A Quantitative-sectoral Approach to Business Risk

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Abstract

A Quantitative-Sectoral Approach to Business Risk

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In the evolution of bank regulation over the last thirty years, the Value-at-Risk (VaR) measure has been a key metric in determining the amount of regulatory capital a bank must hold to deal prudently with its exposure to market, credit and operational risk. The security supposedly provided by VaR was certainly challenged by the financial crisis in 2008. The risk analysis in place at the time appeared to be too narrowly focused, as other issues (particularly liquidity risk) came to the fore.

This thesis has maintained the VaR objective, but extends the traditional analysis along two dimensions. First, we have analyzed a notion of business risk associated with fluctuations in a bank's business income that are not tied to specific market, credit or operational events. Rather the fluctuations that we analyze are more the consequences of ongoing strategic decisions. Second, we have attempted to operationalize a sectoral approach where the losses potentially faced by a particular bank are those that are shared by its competitors.

We first develop in Chapter 2 a general framework for analyzing the core notion *Residual Profit & Loss* (RPL) using the income statements as reported in Capital IQ which also provides data on Interest Earning Assets (IEA). We then construct a business income data set based on *RPL/IEA* for a US Retail Banking Sector. There are twenty-two banks in the sector. *RPL/IEA* is determined for these banks over the period 2002-2015. Using more recent data, we will be able in the thesis to focus on the post-crisis 2008 period.

A data set is also constructed in Chapter 2 for the Canadian banking sector. It is more concentrated than the US sector studied and was less severely affected by the 2008 crisis. But the methodological approach followed in this chapter faces an additional complexity in so far as accounting standards were significantly changed in 2011. Moreover, it is not

possible to reconstruct income statements prior to 2011 using the new standards. We pursue several avenues of adjustment to render the treatment of the data over the entire sample as coherent as possible. We then construct *RPL/IEA* for this typical banking sector following the same methodology as used for the US retail sector.

The remainder of Chapter 2 transforms the time series of business returns (RPL/IEA ratio) for each bank into the US and Canadian sectoral loss datasets. A loss (gain) for a particular bank is characterized as the deviation from its expected return defined as its average return over the sample.

Chapter 3 proposes two approaches to determine the values of VaR corresponding to two ways of looking at the loss datasets. One approach assumes that an individual bank's loss time series follows a sectoral moving average process. The common parameter is estimated across the time series using maximum likelihood. The VaR for an individual bank can readily be retrieved in this multivariate characterization. The second approach ignores the time series dimension and pools the data into a single sample for each sector. In this context, we propose to use the saddlepoint approximation technique that involves the use of sample moments to estimate the percentiles of the underlying loss distribution.

The saddlepoint approach is not commonly use in the applied financial literature. The basic features of this technique are reviewed in Chapter 3 along with several examples to illustrate how it has been applied in finance. The second part of the Chapter presents an extensive Monte Carlo simulation study that contrasts the performance of the saddlepoint percentile estimates with those obtained by the maximum likelihood structural approach.

Chapter 4 returns to the calculation of business risk faced by the US and Canadian sectors considered in the thesis. For each of the associated business loss data sets, there are the two estimation procedures that were introduced in the previous chapter. The VaRs for different confidence levels are determined and contrasted across the two models for each of the two sectors. We include several comparisons with the economic capital held by specific banks in the Canadian sector.

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Chapter 1

A Quantitative-sectoral Approach to Business Risk: Overview

1.1 Motivation and Objectives

Basel I and Basel II have specified a variety of models for determining the levels of regulatory capital that financial institutions must hold as safeguards to their exposure to three particular types of risks; namely, credit risk, market risk and operational risk. The focus as a consequence has been on the impact of potential losses that may arise due to changes in short-term market valuations, defaults or downgrades over a period of time within the credit portfolio, or execution failures in the day-to-day conduct of business operations that result in losses. Regulatory capital is intended to act as buffer to absorb such losses. Within each category, the most sophisticated approach from a modeling perspective has been to estimate a potential distribution of losses over a predetermined horizon based on related loss data. Within this formal framework, requisite regulatory capital is then determined in the final step as function of the tail quantiles of the distribution, or what is termed the VaR (value at risk) for the specific risk category.

Financial institutions hold as well extra economic capital to deal with other challenges. These may include changing liquidity conditions, shifts in the competitive landscape or even adverse shocks to the institution‘s reputation. These factors may not translate into immediate balance-sheet losses. In so far as the bank must remain sufficiently agile to deal successfully with such examples of a changing business environment (business risk) it holds economic capital. For example, regulators may ask the bank how it would react to a scenario whereby it loses several of its main clients. In which case, there would be an immediate loss of income; how would the bank manage or even survive in this context? In other words, does the bank hold sufficient economic capital to deal with such a challenge? One regulatory approach has involved hypothetical scenario assessments of the bank‘s capacity to deal with challenges other than those originating from market, credit or operational losses. The Basel Committee on Banking Supervision (BCBS) has not given any particular direction regarding the quantitative determination of economic capital. None the less its importance is underscored in a variety of its directives:

- Operational Risk is defined as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk. *Principles for the Sound Management of Operational Risk, Bank for International Settlements (2011).*

- The supervisor requires banks to have appropriate policies and processes for assessing other material risks not directly addressed in the subsequent Principles, such as reputational and strategic risks. *Core Principles for Effective Banking Supervision, Bank for International Settlements (2012).*

- Under the Basel Framework, all supervisors require banks to consider all risks, including both traditional financial risks (eg credit risk, market risk, interest rate

> risk) as well as reputational, legal, and strategic risks. Supervisors require banks to consider risks that may not appear to be significant in isolation but when combined with other risks could lead to material losses. Supervisors expect banks to consider all risks that can affect capital, including the ones that might have an indirect or slowly developing impact.... *Overview of Pillar 2 supervisory review practices and approaches, Bank for International Settlements (2019).*

Banks in practice, as described by John Hull in his book *Risk Management and Financial Institutions*, have used two broad approaches to determine the adequate level of economic capital that should be held; these are described as bottom up and top down. On the latter approach, to take one example, the volatility of the financial institution's assets is first estimated and used as a parameter in a stochastic model (for example, the Merton model) that gives the probability that the value of its assets falls below the bank's obligations within a one-year time frame. It is more usual that the bottom up approach is used. The bank determines its total risk as an aggregate of market, credit and operational risks along with an add on to cover business risk. However, as Hull remarks, business risk is difficult to quantify (even less tractable than operational risk) and he observes without detailed elaboration: "it is important that senior risk managers... have a good understanding of the portfolio of business risks being taken."

By contrast, the primary objective in the work that follows is to arrive at a formal VaR analysis of what we may term business risk. The advantage of this approach is that the business-risk issues faced by the bank are assumed to have a probabilistic context that is left to large extent unanalysed in other approaches. The notion of business risk in our thesis is simply and broadly characterized as potential fluctuations in income other than those resulting from market, credit or operational loss events. These are annual changes to what we term in the thesis as Residual Profits & Losses (RPL). A formal definition is given in the next section. Here it suffices to observe that RPL for a particular bank is derived from the bank's income statement.

A second distinguishing feature of our VaR approach to business risk is that it is sectoral in nature. On our view, the business risk faced by a bank is certainly related to strategic decisions adopted by other banks operating in similar business contexts; how the competition is fairing has an important impact on the home bank's income statement. In this broader market environment, the thesis considers two distinct bank universes: a group of retail banks in the United States as well as the complete group of Canadian banks. We quantify business risk for these two banking sectors covering the period from 1994 to 2015. The US sector is composed of eighteen medium-size retail banks; the Canadian banking sector has eight banks.

We should indicate at the outset that a formal approach to the analysis of the business dimension of economic capital is not intended to replace the business judgment of the financial institution's risk managers but rather to inform their decisions. In his presentation of Deutche Bank's calculations in its financial statement for 2013, Hull presents what we think is a useful framework. Business risk is seen to comprise roughly 7% of the capital held as regulatory capital for market, credit and operational risk. It would be useful to compare this number with that obtained from a more formal approach. The comparison would serve as an indication even diagnostic of whether or not the bank is conservative in its determination of its business risk buffer.

To conclude this introductory section, we describe in general terms the major research questions that are addressed in the thesis:

- Can formal techniques be extended to analyze the notion of business risk faced by a financial institution?
- Can the formal analysis reflect the sectoral context of the individual bank's strategic decision making?
- Is it practically possible to implement these formal methods in a way that could be used by risk managers on an annual basis?

1.2 Core Measure of Business Income: Residual Profit & Loss (RPL) and Normalized RPL

In this thesis we analyse fluctuations in a financial institution's business income where the impact of credit, market and operational effects has been removed from the accounting definition of its net income (NI). Also, we smooth the balance-sheet characterization of NI to mitigate the impact of ad hoc accounting adjustments; the intent is to capture their impact in a more realistic manner.

We call the resulting measure of business income *Residual Profits & Losses* (RPL), calculated as follows:

$$RPL = \mathit{Adjusted\ Net\ Income} + \mathit{Credit\ Losses} - \mathit{Market\ Gains} + \mathit{Operational\ Losses}$$

The motivating idea is straightforward: specific credit losses or market gains or losses due to failures in execution should not be counted in defining a notion having a broader, more strategic sense. As indicated above, the ANI component is intended to capture the core elements of the year's income with some smoothing over several years to accommodate *ad hoc* accounting decisions.

A bank's business strategies are most likely influenced by its competitors. We take the position that business performance and its associated risk are better assessed in the relevant competitive environment; we need to compare bank performance with those of its rivals within a sector facing similar regulatory, jurisdictional and economic conditions. To some extent, core strategy may be somewhat homogeneous across banks within a sector. On the other hand, other strategies may differentiate banks in essential ways; for example, a bank may expand into different geographic and product markets to pursue long - term growth objectives and to strengthen its competitive position. Another route to growth is via mergers and acquisitions which in turn have balance sheet consequences. Mergers and acquisitions differ case by case. Banks as well differ as well in their rates of adoption of new technologies which have changed the landscape of banking competition. Banks have adopted at different rates digital technology (such as artificial intelligence, machine learning, robotics, etc.) to enhance customer experience and to improve efficiency; these bring in their wake increasing costs and associated risks. Strategic heterogeneity may yield dissimilar economic results within a sector. In short, banks within a sector face shocks: some are positive, contributing to sustainable growth; but some disruptively weaken a bank's competitive position.

In this context, we need to introduce a business income measure that enables comparison across banks and over different periods: our approach uses RPL normalized by the financial institution's average *Interest Earning Assets (IEA)*. The *RPL/IEA* ratio is interpreted as the strategic balance–sheet return on the bank's holdings. In what follows, we assume that the basic unit of business risk analysis is the *RPL/IEA* ratio.

Unpublished work by Campbell and Stam (2012) proposed to use this notion in a VaR context to analyse business risk. The authors first conducted the analysis on the US retail banking sector covering the period 1994–2011 with yearly accounting data taken from Capital IQ. The sector was composed of 21 medium-sized retail banks. The entire sector's strategic returns as described by *RPL/IEA* were integrated into a single univariate distribution of deviations from expected returns. In this context, the value of VaR for a particular institution was determined as a worst case deviation across the sector – as estimated via a t-distribution—that is then applied to the institution.

By contrast, we take two different approaches to the determination of a bank-specific VaR. In one, we preserve the multivariate time series character of the individual Normalized RPL data in a parametric framework and impose a sectoral approach by the simultaneous estimation of a shared parameter across the parameterization. The VaR for a particular bank is then calculated directly from the estimated parameters for the banks in question estimated in a sectoral context. In the second approach, we pool the data and compute a global sectoral VaR using a non-parametric estimation procedure that is seldom used in the risk management literature. This approach is presented in Chapter 3 where it is contrasted with the time series approach in a comprehensive Monte Carlo study.

1.3 Review of the Business Risk Literature

The formal approach to the calculation of business risk VaR has not to our knowledge been undertaken and, as a consequence, there are no quantitative results that we can refer to by way of comparison. None the less, it is instructive to compare our approach with previous attempts to determine Economic Capital (ECAP) held by financial institutions against business risk in so far as there are shared methodological elements with the work presented in this thesis. To our knowledge, the notion of business risk within the banking sector has been discussed in seven papers Schroeck (2002), Kuritzkes and Scott (2005), Kuritzkes and Schuermann (2008), Böcker (2008), Doff (2008), Chaffai and Dietsch (2015) and Chockalingam, Dabadghao, and Soetekouw (2018) have discussed the notion of business risk within the banking sector.

Schroeck (2002) proposes an earlier version of residual income by removing the gains and losses of trading and credit activities from revenues, also by adjusting for any extraordinary items. But the author does not provide details to illustrate the calculation of residual income from a bank's income statement. From the author's perspective, banks need to hold economic capital against business risk so long as residual income drops below a pre-determined operational threshold that includes fixed costs. In this context, quantitatively determined economic level is derived from a distribution of the residual income using this threshold with a 99.9 per cent confidence level.

Kuritzkes and Scott (2005) along with *Kuritzkes and Schuermann (2008)* has treated a fixed fraction of non-financial risk as a proxy of business risk. Kuritzkes and Scott (2005) is the closest existing work to our thesis, in terms of determining economic capital by integrating material returns from a group of banks into a distribution. This work covers the period 1994-2001 and analyzes non-financial risk with yearly accounting data taken from a group of the top 50 US banks in asset size. These 50 banks include investment banks and retail banks, extra-large banks and medium- sized banks, domestic banks and foreign banks. The authors pool 50 banks' deviations from their own expected value to fix a distribution for further VaR analysis. These authors adopt Schroeck (2002) formula for residual income and derive numbers from banks' income statements. From net income, they simply remove provisions for loan losses and trading income, which are the proxies to credit losses and market gains, respectively. Residual income is normalized using risk weighted assets (RWA).

RWA is commonly used as a measure in estimating the minimum level of regulatory capital to be held by a bank. The framework is based on a weight metric which requires banks to holding more capital for riskier assets and less for safer ones. In the analysis of business risk, we think it is inappropriate to use RWA to normalize returns since it incorporates a risk dimension. On our approach, we use the dispersion of return per interest asset to compare bank performance. We interpret the *RPL/IEA* ratio as the strategic return from the bank's holdings and this return is comparable across banks and across time.

Kuritzkes and Scott (2005) determines ECAP for business risk indirectly by taking around 60 per cent ECAP for non-financial risk while ECAP for non-financial risk is defined as a function of predetermined deviates from the expected normalized residual income. This paper estimates VaR as a quantile of empirical distribution at 99.9 per cent confidence level; Based on the visual observation from the empirical distribution, the 99.9

per cent quantile is approximated by 5 times of the estimated standard deviation.

Böcker (2008) and Chockalingam et al. (2018) quantify business risk using simulated data derived from models such as CAPM and fluctuations characterized by Brownian motion. Böcker (2008) characterizes business risk by the fluctuations in market value as measured by a bank' future cash flow. VaR analysis of the bank's business risk is based on the distribution of its market value. Chockalingam et al. (2018) quantifies business risk for a single bank covering the period of 2012-2015 on a quarterly base. Residual income based on a particular bank's business characteristics within a short period of four years is used to estimate the parameters in a model of a Brownian motion. This work also measures fluctuations in simulated income. But as with Schroeck (2002), it requires banks to hold ECAP as long as the bank has an income lower than costs. Accordingly, with simulated residual income and a threshold for costs, an ECAP distribution is obtained. In contrast with our work, VaR analysis is based on a distribution of ECAP rather than on income and returns. Similar to Schroeck (2002) and Doff (2008), these authors acknowledge that the bank operates in a competitive environment, but leave open the issue of how this reality has an impact on the bank's income statement.

Doff (2008) groups existing methods to determine ECAP associated with business risk using a combination of peer group analysis, statistical methods and scenario analysis. Peer group analysis which had been proposed as well by Schroeck (2002) associates the bank's residual income with a matched non-financial company that has no credit and market risk exposure. The challenge here is to find the appropriate match. The scenario approach proposed in this work differs from that used by the regulators and is based on the opinions of internal experts who are privy to the bank's most recent data. The paper briefly addresses the statistical approach; but it does not actually implement any methodology to arrive at a bank's ECAP.

Chaffai and Dietsch (2015) discusses the notion of business risk based on accounting data from 90 French retail banks covering the period 1993-2011. In this regard, it shares the sectoral perspective adopted by the thesis. But the focus of this work is to test the resilience of their retail banking model rather than to quantify business risk. And in contrast to our quantitative approach, the authors propose a qualitative efficiency frontier methodology to analyse the business risk faced by retail banks.

1.4 Organization of the Thesis

We extend the data related to the US retail banking sector used in Campbell and Stam (2012) until 2015. An important contribution of the thesis is to analyze the business risk faced by the Canadian banking sector. More specifically, the second chapter includes the following material:

- *A general framework for the construction of RPL and IEA*. We base our work on the income statements as reported in Capital IQ and show in considerable detail the methodology for constructing RPL and the normalizer IEA (average interest earning assets) from the income statements.

- *RPL/IEA for a US Retail Banking Sector.* There are twenty-two banks in the sector. *RPL/IEA* is determined for these banks over the period 2002-2015. Using more recent data, we will be able in the thesis to focus on the post-crisis 2008 period.

- *RPL/IEA for the Canadian Banking Sector.* We work with the domestic banking sector that has distinctive features. It is more concentrated than the US sector studied and was less severely affected by the 2008 crisis. But the methodological approach followed in this chapter faces an additional complexity in so far as accounting standards were significantly changed in 2011. Some assets that were treated as off-balance-sheet under the old standard now had to be disclosed under the new standards on the balance sheet. The income generated by these assets currently figures in the RPL which is the basic building block in our analysis. Moreover, it is not possible to reconstruct income statements prior to 2011 using the new standards. In this subsection of Chapter 2, we pursue several avenues of adjustment to render the treatment of the data over the entire sample as coherent as possible. We then construct *RPL/IEA* for this typical banking sector following the same methodology as used for the US retail sector.

- *Sectoral approach to the determination of business risk faced by an individual bank.*

  The remainder of Chapter 2 transforms the time series of business returns (RPL/IEA ratio) for each bank into the US and Canadian sectoral loss datasets. A loss (gain) for a particular bank is characterized as the deviation from its expected return defined as the its average return over the sample.

Chapter 3 proposes two approaches to determine the values of VaR corresponding to two ways of looking at the loss datasets. One approach assumes that an individual bank's loss time series follows a sectoral moving average process. The common parameter is estimated across the time series using maximum likelihood. The VaR for an individual bank can readily be retrieved in this multivariate characterization. The second approach ignores the time series dimension and pools the data into a single sample for each sector. In this context, we propose to use the saddlepoint technique involving sample moments to estimate the percentiles of the underlying loss distribution.

The saddlepoint approach is not commonly use in the applied financial literature. The basic features of this technique are reviewed in Chapter 3 along with several extended examples to illustrate how it has been applied. The second part of the Chapter presents an extensive Monte Carlo simulation study that contrasts the performance of the saddlepoint percentile estimates with those obtained by the maximum likelihood structural approach. In the study, the underlying data generating processes correspond to different parametric specifications of the individual bank's losses that are then aggregated as described above. As a consequence, the relative merits of the saddlepoint approach can be more readily appreciated.

Chapter 4 returns to the calculation of business risk faced by the US and Canadian sectors considered in the thesis. For each of the associated business loss data sets, there are the two estimation procedures that were introduced in the previous chapter. The VaRs for different confidence levels are determined and contrasted across the two models for each of the two sectors. We include several comparisons with the economic capital held by specific banks in the Canadian sector.

Chapter 2

The Construction of Business Loss Datasets for Two Banking Sectors

2.1 Introduction

In this chapter, we construct two loss datasets that will be used in a later chapter to compute VaR measures of business risk. The loss data is derived from the income statements of banks in two different sectors; one American and the other Canadian. The accounting-based methodology is presented in the first section of this chapter. We construct on an annual basis the Residual Profit and Loss (RPL) for each bank in the two sectors, defined as

$$RPL = \textit{Adjusted Net Income} + \textit{Credit Losses} - \textit{Market Gains} + \textit{Operational Losses} \quad (*).$$

This income measure, introduced in Chapter 1, is intended to capture the impact of a bank's overall strategic approach to generate profit. In this context, the income statement is first adjusted to distribute the accounting impact of specific *ad hoc* gains and losses booked in a given period of several years; the result is given as Adjusted Net Income (ANI). The second step involves adjusting income to compensate for gains and losses associated with the Basel regulatory risk categories that are market, credit and operational risk. After these adjustments are made, the resulting RPL is used as the building block to characterize business risk. The ultimate goal is to isolate business risk as one component used to determine economic capital to be held by the bank.

After presenting the framework for determining the annual RPL for an individual bank in the first section, we discuss the specific construction of loss datasets for a sample of medium-sized American retail banks and for the Canadian banking industry as a whole. We use the Capital IQ (a research division of Standards & Poor's) database, which compiles data from income statements of hundreds of financial institutions throughout North America. The data is derived from the banks' annual filings to the SEC (Securities and Exchange Commission) in the United States, and to the OSFI and the AMF in Canada. Along with the task of data collection, Capital IQ organizes accounting data in a uniform presentation which is critical in enabling a comparative assessment of bank performance. All of the banks analyzed in our two sectors can be found in the Capital IQ database.

So defined, the bank's annual business income relative to its *interest earning assets* (IEA) can be used to determine a return for the year. IEA is also collected by Capital IQ on an annual basis, and . IEA is used in the thesis as a normalizing factor that enables us to compare return performance in terms of RPI/IEA across years and, most importantly, across banks.

It should be noted that the construction of the dataset for Canadian banks was complicated by the adoption of new accounting standards in 2011 that affected both the calculation of RPL and IEA. As a consequence, we have been forced to make some *ad hoc* assumptions in the construction of the data in order to achieve coherence across the sample, before and after 2011. These assumptions are described in great detail in the third section.

Whereas an operational loss is booked directly as a dollar entry in the bank's income statement, a business loss in the strategic sense that we are pursuing should be viewed as a deviation from its expected return. In what follows, we estimate the latter notion for each bank as the average return performance over the sample studied. The performance of the American banks declined dramatically in the period after the global financial crisis. The issue then arises as to what estimate of expected return should be used in the post-crisis period in constructing the loss distribution for the US banks. This issue is covered in the second section of this chapter.

