3))f_3(x_3) \end{aligned}$$

with

$$F(x|\boldsymbol{\nu}) = \frac{\partial C_{x,\nu_j|\boldsymbol{\nu}_{-j}}(F(x|\boldsymbol{\nu}_{-j}), F(\nu_j|\boldsymbol{\nu}_{-j}))}{\partial F(\nu_j|\boldsymbol{\nu}_{-j})}.$$

Here $C_{x\nu_j|\boldsymbol{\nu}_{-j}}$ is a bivariate copula and $\boldsymbol{\nu}_{-j}$ is a vector of components of x where the jth component ν_j is removed.

The above three equations can be combined and give a joint density with three dimensions using only two-dimensional copulas

$$\begin{aligned} f(x_1, x_2, x_3) =& f_1(x_1)f_2(x_2)f_3(x_3)c_{1,2}(F_1(x_1), F_2(x_2)) \\ & \times c_{1,3}(F_1(x_1), F_3(x_3))c_{2,3|1}(F(x_2|x_1), F(x_3|x_1)). \end{aligned}$$

All the two-dimensional copulas in the above equation can be independently determined and as a result PCCs can model a broad range of dependence structures in different number of dimensions.

There is an assumption that is usually used in the literature for simplifying purposes which is the pair copula $C_{2,3|1}$ only depends on x_1 through the arguments $F(x_2|x_1)$ and $F(x_3|x_1)$. This assumption is looked into by Stoeber et al. (2013) and Haff et al. (2010).

As mentioned before, vine copulas represent the PCCs graphically. Kurowicka and Cooke (2006) define a *regular vine* (R-vine) on d variables as a sequence of linked trees (connected acyclic graphs) $T_1, \cdots, T_{d-1}$ with nodes N_i and edges E_i for $i = 1, \cdots, d-1$, where T_1 has nodes $N_1 = 1, \cdots, d$ and edges E_1, and for $i = 2, \cdots, d$ tree T_i has nodes $N_i = E_{i-1}$. Moreover, the proximity condition requires that two edges in tree T_i are joined in tree T_{i+1} only if they share a common node in tree T_i .

Bedford et al. (2001) and Kurowicka and Cooke (2006) showed that two nodes, called the *conditioned nodes* and a set of *conditioning nodes* can uniquely describe the edges in an R-vine tree. This means that the edges are denoted by $e = j(e), k(e)|D(e)$ where $D(e)$ is the conditioning set. Figure 2.6 is an example of a seven dimensional R-vine tree.

A two dimensional copula density $c_{j(e),k(e)|D(e)}$ can be associated to each edge $e = j(e), k(e)|D(e)$ in E_i to construct the multivariate copula linked with trees $T_1, \ldots, T_{d-1}$. According to Kurowicka and Cooke (2006) and R-vine copula density can be uniquely

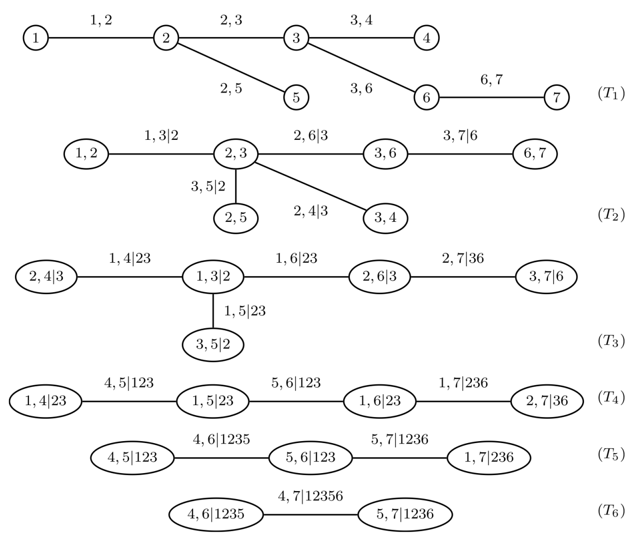


Figure 2.6: An example of a seven dimensional R-vine tree (Dissmann et al., 2013)

described by

$$c(F_1(x_1), \cdots, F_d(x_d)) = \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}(F(x_{j(e)}|\mathbf{x}_{D(e)}), F(x_{k(e)}|\mathbf{x}_{D(e)}))$$

where $\mathbf{x}_{D(e)}$ denotes a subset of the elements of $\mathbf{x} = (x_1, \cdots, x_d)'$ indicated by the indices contained in $D(e)$ (Brechmann and Czado, 2013).

2.6 Modern Portfolio Theory

Markowitz is the founder of modern portfolio theory. He modelled the portfolio optimization as the selection of assets to optimize mean and the variance of the portfolio (Markowitz, 1952). In his approach, the variance can be constant and the expected profit can be maximized, or the expected profit can be constant and the variance of the portfolio can be minimized.

The details of the modern portfolio theory, also known as the mean-variance optimization (MVO), is explained according to Chapter 8 of Cornuejols and Tütüncü (2006). First, some notations are explained.

Let $S_1, \ldots, S_n$ be $n \geq 2$ assets with random returns, μ_i and σ_i be the expected value and standard deviation of the return of the asset S_i and ρ_{ij} be the correlation coefficient of the return of the assets S_i and S_j for $i \neq j$. Then μ can be defined as the random vector of the expected values of the assets, given by $\mu = [\mu_1, \ldots, \mu_n]^T$. Also, $\Sigma = (\sigma_{ij})$ denotes the $n \times n$ covariance matrix given $\sigma_{ii} = \sigma_i^2$ and $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$ for all $i \neq j$. The fraction of the total budget invested in asset S_i is shown by x_i and $x = (x_1, \ldots, x_n)$ denotes the associated portfolio for the assets $S_1, \ldots, S_n$.

With the above notation, the expected return of the portfolio x can be written as

$$\mathbb{E}[x] = \mu_1 x_1 + \ldots, \mu_n x_n = \mu^T x$$

Furthermore, the variance of the portfolio x is given by

$$Var(x) = \sum_{i,j} \rho_{ij}\sigma_i\sigma_j x_i x_j = x^T \Sigma x$$

with $\rho_{ii} \equiv 1$. The variance is always a non-negative value and as a result $x^T\Sigma x \geq 0$. This result shows that the covariance matrix Σ is a positive semi-definite matrix. In addition, it is assumed that there is no redundant asset in the portfolio, which means that the covariance matrix is a definite positive matrix.

Furthermore, the set of feasible portfolios is assumed to be the set $X := \{x|Ax = b, Cx \geq d\}$. Here, A is an $m \times n$ matrix, b is an m-dimensional vector, C is a $p \times n$ matrix and d is a d-dimensional vector. This set is the set of constraints on the portfolio. One of the most common constraints is

$$\sum_{i=1}^{n} x_i = 1$$

which implies x can be understood as weights. This constraint alongside the sign constraint $x \geq 0$ (which means no short-sale is allowed), forces the optimization problem to allocate positive fractions of the total fund to different assets. A similar constraint is given by

$$\sum_{i=1}^{n} x_i = B$$

where B is the total budget or total fund of the portfolio.

Solving the mean-variance optimization problem results in building an efficient frontier. To define the efficient frontier, one needs to know the definition of an efficient portfolio. An efficient portfolio is a portfolio with maximum return in the set of portfolios with the same variance or has the minimum variance in the set of portfolios with equal returns. The set of efficient portfolios builds an efficient frontier. Figure 2.7 illustrates an efficient frontier. It can be seen that as the profit increases, the variance of the portfolio increases as well. All the feasible portfolios below the curve are not efficient. As explained before, Σ is a definite

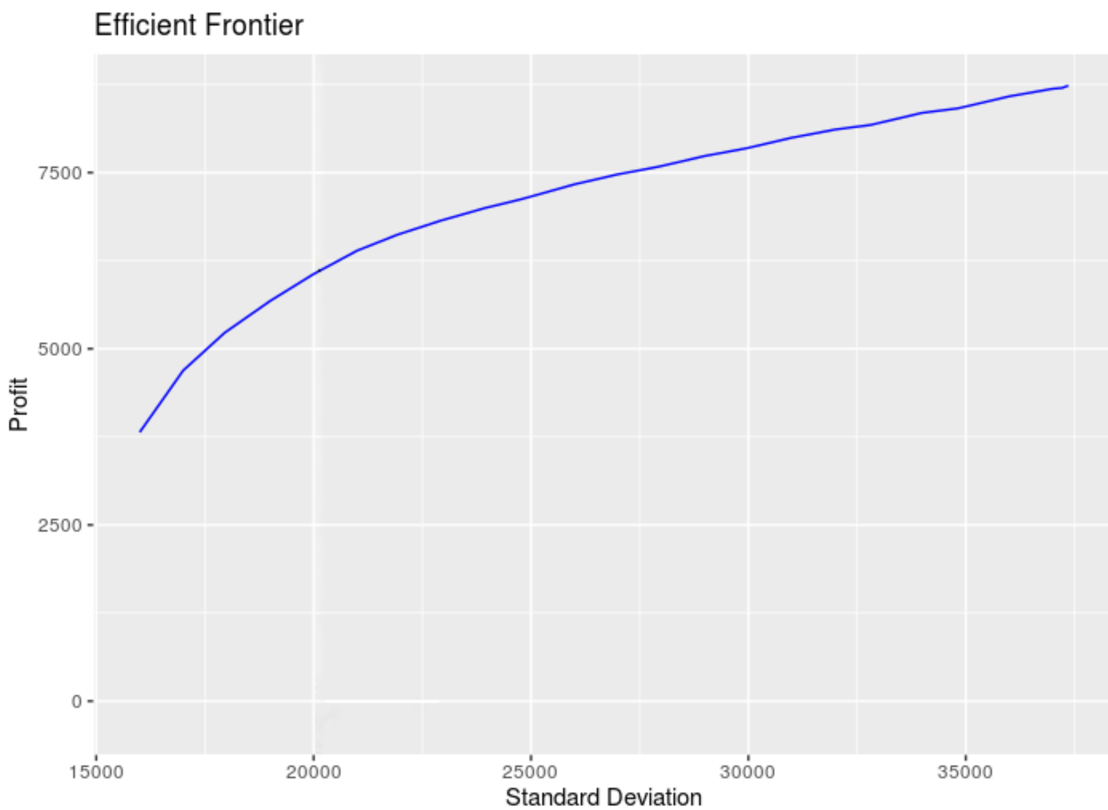


Figure 2.7: An example of an efficient frontier

positive matrix and as a result, the variance is a convex function of x. Consequently, there exists a unique feasible portfolio that has minimum variance.

Cornuejols and Tütüncü (2006) formulated the Markowitz' mean-variance optimization (MVO) in three equivalent ways. In the first mathematical formulation, the problem is trying to minimize the variance of the portfolio while there is a lower bound for the return of the portfolio. This formulation is a quadratic convex programming problem and is given by

$$
\begin{aligned}
\max \quad & \frac{1}{2}x^T\Sigma x \\
s.t. \quad & \mu^T x \geq R && (1) \\
& Ax = b && (2) \\
& Cx \geq d. && (3)
\end{aligned}
$$

Constraint (1) sets a lower bound, R, for the expected return of the portfolio. To obtain the efficient frontier, one can set different values for R, ranging between the expected return of a portfolio with the minimum variance and the expected return of a portfolio with the maximum expected return. The constant coefficient $\frac{1}{2}$, does not affect the optimal solution and it is only added for the sake of simplicity in the optimality conditions.

The second equivalent formulation for MVO is given by

$$
\begin{aligned}
\max \quad & \mu^T x \\
s.t. \quad & x^T\Sigma x \leq \sigma^2 && (4) \\
& Ax = b && (5) \\
& Cx \geq d. && (6)
\end{aligned}
$$

This formulation is attempting to maximize the expected return of the portfolio where the variance of the portfolio is not more than σ^2 which is an upper bound for the variance of the portfolio.

The other form of this formulation can be written as

$$
\begin{aligned}
\max \quad & \mu^T x - \frac{\delta}{2}x^T\Sigma x \\
s.t. \quad & Ax = b && (7) \\
& Cx \geq d. && (8)
\end{aligned}
$$

Here the variance of the portfolio is added to the objective function as a penalty term with a constant coefficient δ which acts as a risk-aversion constant. This objective function is called a *risk-adjusted return* function. It is equivalent because of Lagrange multiplier based solutions to the original problem.

Chapter 3

Portfolio Allocation Model

This chapter explains the process of the formulation of the optimization problem. The main goal of this study is to allocate the budget of the electricity trading firm to different trading strategies such that the risk measures, the variance of the portfolio, and CVaR stay lower than the associated upper bound.

The P&L of the firm in a period is an unknown value. This time horizon can be a day, a week, a month, etc. Each firm is trying to maximize its P&L in a certain period and this period can be determined by the firm itself. As in the mean-variance optimization problem, the expected P&L of the firm's portfolio is going to be maximized in this research.

Furthermore, one of the main concerns that firms have is how much will they be exposed to risk when they try to maximize their P&L. Therefore, when the expected P&L is being optimized, at the same time the risk should be controlled. Commonly in portfolio optimization in the literature, only one risk measure is considered in the problem. However, in this study, two risk measures are considered and they represent the partnering firm's risk limits. To the best of the author's knowledge, using two risk measures in the same optimization problem has never been done in electricity markets. In the next section, more technical details about the problem are going to be provided.

3.1 Formulation

In order to formulate the proposed optimization problem, a selection of notations is explained first.

Let each trading strategy be denoted by s and the set of trading strategies be denoted by S. Furthermore, since trading strategies are sometimes applied in multiple zones, i denotes the zone in which the strategy is applied. In addition, let P denote the total profit of the firm's portfolio in a determined period. P_{si} denotes the P&L of the trading strategy s in zone i per MWh and all of these notations are in the determined period that the firm has chosen. Let I_s denotes the set of the zones in trading strategy s. With this notation, I_s for the trading strategies which only apply to a single zone has only one element. Now, μ_{si} can be defined as the expected return of P_{si}, which can be written as $\mathbb{E}[P_{si}] = \mu_{si}$. Let μ be the vector of μ_{si}'s.

Also, let x be the portfolio of the firm which is a vector of x_{si}'s where x_{si} denotes the allocated budget to the trading strategy s in zone i. Then, one can define the expected P&L of the firm's portfolio as follows

$$\mathbb{E}[P] = \sum_{s \in S} \sum_{i \in I_s} \mathbb{E}[x_{si} P_{si}] \tag{9}$$

$$= \sum_{s \in S} \sum_{i \in I_s} x_{si} \mathbb{E}[P_{si}] \tag{10}$$

$$= \sum_{s \in S} \sum_{i \in I_s} x_{si} \mu_{si} \tag{11}$$

$$= \mu^T x \tag{12}$$

The value that is trying to be maximized in this research is $\mathbb{E}[P] = \mu^T x$ since the actual value of P is unknown in advanced. This linear combination is the objective function of the optimization problem formulated in this study. The goal is to find x such that the objective function is maximized under some certain constraints. The constraints are divided into four categories of constraints. A first budget constraint is added because the allocated budget to different trading strategies cannot be more than the total budget of the firm.

There is a limit based on market credit on the number of megawatt-hours (MWh) that can be traded. To become a market participant and in order to be able to trade a minimum volume, the ISO will ask for depositing a minimum amount into a collateral account. If the market participants want to trade more, they need to deposit more money into the collateral account, and the market will let them trade a specific volume based on the amount of money in the account. In this research, the budget is the number of MWhs that the firm can trade in the market.

The risk constraints are added because the problem entails maximizing the P&L while controlling the risk. The sign constraints are added because short positions are not allowed and only a positive number of MWhs can be traded. However, in the sign constraint, in addition to the lower bound zero for decision variables, an upper bound is set for each decision variable. The upper bound exists because in each trading strategy the number of MWhs cannot exceed a certain amount according to the firm's risk tolerance. Finally, the integrality constraints are added because only an integer number of MWhs can be traded. To summarize it all, one can write the following formulation

$$\begin{aligned} \max \quad & \mathbb{E}[P] \\ s.t. \quad & \text{Budget Constraint} \\ & \text{Risk Constraints} \\ & \text{Sign Constraints} \\ & \text{Integrality Constraints} \end{aligned}$$

Now, each category of constraints is going to be explained one by one and replaced in the formulation. In its mathematical formulation, the budget constraint can be written as $\sum_{s\in S}\sum_{i\in I_s} x_{si} \leq B$ where B denotes the total available budget of the firm in the determined period. Therefore, the formulation becomes

$$
\begin{aligned}
\max \quad & \mathbb{E}[P] \\
s.t. \quad & \sum_{s \in S} \sum_{i \in I_s} x_{si} \leq B && (13) \\
& \text{Risk Constraints} && (14) \\
& \text{Sign Constraints} && (15) \\
& \text{Integrality Constraints} && (16)
\end{aligned}
$$

One of the risk constraints that is requested by the firm is that no more than 35% of the total budget, B, can be assigned to a single trading strategy. Mathematically, it is given by $\sum_{i \in I_s} x_{si} \leq 0.35B$, $\forall s \in S$. The other risk constraints are for setting an upper bound for the variance and the CVaR of the portfolio. Let m_1 and m_2 be the upper bound for the variance of the portfolio and the CVaR of the portfolio, respectively. Then the formulation can be written as follows

$$
\begin{aligned}
\max \quad & \mathbb{E}[P] \\
s.t. \quad & \sum_{s \in S} \sum_{i \in I_s} x_{si} \leq B && && (17) \\
& \sum_{i \in I_s} x_{si} \leq 0.35B && \forall s \in S && (18) \\
& Var(x) \leq m_1 && && (19) \\
& CVaR(x) \leq m_2 && && (20) \\
& \text{Sign Constraints} && && (21) \\
& \text{Integrality Constraints} && && (22)
\end{aligned}
$$

For the calculation of $Var(x)$, the covariance matrix of the trading strategies is needed. Let Σ denote the covariance matrix. Then the variance of the portfolio is given by $Var(x) = x^T \Sigma x$.

To calculate CVaR, the approximation method with a scenario generation approach introduced in Chapter 2 is used. Recall that an auxiliary function is used to be optimized instead of CVaR which is given by

$$F_\alpha(x,\gamma) := \gamma + \frac{1}{1-\alpha}\int_{f(x,y)\geq\gamma}(f(x,y)-\gamma)p(y)d(y)$$

and an approximation of this function can be made by discretizing as follows

$$\tilde{F}_\alpha(x,\gamma) := \gamma + \frac{1}{1-\alpha}\sum_{k=1}^{K} p_k(f(x,y_k)-\gamma)^+$$

where K is the number of scenarios, y_k's are the possible values for the random variable y in different scenarios and p_k is the probability of scenario k. In order to use this function in the formulation, it is transformed into three linear constraints by using auxiliary variables, η_k. These auxiliary variables are needed to be optimized as well as the original decision variables. Then this function can be added as three linear constraints to the formulation as follows (Cornuejols and Tütüncü, 2006)

$$\gamma + \frac{1}{1-\alpha}\sum_{k=1}^{K} p_k\eta_k \leq m_2 \tag{23}$$

$$\eta_k \geq f(x,y_k) - \gamma \qquad \forall k \in \{1,\cdots,K\} \tag{24}$$

$$\eta_k \geq 0 \qquad \forall k \in \{1,\cdots,K\} \tag{25}$$

In this study, the loss function is considered to be $-\sum_{s\in S}\sum_{i\in I_s}\mu_{si}x_{si}$. The negative values of this function mean gain and the positive values are losses. With this loss function constraint (28) becomes $\eta_k \geq -(\sum_{s\in S}\sum_{i\in I_s}P_{sik}x_{si}+\gamma)$, $\forall k \in \{1,\cdots,K\}$ where P_{sik} is the realization of μ_{si} in scenario k. In addition, the sign constraints is given by $0 \leq x_{si} \leq u_{si}$, $\forall s \in S, i \in I_s$ where u_{si} denotes the upper bound for x_{si}. The integrality constraints can be written as $x_{si} \in \mathbb{Z}$, $\forall s \in S, i \in I_s$. The following optimization model wraps up the formulation:

$$\max \quad \sum_{s\in S}\sum_{i\in I_s}\mu_{si}x_{si}$$

$$s.t. \quad \sum_{s\in S}\sum_{i\in I_s}x_{si} \leq B \tag{26}$$

$$\sum_{i\in I_s}x_{si} \leq 0.35B \qquad \forall s \in S \tag{27}$$

$$x^T \Sigma x \leq m_1 \tag{28}$$

$$\gamma + \frac{1}{1-\alpha} \sum_{k=1}^{K} p_k \eta_k \leq m_2 \tag{29}$$

$$\eta_k \geq -(\sum_{s \in S} \sum_{i \in I_s} P_{sik} x_{si} + \gamma) \qquad \forall k \in \{1, \cdots, K\} \tag{30}$$

$$\eta_k \geq 0 \qquad \forall k \in \{1, \cdots, K\} \tag{31}$$

$$0 \leq x_{si} \leq u_{si} \qquad \forall s \in S, i \in I_s \tag{32}$$

$$x_{si} \in \mathbb{Z} \qquad \forall s \in S, i \in I_s \tag{33}$$

Chapter 4

Solution and Results

In this chapter, the steps taken to solve the optimization model are explained. The solution to the problem is assessed on the data provided by the partnering electricity trading firm and the results are discussed. First, the details about the trading strategies of the electricity trading firm are provided.

The firm with which this collaboration is done is a market participant in one of the ISO's in North America. As explained earlier, ISO's have similar characteristics to each other. In order to keep the operations of the firm confidential, the exact ISO in which they operate will not be mentioned. There are both physical and financial approaches in the firm's portfolio. Moreover, each trading strategy has its own unique characteristics which are explained in Appendix A.

This research is trying to allocate the limited number of MWhs to four main trading strategies of an electricity trading firm. Recall that each strategy can be implemented in multiple locations. The reason for selecting this portfolio for this study is that it reflects a real-life portfolio. Two of the trading strategies have a physical approach and two of them are virtual trading strategies.

In order to keep the operations of the firm confidential, the trading strategies are called A, B, C, and D. The assumption here is that these four strategies are already selected and this study is more of a budget allocation project rather than a portfolio selection. Portfolio selection can be done in future research.

The limited budget of the firm should be allocated to the four trading strategies. Trading strategy C is applied to two zones, trading strategy D is applied to seven zones and different budgets should be allocated to different zones in these trading strategies. The reason that trading strategies C and D are considered as two different trading strategies is based on the firm's decisions.