2.2 The Calculation of Business Income from Standardized Income Statements

2.2.1 A First Look at the Standardized Income Statement in Capital IQ

The banks that comprise the US sector studied in this chapter are provided in Table 2.1. To illustrate the steps that we follow in order to determine RPL, we present several examples of income statements for these banks for the selected years (see Tables 2.2a-2.2c).

We first consider Table 2.2a: the income statement for People's United Financial Inc. (PBCT) in 2015. The two major contributions to operational income, net interest income (line 8) and total non-interest income (line 17), can be found on the left-hand side of the table. The sum of these two is Revenue Before Loan Losses, more usually termed as Profit Before Costs. The following lines (from line 24 to 29) describe non-interest expenses, including fixed operational costs. On the right-hand side, there is a section for unusual items found between EBT (Earning Before Tax) and EBT Including Unusual Items (line 12). Line 14 gives the tax contribution. From the $1,251.1 million total revenue, we deduct $850.1 million of non-interest expenses, and $10.5 million un-usual items and $130.4 million for taxes; resulting in $260.1 million Net Income (NI) (see the end of the table).

The section dealing with unusual items is essential for the calculation of RPL. However, it is not generally presented on the banks' income statement. Capital IQ has put considerable effort into these the collection of this information. Items are mostly extracted from the sections on non-interest income or expenses found in banks's financial reports. For instance, in Table 2.2a, the Asset Write-down (line 8 on the right) has a record of $10.5 million, derived as follows: in 2015, PBCT reported $860.6 million non-interest expense versus $850.1 million (line 30 on the left) in the sample table. Capital IQ extracts $10.5 million from the non-interest expense and collects it as an Asset Write-down (On page 28 in PBCT 2015 annual reports, a non-interest expense of $10.5 million is a write-down of banking house assets.).

In Table 2.2a, we note that the figures for non-interest expense do not exactly match those that are actually reported by the bank for that year. Our concern was that the modification by Capital IQ would affect the reported Net Income (NI). We determined that Capital IQ only made some simple changes in the order of presentation (as in the example above); indeed, the extraction of unusual items only affects the presentation. Capital IQ deducts the same entries of total cost and adds the same number of non-daily income to the revenue to get NI. We have verified that the figure of NI from the standardized income statements is identical to the figure reported by the bank.

In Table 2.2a, the line items in the calculation of RPL are indicated in square brackets with "Credit, Market, Op and ANI", referring to the calculation of credit losses, market gains, operational losses and adjusted net income, respectively. PBCT in 2015 had no market gains or operational losses. In order to demonstrate all of the calculations in RPL, two tables, Table 2.2b and 2.2c are included to illustrate the calculation of market gains and operational losses, respectively.

2.2.2 The Calculation of the Residual of Profit & Loss (RPL)

We illustrate in turn the several components in the characterization (*) of RPL, focusing on Table 2.2a.

Credit Losses

In this table, there are five items contributing to credit losses. Four are taken from the sections in the standardized income statements describing non-interest income and unusual items. These are highlighted in Table 2.2a (left-hand column, lines 11, 12, 20; right-hand column, line 8), including Provision for Loan Losses (PLL), Asset Write-down, and two other items. The fifth item, Opportunity Cost of Non-Performing Assets, is taken from the bank's balance sheet; however, this is not presented in this table (see below).

Credit losses on financial instruments are recognized when the value of the underlying collateral has declined due to deteriorating credit quality. These credit losses are mostly from problem loan portfolios. Banks make provisions and specific reserves allocated to these loans. Provision for Loan Losses (PLL) reflects the charge to income by write-offs which are added to the reserve of loan losses. When the credit losses of a problem loan portfolio spills over the bank's reservation limit, the charge is the direct write- offs to income; that is, the assets are written-down. During economical up-trends, PLLs are steady and direct write-downs are insignificant; by contrast, PLLs in downturns increase significantly and direct write-downs are also substantial. Capital IQ especially collects disclosures related to asset write-downs as one of the unusual items on the Income Statement (right-hand column, line 8).

Another item counted toward credit income is the gain from the sale of loans and other assets, including the gains from loan securitization. In general, banks sell poorly per- forming loans (assets) to reduce credit risk or sell loans (assets) in order to diversify their portfolios or to raise capital under constrained conditions. Securitization allows banks to earn revenue from originating and distributing loans rather than from holding loans. Nonetheless, one of the determinants of the gain is the credit quality of these assets. Campbell and Stam (2012) included these items in the block of credit losses.

As indicated above, we take the opportunity cost of non-performing assets into account in the computation of credit losses. According to accounting criteria, once assets (including loans) have lost their original earning ability, they are recognized as non-performing. However, these assets still impact the banks' funding sources on the liability side; that is, banks still need to pay interest to depositors in the reporting period. Campbell and Stam (2012) used the yield on the earning assets to determine opportunity cost on non- performing assets. The cost is calculated as non-performing assets multiplied by the yield.

To summarize: we compute the five items in the Credit Loss component of RPL as:

$$
\begin{aligned}
Credit\ Losses = -\{&-Provision\ for\ Loan\ Losses + Asset\ Write-downs \\
&+ Gain\ (Loss)\ On\ Sale\ of\ Assets\ (Rev) \\
&+ Gain\ (Loss)\ On\ Sale\ of\ Loans\ (Rev)\} + Opportunity\ Cost.
\end{aligned}
$$

The added pair of curly brackets is to show that in determining RPL, credit losses are added back into the RPL as a positive number (if, indeed, a loss). Note that PLL is booked as a positive number, assets write-downs are negative and the other two gains are positive (if, indeed, gains). We use Table 2.2a to exemplify the computation: in 2015 PBCT 2015 had \$34.5 million in credit losses, which is the sum of \$33.4 million PLL, \$10.5 million assets write-downs and a \$7 million opportunity cost reduced by the \$9.2 million and \$7.2 million gains on the sale of loans and assets.

Market Gains

Lines 13 and 14 on the left-hand side of the income statement give the market gains for the institution and year in question. Table 2.2a (PBCT in 2015) has zero for the entries for market gains. Table 2.2b (Fifth Third Bancorp (FITB) in 2015) records 483.0 million dollars for the Gain on Sale of Investments and Securities (line 13) and 63.0 million dollars for Income (loss) on Equity Investments (line 14). The RPL component for Market Gains is then computed as the sum of these two items:

$$\begin{aligned} Market\ Gains &= Gain\ on\ Sale\ of\ Invest.\&Secur(Rev) \\ &\quad + Income\ (Loss)\ On\ Equity\ Invest.\ (Rev). \end{aligned}$$

So, Market Gains for (FITB) in 2015 is the sum of \$483 million and \$63 million.

Operational Losses

In the income statement, operational losses are recorded under the two headings, Total Insurance Settlemen with the definition of operational risk given in Basel II. One of the banks in the American sector that is studied in this thesis, First Horizon Ntl. Corp. (FHN), rts and Total Legal Settlements (right-hand column, lines 9 and 10) in accordanceecords in 2014 (Table 2.2c) operational losses under both categories (respectively, \$122.1 million and -\$110.9 million). Total operational losses for the bank is the sum of these two numbers. In general, Operational Losses are computed as:

$$Operational\ Losses = -\{Total\ Insurance\ Settlements + Total\ Legal\ Settlements\}.$$

Adjusted Net Income (ANI)

The adjustments to NI are intended in general to smooth one-off items in the income statement in order to more realistically reflect the impact of one-off accounting decisions that are reported in a single period. More specifically, ANI incorporates three general objectives. The first is to bring net income back to the pre-tax level, and to remove income (expenses) associated with discontinued operations that have no future effect on business income. The second step is to smooth income shocks associated with one-off items. According to accounting principles, these are categorized as non-recurring items: they occur infrequently, but may be reported with large gains or losses. One-off items from the section of unusual items include Gain (Loss) on Sale of Assets, Gain (Loss) on Sale Of Investments and Other Unusual Items. The final step in determining ANI is to amortize Impairments under Goodwill over a five-year period.

Banks generally report Goodwill Impairments on two occasions: when intangible assets carry a value lower than the market value (equity price is a major indicator in this instance), and a write-off is required in this instance by accounting rules; second, when the Impairment of Goodwill discloses those gains (losses) from acquisitions (divestitures) and mergers, which result from the bank's strategic decisions. The booking of impairment or strategic gain reflects to some extent *ad hoc* accounting decisions. We assume that actual gains or losses take place over a fixed period. In the thesis, we smooth the recorded impairment over the five years following the initial booking in the income statement.

Accordingly, the impairment of goodwill is computed in this thesis as:

$$\begin{aligned}Adj\_Gw_{year} = &-Impairment\ of\ Goodwill_{year} + \frac{1}{5}Impairment\ of\ Goodwill_{year} \\ &+ \frac{1}{5}Impairment\ of\ Goodwill_{year-1} + \frac{1}{5}Impairment\ of\ Goodwill_{year-2} \\ &+ \frac{1}{5}Impairment\ of\ Goodwill_{year-3} + \frac{1}{5}Impairment\ of\ Goodwill_{year-4}.\end{aligned}$$

Or, briefly,

$$Adj\_Gw = -Impairment\ of\ Goodwill + amortized\ Impairment\ of\ Goodwill.$$

Finally to summarize, ANI is computed in three steps:

$$\begin{aligned}Adjusted\ Net\ Income\ (ANI) = &\ Net\ Income\ (NI) - Earning\ of\ Discontinued\ Ops. \\ &+ Income\ Tax\ Expenses + Adj\_Gw + One - off\ Items.\end{aligned}$$

To return to Table 2.2a, the items in ANI are highlighted on the right-hand side of Table 2.2a. ANI, in this illustration, is the sum of \$260.1b million NI and \$130.4 million income tax expense, with the other items being zero. In Table 2.2b, the ANI for FITB in 2015 is \$2,371 million; with ANI from \$1,712 million NI and \$659 million income tax expense. In Table 2.2c, the ANI is \$311.9 million for FHN in 2014; that is, \$222.5 million NI plus \$84.2 million for income tax expenses, plus \$5.2 million for other unusual items.

2.2.3 Choice of RPL's Normalizer: Interest Earning Assets (IEA)

The sectoral nature of our proposal to quantify business risk in a competitive environment requires a comparable measure across banks and across time. RPL must be adjusted to account for differences in the size of banks across the sector. We adopt a normalizing factor based on a measure—Interest Earning Assets—that can be computed from the bank's income statement and, most importantly, is included in the Capital IQ presentation. Our perspective is that when business return is computed as normalized, RPL can therefore be used to evaluate over time the impact of a bank's strategic decisions. Accordingly, the assets generating profits should not count non-strategic holdings nor should they reflect any risk assessment at this stage of the analysis.

Different choices are possible for the normalizing factor; for example, total assets or total deposits. After some experimentation, we have chosen average interest earning assets (IEA), as reported by the individual bank and collected by Capital IQ for each bank for each year.

The RPL/IEA ratio enables us to compare a bank's performance relative to its peers by its business return within a sector and over time. We interpret the RPL/IEA ratio as a strategic return on the bank's holdings.

2.3 Business Income in US Medium-sized Retail Banking Sector

In the previous section, we have presented the general approach used in the calculation of the four components of Residual Profit & Loss (RPL). They are derived from the standardized income statement as provided by Capital IQ. We also introduced the normalized series RPL/IEA. The RPL/IEA ratio measures RPL, relative to Interest Earning Assets (IEA), to give a measure of a bank's performance that enables us to make comparisons across banks and across time. In this section, we implement these general procedures in order to construct a specific business return dataset for a sector consisting of 18 medium-sized US retail banks. Before computing the RPL/IEA sample, we discuss our focus on this particular sector of the American banking industry.

2.3.1 Banks in the US Medium-sized Retail Banking Sector

As we have discussed in the first chapter, it is appropriate to assess the business risk faced by a bank in a competitive environment. In their study, Campbell and Stam (2012) chose a subset of medium-sized retail US banks over the period 1994-2011 as a coherent sector to study business risk. These banks share significant characteristics across their balance sheets: first, they have a large proportion of retail loans in their portfolio of assets; second, their, net interest income comprises a large share of revenue; and third, a strong reliance on deposits are their primary funding source. The sample of banks studied includes 18 banks over the period 1994-2015. Capital IQ provides data beginning in 1994. The banks studied are listed in Table 2.1.

By focusing on this sector, we have also been guided by a general description provided by the Federal Reserve Reference. These are medium-sized domestically-owned banks with a large number of branches. Four large banks are excluded from the benchmark sector: JPMorgan Chase, Bank of America, Wells Fargo and Citigroup. Small banks have also been excluded on the grounds that they are not genuine competitors. Capital One is also excluded since its revenue is primarily from its credit card business unit. We do include-to round out the competitive flavor- State Street (STT) as a peer bank, even though it follows an investment banking model. Another inclusion that may be controversial is the relatively small Zions Bancorporation (ZION), which has been designated as a domestically important bank since 2014. The 18 banks in our sample include all of the retail banks listed as domestically important banks (D - SIBS). Table 2.1 presents the size of assets for these banks in 2001 and 2015. As can be readily observed, this sector has expanded considerably in the past 15 years. We refer to these banks named by their trading ticket name on the NYSE or NASDAK.

Figure 2.1 describes their balance sheets in 2015. These 18 banks hold around $2.3 trillion worth of assets, ranging from $26 billion for FHN to $422 billion held by U. S . Bankcorp (USB). Loans take over half of their assets (except for STT). Deposits range from 56 to 84 per cent of their liabilities. Net interest income is the main source of revenue (except STT); the ratio of net interest income to revenue is 60 per cent on average. These ratios presented in Figure 2.1 are similar across the banks, an indication of the relative coherence of this sample of banks that reflects reasonable disparity in size.

2.3.2 RPL, IEA and Normalized RPL in US Medium-sized Retail Banking Sector

For this particular sector, we follow the fundamental formula given in the previous section in order to determine the normalized RPL. Tables 2.3 and 2.4 document RPLs and IEAs for this sector of the 18 banks from 1994 to 2015. We highlight some interesting features in these calculations.

Normalized Adjusted Net Income (ANI)

We make adjustments on one-off items, impairments of Goodwill and others. We note that there are only 43 instances of one-off items from the gains on sale of assets and from the gains on sale of investments for the 18 banks during the sample 22- year period. It appears that banks in the sector did not sell assets or investments during the global financial crisis to offset the severity of losses. One exception may be FITB's sale of its interest in Vantiv, LLC for $1.8 billion in 2009 (equivalent to 1.7 per cent of IEA). We have included these one-off events, as well as those related to debt extinguishments in the calculation of ANI.

With regard to Goodwill: Capital IQ recorded instances of Goodwill Impairment beginning in 2001 when amortization of Goodwill was eliminated from the balance-sheet. Goodwill and other indefinite intangibles have been subsequently subjected to an annual test imposed by the regulator for the impairment of value. During the sample period, a bank reports Impairment of Goodwill usually under two circumstances: acquiring (or being acquired by) another business. There are 27 examples in our sample. Among the seven gains, two occurred during the crisis and five in its aftermath. The impact of the crisis was considerable and extends to intangible assets as well. The losses in 2008 and 2009 are very large, from 0.5% to 5.7% of the bank's IEA (worth from $15 million up to $6 billion). Five-year forward amortization smooths such losses associated with large write-offs on intangible assets and is directly charged to income. During the earlier global financial downturn between 2001 and 2002, such impairment charges were much smaller in size.

Normalized Credit Losses

Credit losses are affected dramatically by economic conditions. In normal economic times, the credit losses relative to IEA (average interest earning assets) in the sector are stable. However, during major economic downturns, banks have suffered severe credit losses. This sudden severity brings huge volatility to normalized credit losses over the sample period. We graph normalized credit losses (the ratio of credit losses to average interest earning assets) in Figure 2.2. The figure presents two spikes caused by large losses that occurred first in the dotcom recession (years 2001 and 2002) and in the financial crisis (year 2008 and 2009); the latter spike shows the sharpest increase in credit losses. There are 18 bank years where the credit losses over two per cent of IEA in 2008 and 2009, with the worst up to 5.9% of IEA incurred by Synovus Financial Corp.(SNV) in 2009. During the dotcom recession, the spike is not as pronounced; yet there are six bank years where credit losses are more than one per cent of IEA and the worst loss reaches 1.8% of IEA (KeyCorp. (KEY) in 2001).

It is worth noting (the details are not reported) that the different components that contribute to credit losses have played different roles during the two crises. During the earlier crisis in 2001-2002, there was considerable activity in selling assets which dampened the impact of the credit challenge. By contrast, during the latter crisis in 2008-2009, assets in credit trouble were write-downs.

Normalized Market Gains

Market gains in the sector are within a range from a 0.5% gain to a 0.5% loss. There are seven bank years outside the range. For example, PNC Fin. Services Group (PNC) gained .7% (equal to $1.6 billion) in 2009, and. Suntrust Banks Inc. (STI) gained .7% (equal to $1.2 billion) in 2008. In the meantime, we observe that gains and losses vary across banks: some banks gain and some lose within the same year, a situation quite dissimilar to the pattern of credit losses.

The data used to compute market gains is mostly based on the records of gain on the sale of investment and securities. The other component of income (loss) on equity investment does not figure significantly for this sector (in contrast to the Canadian sample of banks to be discussed in the next section).

Normalized Operational Losses

Generally, operational losses are within the range given plus/minus .2 percentage points. The data on operational losses contains some negative numbers. These gains are related to legal matters, resulting from reversals in specific law suits. There are several significant operational loss events in the sample: FHN booked .8% (equivalent to $187 million) in 2015 and .3% (equivalent to $64 million) in 2013, which were associated with the settlement of potential claims related to the underwriting and origination of mortgage loans. STT had .6% (equivalent to $600 million) in 2007 and .2% (equivalent to \$250 million) in 2009, in relation to the settlement of claims that SSGA had adopted inappropriate fixed- income strategies during 2007 and 2009.

What we term business annual returns (i.e., RPL/IEA ratios) for this sector from 1994 to 2015 are presented in Table 2.5 and graphed in Figure 2.3. This figure shows that the financial crisis is a transition period during which returns keep on dropping and are extremely volatile. The median of 18 banks' annual returns has been dropping since 2007; it stabilized around 2010 but at a much lower level. During this period, we found at seven huge significant drops in returns from those of the previous year. Afterwards, the volatility becomes low except for two large drops, one in 2012 and the other in 2015.

2.4 The Analysis of Business Income in the Canadian Banking Sector

In the previous section, we have implemented general procedures to construct a business return dataset for the US retail banking sector. In this section, we apply the same procedures to construct a separate dataset for the Canadian banking sector. However, a different challenge arises for this sector, as accounting standards were significantly changed in 2011. Some assets that were previously treated off - balance sheet under the old standards must now be disclosed on the balance sheet under the new standards. The changes in the income generated by these assets certainly impact the calculation of RPL. As well, the calculation of interest earning assets (IEA) is affected. Moreover, it is not possible to directly reconstruct the income statements prior to 2011 using the new standards. We take up these issues after a brief description of the sector.

2.4.1 Banks in the Canadian Banking Sector

Relative to the US banking sector, the Canadian banking industry has some distinctive features. It is much more concentrated, with the six largest banks accounting for 90% of the combined assets in the DTIs (deposit-taking institutions) sector. We consider a Canadian banking universe with eight banks, including the six largest banks and another two banks. These eight banks form the competitive Canadian banking environment. The six largest banks are known as the “Big Six”, which refers to the Bank of Montreal (BMO), the Canadian Imperial Bank of Commerce (CM), the Toronto Dominion Bank (TD), the Bank of Nova Scotia (BNS), the Royal Bank of Canada (RY), and the National Bank of Canada (NA), which are all regulated by OSFI. In 2015, The Big Six had combined assets of C$4.36 trillion, which accounts for 93% of the C$4.70 trillion held by the entire Canadian banking industry as of 2015. The other two banks included in our study are the Laurentian Bank of Canada (LB) and the Desjardins Group (DES, regulated by the Autorite des marches financiers (AMF) in Quebec).

The Desjardins Group holds C$248 billion in total assets (falling between the CIBC's C$463 billion and National's C$216 billion). Even though Desjardins derives a good part of its revenue from its insurance business, it is a major Quebec and Canadian financial institution that should be included in the Canadian sample. The Laurentian Bank of Canada holds C$40 billion in total assets and is the smallest bank in the Canadian banking universe. We exclude two institutions from our sample: the HSBC Bank Canada, which is the operation of the British banking group in Canada (C$94.6 billion in assets), and the relatively small Canadian Western Bank (C$25.2 billion in assets), which operates primarily in western Canada.

Some features of the sector in 2015 are depicted in Figure 2.4:

- the loans-to-assets ratio is 57% on average, ranging from 44% to 76 %;
- on average, 49% of revenue is derived from interest income. Seven of the eight banks have a net interest income greater than 40% of revenue. Desjardins has a lower weight of net interest income because it has C$6.9 billion in earnings from its insurance business unit (45% of its revenue);
- 67% of funding comes from deposits;
- there is some size disparity presented in the sector, which has C$4.6 trillion com-

bined assets and C$580 billion on average. The biggest two (TD and RY) each holds more than C$1 trillion in assets. The smallest bank is the Laurentian, holding around forty billion in assets.

2.4.2 The Analysis of Business Income for Canadian Banking Sector

In 2006, the Canadian Accounting Standards Board announced that the Canadian banks must adopt the International Financial Reporting Standards (IFRS), as of January 1, 2011. Prior to the adoption of IFRS, banks prepared their financial reports under the Canadian Generally Accepted Accounting Principles (CGAAP). These changes to accounting standards are reflected in the standard income statements compiled by Capital IQ, where statements for the years 2011 and later are based on IFRS standards, whereas those for prior years use CGARP principles. Accordingly, in an effort to ensure the consistency of our construction of the return ratio RPL/IEA over the sample, we must take account of the impact of the accounting changes on this measure.

For our purposes, the material changes are be summarized under three primary impacts. The first two relate to the calculation of RPL, and the third to IEA. In 2011, the banks produced the 2011 (and only for 2011, as required by the regulator) statements reflecting both the old and new standards. Subsequently, Capital IQ presents data based solely on the latter.