The main question that this research is attempting to answer is that how many MWhs should be traded in each of these four trading strategies and how many should be traded in each zone of each trading strategy in order to have the maximum profit with a controlled risk in a determined time horizon. The determined time horizon can be a day, a week, or any number of days which means that the trade volume would be different on the determined time horizon for each trading strategy.

There are two risk measures considered in this problem: the portfolio variance and the conditional value at risk (CVaR). The firm wants to maximize its future profit which is unknown in advance. Also, the portfolio variance and CVaR are unknown variables. All the available information at the time of the formulating the problem is the historical P&L of the firm.

4.1 Data

There are different amounts of data available for each trading strategy. The data used in this research has been retrieved from the partnering firm's database. The data for trading strategies A and B is available since "2017-04-13" and "2016-10-04", respectively. The data points are daily and for each day, the sum of the hourly P&L and the sum of the hourly trade volumes are obtainable. For trading strategies C and D, the available data is since "2012-01-01" and the data points are the hourly P&L in $/MWh. Moreover, there are no missing points in the data for any of the four trading strategies.

As mentioned above the data points for two of the trading strategies are daily and for the other two, the data points are hourly. Therefore, the data needs to be transformed for all four of the strategies to the same kind. In order to do this for all of the trading strategies, trading strategies A and B's P&L data are divided by the sum of the daily trade volume. As a result, the new data points are the daily average of P&L per MWh. Furthermore, in the data for trading strategies C and D, the sum of the hourly P&L in each day is divided by 24, so that the data points are the daily average of P & L per MWh. For example, if the data point is 10$ on a day, such as "2012-01-01" for trading strategy D, it means that on the first day of January in 2012, the firm made 10$/MWh at each hour of the day on average.

4.2 Solving the Optimization Problem

In order to solve the optimization problem, there are various aspects that are needed to be introduced and discussed. First, it needs to be identified which parameters of the model are known and given and which parameters need to be estimated. The second aspect is the procedure of the parameter estimation for the unknown parameters. The next aspects are the assumptions made to solve the problem. Another aspect is the procedure for the scenario generation in the calculation of CVaR and the final aspect is handling the quadratic term in the constraints.

The known parameters of the problem are the total available budget B, the upper bounds for the risk measure m_1 and m_2, the number of scenarios K, and the confidence level of CVaR α. Recall that the scenarios are the possible values for the P&L of different trading strategies in the determined time horizon. All of these values are given at the time that the optimization problem is needed to be solved. As mentioned before, this problem can be solved for the next day, next week, etc. The unknown parameters of the problem are the μ vector, p_k, μ_{sik}'s and Σ.

A few assumptions were made before the start of the parameter estimation procedure and solving the problem. The next section provides a list of these assumptions.

4.2.1 Assumptions

The several assumptions that were made in this study are as follows

1. The distribution of the data for each trading strategy and each zone in the future would be the same as historical data.

2. The dependence structure of the data would not change in the future.

3. The sample data of each trading strategy is independent and identically distributed (i.i.d.) across periods.

4. The probability distribution of the scenarios is uniform and as a result, each scenario has the same probability which can be written as $p_k = \frac{1}{K}$

4.2.2 Estimating the Parameters of the Optimization Problem

The selected period length for this research is one month and the optimization problem is solved at the beginning of each month in order to find the number of MWhs for each trading strategy on the selected month.

All the numbers provided in this section are for "July". The final results for the other months are provided in Appendix B.

To estimate μ_{si}'s, Johnson SU distribution is fitted to the collected data from all years from July for each trading strategy s and each zone i. The parameters of the distribution are estimated by applying the maximum likelihood estimation method. Afterwards, the mean of the distribution is taken as the expected P&L or μ_{si}. The goodness-of-fit of the fitted distribution is tested using the Kolmogorov Smirnov test which tests the goodness-of-fit of the distribution by measuring the maximum distance between the empirical cdf and model distribution cdf. The fitted distribution to different trading strategies and the p-values are provided in the Table 4.1.

The fitted distribution to all the trading strategies is Johnson SU but the parameters of the distribution for each trading strategy are different. The mean of the distribution for different trading strategies are provided in Table 4.2. The values for the mean of the distribution in Table 4.2 are the objective coefficients of the optimization problem (values for μ_{si}). The difference between the mean of the fitted distribution and the average of the realized P&L for trading strategy C and the first two zones of trading strategy D is very small. However, for the other trading strategies this difference is high. The reason of the disparity between estimated and realized mean is found by investigating the available data. There are big negative spreads for zones 3 to 7 in trading strategy D in July 2019. The negative spreads

Trading Strategy	Zone	Fitted distribution	p-value
A	-	Johnson SU	0.20
B	-	Johnson SU	0.83
C	1	Johnson SU	0.83
C	2	Johnson SU	0.82
D	1	Johnson SU	0.68
D	2	Johnson SU	0.94
D	3	Johnson SU	0.83
D	4	Johnson SU	0.69
D	5	Johnson SU	0.94
D	6	Johnson SU	0.94
D	7	Johnson SU	0.13

Table 4.1: The fitted distribution to different trading strategies and p-values of the goodness of fit test

Trading Strategy	Zone	Mean of the distribution	The average of the realized P&L
A	-	-1.62	0.97
B	-	-0.50	-2.72
C	1	2.37	2.40
C	2	1.65	1.85
D	1	2.22	2.59
D	2	2.62	2.13
D	3	28.50	0.78
D	4	8.48	0.74
D	5	5.38	0.03
D	6	36.33	-0.69
D	7	1.49	-5.53

Table 4.2: The mean of the fitted distribution to different trading strategies

caused by an increase in the temperature of these zones in 2019 which has not happened before, in 2017 and 2018. These natural phenomena's cannot be foreseen. However, factors such as global warming provide some guidelines for the foreseeable future. Multiple other unpredictable events have happened in the past that prove that certain circumstances cannot be considered in algorithms, such as whole power plants losing complete functionality due to malfunctions.

Furthermore, the Σ matrix is estimated by the sample covariance between the average hourly P&Ls. The covariance matrix is given in Table 4.3. The correlation matrix is provided in Appendix C.

	A	B	C/1	C/2	D/1	D/2	D/3	D/4	D/5	D/6	D/7
A	38.56	15.32	-7.92	-31.74	-19.25	-2.69	-3.85	-3.27	-3.16	-2.42	19.54
B	15.32	91.85	19.82	45.30	-2.80	25.32	21.70	21.50	21.45	24.82	59.18
C/1	-7.92	19.82	42.36	39.68	41.40	42.59	43.38	43.46	42.42	41.98	68.82
C/2	-31.74	45.30	39.68	170.14	40.17	43.75	41.71	42.21	39.49	42.66	51.38
D/1	-19.25	-2.80	41.40	40.17	134.09	36.47	34.59	34.43	32.61	18.96	42.53
D/2	-2.69	25.32	42.59	43.75	36.47	50.38	48.03	47.49	47.00	45.80	80.44
D/3	-3.85	21.70	43.38	41.71	34.59	48.03	49.41	49.64	48.80	50.01	78.03
D/4	-3.27	21.50	43.46	42.21	34.43	47.49	49.64	50.57	49.30	51.12	75.28
D/5	-3.16	21.45	42.42	39.49	32.61	47.00	48.80	49.30	49.06	50.86	76.00
D/6	-2.42	24.82	41.98	42.66	18.96	45.80	50.01	51.12	50.86	65.32	80.17
D/7	19.54	59.18	68.82	51.38	42.53	80.44	78.03	75.28	76.00	80.17	210.88

Table 4.3: The sample covariance matrix between the average hourly P&L

In order to generate the scenarios, different families of bivariate copulas are fitted to a different selection of two trading strategies. The pair of copulas are selected by looking at the empirical copula and choosing the right family of copulas. The parameters of the fitted bivariate copula to each pair of trading strategies are estimated using the maximum likelihood estimation method. Then, for each pair of trading strategies, the best copula is selected based on the Akaike Information Criteria (AIC) which is calculated for all available bivariate copula families. The AIC of a bivariate copula family c with parameter vector θ is given by (Schepsmeier et al., 2015)

$$AIC := -2\sum_{i=1}^{N} ln\left(c(u_{i,1}, u_{i,2}; \theta)\right) + 2k.$$

Afterwards, a goodness-of-fit test is done. This test is looked into by Huang and Prokhorov (2014) and it uses the information matrix equality of White (1982). This test does not involve kernel weighting, bandwidth selection, or any other strategic choices and it avoids parametric specification of marginal distributions. Furthermore, the test is asymptotically pivotal with a standard distribution and it is easy to complete when compared to other alternatives (Huang and Prokhorov, 2014) .

The families selected in this procedure for the bivariate copulas are Independent copula, Gaussian, t-student, Frank, Clayton, Gumbel, and Joe. These chosen families build the Tree 1. In the next step, they are used to calculate pairwise Kendall's τ for all the edges that keep the required proximity condition for the R-vine tree structure. Kendall (1938) introduced a measure for rank correlation which is given by

$$\tau(X_1, X_2) = 4\int_0^1\int_0^1 \{C(u_1, u_2) - \Pi(u_1, u_2)\} dC(u_1, u_2)$$

where $\Pi(u_1, u_2) = u_1 u_2$ is the independence copula. Repeatedly, the corresponding copula families are chosen using the AIC. By using this method and the selection of the strongest pairwise conditional dependencies first, an eleven-dimensional RVine copula is fitted to the data (Czado et al., 2013). The details of this method are given in the preliminaries chapter. The p-value of the goodness-of-fit test (the same test used for bivariate copulas) is 0.14 and the fitted copula is used to generate scenarios.

In order to solve the optimization model, the solver *MOSEK* is used. The programming has been done in *R* using the *Rmosek* package. One drawback of using Rmosek is that it cannot solve an optimization problem with a quadratic term in the constraints. As a result, a reformulation is done to transform the quadratic constraint into a conic term. Since the covariance matrix is positive semi-definite, there exists a matrix G such that

$$\Sigma = GG^T$$

This decomposition is not unique. The decomposition used in this research is the *Cholesky decomposition*. The Cholesky decomposition or Cholesky factorization is a decomposition of a Hermitian, positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose (Golub and Van Loan, 1996). Let CC^T denotes the Cholesky decomposition of Σ. Then

$$x^T\Sigma x = x^T CC^T x = ||C^T x||^2$$

With this reformulation, the Var constraint can be written as

$$||C^T x|| \leq \sqrt{m_1}$$

or equivalently $(\sqrt{m_1}, C^T x) \in Q^{n+1}$ where Q^{n+1} is the $(n+1)$-dimensional quadratic cone and $n = \sum_{s\in S} |I_s|$ (ApS, 2020).

4.3 Results

The results provided in this section are for the month of July. The estimation of the unknown parameters of the optimization model is done based on data from 2017 and 2018, then the performance is assessed on data from 2019 which is an out of sample testing.

Figure 4.1 shows the efficient frontier surface of the firm's portfolio based on the data from 2017 and 2018 considering both risk measures. The selected level of confidence for CVaR is $\alpha = 0.95$ and the number of scenarios is equal to 10000. The upper bound for the standard deviation of the portfolio, $\sqrt{m_1}$ changes between 1000 and 5000 and the upper bound for the CVaR, m_2 changes between 20000 and 200000.

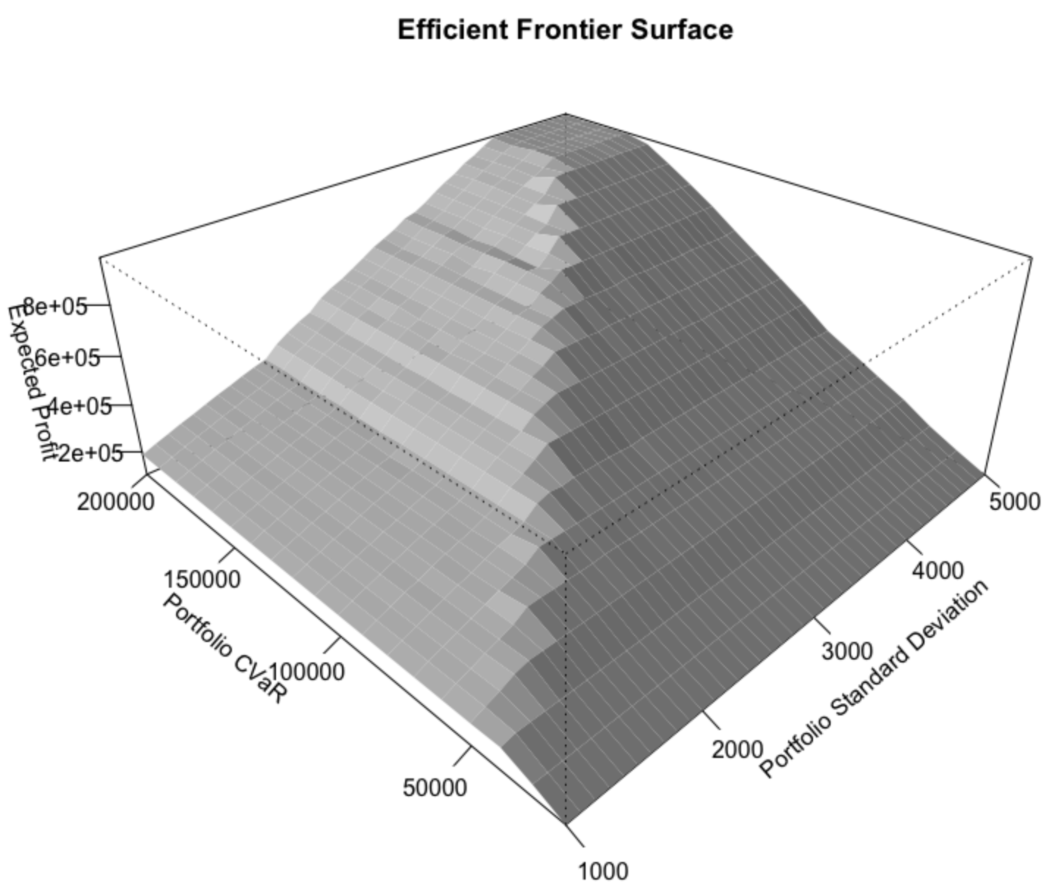


Figure 4.1: Efficient frontier surface of the firm's portfolio based on the data from 2017 and 2018

Any portfolio on this surface is an optimal portfolio. The first optimal portfolio is selected by setting $\sqrt{m_1} = 1000$ and $m_2 = 20000$ which is a very conservative portfolio with a small risk.

The benchmark is the P&L of the electricity trading firm in the selected month. The P&Ls for one day are obtained by the sum of the hourly P&Ls. The trade volumes that used to obtain the benchmark are decided by the firm.

A backtest with real 2019 data is done by using the optimal trade volumes. These optimal trade volumes are found by using the optimization model and they remain fixed for the month. The daily P&Ls with the optimal trade volumes are also calculated. The firm's trade volumes and the optimal trade volumes are given in Table 4.4. The optimization model allocates the

Trading Strategy	Zone	Firm's Trade Volume	Optimal Trade Volume
A	-	277	0
B	-	266	0
C	1	45	0
C	2	15	0
D	1	38	0
D	2	38	0
D	3	38	67
D	4	38	0
D	5	38	0
D	6	38	11
D	7	38	0

Table 4.4: The trade volume allocated to each trading strategy by the firm and the optimal trade volumes with $\sqrt{m_1} = 1000$ and $m_2 = 20000$

most possible budget MWhs to the trading strategies which has the most expected returns to maximize the profit while satisfying the problem constraints. The budget is only allocated to trading strategy D, zones 3 and 6. However, it did not allocate the total budget as the risk measure constraints need to be satisfied. This concentration is not desirable since an unpredicted event such as power outage in zones 3 and 6 might cause a big loss for the firm. To avoid this, a more diverse allocation can be done by setting higher upper bounds for the risk measures.

The total number of MWhs allocated to trading strategies by the optimization model is lower than the firm's allocation due to the conservatism in the optimization model.

Figure 4.2 shows the cumulative P&L in July with an optimal portfolio vs the selected portfolio by the firm. The real trades are the P&Ls obtained by using the firm's selected portfolio and the optimal trades are the ones obtained by using the optimal portfolio.

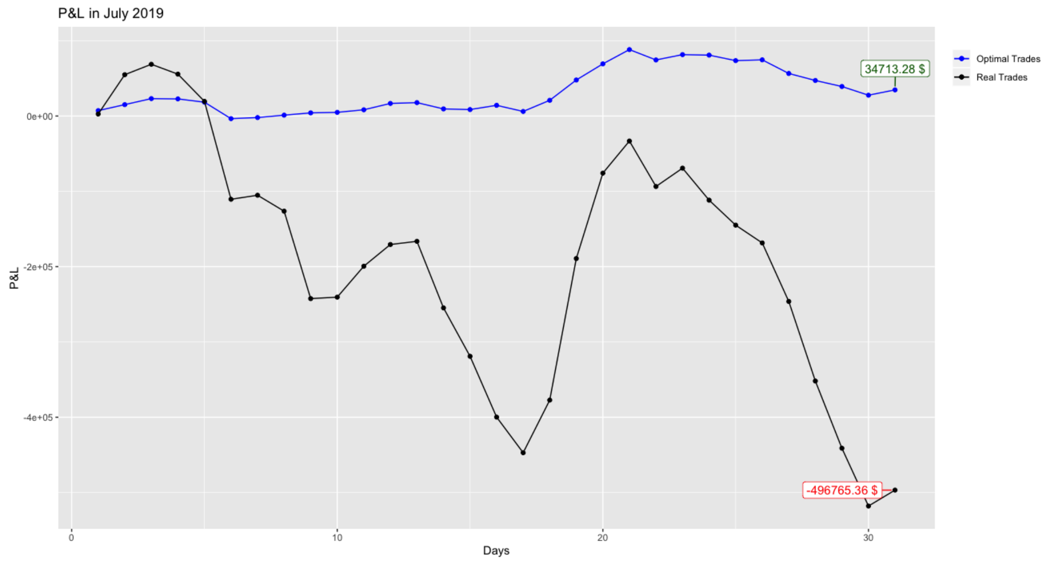


Figure 4.2: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 1000$ and $m_2 = 20000$

The firm's selected portfolio caused a big loss in July 2019. A high trade volume allocation to trading strategy B which has a negative average P&L resulted in a big loss at the end of the month in 2019. In addition, an allocation of a high trade volume to trading strategy A which has a low average P&L in 2019 and negative correlation with most of the other trading strategies caused an undesirable result. Furthermore, the average of the possible values for the P&Ls of the trading strategies A and B are $-45.11\$$ and $-13.62\$$, respectively. This indicates that there is a high possibility that big negatives happen in these two trading strategies. As a result, the optimization model would not allocate any budget to these two trading strategies.

There are a few measures that the firm uses in order to compare two different portfolios. The first and most obvious one is how much the P&L of the portfolio changes at the end of the month when the two portfolios (the optimal portfolio and the firm's portfolio) are compared. The P&L of July 2019 is changed by 107% with the optimal portfolio. This is the ratio of the P&L of the optimal portfolio over the firm's portfolio. The other measure is how much is the average of the losses and average of the three worst losses of each portfolio in the selected month. Moreover, what is the percentage of the winning days of each portfolio in the determined month. A winning day is a day on which the firm made a profit. The values for these measures and also the standard deviation of the portfolios are provided in Table 4.5.

Measure	Optimal Trades	Real Trades
Percentage of change in the P&L	106.99 %	-
P&L at the end of the month	34713.28 $	-496765.40 $
Average loss	-8671.13 $	-65082.2 $
Three worst losses average	-17916.21 $	-117261.8 $
Percentage of winning days	58.06%	45.16 %
Standard deviation of the portfolio	458.58	3291.92

Table 4.5: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 1000$, $m_2 = 20000$ and the firm's selected portfolio

The optimal portfolio, even when it is a conservative one, has a better P&L compared to the selected portfolio by the firm. Furthermore, the worst three losses average is lower and the percentage of the winning days is higher for the optimal portfolio. The optimization model would not allocate budget to the strategies with high variance and as a result the average of the worst three losses is lower in the optimal portfolio.

If the firm is willing to increase the upper limit for CVaR to 50000 but keep the upper limit for the standard deviation of the portfolio at 1000, the results would be as it is shown in Figure 4.3.

The optimal solution for this optimization model is given in Table 4.6 alongside with the firm's trade volumes.

This time the optimization problem allocated more weight to Zone 6 of trading strategy D as the upper bound for CVaR increased. It also decreased the allocated trade volume to trading strategy D Zone 3 to keep the variance constraint satisfied.

However, the profit of the optimal portfolio becomes worse. The reason is that the optimization model increases the weights for the riskier trading strategies which do not increase the variance of the portfolio. Furthermore, the expected P&L of trading strategy

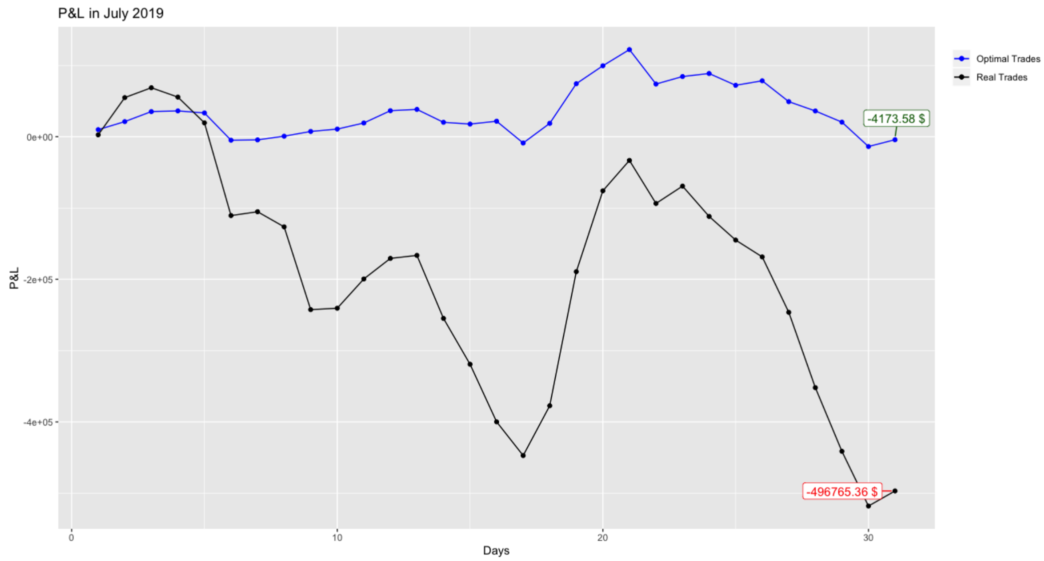


Figure 4.3: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 1000$ and $m_2 = 50000$

Trading Strategy	Zone	Firm's Trade Volume	Optimal Trade Volume
A	-	277	0
B	-	266	0
C	1	45	0
C	2	15	0
D	1	38	0
D	2	38	0
D	3	38	57
D	4	38	0
D	5	38	0
D	6	38	78
D	7	38	0

Table 4.6: The trade volume allocated to each trading strategy by the firm and the optimal trade volumes with $\sqrt{m_1} = 1000$ and $m_2 = 50000$

D Zone 6 is 36.33$ while the realized average P&L is -0.69$. This shows that an unforeseen event happened in Zone 6 in 2019 which has not been happened in the previous data. As a result, the trading strategy which has a higher risk but made a profit in 2018 and 2017, lost money in 2019.