Impact A

The measurement of credit losses is substantially changed across banks. Under the new standards, the income corresponding to loan securitization (previously booked under the entry Gain (Loss) On Sale of Loans (Rev.)) has been eliminated from the bank's consolidated income statement.

Under the new standards, the income associated with these loans is booked over a certain period, and is replaced by the income floating with the underlying assets in the subsequent period. It is impossible to determine the connections between this income and the value of the securitization.

Impact B

Under the old standards, the extent to which market gains are calculated has varied across banks. The new standard applies the equity accounting method to determine income from joint ventures which requires an additional disclosure in the bank's consolidated income statement. However, applying the equity accounting method is optional for Canadian banks. Four out of the eight Canadian banks in our sample are subject to this disclosure and reported gains ranging from several million to C$6 hundred million.

Impact C

The impact of the new standards on IEA (average interest earning assets) is also significant. This is related to the securitized assets as discussed under *Impact A*. The assets that were previously treated as off-balance sheet items must now be included in the income statement. This means that the figure for IEA generally increases in terms under the new standards.

These changes to the banks's income statements compromise the basis of our calculation of RPL/IEA from Capital IQ over the whole sample. This figure should now be calculated incorporating the changes on the income statements prior to 2011, if comparisons before and after the changes are to be undertaken. We also note that the banks themselves have not provided any measures of the impact on their pre-2011 balance sheets. A direct comparison is available only for the year 2011.

Figure 2.5 illustrates the impact on the different components of RPL in 2011. *Impact A*, described above, is clearly the most significant for the majority of the banks in the sample. However, this impact can be directly approximated by eliminating the income from loan securitization from the pre-2011 income statements. On the other hand, *Impact B*(related to Market Gains) cannot be so directly dealt with, as the calculation of its impact would go considerably beyond the information provided in the income statements.

Thus, we propose to approximate the impact of the new standards on RPL solely by incorporating changes related to the credit component on the pre-2011 statement. This approximation is compared with the actual impact on RPL for 2011 in Table 2.6. The third and forth fourth columns are the actual and approximated impacts, respectively, with their differences in the last column. These differences indicate that the approximated impact can be quite close to the actual impact: CIBC's, for example, is C$1016 million versus C$1063 million.

However, we note that the last column of Table 2.6 has two large numbers: C$504 million from BNS, a gap between negative 268 to positive 236 in C$ millions, and C$640 million from RY, a difference between 1437 and 797 in C$ millions. This is because, as shown in Figure 2.5, the impact on BNS's RPL in 2011 is largely due to market gains. As per the discussion of *Impact B*, we have no suggestions on how to decrease this impact. On the other hand, the impact on RY is much less concentrated.

The change to pre-2011 values of IEA, as described in Impact C, cannot be readily approximated. In general, off-balance items are reported as footnotes to the income statements that do not figure in the Capital IQ analysis. We have examined at the footnotes in the 2011 income statement to see how close the impact of the changes on IEA could be recaptured, however, there is not sufficient information to suggest a systematic approach.

The comparison of IEA under two accounting standards in 2011 is provided in Table 2.7. With the exception of Desjardins and CIBC, the percentage differences vary from three per cent to nine per cent. We have imposed a systematic adjustment of six per cent on each bank's IEA, calculated under the standards used prior to 2011, to arrive at an approximation of these values that would have been calculated under the new standards.

With the incorporation of these two changes on the Capital IQ pre-2011 income statements, a dataset is now constructed for the whole sample, which approximates RPL/IEA for each bank under the new accounting standards. Canadian RPL/IEA ratios are presented in Table 2.10 and graphed in Figure 2.6, with the components given in Tables 2.8 and 2.9, respectively. From Figure 2.6, we see the RPL/IEA curves from the Desjardins Group (DEJ) and Laurentian Bank of Canada (LB) are evidently different from the other banks. Rearding Desjardins, we have no access to earlier income statements. Desjardins also presents high fluctuations due to its different operation; for example, it has a relatively large presence in the insurance business. Laurentian's shows generally lower level of the ratios due to its small size. In 2015 the bank had less than one fifth of the assets held by the second smallest bank (National Bank of Canada) in 2015.

We conclude this section with several observations about the Canadian sample:

- With regard to *Adjusted Net Income* (ANI). Earnings from the Discontinued Operation recorded some large losses. During 2009 and 2012, Royal (RY) reported losses from the sale of US regional retail banking operations to the PNC Financial Services Group (PNC) and the sale of Liberty Life Insurance;

- With regard to one-off items of Gain (Loss) on Sale of Assets and Gain (Loss) on Sale of Investment. The two lines also recorded big amounts: TD in 1999 disclosed a C$3 billion gain from the sale of its US brokerage business and another C$1.6 billion gain in 2006 from the same business. None of these big gains (losses) is generated from banks' daily operations but they do cause fluctuations in income. They are excluded from RPL by the adjustments on NI;

- With regard to the *Impairment of Goodwill*. Canadian banks have recorded ten such impairments. Half Five of the ten impairments are reported as gains (losses) from acquisitions (divestitures) in the years after the crisis: Desjardins (DEJ) in 2015, Nova Scotia (BNS) in 2010, Laurentian (LB) in 2012, National (NA) in 2012 and Royal (RY) in 2010. During the crisis, only Desjardins (DEJ) reported impairments of C$13 million in 2008 and C$31 million in 2009. The remaining three reported impairments are for certain business units involving CIBC (CM) in 2014, Laurentian (LB) in 2015 and TD in 2013. The scale of write-downs is much smaller: there are only two over C$100 million among the ten reported impairments. The smoothing role of using the five-year forward amortization is less pronounced with Canadian data than with the American sample;

- With regard to *ANI*. Most of the Canadian banks' change over the entire time horizon are gains. Large banks lead the sector with an upward trend which matches the growth in assets in the sector. The sector, however, has two notable losses during the dotcom recession and the global financial crisis: a TD loss of a C$552 million due to C$2.5 billion loss in loans in 2002 and CIBC's C$4.3 billion loss largely caused by a C$6.8 billion loss in trading revenues in 2008;

- With regard to *Normalized Credit Losses*. Figure 2.7 depicts Canadian banks' normalized credit losses over the entire sample period (in an attempt to make a fair comparison, the y-axis has the same scale as the Figure 2.6 depicting RPL/IEA ratios. The Canadian credit losses (as per the US sector) have two peaks corresponding to the two economic downturns. However the scales are quite different. During the global financial crisis, Canadian banks suffered less disruptive credit losses than did the US banks: the largest losses was 0.58% incurred by CIBC in 2009, with over C$1.5 billion from each of the five largest banks and with the greatest at C$2.6 billion from TD. It appears that the dotcom recession was costlier for some Canadian banks than the later financial crisis: TD at 1.21% (worth C$3 billion) in 2002 versus 0.53% per cent (worth C$2.6 billion) in 2009; Nova Scotia's at 0.9% (worth C$2.3 billion) versus 0.42% (worth C$1.9 billion);

- With regard to *Market Gains*. Among the eight Canadian banks, Desjardins' results fluctuated. The bank had a loss of C$446 million in 2013, but gains of over C$2 billion in 2011 and 2014. Another bank with volatile market gains is Nova Scotia with a gain C$ 1.8 billion in 2014 compared with smaller returns in other years. It should be noted that during the global financial crisis, CIBC had a gain of around

C$ 1 billion, Royal had a loss of C$ 632 million, and National's loss was C$409 million;

- With regard to *Operational Losses*. The records of Canadian operational losses are sparse. We have a total of six over the entire period. The most prominent booked event is related to the Enron litigation matters in 2005, which cost CIBC$2.83 billion, Royal Bank C$591 million and TD C$365 million.

In summary, Figure 2.6 indicates that Canadian business returns (measured by RPL/IEA ratios) over the entire sample are quite different from those in the US sample, they have no obvious downtrend or high volatility. This pattern continues over the sample period and is not interrupted by the global financial crisis, except for a huge single drop to negative 1.29% by CIBC in 2008.

2.5 Sectoral Business Income Loss Datasets: Construction and VaR Estimation

In the last sections we have taken the crucial steps towards the formal analysis of business risk. A measure of business income that abstracts from market, credit and operational risk has been elaborated and constructed for two different banking sectors. Business income has been defined relative to bank size (i.e., RPL/IEA) to permit inter-year and intra-bank comparisons. The next step towards a comprehensive risk analysis is to characterize the fluctuations in such income to permit a VaR analysis with estimation of extreme fluctuations.

We propose to demean the business income data using a bank's own expected mean. With no suitable theory at hand, we use the bank's sample mean over the sample as an estimate of its expected return. This section then investigates the empirical properties of the business income fluctuations so defined for the two sectors.

2.5.1 Fluctuations in Business Income

US Banking Sector

The US data has 18 time series (the individual banks) over a sample from 1994 to 2015. Figure 2.3 indicates strongly that there has been a downward shift in business income returns during the period of the financial crisis (2007-2009) for the retail banking sector and suggests that the estimate of expected returns must be distinguished for the two periods demarcated by the financial crisis. The breakpoint for returns could be taken to be either 2009 or 2010. It is instructive to look at the statistical properties of demeaning the data on both approaches. Table 2.11 provides sample statistical properties for the two cases. In either case, the two-sample t-test strongly rejects the hypothesis that the means are equal across the split samples. The standard deviations for the demeaned series are not affected by the choice of break point. Most importantly, on either approach, the data indicates that the standard deviation does not change from one sub-sample to the other. In the empirical analysis presented in Chapter 4, the break is taken to be at 2010 rather than at 2009 as it corresponds better to the reality of the financial crisis. Across the whole sample so demeaned, we note that there is slight asymmetry and excess kurtosis.

The time series correlation structure for each bank's demeaned business returns is examined using a moving average process (MA) of different orders. Under this structure of the MA process, the inter-year autocorrelations are captured by the estimated weights. We see from Panel A in Table 2.12 that the parsimonious MA(1) structure appears to be appropriate: the coefficient estimates (denoted $\hat{\theta}$) in MA(1) representation display in general statistical significance with a median value around .5; half of the estimates are between .32 and .72. The significance of the additional MA parameters in the other representations is not as extensive across the US universe of retail banks in this sample.

We should emphasize at this point that these observations are intended to give broad brush strokes of the business return data we have constructed in this chapter. The observations are intended to lend support to a particular parametric structure which will be introduced in the next section.

Using the median of the MA(1) coefficients so obtained, the time-t shock is estimated for each bank. The cross-correlation structure of these shocks is presented in Table

2.13. Table 2.13 shows that high cross-correlations (with absolute value above 0.5) are scattered around the whole matrix without a pattern; it is hard to discern a pattern of inter-bank correlations. US Bankcorp (USB), for example, the largest bank in the sector in terms of total assets in 2015, is highly correlated to the second largest PNC Fin Services Group (PNC) but also is highly correlated to the much smaller BB&T Corp. (BBT). By contrast, the other 138 among 153 elements in this cross-correlation matrix have a small median and 34 have absolute values under 0.1. Within this sectoral context, it appears the contemporaneous correlations between the US banks are generally insignificant or have a common correlation which is not particularly strong.

Canadian Banking Sector

We quickly review the results of a similar analysis for the eight Canadian banks that is presented in the second panels of Tables 2.11 and 2.12, and in Table 2.14.

Canadian banking sector has RPL/IEA ratios graphed in Figure 2.6. The data is certainly less volatile than the US counterpart. There is no clear regime break in returns, while most returns stay within a smaller range of deviations from 0.5 per cent to 1.5 per cent, except for a big CIBC drop in 2008 and relatively larger fluctuations for Desjardins. These features suggest there no need to set a breakpoint for the Canadian sector, an intuition that is confirmed by Panel B in Table 2.11. Each Canadian bank's income is accordingly demeaned using its own sample average over the entire sample period.

We use the same approach as was applied to the US sector to analyze the correlation structure for the Canadian banks. Panel B in Table 2.12 presents the autocorrelations under the structure of the MA process and the cross-correlations are shown in Table 2.14.

We observe the similarity with the US results: the MA(1) representation appears to be preferable to the MA(2) and MA(3) characterizations. In general, the contemporaneous correlations among the banks using the residuals obtained from a characterization using the median estimate of the MA(1) parameter is not pervasive. In Table 2.14 , we see 4 correlations above 0.5 (in absolute values) among the 21 with 6 below .1, and the other 11 in between. Based on this evaluation, we may also conclude in general that the contemporaneous correlations among Canadian banks are weak.

2.5.2 Towards a Sectoral VaR Analysis: Two Estimation Methodologies

The demeaned business income across the banks in the two sectors is the point of departure for the empirical VaR analysis undertaken in Chapter 4. The demeaned data are viewed as losses that are the basis for traditional VaR analysis. We take two modeling approaches to analyze these fluctuations. These are briefly presented in this section and are studied and contrasted more systematically in an extensive Monte Carlo study presented in the next Chapter. The one approach preserves the time character of the data and models the sectoral inter-relationship among the banks in a multivariate MA(1) framework where each bank shares the same MA(1) parameter. The contrasting approach is non-parametric and ignores the time series character of the data by viewing the data for each bank as a draw from the same distribution governing business income losses. On this approach, the data is pooled.

It should be emphasized that both approaches incorporate sectoral perspective in an essential way. The time series approach imposes common parameters across the individual bank's losses; in the pooled approach, each bank faces the same loss shock.

Time Series Approach: VaR Estimation via Maximum Likelihood

Consider two banks labelled a and b:

- Bank a has a loss series $\{R_{a,t}\}$ over T_a periods, written as

$$R_{a,t} = \epsilon_{a,t} + \theta_a \epsilon_{a,t-1}, \quad \epsilon_{a,0} = 0, \quad t = 1, \ldots, T_a.$$

- Bank b has a loss series $\{R_{b,t}\}$ over T_b periods, written as

$$R_{b,t} = \epsilon_{b,t} + \theta_b \epsilon_{a,t-1}, \quad \epsilon_{b,0} = 0, \quad t = 1, \ldots, T_b.$$

- In both series, for $k = \{a, b\}$, $\{\epsilon_{k,t}\}$ is a series of disturbance and $\epsilon_{k,t}$ is independently and identically distributed with a scale parameter σ_k (denoted $\epsilon_{k,t} \overset{iid}{\sim} (0, \sigma_k^2)$), $\epsilon_{k,0}$ is the initial disturbance, θ_k is the MA coefficient and $|\theta_k| < 1$.

The time-series approach fixed a common structure for the two banks: there is imposed a common MA(1) parameter–θ–and a common variance of the disturbances σ^2, written as

$$R_{k,t} = \epsilon_{k,t} + \theta \epsilon_{k,t-1}, \quad \epsilon_{k,0} = 0, \quad t = 1, \ldots, T_k, \quad \text{for} \quad k = \{a, b\}, \quad \text{with} \quad \epsilon_{k,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2).$$

Under the assumption of normality, each bank has mean zero losses that are normally distributed with variance

$$\sigma^2(1 + \theta^2)$$

ML estimation is used to estimate the parameters of this shared structure. The VaR at a statistical confidence level $1-p$ are then readily obtained via the formula

$$\hat{\sigma}\sqrt{1+\hat{\theta}^2}\Phi^{-1}(p),$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of cumulative probability function of $\mathcal{N}(0,1)$.

Pooled Data Approach: VaR Estimation via Saddlepoint Approximation

When the data are pooled in the example just considered, the new sample of losses has size $n = T_a + T_b$ with a loss denoted R_i for $i = 1, \cdots, n$. These points are assumed to be drawn from a distribution meeting some general moment conditions. Based on this data, the saddlepoint approach (SPA) uses these first-four sample moments to estimate the VaR. The VaR is estimated via the formula

$$\hat{F}(R) = \Phi(\omega) + \phi(\omega)(\frac{1}{\omega} - \frac{1}{\lambda}),$$

where $\hat{F}(R)$ denotes the approximated loss distribution function. $\Phi(\cdot)$ and $\phi(\cdot)$ are respectively cumulative probability function and density function from $\mathcal{N}(0,1)$. The parameters ω and λ are determined by the saddlepoint methodology that is presented in the next Chapter. Here we can simply observe that the VaR at a statistical confidence level $1-p$ could be obtained from this formula via interpolation.

Both estimation procedures will be pursued in our empirical analysis. This approach is exploratory. On the one hand, a specific time series framework has been proposed. There is insufficient data to establish whether this sectoral model is correct in a formal sense. It can be argued that the data analysis presented in the previous section supports in an informal way. On the other hand, the saddlepoint methodology, while avoiding this modeling problem, does make strong general distributional assumptions. Its general applicability may not yield particularly informative results in this specific context. A comprehensive Monte Carlo study is undertaken in the next Chapter to shed some light on the relative merits of these two approaches under a variety of stochastic specifications.

Table 2.1 Banks in the US Retail Banking Sector

Institution	Banks	Location	Total Assets ($ in millions)	
			2001	**2015**
Fifth Third Bancorp	FITB	Cincinnati, Ohio	23,444	141,082
Associated Bancorp	ASB	Green Bay, Wisconsin	9,649	27,715
BB&T Corp.	BBT	Winston-Salem, North Carolina	54,700	209,947
BOK Fin. Corp.	BOKF	Tulsa, Oklahoma	9,083	31,476
Comerica	CMA	Dallas, Texas	4,112	71,877
First Horizon Ntl. Corp.	FHN	Memphis, Tennessee	20,600	26,195
Huntington Bancshares	HBAN	Columbus, Ohio	28,441	71,045
KeyCorp.	KEY	Cleveland, Ohio	7,360	95,133
M&T Bank Corp.	MTB	Buffalo, New York	30,804	122,788
NY Community Bancorp Inc.	NYB	Westbury, New York	9,200	50,318
People's United Fin. Inc.	PBCT	Bridgeport, Connecticut	-	38,877
PNC Fin. Services Group	PNC	Pittsburgh, Pennsylvania	62,610	358,493
Regions Financial	RF	Birmingham Alabama	42,002	126,050
State Street Corp.	STT	Boston, Massachusetts	65,410	245,192
Suntrust Banks Inc.	STI	Atlanta, Georgia	102,377	190,817
Synovus Financial Corp.	SNV	Columbus, Georgia	3,963	28,793
U.S. Bancorp	USB	Minneapolis, Minnesota	166,949	421,853
Zions Bancorporation	ZION	Salt Lake City, Utah	8,669	59,670

A bank is denoted by its stock trading ticker on the NYSE or NASDAK.

Table 2.2a People's United Fin. Inc. (2015): Standardized Income Statement

1	Interest Income On Loans	946.9	**EBT Excl. Unusual Items**	**401.1**
2	Interest Income On Investments	122.0		
3	**Total Interest Income**	**1068.9**	Restructuring Charges	0.0
4			Total Merger & Rel. Restruct. Charges	0.0
5	Interest On Deposits	95.5	Impairment of Goodwill [ANI]	0.0
6	Total Interest On Borrowings	41.3	Gain (Loss) On Sale Of Assets [ANI]	0.0
7	**Total Interest Expense**	**136.8**	Gain (Loss) on Sale of Invest. [ANI]	0.0
8	**Net Interest Income**	**932.1**	Asset Write-down [Credit]	-10.5
9			Total Insurance Settlement [Op]	0.0
10	Trust Income	43.7	Total Legal Settlements [Op]	0.0
11	Gain (Loss) On Sale Of Assets (Rev) [Credit]	9.2	Other Unusual Items [ANI]	0.0
12	Gain (Loss) On Sale Of Loans (Rev) [Credit]	7.2	**EBT Incl. Unusual Items**	**390.5**
13	Gain on Sale of Invest. & Secur (Rev) [Market]	0.0		
14	Income (Loss) On Equity Invest. (Rev) [Market]	0.0	Income Tax Expense [ANI]	130.4
15	Total Other Non-Interest Income	292.3	**Earnings from Cont. Ops.**	**260.1**
16	Non-Oper. Income (Exp.)	0.0		
17	**Total Non-Interest Income**	**352.4**		
18	**Revenue Before Loan Losses**	**1284.5**	Earnings of Discontinued Ops. [ANI]	0.0
19			Extraord. Item & Account. Change [ANI]	0.0
20	Provision For Loan Losses [Credit]	33.4		
21			**Net Income to Company**	**260.1**
22	**Total Revenue**	**1251.1**		
23			Minority Int. in Earnings	0.0
24	Salaries and Other Empl. Benefits	426.9	**Net Income** [ANI]	**260.1**
25	Amort. of Goodwill & Intang. Assets	23.9		
26	Occupancy Expense	90.6		
27	Federal Deposit Insurance	35.1		
28	Selling General & Admin Exp., Total	198.6		
29	Total Other Non-Interest Expense	75.0		
30	**Total Non-Interest Expense**	**850.1**		

Source: Standard & Poor's Capital IQ, $ in millions.
The contributions of credit losses and adjusted net income (ANI) to RPL are highlighted.

Table 2.2b Fifth Third Bancorp (2015): Standardized Income Statement

1	Interest Income On Loans	3,151.0	**EBT Excl. Unusual Items**	**2506.0**
2	Interest Income On Investments	877.0		
3	**Total Interest Income**	**4,028.0**		
4				
5	Interest On Deposits	186.0	Impairment of Goodwill	0.0
6	Total Interest On Borrowings	309.0		
7	**Total Interest Expense**	**495.0**		
8	**Net Interest Income**	**3,533.0**	Asset Writedown	-141.0
9			Total Insurance Settlement	0.0
10	Service Charges On Deposits	563.0	Total Legal Settlements	0.0
11	Total Mortgage Banking Activities	344.0	Other Unusual Items	
12	Gain (Loss) On Sale Of Assets (Rev)	8.0	**EBT Incl. Unusual Items**	**2365.0**
13	Gain on Sale of Invest. & Secur (Rev) [Market]	483.0		
14	Income (Loss) On Equity Invest. (Rev) [Market]	63.0	Income Tax Expense	659.0
15	Total Other Non-Interest Income	1,538.0	**Earnings from Cont. Ops.**	**1706.0**
16	Non-Oper. Income (Exp.)			
17	**Total Non Interest Income**	**2,999.0**		
18	**Revenue Before Loan Losses**	**6,532.0**	Earnings of Discontinued Ops.	0.0
19			Extraord. Item & Account. Change	0.0
20	Provision For Loan Losses	396.0		
21			**Net Income to Company**	**1706.0**
22	**Total Revenue**	**6,136.0**		
23			Minority Int. in Earnings	6.0
24	Salaries and Other Empl. Benefits	1,748.0	**Net Income**	**1712.0**
25	Amort. of Goodwill & Intang. Assets			
26	Occupancy Expense	211.0		
27				
28	Selling General & Admin Exp., Total	668.0		
29	Total Other Non-Interest Expense	1,003.0		
30	**Total Non-Interest Expense**	**3,630.0**		

Source: Standard & Poor's Capital IQ, $ in millions.
The contributions of market gains to RPL are highlighted.