The values for the comparison measures is given in Table 4.7. Although, the percentage of the winning days have increased, the final profit of the optimal portfolio became worse by increasing the upper bound for CVaR.

Measure	Optimal Trades	Real Trades
Percentage of change in the P&L	99.16 %	-
P&L at the end of the month	-4173.58$	-496765.40 $
Average loss	-22665.86 $	-65082.2 $
Three worst losses average	-40266.74 $	-117261.8 $
Percentage of winning days	64.52%	45.16 %
Standard deviation of the portfolio	898.41	3291.92

Table 4.7: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 1000$, $m_2 = 50000$ and the firm's selected portfolio

The optimal portfolio with the highest expected profit can be obtained by setting $\sqrt{m_1} = 5000$ and $m_2 = 200000$. This optimal portfolio is the riskiest portfolio amongst all the optimal portfolios. The allocated trade volumes using this optimization model is given in Table 4.8. The optimization model allocated more of the total budget to the trading strategy as the

Trading Strategy	Zone	Firm's Trade Volume	Optimal Trade Volume
A	-	277	0
B	-	266	0
C	1	45	200
C	2	15	104
D	1	38	0
D	2	38	24
D	3	38	80
D	4	38	40
D	5	38	80
D	6	38	80
D	7	38	0

Table 4.8: The trade volume allocated to each trading strategy by the firm and the optimal trade volumes with $\sqrt{m_1} = 2000$ and $m_2 = 200000$

upper bounds for the risk measures increase. The result for this portfolio is shown in Figure 4.4.

This portfolio changes the P&L of the firm by 165%. The average of the worst three losses is -125577.4\$ which is higher than the average three losses of the selected portfolio by the firm, however, The P&L is also much higher. The values for the comparison measures is given in Table 4.9.

The last portfolio that is explored in this section is the portfolio obtained by setting $\sqrt{m_1} = 4566$ and $m_2 = 153270$. These numbers are the estimated variance and the estimated

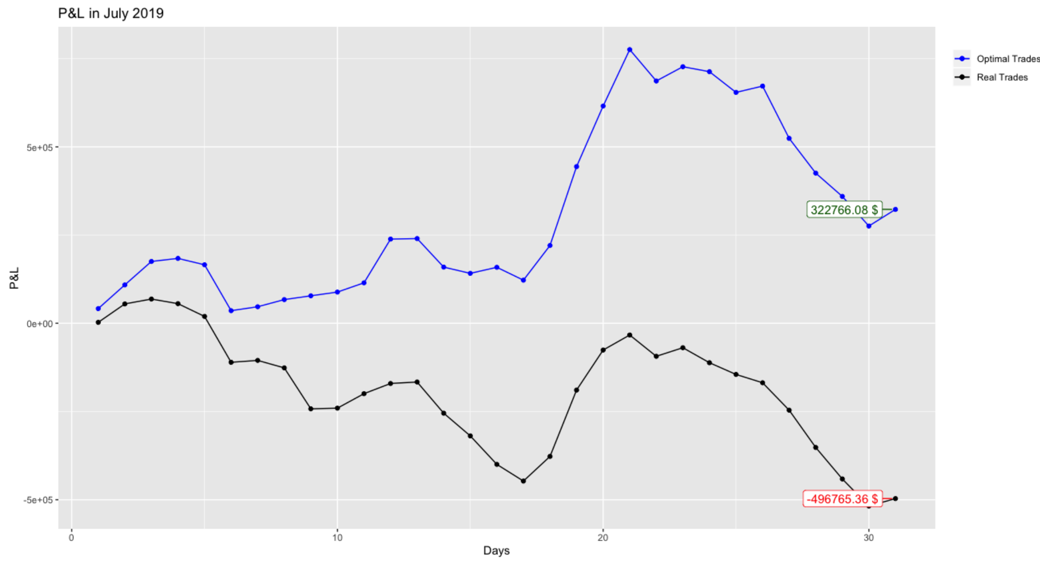


Figure 4.4: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 5000$ and $m_2 = 200000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P&L	164.97 %	-
P&L at the end of the month	322766.1$	-496765.40 $
Average loss	-70134.24 $	-65082.2 $
Three worst losses average	-125577.4 $	-117261.8 $
Percentage of winning days	61.30%	45.16 %
Standard deviation of the portfolio	3595.65	3291.92

Table 4.9: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 5000$, $m_2 = 200000$ and the firm's selected portfolio

CVaR of the electricity trading firm. The allocated budget to different trading strategies using these upper bounds are shown in Table 4.10. In this portfolio, the optimization model allocated less budget to trading strategy C, zone 2. When the upper bounds for the risk constraints are set to lower amounts, the algorithm allocated less budget to the trading strategy with more variance. The results of this portfolio is illustrated in Figure 4.5. By allocating less budget to trading strategy C, zone 2 the profit becomes less, however the optimal portfolio outperforms the firm's portfolio. The comparison measures are given in Table 4.11.

In all the four optimal portfolios, the allocated trade volume to trading strategies A,B, zone 1 and zone 7 of trading strategy D is 0. This is due to the fact that the expected P&L

Trading Strategy	Zone	Firm's Trade Volume	Optimal Trade Volume
A	-	277	0
B	-	266	0
C	1	45	200
C	2	15	16
D	1	38	0
D	2	38	24
D	3	38	80
D	4	38	40
D	5	38	80
D	6	38	80
D	7	38	0

Table 4.10: The trade volume allocated to each trading strategy by the firm and the optimal trade volumes with $\sqrt{m_1} = 4566$ and $m_2 = 153270$

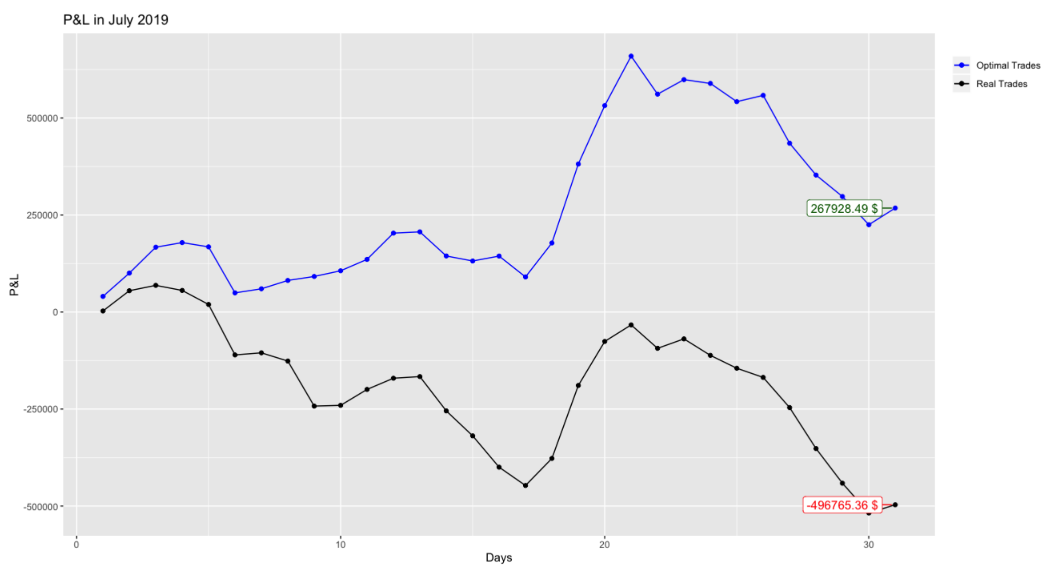


Figure 4.5: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 4566$ and $m_2 = 153270$

for trading strategies A and B are negative. The reason that the model allocates no trade volume to zones 1 and 7 of trading strategy D is that they have higher variance amongst all zones in trading strategy D, according to Table 4.3.

Measure	Optimal Trades	Real Trades
Percentage of change in the P&L	153.93 %	-
P&L at the end of the month	267928.5 $	-496765.40 $
Average loss	-62208.3 $	-65082.2 $
Three worst losses average	-113351.5 $	-117261.8 $
Percentage of winning days	61.29%	45.16 %
Standard deviation of the portfolio	3125.61	3291.92

Table 4.11: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 4566$, $m_2 = 153270$ and the firm's selected portfolio

Chapter 5

Conclusion

In this research, a mathematical solution is proposed to solve the portfolio optimization problem for a partnering electricity trading firm which has four different trading strategies in their portfolio. The main goal was to allocate the limited number of MWhs (the budget) to these four trading strategies such that two risk measures are controlled.

The portfolio optimization problem was formulated as an integer stochastic optimization problem. The model parameters were obtained by using the mean of the fitted distribution to different trading strategies. The risk measures used in this study were the portfolio variance, which is a quadratic term, and conditional value at risk of the portfolio. The two risk measures were added to the optimization problem as constraints with upper bounds. The quadratic term was reformulated as a conic term and the method proposed for the calculation of CVaR was scenario generation. The scenarios were the possible values for the P&Ls of different trading strategies and they were generated from the joint distribution of the trading strategy returns. Furthermore, the joint distribution was modelled by fitting an eleven-dimensional vine copula. One of the challenges in this study was the limited number of the data points since all of the data used was real data and no simulation was done. As a result, the classical statistical methods such as distribution fitting was used instead of more modern methods such as machine learning.

The optimization model was formulated for a determined time horizon where the period could be selected by the firm. The selected period for this study is one month. This problem was solved by using a solver called Rmosek and the efficient frontier surface was built for the month of July. Different optimal portfolios were assessed on the unseen data.

The expected P&L obtained from the training set, was different from the realized average P&L in the test set, for some of the trading strategies. The reason was an increase in the temperature of some zones in those trading strategies in 2019 and the fact that it had not happened in 2017 and 2018. The effect of this issue can be seen in Figure 4.3 which illustrates the cumulative return of the firm both with optimal trade volumes and the firm's selected trade volumes with $\sqrt{m_1} = 1000$ and $m_2 = 50000$. Recall that m_1 and m_2 were the upper bounds for the variance of the portfolio and the CVaR, respectively. This figure demonstrates that even when the upper bound for the CVaR of the portfolio increased, the realized profit became worse. The optimization problem allocated higher trade volume to the trading strategy for which the expected P&L was high but the average realized P&L was low. The results showed that there is always a trade-off between the taken risk and profit.

In the results section, the result of the four optimal portfolios is provided. These optimal portfolios are obtained by setting different upper bounds for the optimization model constraints. Note that each of these has its pros and cons. The optimal portfolio which is going to be used as the firm's portfolio can be decided by the firm by considering all of the negative and positive facts about each portfolio. More profit can be made if the firm is willing to take more risk, however, this is not always the case. Taking more risk in the second optimal portfolio resulted in less profit.

The electricity markets are an interesting area for studying. There is a lot of potential for further research. The expected P&L of the firm is estimated by the mean of the distribution in this study. The estimation can also be done by using forecasting methods in time series and machine learning including GLMs, GAMs, Neural Networks, etc. Furthermore, the marginal distributions used in this study comes from the Johnson family of distributions. The assumption here was that the marginals would not change but one can remove this assumption and estimate the parameters of the marginals for future data. A multivariate time-series model can be used to estimate the parameters of the marginal distributions at each time step. As a result, the parameters of the marginals can be estimated dynamically.

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Appendix A

Trading Strategies

In order to keep the operations of the firm confidential, the ISO's in which the trading strategies apply is called ISO 1, ISO 2, ISO 3, ISO 4 and, ISO 5. ISO's 2, 3, 4 and, 5 have interfaces in ISO 1. Furthermore, ISO 1 has an interface in ISO's 2 to 5.

1. **Trading strategy A** is a physical trading strategy. Let say an offer is submitted to the ISO 1's day-ahead market to deliver a certain amount of electricity at the day-ahead price. If the offer clears in the day-ahead market, there is an obligation to deliver that certain amount of energy and will receive the day-ahead ISO 1's price at the ISO 2's interface of ISO 1. In order to buy that amount, there are two options, either it can be bought from ISO 2 at the real-time price of the ISO 1's interface of ISO 2, or it can be bought at the real-time price of the ISO 2's interface in ISO 1. The bid can be submitted 90 minutes before the operation hour in the real-time market.

   There is a prediction for the real-time price from ISO 2 at the ISO 1's interface in ISO 2, which is being updated every 30 minutes before the operation hour. A bid must be submitted no later than 90 minutes before the operating hour, such that if the last prediction of the price were lower than a certain amount, a certain amount of energy would be bought. After the bid is submitted, if the condition is satisfied and the flow is physically feasible, the bid will clear, the electricity will flow and the amount of electricity needs to be delivered from ISO 2 will be imported. If the bid does not clear in ISO 2, there is an obligation to buy that certain amount from ISO 1 at the real-time

ISO 1's price. It is also possible that the bid clears partially in ISO 2.

For example, a bid is submitted such that 100 MWhs will be sold at the day-ahead price at 2:00 pm. This bid is submitted the day before the operating day in the day-ahead market. After the day-ahead market is closed and the day-ahead price is released, the day-ahead price at the ISO 2's interface in ISO 1 is 10$/MWh. Suppose that the offer is cleared in the day-ahead market and the firm has the obligation to deliver 100 MWhs of electricity to the ISO 2's interface in ISO 1 and sell it for 10$/MWh on the operating day. To deliver this amount of electricity, the firm submits a bid before 12:30 pm to the ISO 2's market to buy 100 MWhs of energy at the ISO 1's interface if the predicted price at 1:30 pm was less than a certain amount determined by the firm. At 1:30 pm if the condition is satisfied and the flow is physically feasible, the bid is cleared in the ISO 2's market. At 2:00 pm, if the real-time price at the ISO 1's interface is greater than 10$/MWh, the firm would lose money and if it is less than 10$/MWh the firm would make profit.

However, it is possible that the flow is not physically feasible due to the transmission constraints or the bid is not cleared because the condition is not satisfied. In this case, the firm has the obligation to buy the electricity at real-time price at the ISO 2's interface in ISO 1. Again if the real-time price at the ISO 2's interface in ISO 1 is less than 10$/MWh, the firm would make money and if it is more that 10$/MWh the firm would lose money.

2. **Trading strategy B** is also a physical trading strategy. A bid is submitted to the real-time market no later than 90 minutes before the operating hour to the ISO 3's market at the ISO 1's interface to buy energy from ISO 3 and sell it in ISO 1. The settlement of the bid is the same as the one in trading strategy A, and if the bid is cleared and the flow of the energy is physically feasible, the real-time price at the ISO 1's interface in ISO 3 is paid and the price at the ISO 3's interface at ISO 1 is received.

   For example, the firm submits a bid to ISO 3's market at 12:30 pm to buy 100 MWhs of energy at 2:00 pm if the predicted price at 1:30 pm was less than a certain amount, determined by the firm. Also the firm determines that this amount of energy would be sold at the ISO 3's interface in ISO 1 at the real-time price. If the condition is satisfied and the flow is physically feasible, the bid is cleared the firm buys 100 MWhs of electricity at the real-time price, at the ISO 1's interface in ISO 3 and sells it at the ISO 3's interface in ISO 1 at the real-time price. If the real-time price of the ISO 1's interface at ISO 3 is higher that the real-time price at the ISO 3's interface in ISO 1, the firm will lose money and otherwise, the firm will make profit.

3. **Trading strategy C** is a virtual (financial) trading strategy that trades on the day-ahead price of the ISO 4 and the ISO 5's interfaces in ISO 1 which will be denoted as zone 1 and zone 2 in the following chapters. As it is mentioned before, virtual traders can buy or sell electricity virtually in the day-ahead market, and the amount should be sold back or repurchased at the same hour in the real-time market. A long or short position can be taken in the day-ahead market, and the difference of the real-time price and the day-ahead price will be lost or gained based on the taken position in the market. For instance, let's say the real-time price at 2:00 pm at the zone 1 is 20$/MWh and the day-ahead price at 2:00 pm is 30$/MWh, if a short position is take in the day-ahead market, the firm will make 10$/MWh of profit and if a long position is taken in the market, the firm will lose 10$/MWh.

4. **Trading strategy D**

   The trade in this strategy happens the same as in trading strategy C. The only difference between these strategies is that trading strategy D trades on seven zones in ISO 1 which will be called zones 1 to 7.

Appendix B

Results for different months in 2019

B.1 January

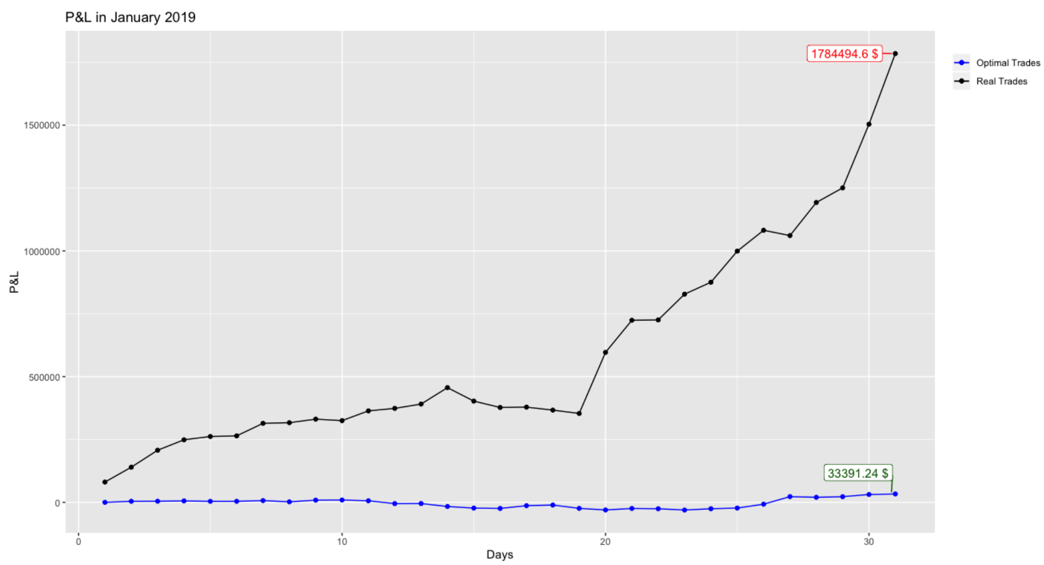

Figure B.1: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 1000$ and $m_2 = 50000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	-98.13 %	-
P&L at the end of the month	33391.24 $	1784495 $
Average loss	-5766.54 $	-21808.92 $
Three worst losses average	-12068.01 $	-33299.96 $
Percentage of winning days	61.29 %	80.64 %
Standard deviation of the portfolio	345.32	4065.13

Table B.1: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 3000$, $m_2 = 100000$ and the firm's selected portfolio in January 2019

B.2 February

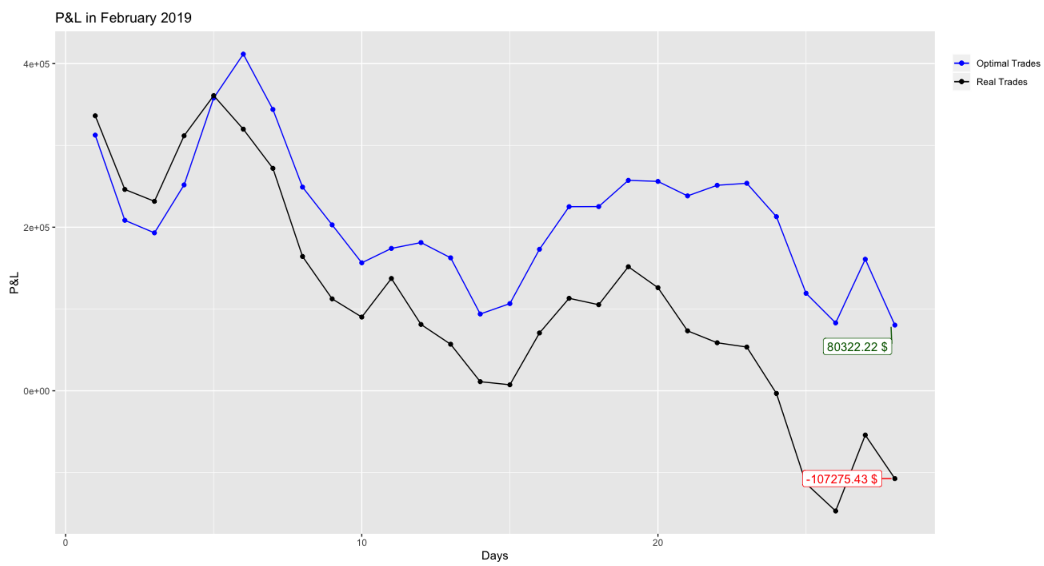

Figure B.2: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 4000$ and $m_2 = 100000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	174.87 %	-
P&L at the end of the month	80322.22 $	-107275.4 $
Average loss	-52288.05 $	-43239.69 $
Three worst losses average	-97569.69 $	-102617.2 $
Percentage of winning days	50 %	28.57 %
Standard deviation of the portfolio	3425.52	3519.79

Table B.2: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 4000$, $m_2 = 100000$ and the firm's selected portfolio in February 2019