Table 2.2c First Horizon Ntl. Corp. (2014): Standardized Income Statement

1	Interest Income On Loans	583.0	**EBT Excl. Unusual Items**	**316.9**
2	Interest Income On Investments	126.3		
3	**Total Interest Income**	**709.2**	Restructuring Charges	-4.7
4				
5	Interest On Deposits	26.8	Impairment of Goodwill	0.0
6	Total Interest On Borrowings	54.7	Gain (Loss) On Sale Of Assets	0.0
7	**Total Interest Expense**	**81.5**		
8	**Net Interest Income**	**627.7**		
9			Total Insurance Settlements [Op]	122.1
10	Service Charges On Deposits	112.0	Total Legal Settlements [Op]	-110.9
11	Trust Income	27.8	Other Unusual Items	-5.2
12	Total Mortgage Banking Activities	71.3	**EBT Incl. Unusual Items**	**318.2**
13	Gain (Loss) On Sale Of Assets (Rev)	0.0		
14	Gain on Sale of Invest. & Secur (Rev)	-5.1	Income Tax Expense ANI	84.2
15	Total Other Non-Interest Income	340.4	**Earnings from Cont. Ops.**	**234.0**
16	Non-Oper. Income (Exp.)	0.0		
17	**Total Non Interest Income**	**546.2**		
18	**Revenue Before Loan Losses**	**1173.9**	Earnings of Discontinued Ops.	0.0
19			Extraord. Item & Account. Change	0.0
20	Provision For Loan Losses	27.0		
21			**Net Income to Company**	**234.0**
22	**Total Revenue**	**1146.9**		
23			Minority Int. in Earnings	-11.5
24	Salaries and Other Empl. Benefits	465.8		
25	Amort. of Goodwill & Intang. Assets	4.2	**Net Income**	**222.5**
26	Federal Deposit Insurance	11.5		
27	Selling General & Admin Exp., Total	240.4		
28	(Income) Loss on Real Estate Property	(1.8)		
29	Total Other Non-Interest Expense	110.0		
30	**Total Non-Interest Expense**	**830.1**		

Source: Standard & Poor's Capital IQ, $ in millions.
The contributions of operational losses to RPL are highlighted.

Table 2.3 US Banking Sector: RPL (Residual Profit & Loss)
($ in millions)

Banks	**2015**	**2014**	**2013**	**2012**	**2011**	**2010**	**2009**	**2008**	**2007**	**2006**	**2005**
FITB	2,376	2,400	2,182	2,385	2,396	2,326	2,234	2,651	2,403	2,409	2,519
ASBC	304	288	279	298	264	349	515	501	444	474	477
BBT	3,485	3,431	3,760	3,876	2,773	3,193	3,777	3,472	3,084	2,810	2,702
BOKF	458	430	466	492	413	494	526	416	384	348	328
CMA	895	893	921	908	799	923	835	1,089	1,270	1,167	1,192
FHN	295	341	140	7	177	291	467	671	88	445	615
HBAN	1,057	982	392	423	363	498	593	1,059	762	668	640
KEY	1,356	1,338	1,249	1,181	1,150	1,195	686	1,100	1,362	1,673	1,546
MTB	1,845	1,831	1,982	1,897	1,484	1,617	1,329	1,386	1,328	1,302	1,297
NYCB	- 1	733	806	905	880	1,003	657	428	389	377	434
PBCT	425	413	394	422	355	202	172	229	285	208	190
PNC	5,424	5,447	6,188	4,863	5,300	6,573	6,427	3,018	2,380	2,233	1,906
RF	1,672	1,698	1,771	687	643	613	604	1,086	2,745	2,177	1,533
STT	2,738	2,625	2,665	2,379	2,471	2,731	2,783	2,746	2,530	1,756	1,417
STI	2,870	2,911	2,432	2,120	2,114	2,529	2,178	2,617	2,771	3,235	3,045
SNV	391	350	349	241	241	228	128	388	737	708	649
USB	9,606	9,752	9,926	10,427	9,487	9,480	9,023	8,261	7,361	7,432	7,435
ZION	606	522	485	458	470	416	816	963	987	887	703
Banks	**2004**	**2003**	**2002**	**2001**	**2000**	**1999**	**1998**	**1997**	**1996**	**1995**	**1994**
FITB	2,881	2,749	2,343	1,727	1,828	1,622	952	906	670	472	403
ASBC	389	373	354	281	237	252	238	183	213	188	71
BBT	2,585	2,149	1,915	1,493	1,400	1,296	1,075	737	578	407	
BOKF	301	255	252	202	169	141	122	104	83	65	64
CMA	1,206	1,312	1,589	1,386	1,422	1,207	1,037	939	748	714	649
FHN	635	785	659	512	258	435	403	371	323	274	217
HBAN	591	650	514	369	482	648	514	566	522	463	380
KEY	1,429	1,653	1,835	1,736	1,760	1,571	1,658	1,555	1,304	1,265	1,446
MTB	1,168	994	813	669	498	467	337	333	301	265	262
NYCB	682	429	320	149	31	49	46	38	38	33	34
PBCT	135	90	119	168	205	206	156	128	133	142	121
PNC	1,673	1,670	2,116	1,542	1,860	1,499	1,791	1,576	1,540	1,213	1,535
RF	1,326	1,046	967	881	844	893	711	676	525	331	284
STT	1,148	744	988	898	913	701	675	580	455	374	351
STI	2,453	2,115	2,118	2,193	2,080	1,995	1,669	1,737	1,420	945	942
SNV	754	669	605	514	448	383	332	302	255	208	168
USB	7,186	6,935	6,192	4,307	5,282	4,405	1,374	1,370	732	398	373
ZION	570	561	521	413	302	247	178	167	128	104	84

See Section 2.3 for details of the calculation.

Table 2.4 US Banking Sector: IEA (Average Interest Earning Assets)

($ in millions)

Banks	**2015**	**2014**	**2013**	**2012**	**2011**	**2010**	**2009**	**2008**	**2007**	**2006**	**2005**
FITB	123,584	115,993	107,954	101,636	97,682	98,931	101,526	99,880	90,382	94,799	92,736
ASBC	24,571	22,760	20,980	19,614	19,442	20,568	21,337	19,840	18,645	19,230	19,182
BBT	172,673	161,252	156,711	153,426	139,092	135,330	135,665	120,852	112,305	101,572	92,703
BOKF	27,722	25,329	24,792	23,606	21,959	20,771	20,246	18,915	16,935	14,719	13,431
CMA	65,129	61,560	59,091	57,483	52,121	51,004	58,162	60,422	54,688	52,291	48,232
FHN	23,456	21,825	21,772	22,225	21,959	22,960	25,374	30,426	33,405	34,042	31,976
HBAN	63,023	57,705	51,598	50,709	48,574	47,420	46,105	47,787	39,356	31,451	29,308
KEY	82,519	78,091	75,394	71,849	72,958	78,438	85,095	86,815	79,612	79,523	76,067
MTB	91,187	81,681	74,013	70,299	64,732	59,732	59,551	58,016	51,951	49,652	48,118
NYCB	43,622	42,726	38,677	36,116	34,688	34,173	29,043	27,199	25,894	24,741	21,697
PBCT	33,228	30,181	27,360	24,366	22,497	18,989	18,157	17,676	11,806	9,875	10,048
PNC	308,825	283,305	260,645	248,554	224,340	224,749	238,487	114,484	98,010	77,692	73,001
RF	107,871	104,097	103,707	107,795	112,214	117,507	123,985	120,130	116,964	83,109	74,385
STT	220,456	209,054	178,101	167,615	147,657	126,256	122,923	132,625	104,550	92,665	87,786
STI	168,813	162,189	153,728	153,479	147,802	147,187	150,908	152,749	155,204	158,429	146,640
SNY	25,992	24,300	23,909	24,505	26,462	29,499	31,873	31,232	29,114	26,523	23,269
USB	367,445	340,994	315,139	306,270	283,290	252,042	237,287	215,046	194,683	186,231	178,425
ZION	54,374	52,007	51,007	48,999	47,170	46,941	48,770	47,691	43,048	38,668	30,154
Banks	**2004**	**2003**	**2002**	**2001**	**2000**	**1999**	**1998**	**1997**	**1996**	**1995**	**1994**
FITB	87,639	81,361	69,124	64,826	61,521	55,880	33,851	31,746			
ASBC	15,203	13,947	13,295	12,272	12,047	10,955	10,062	9,897	9,135	8,632	
BBT	84,946	75,463	68,230	62,905	57,616	53,258	42,530	31,210	24,946	22,799	17,938
BOKF	12,427	11,542	10,299	9,201	7,773	6,727	5,280	4,670	3,835	3,603	
CMA	46,975	48,841	47,053	45,722	43,364	39,247	32,113	32,025	31,370	31,537	
FHN	23,718	21,329	17,397	16,125	16,096	15,584	14,321	11,512	11,062	10,095	
HBAN	27,697	24,593	20,845	21,903	22,648	25,547	24,116	23,391	21,674	20,471	15,428
KEY	71,475	73,025	72,300	75,441	74,575	71,656	66,678	61,300	57,845	60,203	
MTB	45,200	39,531	28,921	27,818	21,500	19,076	16,900	12,795	12,030	11,078	
NYCB	21,628	12,820	8,658	5,732	2,193	1,820	1,615	1,401			
PBCT	9,843	9,834	9,815	10,173	9,855	9,303	8,280	7,188	6,547		
PNC	61,821	55,172	55,345	59,341	59,875	61,301	63,077	64,017	64,725	69,535	
RF	59,603	44,233	42,136	41,334	40,085	36,759	31,678	27,359	23,636	15,234	
STT	83,876	73,574	73,247	65,648	57,837	49,489	41,406				
STI	117,969	109,257	96,371	92,034	88,609	82,256	74,881	66,944	61,644	0	
SNY	20,574	18,078	15,564	13,701	12,094	10,230	8,799	8,154	7,308	6,740	6,082
USB	168,123	160,808	147,410	143,501	140,606	133,757	64,171	55,498	27,886		
ZION	27,709	25,115	23,176	20,911	19,223	17,560	12,811	8,842	6,052	5,274	

See Section 2.4 for details.

Table 2.5 US Banking Sector: Normalized RPL (RPL/IEA Ratio) (%)

Banks	**2015**	**2014**	**2013**	**2012**	**2011**	**2010**	**2009**	**2008**	**2007**	**2006**	**2005**
FITB	1.92	2.07	2.02	2.35	2.45	2.35	2.20	2.65	2.66	2.54	2.72
ASBC	1.24	1.26	1.33	1.52	1.36	1.69	2.41	2.53	2.38	2.46	2.49
BBT	2.02	2.13	2.40	2.53	1.99	2.36	2.78	2.87	2.75	2.77	2.91
BOKF	1.65	1.70	1.88	2.08	1.88	2.38	2.60	2.20	2.27	2.37	2.44
CMA	1.37	1.45	1.56	1.58	1.53	1.81	1.44	1.80	2.32	2.23	2.47
FHN	1.26	1.56	0.64	0.03	0.81	1.27	1.84	2.20	0.26	1.31	1.92
HBAN	1.68	1.70	0.76	0.83	0.75	1.05	1.29	2.22	1.94	2.12	2.18
KEY	1.64	1.71	1.66	1.64	1.58	1.52	0.81	1.27	1.71	2.10	2.03
MTB	2.02	2.24	2.68	2.70	2.29	2.71	2.23	2.39	2.56	2.62	2.69
NYCB	0.00	1.72	2.09	2.51	2.54	2.93	2.26	1.57	1.50	1.52	2.00
PBCT	1.28	1.37	1.44	1.73	1.58	1.06	0.95	1.30	2.42	2.11	1.89
PNC	1.76	1.92	2.37	1.96	2.36	2.92	2.70	2.64	2.43	2.87	2.61
RF	1.55	1.63	1.71	0.64	0.57	0.52	0.49	0.90	2.35	2.62	2.06
STT	1.24	1.26	1.50	1.42	1.67	2.16	2.26	2.07	2.42	1.90	1.61
STI	1.70	1.79	1.58	1.38	1.43	1.72	1.44	1.71	1.79	2.04	2.08
SNV	1.50	1.44	1.46	0.98	0.91	0.77	0.40	1.24	2.53	2.67	2.79
USB	2.61	2.86	3.15	3.40	3.35	3.76	3.80	3.84	3.78	3.99	4.17
ZION	1.12	1.00	0.95	0.93	1.00	0.89	1.67	2.02	2.29	2.29	2.33
Banks	**2004**	**2003**	**2002**	**2001**	**2000**	**1999**	**1998**	**1997**	**1996**	**1995**	**1994**
FITB	3.29	3.38	3.39	2.66	2.97	2.90	2.81	2.85			
ASBC	2.56	2.68	2.66	2.29	1.97	2.30	2.37	1.85	2.33	2.18	2.26
BBT	3.04	2.85	2.81	2.37	2.43	2.43	2.53	2.36	2.32	1.79	
BOKF	2.42	2.21	2.44	2.20	2.18	2.09	2.31	2.22	2.16	1.80	
CMA	2.57	2.69	3.38	3.03	3.28	3.07	3.23	2.93	2.38	2.26	
FHN	2.68	3.68	3.79	3.18	1.60	2.79	2.82	3.22	2.92	2.71	
HBAN	2.13	2.64	2.46	1.69	2.13	2.54	2.13	2.42	2.41	2.26	
KEY	2.00	2.26	2.54	2.30	2.36	2.19	2.49	2.54	2.25	2.10	
MTB	2.58	2.52	2.81	2.41	2.32	2.45	2.00	2.60	2.50	2.40	
NYCB	3.15	3.35	3.70	2.60	1.43	2.67	2.83	2.74			
PBCT	1.37	0.92	1.21	1.65	2.08	2.21	1.89	1.79	2.03		
PNC	2.71	3.03	3.82	2.60	3.11	2.45	2.84	2.46	2.38	1.74	
RF	2.22	2.36	2.30	2.13	2.11	2.43	2.25	2.47	2.22	2.17	
STT	1.37	1.01	1.35	1.37	1.58	1.42	1.63				
STI	2.08	1.94	2.20	2.38	2.35	2.43	2.23	2.59	2.30		
SNV	3.67	3.70	3.89	3.75	3.71	3.75	3.78	3.71	3.48	3.09	2.77
USB	4.27	4.31	4.20	3.00	3.76	3.29	2.14	2.47	2.62	2.38	
ZION	2.06	2.23	2.25	1.97	1.57	1.41	1.39	1.88	2.12	1.97	

See Section 2.5 for details.

Table 2.6 The Impact of New Accounting Rules on Canadian RPL in 2011
(C$ in millions)

Banks	RPL 2011 Under Two Accounting Standards			Approximation	
	RPL Under IFRS	RPL Under CGAAP	Actual Impact	Approximted Impact	Difference from the Actual Impact
DEJ	868	945	-77	0	77
BMO	5,025	4,111	914	821	-93
BNS	7,047	7,315	-268	236	504
CM	4,483	3,467	1,016	1,063	47
LB	215	182	33	36	2
NA	1,584	1,200	384	338	-46
RY	10,099	8,662	1,437	797	-640
TD	8,515	7,945	570	450	-120

Sources: Banks' income statements.
CGAAP stands for Generally Accepted Accounting Principles.
IFRS stands for International Financial Reporting Standards.

Table 2.7 The Impact of New Accounting Rules on Canadian IEA in 2011
Measured by the change of IEA in 2011 under IFRS from being under CGAAP

Banks	Under IFRS	Under CGAAP	Impact	
	C$ million	C$ million	C$ million	%
BMO	404,195	374,390	29,805	8.0
BNS	518,100	501,000	17,100	3.4
CM	347,634	316,533	31,101	9.8
LB	27,144	23,980	3,164	13.2
NA	145,671	134,858	10,813	8.0
RY	608,987	577,643	31,344	5.4
TD	593,141	542,377	50,764	9.4

Sources: Banks' income statements.

Table 2.8 Canadian Banking Sector: RPL (Residual Profit & Loss)
(C$ in millions)

Banks	2015	2014	2013	2012	2011	2010	2009	2008	2007	2006	2005
DEJ	1,531	-8	2,687	1,167	261	868	1,695	478	1,678	1,545	1,606
BMO	5,789	5,628	5,554	5,654	5,025	4,461	4,148	3,647	3,272	3,444	3,244
BNS	9,907	9,059	8,410	7,640	7,048	7,034	7,008	4,977	5,006	4,404	3,965
CM	4,596	4,552	4,936	4,926	4,483	4,680	3,025	-3,425	3,488	3,792	3,606
LB	250	236	195	201	215	241	201	193	158	135	127
NA	1,921	1,909	1,819	1,550	1,584	1,296	1,519	1,144	1,143	1,055	1,098
RY	13,223	12,485	11,277	10,734	10,089	9,130	10,671	8,272	7,655	6,528	5,893
TD	11,164	10,563	9,124	9,004	8,515	7,559	6,379	4,714	4,911	3,971	2,997
Banks	**2004**	**2003**	**2002**	**2001**	**2000**	**1999**	**1998**	**1997**	**1996**	**1995**	**1994**
DEJ	1,605	1,217									
BMO	3,031	3,010	2,914	2,959	3,022	2,569	2,243	2,429	2,201	1,903	
BNS	3,716	4,112	4,617	4,576	3,333	2,875	2,594	2,052			
CM	3,327	3,575	1,953	2,372	2,726	1,897	1,856	2,964	2,842	2,256	
LB	88	124	200	181	146	92					
NA	1,052	1,097	1,082	998	818	884	813				
RY	4,633	5,144	5,374	4,706	4,548	3,513	3,174	3,227			
TD	2,356	2,073	2,461	1,613	1,477	2,196	1,841	1,792	1,515	1,423	1,389

See Section 2.4 for details of the calculation.
Data is not available for Desjardins prior to 2003.

Table 2.9 Canadian Banking Sector: IEA (Average Interest Earning Assets)
(C$ in millions)

Banks	**2015**	**2014**	**2013**	**2012**	**2011**	**2010**	**2009**	**2008**	**2007**	**2006**	**2005**
DEJ	170,922	160,600	151,009	143,266	134,709	134,295	123,500	119,027	109,239	100,995	94,144
BMO	536,741	489,794	449,413	427,023	374,390	396,853	352,416	362,359	346,411	322,739	277,149
BNS	740,718	697,300	651,078	559,116	501,000	531,060	485,056	466,612	422,834	383,508	332,310
CM	360,222	330,522	320,223	310,541	316,533	335,525	312,094	302,697	309,689	365,547	308,754
LB	27,606	26,377	27,556	27,047	23,980	25,419	23,554	20,943	18,863	17,308	17,692
NA	179,987	169,667	157,278	146,979	134,858	142,949	129,476	124,448	118,921	118,313	102,372
RY	820,244	720,496	667,524	598,625	577,643	612,302	550,000	518,549	562,292	529,203	460,154
TD	835,596	738,804	668,254	616,418	542,377	574,920	519,662	473,031	408,130	355,888	333,852
Banks	**2004**	**2003**	**2002**	**2001**	**2000**	**1999**	**1998**	**1997**	**1996**	**1995**	**1994**
DEJ	98,794	90,459									
BMO	257,788	236,440	224,436	210,591	212,863	214,223	210,585	216,811	187,878	155,262	141,067
BNS	292,454	265,848	267,120	275,812	252,492	221,544	210,092	196,206	165,890		
CM	306,176	240,981	242,550	253,814	245,507	241,354	237,200	246,041	216,368	181,647	
LB	17,308	17,307	19,661	19,684	18,207	15,240	13,689				
NA	89,401	74,800	68,556	66,712	65,604	70,984	67,296	65,164			
RY	401,608	374,166	342,440	331,093	300,547	261,928	248,350	235,510	219,413		
TD	305,442	270,691	266,915	271,954	268,658	248,062	197,316	170,919	134,511	108,969	99,906

See Section 2.5 for details.
Data is not available for Desjardins prior to 2003.

Table 2.10 Canadian Banking Sector: Normalized RPL (RPL/IEA Ratio) (%)

Banks	**2015**	**2014**	**2013**	**2012**	**2011**	**2010**	**2009**	**2008**	**2007**	**2006**	**2005**
DEJ	0.90	-0.01	1.78	0.81	0.19	0.65	1.37	0.40	1.54	1.53	1.71
BMO	1.00	1.06	1.14	1.23	1.24	1.07	0.89	0.90	0.92	1.21	1.21
BNS	1.29	1.26	1.25	1.32	1.36	1.42	1.41	1.15	1.30	1.31	1.33
CM	1.16	1.25	1.40	1.44	1.29	1.30	0.83	-1.29	0.82	1.07	1.07
LB	0.80	0.79	0.63	0.66	0.79	1.00	0.80	0.83	0.88	0.70	0.67
NA	0.99	1.04	1.07	0.98	1.09	0.78	0.94	0.77	0.81	0.86	1.01
RY	1.53	1.64	1.60	1.70	1.66	1.52	1.83	1.39	1.40	1.36	1.40
TD	1.22	1.31	1.25	1.34	1.44	1.36	1.25	1.10	1.27	1.09	0.85
Banks	**2004**	**2003**	**2002**	**2001**	**2000**	**1999**	**1998**	**1997**	**1996**	**1995**	**1994**
DEJ	1.62	1.35									
BMO	1.21	1.23	1.23	1.23	1.25	1.08	0.96	1.28	1.42	1.35	
BNS	1.36	1.49	1.62	1.73	1.41	1.29	1.30	1.24			
CM	1.30	1.38	0.69	0.87	1.03	0.72	0.71	1.35	1.56		
LB	0.48	0.63	0.93	0.89	0.81	0.49					
NA	1.17	1.30	1.32	1.28	1.01	1.21	1.20				
RY	1.18	1.45	1.57	1.52	1.69	1.32	1.26	1.47			
TD	0.73	0.68	0.82	0.50	0.50	1.07	1.06	1.33	1.39	1.42	1.51

See Section 2.6 for details.
Data is not available for Desjardins prior to 2003.