B.3 March

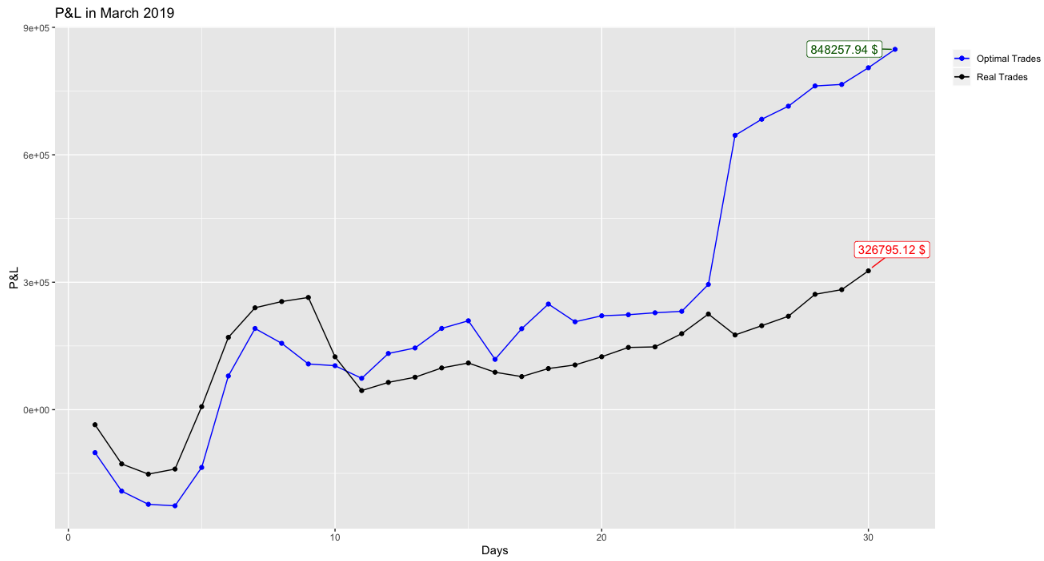

Figure B.3: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 3000$ and $m_2 = 100000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	159.57 %	-
P&L at the end of the month	848257.9 $	326795.1 $
Average loss	-47626.45 $	-56520.99 $
Three worst losses average	-94222.54 $	-103813.9 $
Percentage of winning days	67.74 %	73.33 %
Standard deviation of the portfolio	3630.42	3756.23

Table B.3: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 3000$, $m_2 = 100000$ and the firm's selected portfolio in March 2019

B.4 April

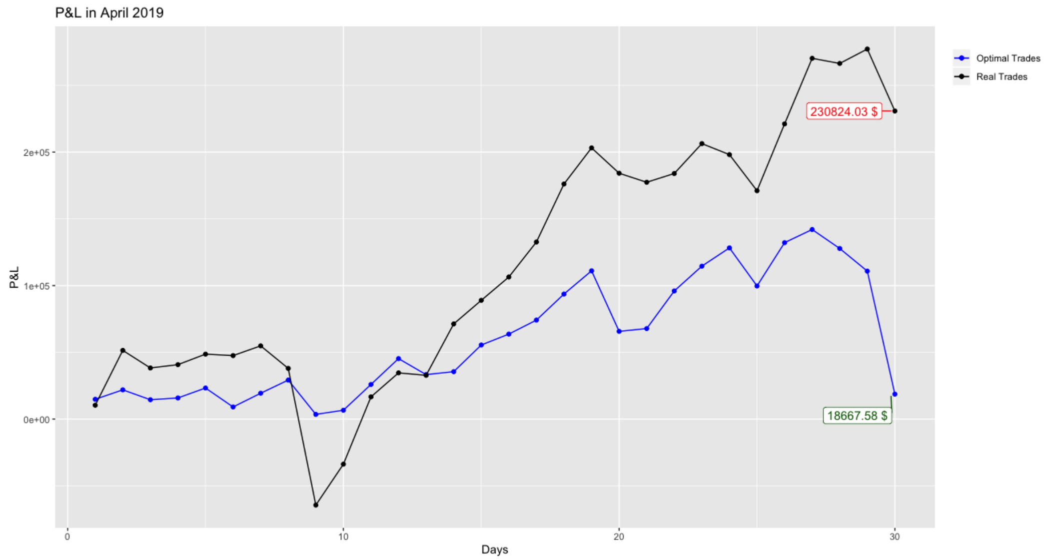

Figure B.4: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 1000$ and $m_2 = 50000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	91.91 %	-
P&L at the end of the month	18667.58 $	230824 $
Average loss	-28506.8 $	-22434.97 $
Three worst losses average	-55359.9 $	-58642.61 $
Percentage of winning days	70 %	63.33 %
Standard deviation of the portfolio	1036.38	1626.38

Table B.4: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 1000$, $m_2 = 50000$ and the firm's selected portfolio in April 2019

B.5 May

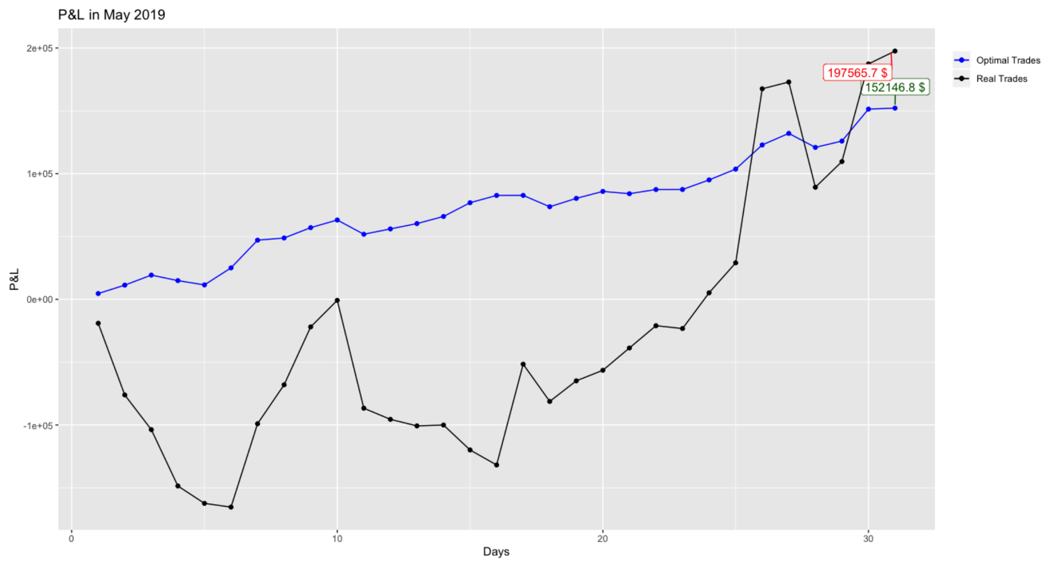

Figure B.5: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 1000$ and $m_2 = 50000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	-23 %	-
P&L at the end of the month	152146.8 $	197565.7 $
Average loss	-6860.19 $	-29481.95 $
Three worst losses average	-10517.19 $	-75542.54 $
Percentage of winning days	80.64 %	54.84 %
Standard deviation of the portfolio	348.32	2351.42

Table B.5: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 1000$, $m_2 = 50000$ and the firm's selected portfolio in May 2019

B.6 June

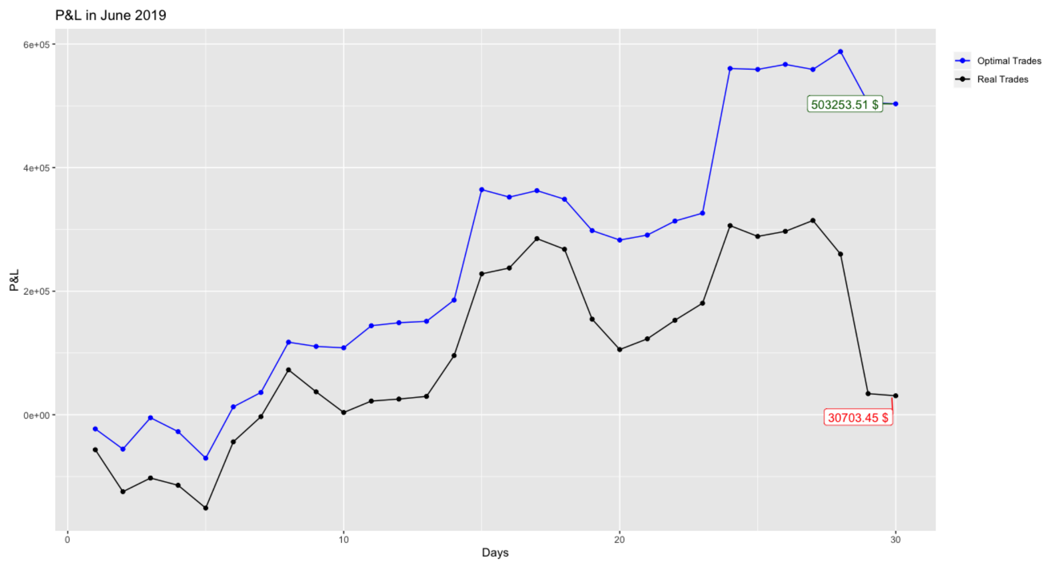

Figure B.6: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 3000$ and $m_2 = 70000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	1539.08 %	-
P&L at the end of the month	503253.5 $	30703.45 $
Average loss	-22617.09 $	-55614.87 $
Three worst losses average	-58865.15 $	-135637.7 $
Percentage of winning days	53.33 %	56.67 %
Standard deviation of the portfolio	2604.86	4511.92

Table B.6: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 3000$, $m_2 = 70000$ and the firm's selected portfolio in June 2019

B.7 August

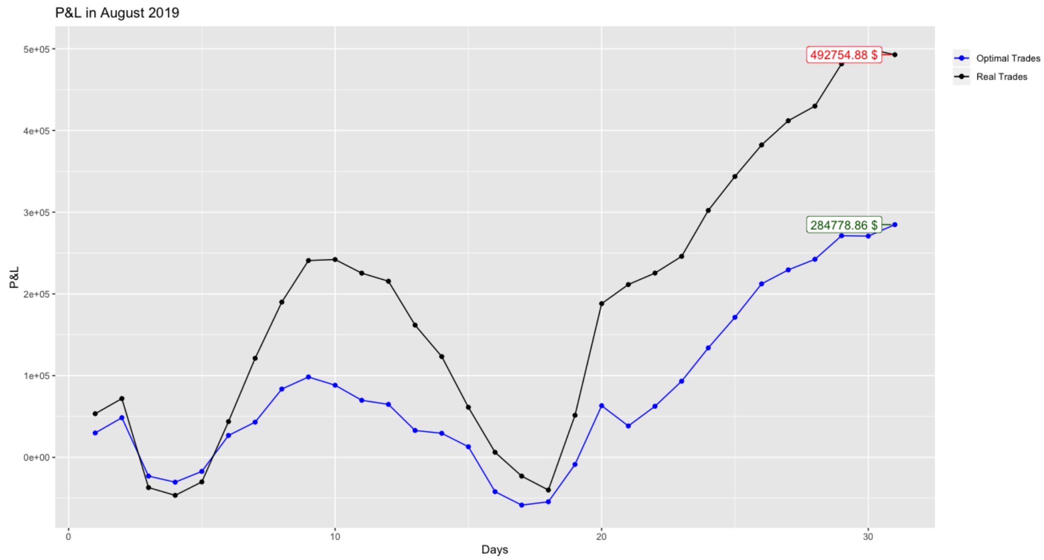

Figure B.7: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 3000$ and $m_2 = 90000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	-42.21 %	-
P&L at the end of the month	284778.9 $	492754.9 $
Average loss	-21753.86 $	-37047.5 $
Three worst losses average	-52799.58 $	-75366.46 $
Percentage of winning days	61.29 %	64.52 %
Standard deviation of the portfolio	1289.33	2353.75

Table B.7: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 3000$, $m_2 = 90000$ and the firm's selected portfolio in August 2019