Table 2.11 Statistics Properties of US and Canadian Samples

Panel A: US sample								
Setting a breakpoint for seriess of RPL/IEA ratio:								
	Breakpoint at 2009				Breakpoint at 2010			
	sub-sample1 (1994-2008) obs.245		sub-sample2 (2009-2015) obs.126		sub-sample1 (1994-2009) obs.263		sub-sample2 (2010-2015) obs.108	
	mean	std	mean	std	mean	std	mean	std
	2.44%	0.65%	1.72%	0.72%	2.40%	0.68%	1.70%	0.69%
	test on equal mean of two samples				test on equal mean of two samples			
t-stats	9.64				8.90			
p-value	0.00				0.00			
Conclusion	rejected at 5% significance level				rejected at 5% significance level			
Descriptive statistics of loss dataset, choosing breakpoint at 2010, obs.371.								
	Min		-2.65%		Std		0.48%	
	Max		1.33%		Skewness		-1.17	
	Mean		0.00		Kurtosis		7.45	
Panel B: Canadian sample								
Setting a breakpoint for seriess of RPL/IEA ratio:								
	Breakpoint at 2009				Breakpoint at 2010			
	sub-sample1 (1994-2008) obs.94		sub-sample2 (2009-2015) obs.56		sub-sample1 (1994-2009) obs.102		sub-sample2 (2010-2015) obs.48	
	mean	std	mean	std	mean	std	mean	std
	1.12%	0.40%	1.15%	0.36%	1.12%	0.40%	1.15%	0.36%
	test on equal mean of two samples				test on equal mean of two samples			
t-stats	0.46				0.35			
p-value	0.64				0.73			
Conclusion	accepted at 5% significance level				accepted at 5% significance level			
Descriptive statistics of loss dataset, without breakpoint, obs. 150.								
	Min		-2.29%		Std		0.32%	
	Max		0.72%		Skewness		-2.65	
	Mean		0.00		Kurtosis		19.45	

The US sample: business return data, 18 banks, length of return series between 18 and 22.
The Candian sample: business return data, eight banks, length of return series between 14 and 22.

Table 2.12 Demeaned Returns: MA Representations

US									
	MA(1)		**MA(2)**			**MA(3)**			
Banks	$\hat{\theta}$	$\hat{\sigma}^2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\sigma}^2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\sigma}^2$
FITB	0.35	0.08	0.46	0.17	0.07	0.41	0.00	-0.20	0.07
ASBC	0.26	0.04	0.27	0.14	0.04	0.23	0.20	0.20	0.04
BBT	0.45	0.06	0.72*	0.46	0.05	0.73*	0.48	0.03	0.05
BOKF	0.34	0.04	0.41	0.12	0.04	0.42	0.10	-0.07	0.04
CMA	0.48	0.16	0.49	0.24	0.15	0.56	0.35	0.21	0.14
FHN	0.41	0.54	0.53	0.33	0.49	0.55	0.36	0.04	0.49
HBAN	0.17	0.12	0.16	-0.01	0.12	0.15	0.02	0.06	0.12
KEY	0.53	0.10	0.74	0.46	0.09	0.76	0.47	0.03	0.09
MTB	0.00	0.05	0.00	-0.13	0.05	0.01	-0.12	0.00	0.05
NYCB	0.42	0.54	0.50	0.16	0.53	0.25	-0.34	-0.49	0.48
PBCT	0.82*	0.09	0.84*	-0.16	0.09	0.42*	-0.57*	-0.49*	0.08
PNC	0.12	0.17	0.19	0.18	0.17	0.19	0.15	-0.07	0.17
RF	0.77*	0.17	0.89	0.19	0.17	0.89	0.19	0.00	0.17
STT	1.00	0.07	1.00	0.00	0.07	0.66*	0.66*	1.00	0.05
STI	0.37	0.06	0.46	0.29	0.05	0.46	0.30	0.05	0.05
SNV	0.71*	0.38	0.89*	0.26	0.35	0.84	0.34	0.19	0.34
USB	0.63*	0.26	0.77*	0.20	0.24	0.47	0.43	0.50	0.20
ZION	0.73*	0.04	0.97*	0.36	0.03	1.13	0.88	0.43	0.03
first quantile	0.32	0.05	0.38	0.09	0.05	0.24	0.01	-0.07	0.05
median	**0.43**	0.10	**0.52**	0.19	0.09	**0.46**	0.25	0.04	0.09
third quantile	0.72	0.20	0.85	0.30	0.19	0.74	0.44	0.20	0.18
Canadian									
	MA(1)		**MA(2)**			**MA(3)**			
Banks	$\hat{\theta}$	$\hat{\sigma}^2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\sigma}^2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\sigma}^2$
DEJ	0.04	0.34	0.03	0.03	0.34	-0.09	0.25	0.40	0.29
BMO	0.70*	0.01	0.91*	0.31	0.01	0.92*	0.31	0.00	0.01
BNS	0.58*	0.01	0.71*	0.23	0.01	0.74*	0.37	0.13	0.01
CM	0.30	0.32	0.25	-0.10	0.32	0.03	-0.60*	-0.43	0.29
LB	0.65*	0.02	0.00	-0.76*	0.01	0.24	-0.79*	-0.32	0.01
NA	0.41	0.02	0.51*	0.76*	0.01	0.49*	0.97*	0.54*	0.01
RY	0.26	0.02	0.22	0.18	0.02	0.28	0.32	0.35	0.02
TD	0.85*	0.04	0.98*	0.19	0.04	1.02*	0.57*	0.36	0.03
first quantile	0.27	0.02	0.08	-0.07	0.01	0.08	-0.38	-0.24	0.01
median	**0.50**	0.01	**0.38**	0.19	0.02	**0.38**	0.32	0.24	0.02
third quantile	0.68	0.34	0.86	0.29	0.25	0.87	0.52	0.39	0.22

Assume that a bank's loss series $\{x_t\}$ over T periods has MA process of order one, two or three; $x_t = \epsilon_t + \theta\epsilon_{t-1}, x_t = \epsilon_t + \theta_1\eta_{t-1} + \theta_2\epsilon_{t-2}$, or $x_t = \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \theta_3\epsilon_{t-3}$, where ϵ_t is the shock at time t and $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$ for $t = 1, \ldots, T$.

The subscript * indicates the estimate is statistically significant at 95 percent confidence level.

Table 2.13 Residual Correlations Across US Banks

Banks	FITB	ASBC	BBT	BOKF	CMA	FHN	HBAN	KEY	MTB	NYCB	PBCT	PNC	RF	STT	STI	SNV	USB	ZION
FITB	1																	
ASBC	**-0.51**	1																
BBT	0.14	-0.12	1															
BOKF	0.27	-0.23	**0.83**	1														
CMA	0.36	-0.41	0.21	0.29	1													
FHN	0.27	-0.10	-0.15	-0.24	0.22	1												
HBAN	0.14	-0.04	0.12	-0.13	0.49	**0.57**	1											
KEY	0.32	-0.40	0.03	0.03	**0.87**	0.39	**0.64**	1										
MTB	-0.07	0.19	0.28	0.22	0.35	0.26	**0.58**	0.35	1									
NYCB	**0.50**	0.05	-0.22	-0.15	0.14	0.22	0.22	0.02	0.28	1								
PBCT	-0.10	0.02	-0.27	-0.15	0.01	0.04	0.08	0.26	-0.09	-0.44	1							
PNC	**0.60**	-0.25	**0.64**	**0.71**	**0.56**	0.01	0.24	0.39	0.11	0.13	-0.13	1						
RF	-0.29	-0.01	-0.19	-0.25	0.49	0.20	0.47	**0.68**	0.35	-0.23	0.33	-0.07	1					
STT	-0.49	0.40	0.19	0.03	-0.22	-0.01	0.12	-0.12	0.39	-0.34	0.13	-0.15	0.35	1				
STI	-0.07	-0.22	-0.20	0.04	-0.05	-0.24	-0.44	-0.08	-0.49	-0.27	0.10	-0.17	0.00	-0.49	1			
SNV	-0.12	0.05	-0.15	-0.42	0.14	0.09	0.08	0.26	-0.28	-0.21	-0.06	-0.14	0.37	-0.12	0.23	1		
USB	0.07	0.42	**0.71**	**0.55**	0.14	-0.15	0.21	-0.05	0.45	0.12	-0.27	**0.59**	-0.13	0.28	-0.41	-0.17	1	
ZION	-0.33	0.30	0.13	-0.04	-0.14	0.04	0.22	0.05	0.43	-0.08	-0.28	-0.06	0.24	0.48	-0.37	0.08	0.33	1

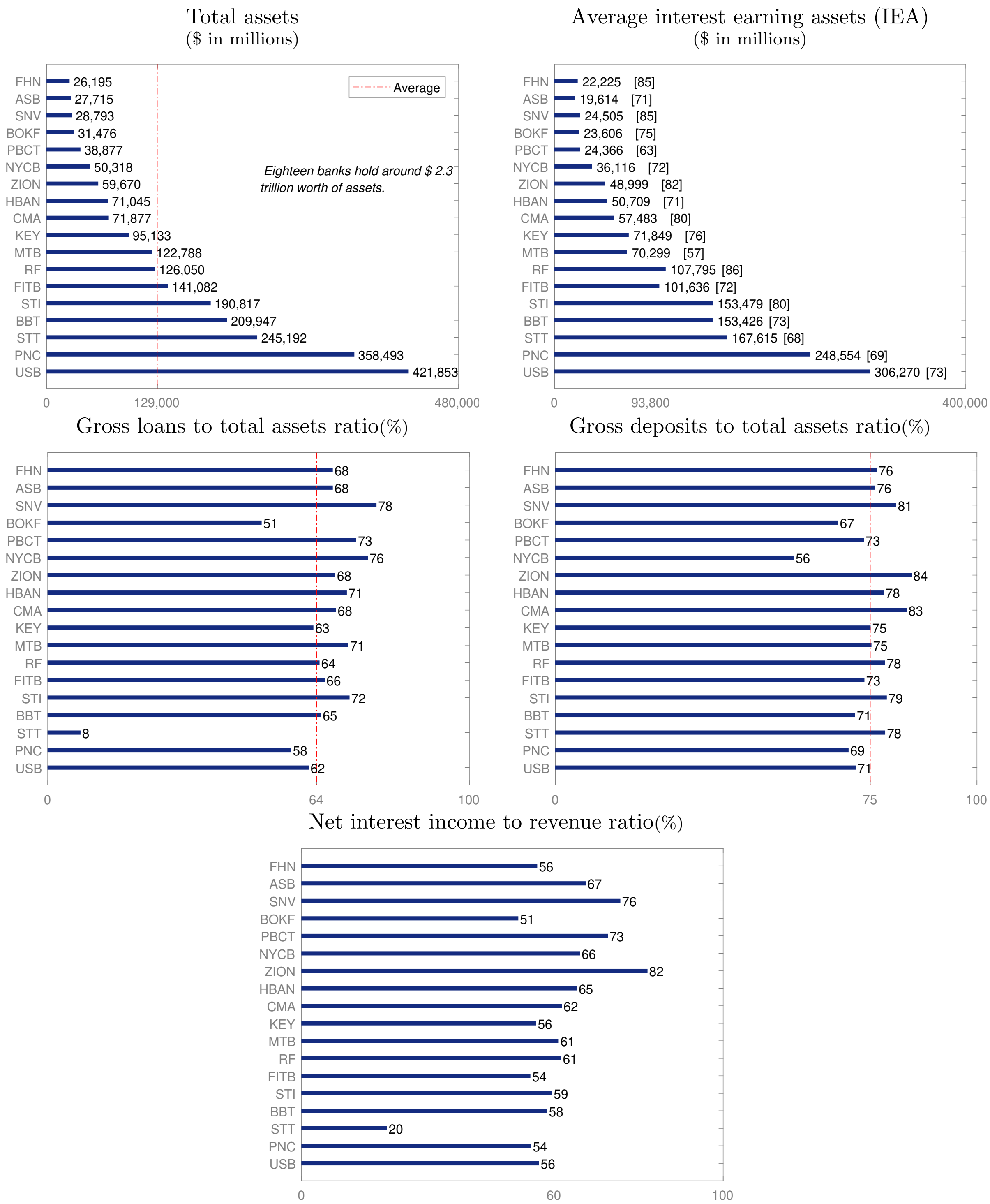
Each bank's residual series is estimated by assuming the bank's demeaned business return series following a MA(1) process with the coefficient set at 0.5. The value of the common weight is taken around the median of 18 estimates, as shown in Table 2.12.

Table 2.14 Residual Correlations Across Canadian Banks

	BMO	BNS	CM	LB	NA	RY	TD
BMO	1						
BNS	-0.35	1					
CM	**0.50**	-0.40	1				
LB	-0.06	-0.33	-0.01	1			
NA	0.18	0.05	**0.55**	-0.01	1		
RY	**-0.65**	0.31	**-0.68**	-0.13	-0.27	1	
TD	0.03	-0.19	0.10	-0.15	0.32	0.27	1

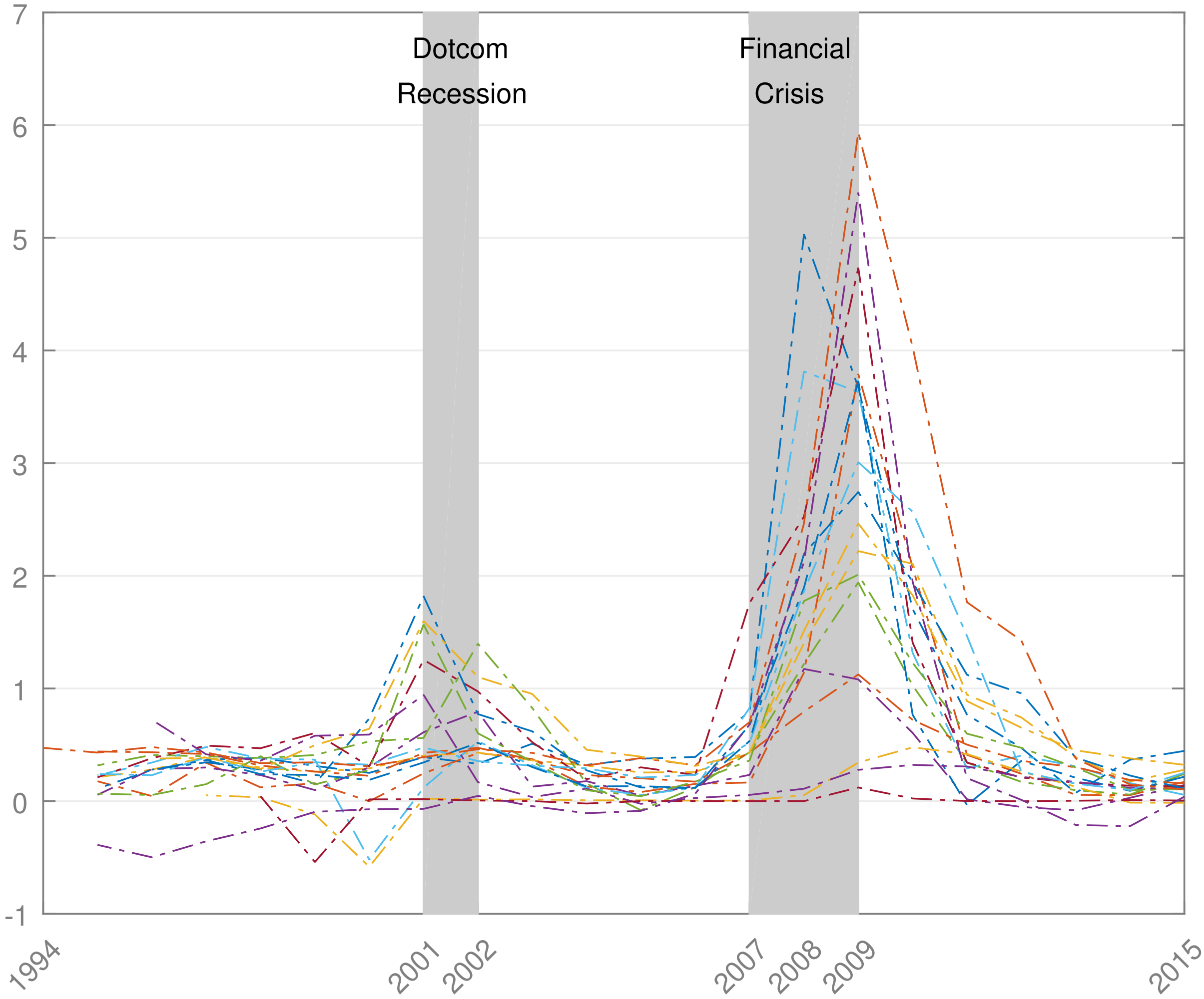
Each bank's residual series is estimated by assuming the bank's demeaned business return series following a MA(1) process with the coefficient set at 0.5. The value of the common weight is taken around the median of eight estimates, as shown in Table 2.12.

Figure 2.1 US Banking Sector in 2015



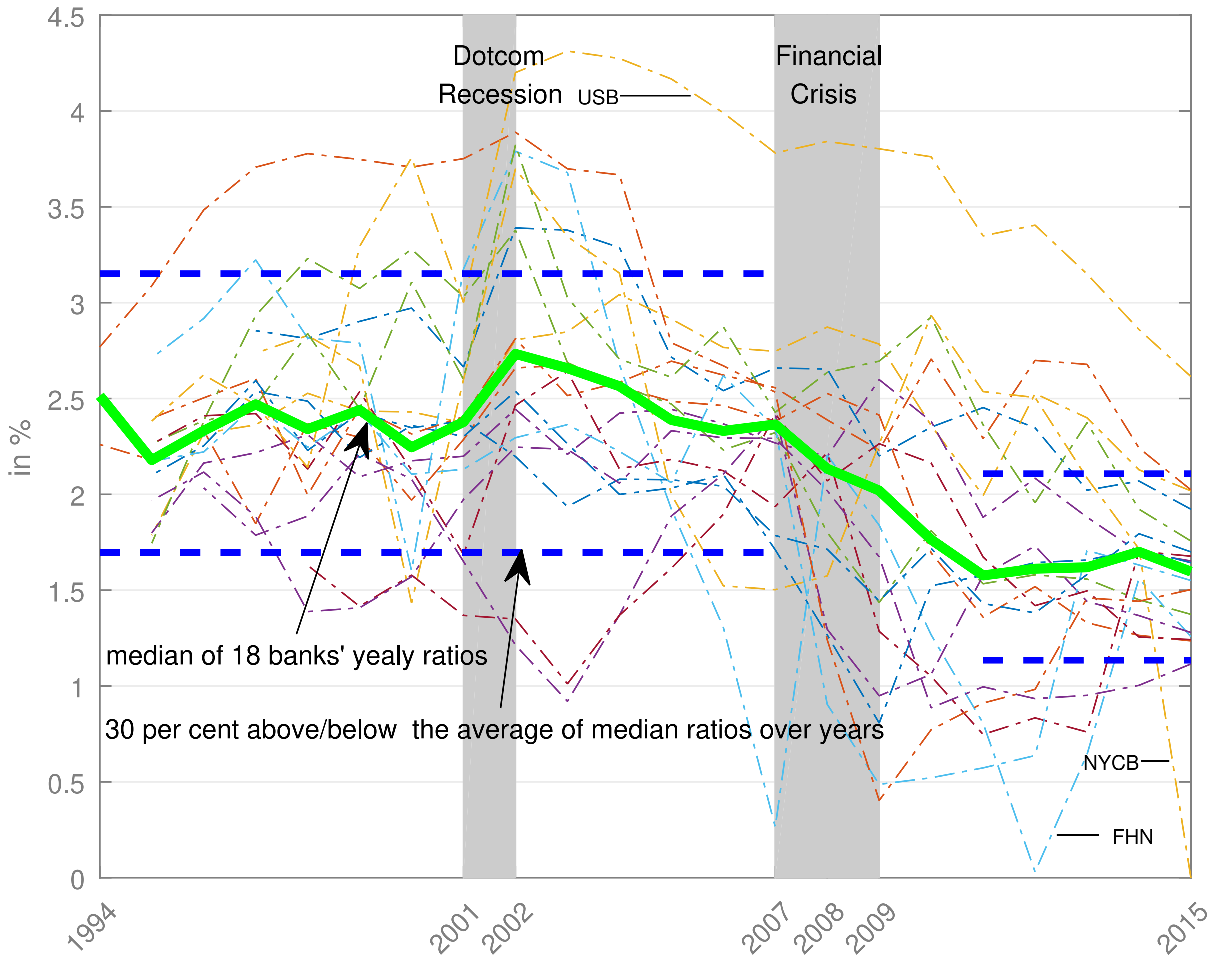
Sources: annual financial reports, as of Oct 31 2015.
Gross loans count loan or lease, include commercial and industrial loans and leases, commercial real estate loans, auto-mobile loans, home equity loans, residential mortgage loans and other consumer loans; gross deposits include interest–bearing, noninterest-bearing and deposits in foreign office; revenue is the sum of net interest income and noninterest income.