B.8 September

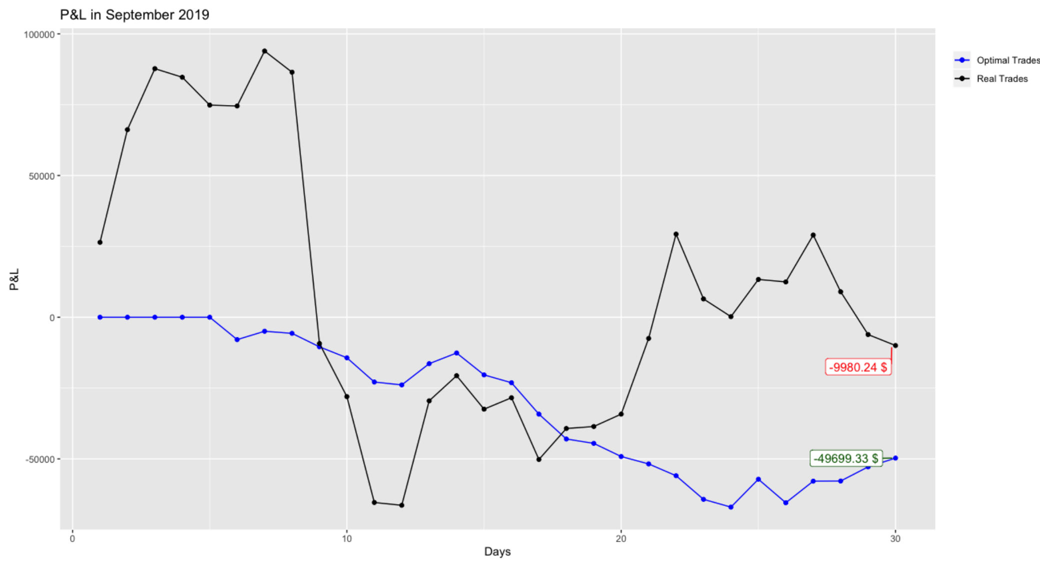

Figure B.8: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 1000$ and $m_2 = 20000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	-397.98 %	-
P&L at the end of the month	-49699.33 $	-9980.24 $
Average loss	-5265.77 $	-17250.99 $
Three worst losses average	-9466.58 $	-51982.72 $
Percentage of winning days	26.67 %	46.67 %
Standard deviation of the portfolio	221.81	1697.93

Table B.8: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 1000$, $m_2 = 20000$ and the firm's selected portfolio in September 2019

B.9 October

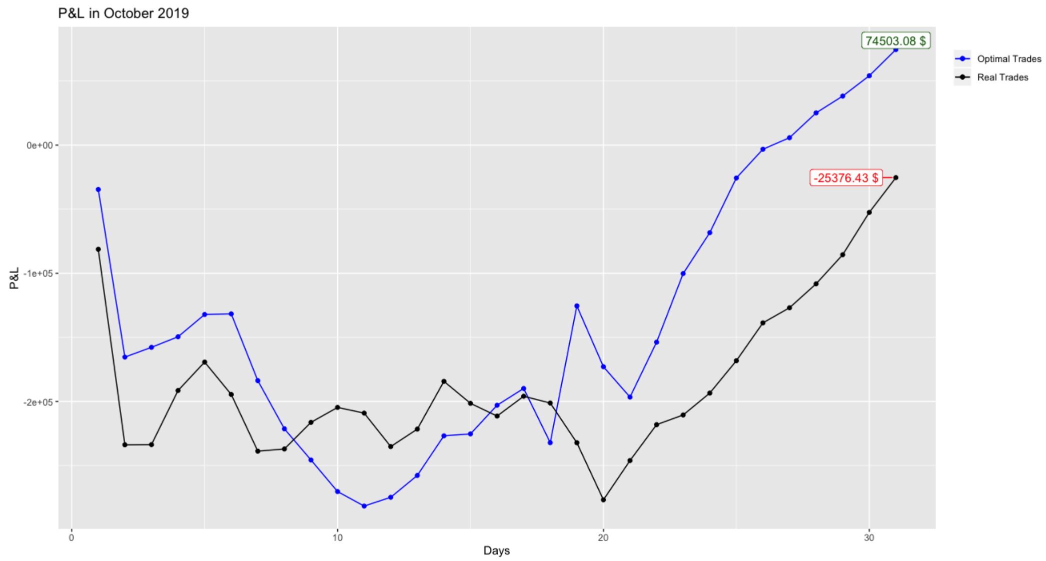

Figure B.9: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 3000$ and $m_2 = 100000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	393.59 %	-
P&L at the end of the month	74503.08 $	-25376.43 $
Average loss	-42878.57 $	-40161.86 $
Three worst losses average	-76771.67 $	-92782.29 $
Percentage of winning days	67.74 %	64.52 %
Standard deviation of the portfolio	1722.9	2138.24

Table B.9: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 3000$, $m_2 = 100000$ and the firm's selected portfolio in October 2019

B.10 November

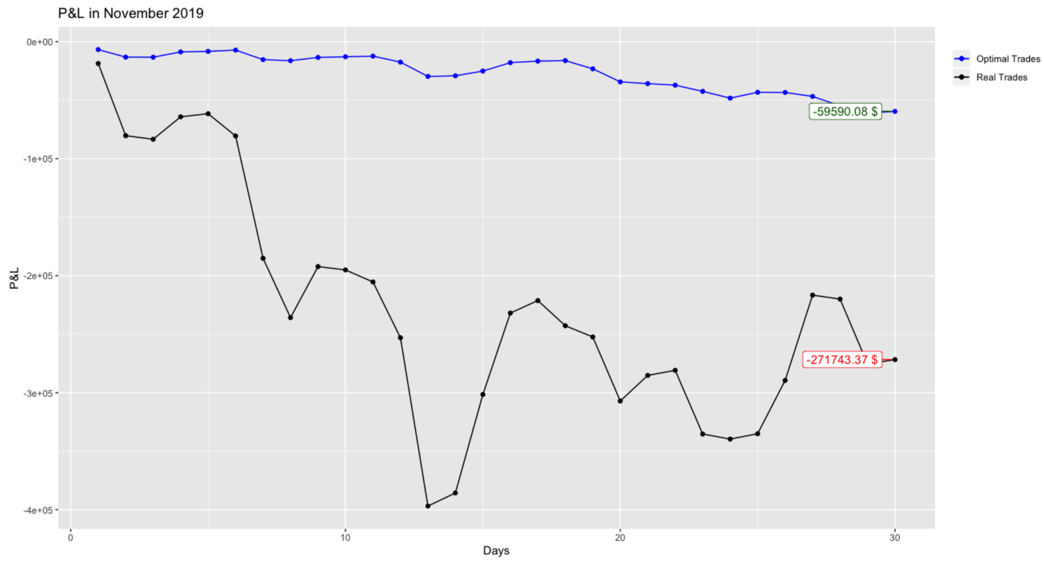

Figure B.10: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 1000$ and $m_2 = 20000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	78.07 %	-
P&L at the end of the month	-59590.08 $	-271743.4 $
Average loss	-5267.29 $	-39164.83 $
Three worst losses average	-10512.36 $	-103365.4 $
Percentage of winning days	43.33 %	43.33 %
Standard deviation of the portfolio	203.74	3558.01

Table B.10: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 1000$, $m_2 = 20000$ and the firm's selected portfolio in November 2019

B.11 December

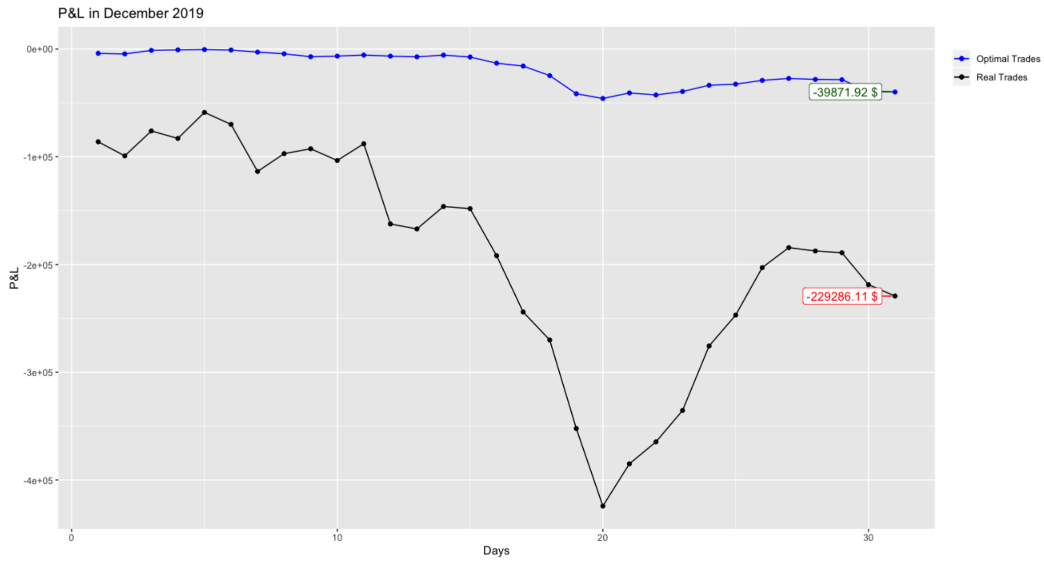

Figure B.11: The cumulative return of the firm both with optimal weights and weights selected by the firm with $\sqrt{m_1} = 1000$ and $m_2 = 20000$

Measure	Optimal Trades	Real Trades
Percentage of change in the P& L	82.61 %	-
P&L at the end of the month	-39871.92 $	-229286.1 $
Average loss	-3556.68 $	-31881.28 $
Three worst losses average	-12074.33 $	-80887.7 $
Percentage of winning days	38.71 %	41.94 %
Standard deviation of the portfolio	189.79	3123.14

Table B.11: The values for the firm's measures to compare the optimal portfolio with $\sqrt{m_1} = 1000$, $m_2 = 20000$ and the firm's selected portfolio in December 2019

Appendix C

Correlation Matrix

	1	2	3	4	5	6	7	8	9	10	11
1	1.00	0.26	-0.20	-0.39	-0.27	-0.06	-0.09	-0.07	-0.07	-0.05	0.22
2	0.26	1.00	0.32	0.36	-0.03	0.37	0.32	0.32	0.32	0.32	0.43
3	-0.20	0.32	1.00	0.47	0.55	0.92	0.95	0.94	0.93	0.80	0.73
4	-0.39	0.36	0.47	1.00	0.27	0.47	0.45	0.46	0.43	0.40	0.27
5	-0.27	-0.03	0.55	0.27	1.00	0.44	0.42	0.42	0.40	0.20	0.25
6	-0.06	0.37	0.92	0.47	0.44	1.00	0.96	0.94	0.95	0.80	0.78
7	-0.09	0.32	0.95	0.45	0.42	0.96	1.00	0.99	0.99	0.88	0.76
8	-0.07	0.32	0.94	0.46	0.42	0.94	0.99	1.00	0.99	0.89	0.73
9	-0.07	0.32	0.93	0.43	0.40	0.95	0.99	0.99	1.00	0.90	0.75
10	-0.05	0.32	0.80	0.40	0.20	0.80	0.88	0.89	0.90	1.00	0.68
11	0.22	0.43	0.73	0.27	0.25	0.78	0.76	0.73	0.75	0.68	1.00

Table C.1: The sample correlation matrix between the average hourly P&L