Figure 2.2 US Banking Sector: Normalized Credit Losses
(% of IEA)



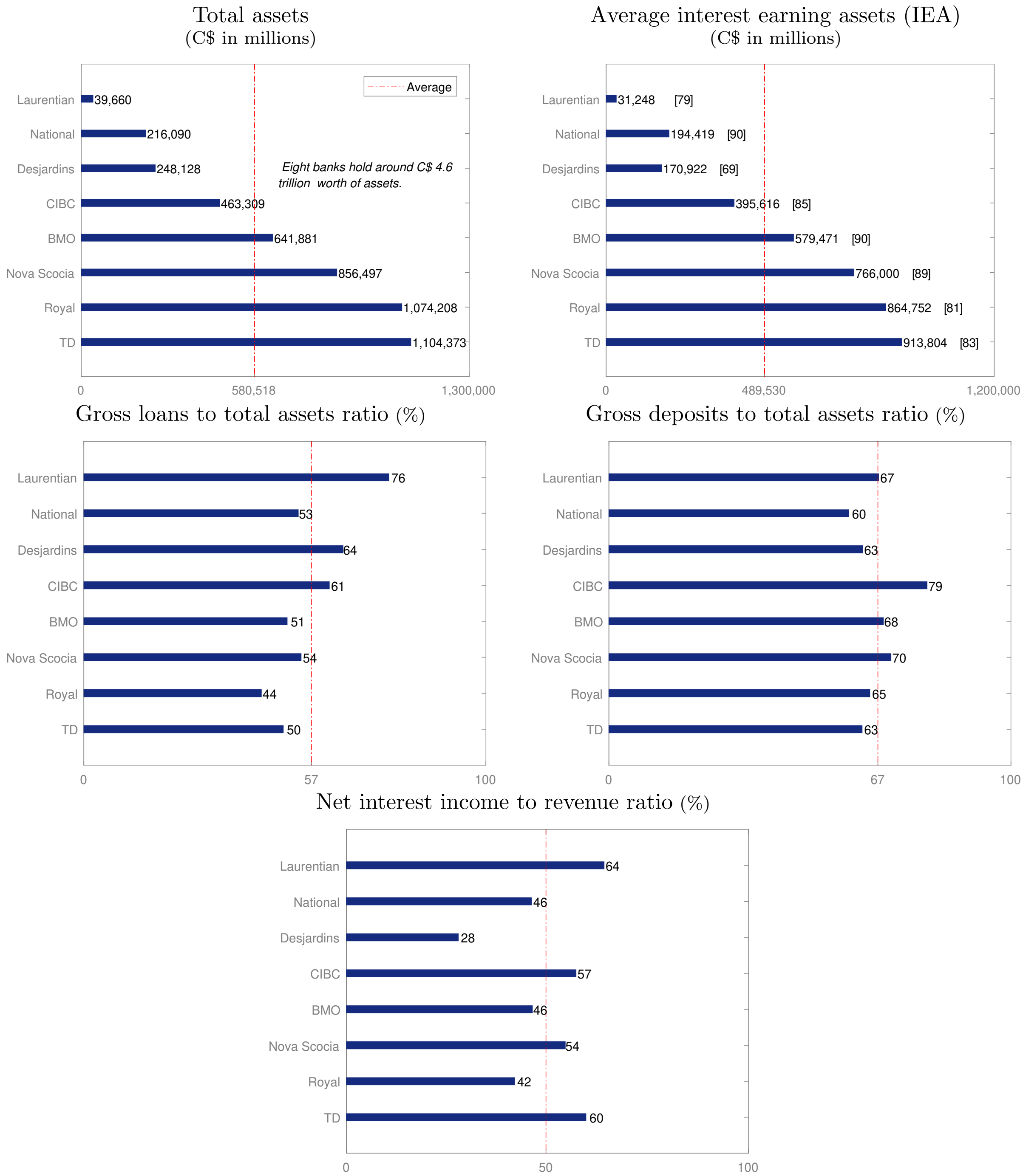
Shaded areas indicate US recessions as defined by National Bureau of Economic Research (NBER).

Figure 2.3 US Banking Sector: RPL/IEA Ratio
(1994-2015)



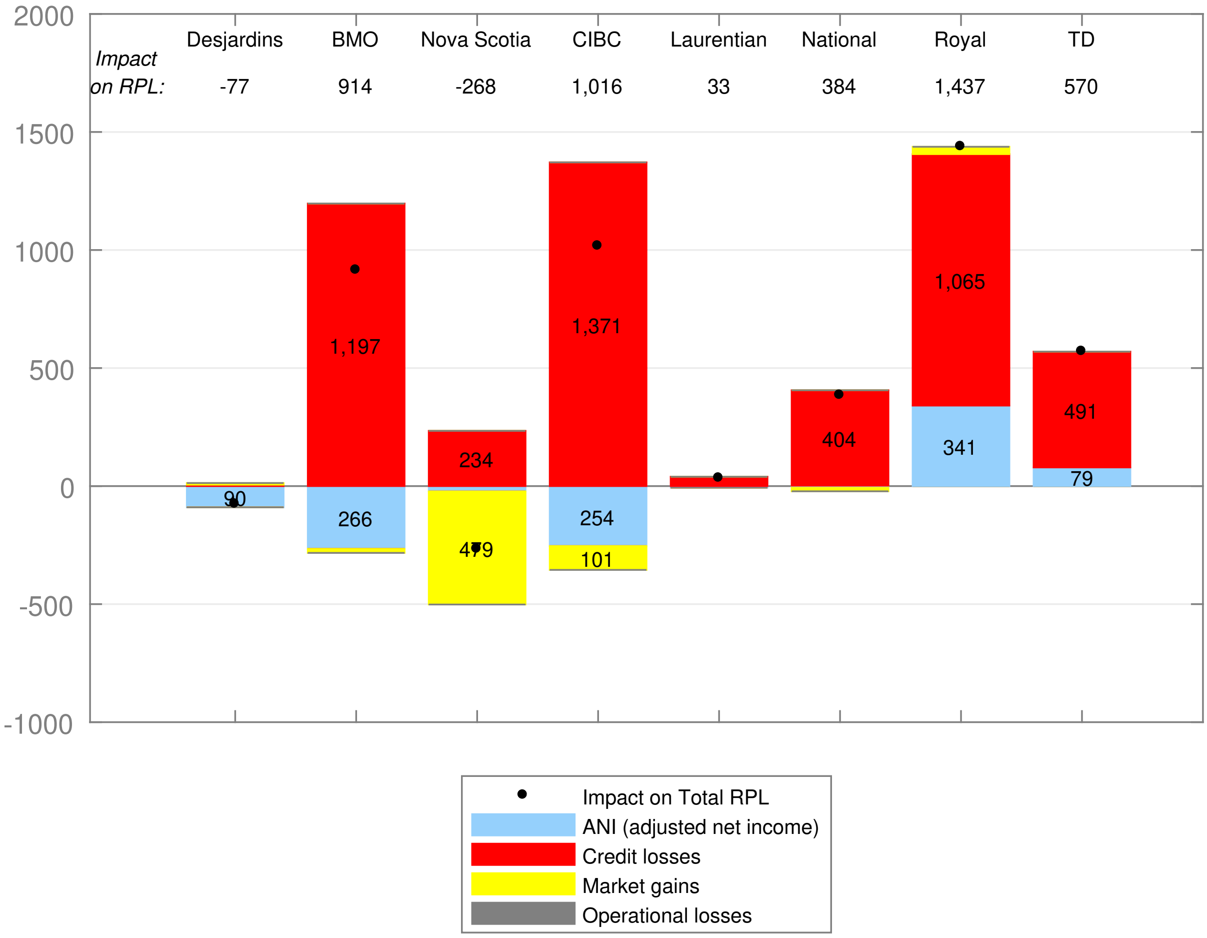
Shaded areas indicate US recessions as defined by National Bureau of Economic Research (NBER).

Figure 2.4 Canadian Banking Sector in 2015



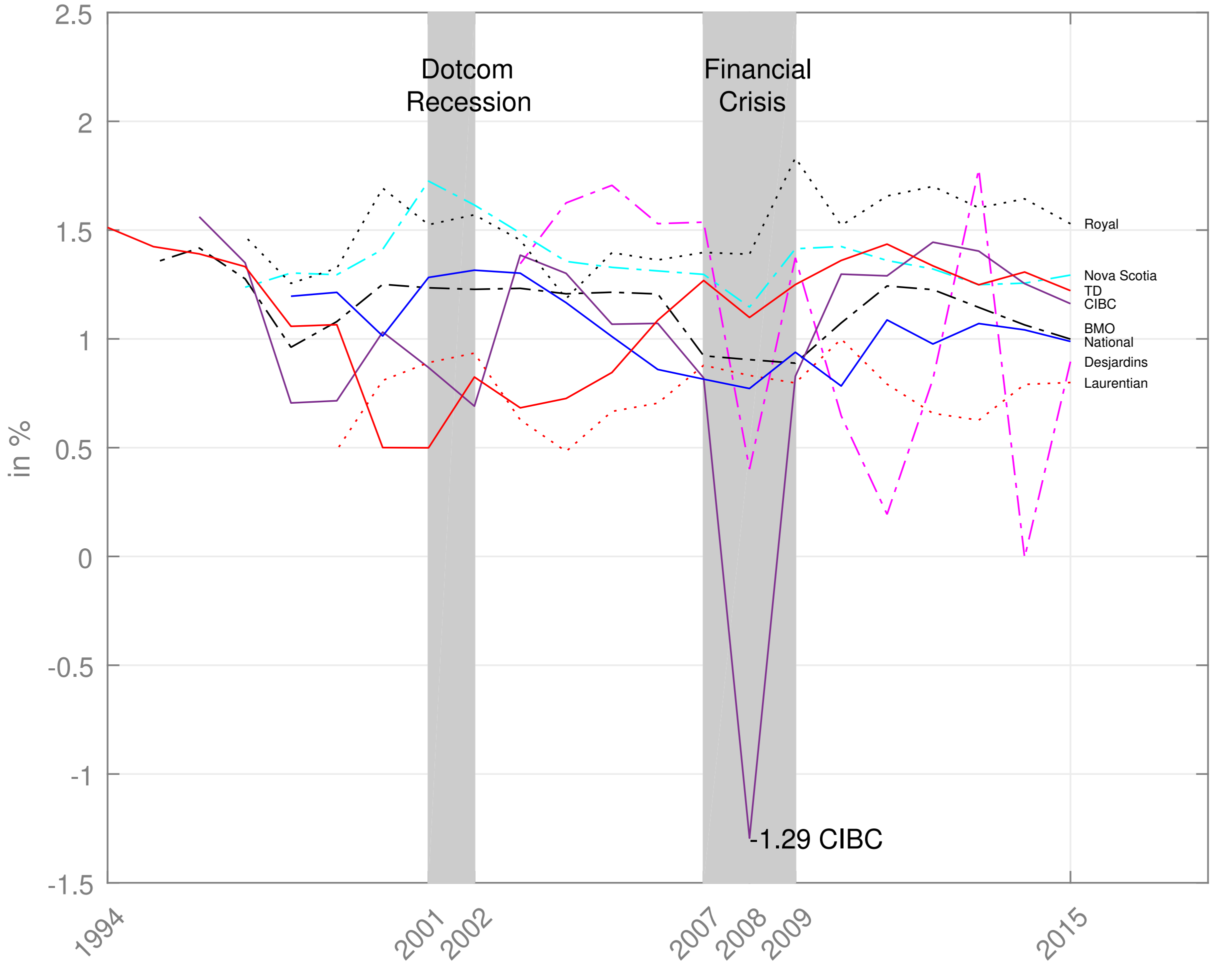
Sources: annual financial reports, as of Oct 31 2015. Desjardins' as of Dec 31, 2015.
Gross loans count loan or lease, include commercial and industrial loans and leases, commercial real estate loans, auto-mobile loans, home equity loans, residential mortgage loans and other consumer loans; gross deposits include interest–bearing, noninterest-bearing and deposits in foreign office; revenue is the sum of net interest income and noninterest income.

Figure 2.5 The Impact of New Accounting Rules on Canadian RPL in 2011
(C$ in millions)



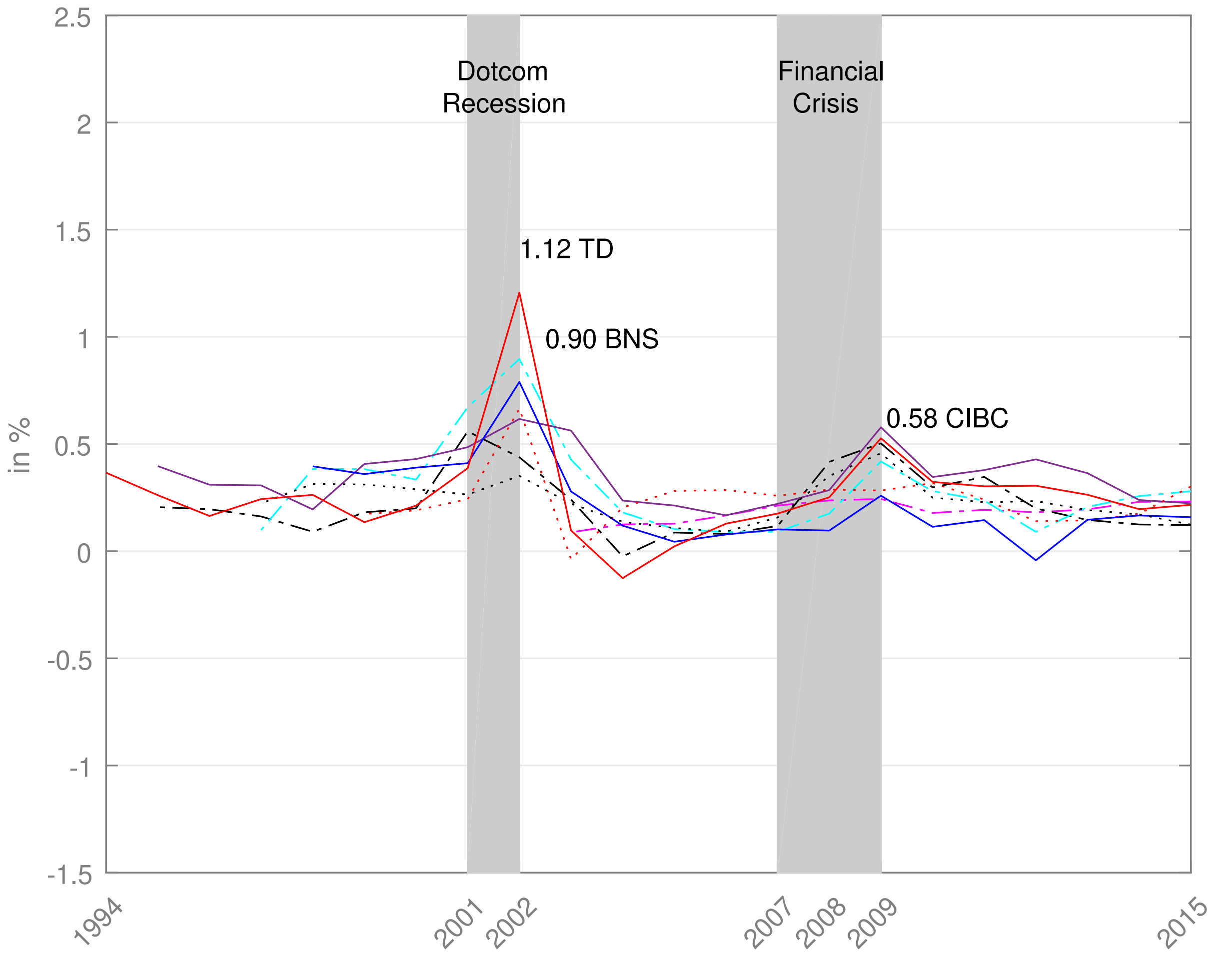
Sources: Banks' income statements.

Figure 2.6 Canadian Banking Sector: RPL/IEA Ratio
(1994-2015)



Shaded areas indicate US recessions as defined by National Bureau of Economic Research (NBER).

Figure 2.7 Canadian Banking Sector: Normalized Credit Losses
(% of IEA)



Shaded areas indicate US recessions as defined by National Bureau of Economic Research (NBER).

Chapter 3

The Saddlepoint Estimation of Business Risk

In this chapter, two approaches to the estimation of business risk are compared. These approaches reflect the sectoral approach to the analysis of business risk that is being explored in this thesis. The first, which is taken to be the benchmark approach, involves the joint time multivariate analysis of business returns for a sector of banks. In this context, the sectoral perspective imposes a common parameter across the equations. The estimation involves traditional maximum likelihood procedures which are described in detail in the Appendix to this chapter.

The second pools the business return data to estimate the common risk profile of the sector in a univariate setting. We implement such a univariate approach using saddlepoint estimation methods. These have had limited exposure in the applied financial literature and, accordingly, the first section of this chapter presents an overview of this statistical method along with a variety of examples to illustrate its application in applied settings; this section also briefly surveys various applications taken from the empirical financial literature. In the second section a simulation framework is constructed to contrast the performance of this approach with the maximum likelihood methods. The simulation results are presented in the final section.

3.1 Saddlepoint Approximation (SPA)

Saddlepoint techniques were introduced into statistics by Daniels (1954) to establish an approximation formula for a density function of sample mean. The techniques use moments of a distribution via its cumulant-generating function to characterize the distribution, and to establish approximations formulas for the density and cumulative distribution function that are derived from an approximation to the density inversion integral. This section serves primarily to present these formulas that will play a crucial role in the computation of economic capital for the financial sectors we are studying. The emphasis in what follows is on the practical application of these approximating formulas in computing tail percentiles of a generally unknown distribution. We also survey several applications in theoretical finance.

3.1.1 SPA: Theory and Illustrations

Let a random variable $X \in \mathbb{R}$ with a distribution function $F(x)$ and a density function $f(x)$. Its m^{th} moment is defined as $\mathbb{E}(X^m)$ and $\mathbb{E}(X^m)$ is finite. Its moment-generating function is defined as

$$\mathcal{M}_X(s) = \mathbb{E}(\exp^{sX}) = \int_{-\infty}^{\infty} \exp^{sx} f(x)dx; \tag{3.1}$$

where s is in a real interval containing the origin, the expectation is taken w.r.t. the true distribution of X, in particular, $\mathcal{M}_X(0) = 1$.

The corresponding cumulant-generating function is defined as a natural logarithm of $\mathcal{M}_X(s)$,

$$\mathcal{C}_X(s) = log\mathcal{M}_X(s). \tag{3.2}$$

The approximation method is developed upon an contour integral of density inversion formula using steepest decent method: Along the contour in question, the SPA technique chooses a path passing through a stable point, at which $\mathcal{C}_X(s) - sx$ reaches a minimum and the modulus of integrand have a maximum, having an image of saddlepoint in mind. The stable point is given by solving the saddle equation

$$\mathcal{C}_X^{(1)}(s) = x, \tag{3.3}$$

where $\mathcal{C}_X^{(1)}$ denotes the first derivative of the cumulant-generating function w.r.t. s, while the solution is called *saddlepoint* denoted by $\hat{s}$.

The approximation density formula is given as

$$\hat{f}(x) = \sqrt{\frac{1}{2\pi\mathcal{C}_X^{(2)}(\hat{s})}} \exp^{\mathcal{C}_X(\hat{s}) - \hat{s}x}, \tag{3.4}$$

where $\mathcal{C}_X^{(2)}$ denotes the second derivative w.r.t. s.

In implementing the VaR measure in business risk quantification, the primary interest is the tail probabilities. For an instance, VaR measure reaches an estimated extreme loss defined as a value related to a low percentile of a distribution. VaR at confidence level $1 - p$ can be defined as being related to the inverse of a left-tail probability function, $\mathbb{P}(X < x_p) = p$, $0 < p < 1$, where the probability function $\mathbb{P}$ is taken with respect to the distribution function $F(x)$ and $F(x_p) = \mathbb{P}(X < x_p)$.

Researchers have established the approximation formulas for tail probabilities in the SPA context. The best known one was developed initially by Lugannani and Rice (1980).

It is written as equations 1.21 and 1.22 in Butler (2007),

$$\hat{F}(x) = \Phi(\omega) + \phi(\omega)(\frac{1}{\omega} - \frac{1}{\lambda}) \tag{3.5}$$

$\omega = sign(\hat{s})\sqrt{2(\hat{s}x - \mathcal{C}_X(\hat{s}))}, \lambda = \hat{s}\sqrt{\mathcal{C}_X^{(2)}(\hat{s})}$, Φ and ϕ are respectively standard normal distribution function and density function, the rest notations are defined in the same manner as in the formula of approximating density. Note that $\mathbb{P}(X > x)$ is approximated with the formula: $1 - \hat{F}(x)$.

The following two examples illustrate how the SPA can be used in theory. In the first, the approximation is exact. The second—which is used further in the next section on more practical applications—highlights the relevance of the approximation.

Example 1 SPA applied to the Normal Distribution

Suppose $X \sim N(\mu, \sigma^2)$. Then, by using the change-of-variable technique with $z = \frac{x-\mu}{\sigma}$, we get moment/cumulant-generating functions:

$$\mathcal{M}_X(s) = \exp^{\mu s} \int_{-\infty}^{\infty} \exp^{z\sigma s} \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{-\frac{z^2}{2}} \frac{dx}{dz} dz$$

$$= \exp^{\mu s + \frac{s^2}{2}\sigma^2},$$

$$\mathcal{C}_X(s) = \mu s + \frac{s^2}{2}\sigma^2.$$

Then saddlepoint equation is $\mu + \sigma^2 s$ at x and is solved at

$$\hat{s} = \frac{x - \mu}{\sigma^2}.$$

Using the approximation formula equation 3.4,

$$\hat{f}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2},$$

which is the exact density of $X \sim N(\mu, \sigma^2)$.

Example 2 SPA applied to the Gamma Distribution

This example is taken from Butler (2007), see page 13. It illustrates how tail probabilities are approximated using equation 3.5 applied to a long right-tailed Gamma (α, β) with parameters $\alpha = 0.5$ and $\beta = 1$. In this specification the distribution has the moment/cumulant-generating functions:

$$\mathcal{M}_X(s) = (1 - \frac{s}{\beta})^{-\alpha},$$

$$\mathcal{C}_X(s) = -\alpha log(1 - \frac{s}{\beta})$$

Given $\alpha = 0.5$ and $\beta = 1$, the saddle equation is $-\frac{1}{2}log(1 - s)$ at x and is solved at $\hat{s} = 1 - \frac{1}{2x}$. Then

$$\omega(x) = sign(\hat{s})\sqrt{2x - 1 - log(2x)},$$

$$\lambda(x) = \frac{1}{\sqrt{2}}(2x - 1).$$

To calculate the probability at x, say $x = 1$, it follows that $\hat{s} = .5$, $\omega(1) = .8360$ and $\lambda(1) = \frac{1}{\sqrt{2}}$. The right-tail probability at $x = 1$ has an approximated value .156.

Table 3.1 presents the corresponding values of the distribution corresponding to specific tail properties.

The development of SPA theory is based on assumptions which have not been highlighted in this overview. These include: (i) the existence of moments (ii) a unique solution to the saddle equation (iii) non-negativity of the second derivative of cumulant- generating function. These issues will be discussed to some extent in the following sections.

3.1.2 Selected Applications in Theoretical Finance

The saddle point approach (SPA) has been used primarily in the theoretical financial applications, particularly in portfolio analysis and credit risk modeling where it is useful, for example, in loss distributions associated with large portfolios or associated with complicated credit risk models.

The numerical studies that are conducted in this literature indicate that saddlepoint technique with such challenges fairly well. We consider some recent illustrative papers that apply SPA to determine a VaR measure.

- *Feuerverger and Wong (2000)* applies SPA to a portfolio market risk model. Assume that returns of underlying assets (or risk factors) over a single period have a multivariate normal distribution; the return of asset (or risk factor) is approximated by a second order Taylor expansion. The convenience of applying SPA in this context is the availability of a cumulant-generating function that is also in a quadratic form (see equation 12 in Feuerverger and Wong (2000)). The paper shows SPA works well when involving a relatively large-size risk matrix. One of the numerical examples is of a portfolio simulated with 400 correlated risk factors. The quality of SPA at low tail (one per cent and five per cent) compares favorably to a standard Monte Carlo method which by comparison is computationally very demanding.

- *Gordy (2002) and Martin (2011)* apply SPA to portfolio credit risk modeling with CreditRiskPlus approach. Here the occurrence of default is driven by risk factors characterized by Gamma distributions. In this context, the cumulant-generating function of the portfolio loss is the sum of cumulant-generating function from each of the underlying obligors (see equation 9 in Gordy, 2002). One of numerical results from the Monte Carlo experiments in Martin (2011) graphically demonstrates the extreme accuracy for a large portfolio (having 1000 underlying obligors).

- *Huang, Oosterlee, and van der Weide (2007)* applies SPA to another portfolio credit risk model characterized by a one-risk factor Vasicek Gaussian copula model. The portfolio loss given the setting of the model is a weighted sum of conditionally independent Bernoulli random variables. The authors use the same moment-generating function developed in Gordy (2002) except that the related probability is replaced by the one from Gaussian Vasicek model, and examine SPA performance with both small and large portfolios and with varied portfolio characteristics. Moreover, as compared to standard method of using Vasicek formula, SPA's performance is more robust.

- *Kim and Kim (2017)* studies one example of VaR analysis on a portfolio with underlying losses following generalized hyperbolic distributions and shows both SPA and Monte Carlo simulation yield high accurate VaR values (see Figure 1 in Kim and Kim (2017)). Another example is of a portfolio associated with a multivariate normal distribution; In this context, authors present the formula for VaR approximation (without numerical study). Rest examples extend the SPA techniques to risk measures such as VaR and expected shortfall sensitivity and to some well-known models; for example, to the delta-gamma model as in Feuerverger and Wong (2000).

These papers illustrate that SPA achieves high performance. Several obtain an analytic form of cumulant generating function related to the underlying model. This is critical to the applicability and the high quality of the approximation.. Since the cumulant-generating function is analytically tractable, the convergence of the approximation is fast; the approximation of tail loss distribution for small portfolios is insignificantly distorted. This has been shown in Martin (2011) with 100 underlying assets, albeit both Gordy (2002) and Martin (2011) suggest that the approximation for large portfolio is much reliable than small portfolio.

As well, the papers examine the conditions that support the application of SPA; namely, the existence of moments, the uniqueness of the real solution to saddle equation and the non-negativity of the second-order derivative of cumulant-generating function.

With regard to dependence and identical distribution of the underlying assets, these applications suggest that both are not big issues. Losses from underlying assets are indeed correlated when implementing some of the above models. It is most evident when modeling with Gaussian copula. The examination of performance in Martin (2011) with various correlations indicates that dependence is unlikely to be relevant. And in implementations of these models, the underlying assets (or risk factors) distributions are not necessarily identical. For example, in Huang et al. (2007) the relevant Bernoulli variates are simulated from distributions with different parameters.

3.1.3 SPA in Empirical Practice

In empirical applications, there is generally little or no information concerning the theoretical cumulant generating function. Our approach follows Wang (1992) who introduces a truncated version of the empirical cumulant generating function. The empirical analogue that we adopt uses the sample moments in applying this truncated version. This section presents the implementation of the procedure used in the remainder of the thesis. We first survey several of the papers that have used sample information to apply SPA methods. We then describe in detail the procedures adopted in the Monte Carlo study in the next section and in the VaR estimations in Chapter 4. The section ends with a continuation of Example 2 that investigates the impact of using these methods in determining the tail probabilities of the Gamma distribution.

- *Wang (1992)* uses the truncated version of the cumulant generating function in a bootstrap study to approximate the distribution of a test statistic. The performance of SPA compares favourably with other standard approximation approaches.
- *Davison and Hinkley (1988)* uses the non-truncated empirical cumulant-generating function and applies SPA also in Bootstrap to characterize the parent distribution. One of numerical experiments is based on a sample having ten observations and to approximate the distribution of the bias related to mean. SPA is compared to one of the methods from the approximation family (Normal Approximation) and SPA performs better.
- *Ronchetti and Welsh (1994)* uses an empirical cumulant-generating function.

In one example, the approximated density is compared to the true density of a sample mean, given 20 observations from a uniform distribution and the approximated left-tail distribution is compared to the simulated distribution. The results indicate that SPA works well.

- *Fasiolo, Wood, Hartig, Bravington, et al. (2018)* uses an empirical cumulant generating function and focuses on approximating the density of a sample distribution. One of the experiments shows good approximations for the density of two highly skewed distributions in the comparison with the true density.

- *Jiang, La Vecchia, Ronchetti, and Scaillet (2020)* uses the empirical cumulant-generating function related to the truncated one and approximate the density which is then used in ML estimation. One of the results, from sample sizes 24 and 100, shows that ML based on SPA performs well comparing with another one based on the assumption of normality.

The Application of Truncation in the Estimation of Business Risk VaR

In our applications of SPA, the following empirical approximation is used to characterize the cumulant generating function.

Let $\mathcal{M}_n(s)/\mathcal{C}_n(s)$ denote empirical moment/cumulant-generating function under empirical distribution $F_n(x)$, n is the size of a sample over which the averages are obtained. The truncated empirical cumulant-generating one use the empirical cumulants estimated from the sample, and it is written as

$$\mathcal{C}_n(s) \approx \kappa_{1n} s + \kappa_{2n} \frac{s^2}{2!} + \kappa_{3n} \frac{s^3}{3!} + \kappa_{4n} \frac{s^4}{4!}, \tag{3.6}$$

in which κ_{mn} is the m^{th} empirical cumulant and $m = 1, \ldots, 4$. Finding the saddlepoint $\hat{s}$ at x needs to solve the saddle equation

$$\kappa_{1n} + \kappa_{2n} s + \frac{1}{2}\kappa_{3n} s^2 + \frac{1}{6}\kappa_{4n} s^3 = x.$$

Empirical cumulants till $m = 4$ could be estimated from the sample and are calculated as

$$\kappa_{1n} = \bar{x}_n, \quad \kappa_{2n} = \mu_{2n},$$

$$\kappa_{3n} = \mu_{3n}, \quad \kappa_{4n} = \mu_{4n} - 3\mu_{2n}^2,$$

where

$$\mu_{mn} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x}_n)^m, \;\; m = 2, 3, 4,$$

and in which $\bar{x}_n$ is calculated as $\bar{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i$.

In the use of the truncated empirical cumulant-generating function, we solve a non-linear saddle equation with the help of the Matlab function fmincon, which may always give us a solution. But it could happen that this solution generates a negative value of $\mathcal{C}_n^{(2)}(\hat{s})$(i.e., $\kappa_{2n} + \kappa_{3n}\hat{s} + \frac{1}{2}\kappa_{4n}\hat{s}^2$) and causes the failure of the approximation formulas. This means that the real saddlepoint is unattainable.

To obtain the estimates of the VaR percentiles, we use the interpolation procedure suggested in Ronchetti and Welsh (1994) and Davison and Mastropietro (2009) that involves a grid search involving solutions of the saddlepoint equation to approximate the distribution. We then interpolate to the desired probability to determine the value of the percentile.

Note: One condition in applying the SPA approximation is the non-negativity of the second derivative of the cumulant generating function. In our Monte Carlo study, the negativity issue has never arisen. It should be mentioned here that Wang (1992) proposes to modify third and fourth order cumulants in the above representation to guarantee that $\mathcal{C}_n^{(2)}(\hat{s})$ positive. A similar idea is proposed in Fasiolo et al. (2018).

Truncation and the Gamma Distribution

We return to Example 2 to investigate (i) the impact of truncation and (ii) the use of sample moments in applying SPA to the Gamma distribution.

In Table 3.2, the known moments of the distribution are used in applying the truncated approximation to three or four moments. We see that the approximation works well and improves with the addition of the fourth moment.

Simulation methods involving 10000 replications of the Gamma distribution are used to assess the consequences of using sample moments in the truncation approximation. The results are presented in Table 3.3. Here we see the importance of including the fourth moment. The approximation of the point on the distribution corresponding to $p = 0.05$ is particularly sharp. We note that the approximation is not particularly affected by sample size.

The approximation results relating to the SPA approximation of tail probabilities for the Gamma distribution are collected in Table 3.4. The conclusion is that SPA performs quite well.

Other Issues

The concern of the existence of saddlepoint has been extensively discussed; for example, Daniels (1954) itself and Kolassa (2006). The existence condition is summarized in a proposition in Kolassa (2006); see theorem 4.3.2, which says that the saddlepoint won't exist when approximating at a value of x that is out of the span of X.

But in some context, the existence condition is not that difficult of being satisfied. For example, the credit risk model in Gordy (2002) assures the unique solution so long as the conditions under the model got satisfied, such as the elements in the matrix of asset weight, default probability, loss given default are non-negative. In our use of the truncated empirical cumulant-generating function, the existence of the solution to saddle equation is not a issue.

A final concern is with the use of normality in the approximation formula 3.5. which contains the leading term from an aymptotic expansion. Higher order terms could theoretically improve performance; yet the numerical study in Broda and Paolella (2012) indicates that higher order terms have no such capacity. Another related discussion is of the 'normal-base' in the approximation formulas. Booth and Wood (1995), Wood, Booth, and Butler (1993) and Zhang and Kuen Kwok (2020) suggest that the 'normal base', is responsible to some extent for the inaccuracy in the tail approximation, and suggest to replace the 'normal base' with a 'non-normal base'.

3.2 Simulation Study of the Relative Performance of the Saddlepoint Approach vs ML: Specification

The goal is to contrast the performance of the saddlepoint approach in the estimation of percentiles described in the previous section with a more standard time series approach. The estimated percentiles determined by either approach will be used in the following Chapter to compute the economic capital held by financial institutions against their exposure to business risk. The structure of the simulation study corresponds to the two sectoral approaches presented in the previous chapter. Here we begin with a description of the general framework for the study that is followed by a more explicit formulation of four specifications that are used to contrast the two estimation approaches.

3.2.1 Framework and Specifications

We introduce the notation BS_i to denote a particular replication designated by the index i of the business returns for a particular sector. The replication involves the specification of returns for K banks over a time horizon T; in a particular replication i, these are denoted $R^i_{k,t}$ for a specific bank k and time period t. In the US retail sector, we are studying $K = 18$ and the longest span $T = 22$; in the Canadian sector $K = 8$ and the the same longest span $T = 22$. In the simulations considered in this section, we fix T to be 30 and let K vary.

In our analysis, BS_i can be viewed in two ways: as K time series (BS^{TS}_i) of length T, say

$$BS^{TS}_i = \{R^i_{k,t} : t = 1, \ldots, T; k = 1, \ldots, K\},$$

or as a single series of pooled data (BS_i^P)

$$BS_i^P = \{R_{k,t}^i : t = 1, \ldots, T \text{ and } k = 1, \ldots, K\}.$$

The next step in describing a particular simulation is to characterize the return processes $R_{k,t}^i$ in a particular replication i. In what follows, we present four different approaches; the replication index i is suppressed to facilitate readability.

(I) *Case a* The returns for each financial institution k follow a MA(1) process given by a fixed parameter θ common to every bank with normal disturbances $\epsilon_{k,t}$ with mean 0 and variance .2 (denoted $\sigma^2 = .2$, corresponding to the scale of the empirical data):

$$R_{k,t} = \epsilon_{k,t} + \theta\epsilon_{k,t-1}, t = 1, \ldots 30.$$

For each k, the initial disturbance is zero; at $t = 1, R_{k,1} = \epsilon_{k,1}$. In this study $K =$ 5, 10 or 20 and $\theta = .6$.

Case b The disturbances are drawn from a Student-t distribution with degree of freedom (denoted ν) given by 10 or 6 and scaled by $\sigma\sqrt{\frac{\nu - 2}{\nu}}$.

(II) *Case a* In this specification, the moving average parameter varies from bank to bank; θ_k is given by a draw from $\mathcal{N}(0.6, 0.09)$ with mean .6 and variance .09 (corresponding broadly to the fitting of each bank's return series to MA(1) in Chapter 2):

$$R_{k,t} = \epsilon_{k,t} + \theta_k\epsilon_{k,t-1}, t = 1, \ldots 30.$$

The remaining parameters are specified as in Ia.

Case b Student-t disturbances as in Ib.

(III) *Case a* This specification adds cross-correlations to the MA(1) models in Case II, by integrating a one-factor approach to financial returns, see Hull (2012), page 244. $\epsilon_{k,t}$ is decomposed into a common disturbance across banks (denoted v_t) and an idiosyncratic one (denoted $w_{k,t}$). We have:

$$\epsilon_{k,t} = cv_t + w_{k,t}, t = 1, \dots 30,$$

where c is one-factor parameter which is fixed across banks.

We use an example to illustrate how the one-factor model is integrated into the specification of the K-dimensional correlated case, say, for $K = 2$, $(v_t \ w_{1,t} \ w_{2,t})' \overset{iid}{\sim} \mathcal{N}(0, \sigma^{*2} I_3)$.

$$\epsilon_{1,t} = cv_t + w_{1,t},$$

$$\epsilon_{2,t} = cv_t + w_{2,t},$$

$$\mathbb{V}ar(\epsilon_{1,t}) = \mathbb{V}ar(\epsilon_{2,t}) = (1 + c^2)\sigma^{*2}$$

$$\mathbb{C}ov(\epsilon_{1,t}, \epsilon_{2,t}) = \mathbb{E}(\epsilon_{1,t}\epsilon_{2,t}) = c^2\sigma^{*2},$$

$$\mathbb{C}orr(\epsilon_{1,t}, \epsilon_{2,t}) = \frac{\mathbb{C}ov(\epsilon_{1,t}, \epsilon_{2,t})}{\sqrt{\mathbb{V}ar(\epsilon_{1,t})\mathbb{V}ar(\epsilon_{2,t})}} = \frac{c^2}{1 + c^2}.$$

The variance-covariance matrix of ϵ_t is written as

$$\Sigma_{\epsilon_t} = \begin{bmatrix} (1 + c^2)\sigma^{*2} & c^2\sigma^{*2} \\ c^2\sigma^{*2} & (1 + c^2)\sigma^{*2} \end{bmatrix} = (1 + c^2)\sigma^{*2} \begin{bmatrix} 1 & \dfrac{c^2}{1 + c^2} \\ \dfrac{c^2}{1 + c^2} & 1 \end{bmatrix}.$$

Let $(1 + c^2)\sigma^{*2}$ equal to σ^2 and ρ equal to $\dfrac{c^2}{1 + c^2}$. ρ is presented not only in the 2 by

2 matrix also implicitly in σ^2. Given the true values of ρ and σ^2, σ^{*2} is calculated from $\sigma^2(1-\rho)$ and c is from $\sqrt{\dfrac{\rho}{1-\rho}}$.

Inter banks correlations (denoted ρ) are set are 0.2 corresponding to the empirical correlations estimated in Chapter 2, σ^{*2} is set at 0.16, σ^2 is remained at 0.2.

Case b v_t and $w_{k,t}$ are draws from a Student-t distribution with degree of freedom (denoted ν) given by 10 or 6 and scaled by $\sigma\sqrt{(1-\rho)\dfrac{\nu-2}{\nu}}$.

(IV) *Case a* This specification has auto-regressive process of order one (AR(1) that

$$R_{k,t} = \epsilon_{k,t} + \gamma R_{k,t-1}, t = 1, \ldots 30,$$

with initial value ($R_{k,0}$) equal to zero and $R_{k,1} = \epsilon_{k,1}$. The time-invariant variance of a AR(1) is the variance of $\epsilon_{k,t}$ times $\dfrac{1}{1-\gamma^2}$. Considering that we will make the comparison across the cases, the values of the parameters are set to remain the time-invariant variance of $R_{k,t}$ as σ^2 times $(1+\gamma^2)$ with σ^2 at 0.2 and γ at 0.6; the disturbances are draws from a distribution with variance σ^2 times $1-\gamma^4$. The rest parameters are specified as in Ia.

Case b Student-t disturbances are scaled by $\sigma\sqrt{(1-\gamma^4)\dfrac{\nu-2}{\nu}}$.

Notes to the specifications:

1. The time-invariant variance of the series $\{R_{k,t}\}$ asymptotically remains at $0.2(1+0.6^2)$ given the coefficient (θ, mean of θ_k distribution or γ) at 0.6 and σ^2 at 0.2 throughout the above four cases.

2. ρ is set at the value 0.2 in Case III. We conduct the simulation with varied ρ at

0.4 and 0.6. The results (not presented in the thesis) shows no impact of changing the value of ρ on the estimation of the percentiles.

3.2.2 Estimation of Percentiles: BM_p, TS_p and P_p

For each specification and parametrization, three sets of percentiles (corresponding to points on the estimated return distribution at $p = 0.01, 0.05$ or 0.10 tail probabilities) are determined. These percentiles are obtained (i) by simulation of the underlying data generating process (DGP); or by (ii) and (iii) by analysis of the simulated distribution of the percentile obtained via an estimation of the demeaned return of each sectoral replication either in the time series form BS_i^{TS} or in the pooled version BS_i^P. We denote these respectively by BM_p, TS_p and P_p ; again for readability we suppress any reference to the particular model specification. More formally, to obtain:

i- BM_p

Consider the return $R_{1,30}$ at the last point in the time series for Bank 1 in a replication. The simulations over all replications determine a distribution for this point. The benchmark percentiles BM_p are the appropriate points on this simulated distribution. Which is simulated in our experiments by 10000 replications. As these are associated with the true DGP, they are interpreted as the benchmark values of the percentiles and are used in the next section to assess the relative performance of the estimated percentiles those obtained from model estimation.

Note: the last point (i.e., the thirtieth) in the time series for the first bank is chosen simply to standardize the simulation procedure. In the simulated data sets, the banks are distinguished solely by the disturbances to the return process.

ii- TS_p

Consider the time series sectoral representation of the demeaned return distributions for K banks in replication i; this is denoted BS_i^{TS}. This sample is used to estimate a K-variate MA(1) model a common parameter θ and the variance of the normally distributed with mean 0 and variance σ^2. The likelihood is given by equation 3.7 as in the Appendix to this Chapter. The percentiles for the particular replication are then determined using:

$$TS_p^i = \hat{\sigma}\sqrt{1+\hat{\theta}^2}\Phi^{-1}(p),$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of cumulative probability function of $\mathcal{N}(0,1)$.

The Monte Carlo study in the next section simulates the distributions of these percentiles iterating over 5000 replications; these are denoted respectively as TS_p.

Note: The estimation in Case III takes into account the contemporaneous correlation across banks. Here the likelihood is estimated using equation 3.11.

iii- P_p

To obtain an estimate of this percentile, we consider the pooled sectoral representation in replication i - BS_i^P. Here we use the SPA estimation procedure presented in the previous sections to determine the estimated percentiles based on this particular sample using the following equations.

$$\hat{F}(R) = \Phi(\omega) + \phi(\omega)(\frac{1}{\omega} - \frac{1}{\lambda}),$$

where $\hat{F}(R)$ denotes the approximated distribution function, Φ and ϕ are respectively standard normal distribution function and density function, the parameters ω and λ are calculated using SPA techniques shown in equations 3.3, 3.4 and 3.5 and using the sample-based cumulant-generating function as equation. 3.6.

The required percentiles are not obtained directly but be interpolated (see Section 3.1.3). The Monte Carlo study in the next section simulates the distributions of these percentiles iterating over 5000 replications; these are denoted respectively as P_p.

3.3 Simulation Study of the Relative Performance of the Saddlepoint Approach vs ML: Results

The specific data generating processes (DGPs) that provide the material (Cases I-IV) for the Monte Carlo study of the two estimation procedures have been described in some detail in the previous section. The relative performance of the structural estimation approach versus the more general univariate approach is of independent interest. However, the ultimate objective remains the empirical analysis of the business risk exposure faced by the two banking sectors analyzed in Chapter Two and, as a consequence, the parameters of the different DGPs have been fixed to reflect features of these two sectors.

Accordingly, the time dimension of the business return series is short with a duration fixed at length 30. The number of banks is the more relevant parameter that varies in the study with the consequence that the sample size reflects the number of banks rather than the time horizon. We have demeaned the return data to permit inter bank and intertemporal comparisons, a feature that is incorporated in the estimation procedures. As well, we do not report any results concerning particular parameter estimates: the focus is entirely on the estimation of tail percentiles. In these respects, the study reflects our objectives.

The simulation results from the four Cases I-IV are presented in four Tables 3.5 - 3.8, respectively. Each table has three panels, from the top to the bottom, representing normal, t_{10} and t_6 distributions that the disturbances are drawn from. In each panel, the number K increases from five to ten and 20, from the left-hand side to the right-hand side of the table.

Case I

Table 3.5 presents the results for Case I with fixed θ across K dimensions and zero cross-correlation. The parameter θ in ML estimation using the equation 3.8 reflects Case I, i.e., the MA(1) process is common across K dimensions.

Across panels, we see that at $p = 0.1$ and $p = 0.05$ both ML estimation (represented by TS_p) and SPA (represented by P_p) perform extremely well relatively to the benchmark, regardless of sample size and the disturbance distributions. For example, at $p = 0.05$ and $K = 5$, TS_p is -.85 and P_p is -.86, compared with BM_p -.85. Both estimates also have similar short ranges; one is from -.93 to -.76 and the other from -.95 to -.76. We also see that as K increases to 20, where disturbances are drawn from a normal distribution, the inter-quartile range of P_p is narrower than TS_p, for example, at $p = 0.1$ (-.70:-.64) versus(-.73:-.59) and at $p = 0.05$ (-.94:-.84) versus(-.94:-.76).

We see a difference in performance when $p = 0.01$. From Panel A to Panel C with the thickness of the tail distribution increasing, we see the benchmark values increasing while the values of $TS_{0.01}$ barely change. At this point, the SPA method appears more robust and performs better; for example, in Panel C when $K = 5$, SPA yields $P_{0.01}$ -1.29 in a range from -1.44 to -1.07, compared to the benchmark -1.28; when $K = 20$ from the larger samples yields -1.37 in a shorter range between -1.48 and -1.30. It is noticeable that the sample size affects the performance of SPA more than that of ML estimation. We also observe SPA tends to overestimate the true value while ML underestimate.

Case II

The results of Case II are presented in Table 3.6. The specification in this case deviates from Case I, with θ_k varied across K dimensions but centered at the value of the fixed θ in Case I. We see that Table 3.6 is virtually identical to Table 3.5. Our observations regarding Table 3.6 also hold for Table 3.5.

Case III

Table 3.7 presents the results of Case III that has added cross-correlation equal to 0.2 to the specifications in Case II. The results for ML estimation with zero correlation are denoted TS_p (see equation 3.8 in the Appendix); ML results for specification with non-zero correlation are denoted TS_p^* (see equation 3.11 in the Appendix). We see the results indicate that at $K = 5$, there is small difference between TS_p and TS_p^*. But the results are similar for $K = 10$ and $K = 20$.

The performance of SPA is satisfactory in small samples ($K = 5$) and in large samples when the disturbance are normal and $p = 0.10$ and $p = 0.05$. When the disturbances are non-normal SPA tends to overestimate for all sample sizes.

Case IV

Table 3.8 shows the results from Case IV, different in DGP involving AR(1) process from Case I. We know that an AR(1) can be transformed into a MA process by iteration but containing higher-order terms. The transformed process can be written as $R_t = \epsilon_t + \gamma\epsilon_{t-1} + \gamma^2\epsilon_{t-2} + \gamma^3\epsilon_{t-3} + \ldots$, with $|\gamma| < 1$ and a smaller γ linked to fast diminishing higher-order terms while the larger γ to the sticky ones. In Case IV γ is set at 0.6.

Under this DGP, the performance of ML estimation deteriorates even under normality. TS_p considerably underestimates benchmark for all sample sizes.

Under normality, SPA performs well for large sample sizes (in smaller sample size it tends to underestimate). For the disturbances are fat-tailed, SPA performs well for $p = 0.1$ and $p = 0.05$. With mixed results with other cases.

The above results can be summarized:

- Both SPA and ML perform well when $p = 0.1$ and $p = 0.05$. Under normality, when $p = 0.01$, SPA tends to overestimate while ML underestimates.

- With t disturbances in Cases I, II and III, when $p = 0.1$ and $p = 0.05$ the ML estimates and SPA estimates perform equally well.

- In the far tail, ML under estimates and SPA overestimates.

- When the model is mis-specified as in Case IV, SPA outperforms ML considerably when $p = 0.1$ and $p = 0.05$.

Table 3.1 Theoretical SPA Approximates Tail Probabilities of Gamma Distribution

$\mathbb{P}(X < x_p)$	x_p	**SPA**	$\mathbb{P}(X > x_p)$	x_p	**SPA**
0.20	0.0321	0.2132	0.20	0.82	0.1977
0.10	0.0079	0.1094	0.10	1.35	0.0997
0.05	0.0020	0.0557	0.05	1.92	0.0502
0.01	0.0000785	0.0114	0.01	3.32	0.0101

The table is an elaboration of an example from Butler (2007) with $\alpha = 0.5$ and $\beta = 1$, using cumulant-generating function based on equation 3.2. The probabilities are calculated from equation 3.5. For the indicated probability, x_p is the value on the Gamma distribution; SPA approximates the probability for this specific point.

Table 3.2 Using Truncated Cumulant-generating Function in Approximating Tail Probabilities of Gamma Distribution

$\mathbb{P}(X > x_p)$	x_p	**Three moments**	**Four moments**
0.10	1.35	0.1423	0.1352
0.05	1.92	0.0675	0.0606
0.01	3.32	0.0042	0.0092

The table is an elaboration of an example from Butler (2007) with $\alpha = 0.5$ and $\beta = 1$, using the truncated cumulant-generating function. The approximated probabilities are calculated from equation 3.5 and use the known theoretical moments. See the notes in Table 3.1.

Table 3.3 Using Truncated Empirical Cumulant-generating Function in Approximating Tail Probabilities of Gamma Distribution

$\mathbb{P}(X > x_p)$	x_p	**Three moments**			**Four moments**		
		$\boldsymbol{n}$**=150**	$\boldsymbol{n}$**=300**	$\boldsymbol{n}$**=600**	$\boldsymbol{n}$**=150**	$\boldsymbol{n}$**=300**	$\boldsymbol{n}$**=600**
0.10	1.35	0.1063 (0.13:0.08)	0.1086 (0.13:0.09)	0.1161 (0.13:0.10)	0.1252 (0.15:0.10)	0.1275 (0.14:0.11)	0.1288 (0.14:0.12)
0.05	1.92	0.0246 (0.05:0.02)	0.0386 (0.10:0.02)	0.0915 (0.10:0.08)	0.0533 (0.07:0.04)	0.0582 (0.07:0.05)	0.0602 (0.07:0.05)
0.01	3.32	0.0122 (0.02:0.01)	0.0124 (0.02:0.01)	0.0125 (0.02:0.01)	0.0046 (0.01:0.00)	0.0060 (0.01:0.00)	0.0067 (0.01:0.00)

The table is an elaboration of an example from Butler (2007) with $\alpha = 0.5$ and $\beta = 1$, using the cumulant-generating function based on equation 3.6. The approximated probabilities are calculated from equation 3.5 and based on samples.
The value of the approximated probability is the median of 10000 replications; these are given along with 25% and 75% percentile from 10000 replications.

Table 3.4 Summary of SPA Approximating Tail Probabilities of Gamma Distribution

			Truncated		**Empirical & Truncated**					
$\mathbb{P}(X > x_p)$	x_p	**Theoretical**			**Three moments**			**Four moments**		
			Three moments	**Four moments**	n**=150**	n**=300**	n**=600**	n**=150**	n**=300**	n**=600**
0.10	1.35	0.0997	0.1423	0.1352	0.1063	0.1086	0.1161	0.1252	0.1275	0.1288
0.05	1.92	0.0502	0.0606	0.0675	0.0246	0.0386	0.0915	0.0533	0.0582	0.0602
0.01	3.32	0.0101	0.0042	0.0092	0.0122	0.0124	0.0125	0.0046	0.0060	0.0067

The table summarizes Table 3.1, Table 3.2 and Table3.3

Table 3.5 SPA versus ML Estimating Percentiles: Specification I

Panel A: $\epsilon_{k,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$

		K=5		K=10		K=20	
p	**BM**$_p$	**TS**$_p$	**P**$_p$	**TS**$_p$	**P**$_p$	**TS**$_p$	**P**$_p$
0.10	-0.67	-0.66 (-0.73:-0.60)	-0.66 (-0.72:-0.60)	-0.66 (-0.73:-0.60)	-0.67 (-0.71:-0.63)	-0.66 (-0.73:-0.59)	-0.67 (-0.70:-0.64)
0.05	-0.86	-0.85 (-0.93:-0.76)	-0.86 (-0.95:-0.76)	-0.85 (-0.93:-0.76)	-0.88 (-0.95:-0.81)	-0.85 (-0.94:-0.76)	-0.88 (-0.94:-0.84)
0.01	-1.19	-1.20 (-1.32:-1.08)	-1.08 (-1.29:-0.83)	-1.20 (-1.32:-1.08)	-1.16 (-1.35:-0.96)	-1.20 (-1.32:-1.08)	-1.23 (-1.44:-1.05)

Panel B: $\epsilon_{k,t} \overset{iid}{\sim} t_{10}$

		K=5		K=10		K=20	
p	**BM**$_p$	**TS**$_p$	**P**$_p$	**TS**$_p$	**P**$_p$	**TS**$_p$	**P**$_p$
0.10	-0.64	-0.65 (-0.73:-0.58)	-0.66 (-0.72:-0.60)	-0.65 (-0.73:-0.58)	-0.66 (-0.70:-0.61)	-0.65 (-0.73:-0.58)	-0.66 (-0.69:-0.63)
0.05	-0.84	-0.84 (-0.94:-0.75)	-0.87 (-0.96:-0.79)	-0.84 (-0.94:-0.75)	-0.87 (-0.93:-0.81)	-0.83 (-0.93:-0.75)	-0.86 (-0.90:-0.82)
0.01	-1.26	-1.18 (-1.33:-1.06)	-1.23 (-1.39:-0.99)	-1.19 (-1.33:-1.06)	-1.32 (-1.43:-1.16)	-1.18 (-1.32:-1.06)	-1.35 (-1.48:-1.26)

Panel C: $\epsilon_{k,t} \overset{iid}{\sim} t_{6}$

		K=5		K=10		K=20	
p	**BM**$_p$	**TS**$_p$	**P**$_p$	**TS**$_p$	**P**$_p$	**TS**$_p$	**P**$_p$
0.10	-0.62	-0.64 (-0.73:-0.57)	-0.65 (-0.71:-0.58)	-0.64 (-0.73:-0.57)	-0.65 (-0.70:-0.60)	-0.64 (-0.73:-0.57)	-0.65 (-0.69:-0.62)
0.05	-0.82	-0.82 (-0.94:-0.73)	-0.86 (-0.96:-0.78)	-0.83 (-0.94:-0.73)	-0.87 (-0.94:-0.81)	-0.83 (-0.94:-0.73)	-0.87 (-0.92:-0.82)
0.01	-1.28	-1.16 (-1.33:-1.03)	-1.29 (-1.44:-1.07)	-1.17 (-1.33:-1.03)	-1.35 (-1.47:-1.24)	-1.17 (-1.33:-1.03)	-1.37 (-1.48:-1.30)

DGP corresponds to specification I described in text Section 3.2; Panel A corresponds to Case Ia; Panels B & C correspond to Case Ib.

The benchmark values of the percentiles (BM_p) are estimated by the medians of 10000 replications; The values of the percentiles TS_p (P_p) are the medians of 5000 estimated values; these are given 25% and 75% percentiles of the distribution from 5000 estimated values.

Table 3.6 SPA versus ML Estimating Percentiles: Specification II

Panel A: $\epsilon_{k,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$

		K=5		K=10		K=20	
p	$\mathbf{BM}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$
0.10	-0.67	-0.66 (-0.76:-0.59)	-0.68 (-0.75:-0.62)	-0.66 (-0.75:-0.59)	-0.69 (-0.73:-0.64)	-0.66 (-0.76:-0.59)	-0.68 (-0.71:-0.65)
0.05	-0.86	-0.85 (-0.97:-0.76)	-0.89 (-0.99:-0.79)	-0.85 (-0.96:-0.75)	-0.90 (-0.97:-0.84)	-0.85 (-0.97:-0.76)	-0.89 (-0.94:-0.85)
0.01	-1.19	-1.21 (-1.37:-1.07)	-1.18 (-1.38:-0.91)	-1.20 (-1.36:-1.06)	-1.30 (-1.44:-1.08)	-1.20 (-1.38:-1.07)	-1.36 (-1.50:-1.18)

Panel B: $\epsilon_{k,t} \overset{iid}{\sim} t_{10}$

		K=5		K=10		K=20	
p	$\mathbf{BM}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$
0.10	-0.64	-0.66 (-0.76:-0.58)	-0.67 (-0.74:-0.6)	-0.66 (-0.76:-0.57)	-0.67 (-0.72:-0.63)	-0.66 (-0.76:-0.58)	-0.68 (-0.71:-0.64)
0.05	-0.84	-0.85 (-0.97:-0.74)	-0.89 (-0.99:-0.81)	-0.85 (-0.97:-0.74)	-0.89 (-0.96:-0.83)	-0.85 (-0.97:-0.74)	-0.89 (-0.94:-0.85)
0.01	-1.26	-1.20 (-1.37:-1.05)	-1.29 (-1.45:-1.06)	-1.20 (-1.37:-1.04)	-1.36 (-1.47:-1.24)	-1.20 (-1.38:-1.05)	-1.39 (-1.48:-1.31)

Panel C: $\epsilon_{k,t} \overset{iid}{\sim} t_{6}$

		K=5		K=10		K=20	
p	$\mathbf{BM}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$
0.10	-0.62	-0.65 (-0.76:-0.56)	-0.66 (-0.73:-0.60)	-0.65 (-0.75:-0.56)	-0.66 (-0.72:-0.62)	-0.66 (-0.75:-0.56)	-0.67 (-0.71:-0.63)
0.05	-0.82	-0.83 (-0.97:-0.72)	-0.89 (-0.99:-0.80)	-0.83 (-0.96:-0.72)	-0.89 (-0.97:-0.83)	-0.84 (-0.97:-0.72)	-0.90 (-0.96:-0.85)
0.01	-1.28	-1.18 (-1.37:-1.02)	-1.34 (-1.49:-1.14)	-1.18 (-1.36:-1.02)	-1.39 (-1.51:-1.28)	-1.19 (-1.37:-1.03)	-1.40 (-1.51:-1.32)

DGP corresponds to specification II described in text Section 3.2; Panel A corresponds to Case IIa; Panels B & C correspond to Case IIb.

The benchmark values of the percentiles (BM_p) are estimated by the medians of 10000 replications; The values of the percentiles TS_p (P_p) are the medians of 5000 estimated values; these are given 25% and 75% percentiles of the distribution from 5000 estimated values.

Table 3.7 SPA versus ML Estimating Percentiles: Specification III

		K=5			K=10			K=20		
Panel A: $v_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2(1-\rho))$, $w_{k,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2(1-\rho)$										
$\boldsymbol{p}$	**BM**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$
0.10	-0.67	-0.68 (-0.73:-0.64)	-0.67 (-0.72:-0.63)	-0.67 (-0.76:-0.60)	-0.69 (-0.72:-0.66)	-0.69 (-0.72:-0.66)	-0.68 (-0.74:-0.62)	-0.69 (-0.71:-0.66)	-0.69 (-0.71:-0.66)	-0.68 (-0.73:-0.63)
0.05	-0.86	-0.88 (-0.94:-0.82)	-0.86 (-0.92:-0.81)	-0.88 (-0.99:-0.77)	-0.88 (-0.92:-0.84)	-0.88 (-0.92:-0.84)	-0.90 (-0.99:-0.83)	-0.88 (-0.91:-0.85)	-0.88 (-0.91:-0.85)	-0.89 (-0.96:-0.83)
0.01	-1.19	-1.24 (-1.32:-1.17)	-1.22 (-1.30:-1.15)	-1.15 (-1.38:-0.89)	-1.25 (-1.31:-1.19)	-1.25 (-1.31:-1.19)	-1.27 (-1.43:-1.05)	-1.25 (-1.29:-1.20)	-1.25 (-1.29:-1.20)	-1.35 (-1.50:-1.15)
Panel B: $v_t \overset{iid}{\sim} t_{10}$, $w_{k,t} \overset{iid}{\sim} t_{10}$										
		K=5			K=10			K=20		
$\boldsymbol{p}$	**BM**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$
0.10	-0.62	-0.68 (-0.73:-0.63)	-0.67 (-0.72:-0.62)	-0.66 (-0.75:-0.59)	-0.68 (-0.72:-0.65)	-0.68 (-0.72:-0.65)	-0.66 (-0.73:-0.60)	-0.69 (-0.72:-0.66)	-0.68 (-0.71:-0.66)	-0.67 (-0.73:-0.61)
0.05	-0.84	-0.87 (-0.94:-0.81)	-0.86 (-0.93:-0.80)	-0.89 (-1.00:-0.79)	-0.88 (-0.93:-0.83)	-0.88 (-0.93:-0.83)	-0.89 (-0.97:-0.81)	-0.88 (-0.92:-0.84)	-0.88 (-0.92:-0.84)	-0.89 (-0.96:-0.83)
0.01	-1.26	-1.24 (-1.33:-1.15)	-1.22 (-1.31:-1.13)	-1.31 (-1.47:-1.09)	-1.24 (-1.31:-1.17)	-1.24 (-1.31:-1.17)	-1.37 (-1.50:-1.24)	-1.24 (-1.30:-1.19)	-1.24 (-1.30:-1.19)	-1.40 (-1.51:-1.30)
Panel C: $v_t \overset{iid}{\sim} t_6$, $w_{k,t} \overset{iid}{\sim} t_6$										
		K=5			K=10			K=20		
$\boldsymbol{p}$	**BM**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$	**TS**$_p$	**TS**$_p$*	**P**$_p$
0.10	-0.64	-0.66 (-0.70:-0.62)	-0.65 (-0.69:-0.61)	-0.65 (-0.73:-0.58)	-0.66 (-0.70:-0.63)	-0.66 (-0.70:-0.63)	-0.62 (-0.71:-0.59)	-0.66 (-0.69:-0.64)	-0.66 (-0.69:-0.64)	-0.65 (-0.70:-0.60)
0.05	-0.82	-0.85 (-0.90:-0.80)	-0.84 (-0.89:-0.79)	-0.87 (-0.97:-0.78)	-0.85 (-0.89:-0.81)	-0.85 (-0.89:-0.81)	-0.86 (-0.94:-0.79)	-0.85 (-0.89:-0.82)	-0.85 (-0.89:-0.82)	-0.86 (-0.93:-0.80)
0.01	-1.28	-1.20 (-1.28:-1.13)	-1.18 (-1.26:-1.11)	-1.25 (-1.42:-1.03)	-1.21 (-1.26:-1.15)	-1.21 (-1.26:-1.15)	-1.33 (-1.46:-1.17)	-1.21 (-1.25:-1.16)	-1.21 (-1.25:-1.16)	-1.37 (-1.49:-1.26)

DGP corresponds to specification III described in text Section 3.2; Panel A corresponds to Case IIIa; Panels B & C correspond to Case IIIb.
The benchmark values of the percentiles (BM_p) are estimated by the medians of 10000 replications; The values of the percentiles TS_p (TS_p^*, P_p) are the medians of 5000 estimated values; these are given 25% and 75% percentiles of the distribution from 5000 estimated values.

Table 3.8 SPA versus ML Estimating Percentiles: Specification IV

Panel A: $\epsilon_{k,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2(1-\gamma^4))$

		K=5		K=10		K=20	
p	$\mathbf{BM}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$
0.10	-0.66	-0.62 (-0.69:-0.55)	-0.66 (-0.74:-0.59)	-0.61 (-0.68:-0.55)	-0.66 (-0.72:-0.61)	-0.62 (-0.69:-0.55)	-0.66 (-0.70:-0.63)
0.05	-0.87	-0.79 (-0.88:-0.71)	-0.85 (-0.96:-0.72)	-0.79 (-0.88:-0.70)	-0.88 (-0.97:-0.79)	-0.79 (-0.88:-0.71)	-0.88 (-0.96:-0.83)
0.01	-1.21	-1.12 (-1.25:-1.00)	-1.02 (-1.27:-0.77)	-1.11 (-1.24:-0.99)	-1.13 (-1.34:-0.92)	-1.12 (-1.25:-1.00)	-1.22 (-1.43:-1.02)

Panel B: $\epsilon_{k,t} \overset{iid}{\sim} t_{10}$

		K=5		K=10		K=20	
p	$\mathbf{BM}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$
0.10	-0.66	-0.61 (-0.69:-0.54)	-0.65 (-0.73:-0.58)	-0.61 (-0.69:-0.54)	-0.65 (-0.70:-0.60)	-0.61 (-0.69:-0.54)	-0.65 (-0.69:-0.61)
0.05	-0.86	-0.78 (-0.88:-0.70)	-0.86 (-0.97:-0.76)	-0.78 (-0.88:-0.70)	-0.86 (-0.94:-0.80)	-0.79 (-0.89:-0.69)	-0.86 (-0.91:-0.81)
0.01	-1.25	-1.11 (-1.25:-0.98)	-1.16 (-1.37:-0.91)	-1.11 (-1.25:-0.98)	-1.28 (-1.42:-1.07)	-1.11 (-1.25:-0.98)	-1.34 (-1.47:-1.21)

Panel C: $\epsilon_{k,t} \overset{iid}{\sim} t_{6}$

		K=5		K=10		K=20	
p	$\mathbf{BM}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$	$\mathbf{TS}_p$	$\mathbf{P}_p$
0.10	-0.63	-0.60 (-0.68:-0.52)	-0.64 (-0.73:-0.57)	-0.60 (-0.69:-0.53)	-0.64 (-0.70:-0.59)	-0.60 (-0.69:-0.53)	-0.65 (-0.69:-0.61)
0.05	-0.83	-0.77 (-0.88:-0.67)	-0.86 (-0.97:-0.76)	-0.77 (-0.88:-0.67)	-0.86 (-0.94:-0.80)	-0.77 (-0.88:-0.68)	-0.86 (-0.92:-0.81)
0.01	-1.27	-1.09 (-1.24:-0.95)	-1.24 (-1.42:-0.99)	-1.09 (-1.25:-0.95)	-1.33 (-1.45:-1.18)	-1.09 (-1.25:-0.96)	-1.36 (-1.47:-1.27)

DGP corresponds to specification IV described in text Section 3.2; Panel A corresponds to Case IVa; Panels B & C correspond to Case IVb.

The benchmark values of the percentiles (BM_p) are estimated by the medians of 10000 replications; The values of the percentiles TS_p (P_p) are the medians of 5000 estimated values; these are given 25% and 75% percentiles of the distribution from 5000 estimated values.

3.A Maximum Likelihood (ML) Estimation

A multivariate Gaussian ML estimation procedure is applied to the different data generating processes considered in the Chapter to obtain estimates of the usual VaR percentiles. The T-period losses of the K banks are assumed to have zero mean and share a common MA(1) parameter with normal disturbances having a fixed across the banks. In this context, it is straightforward to estimate the VaR percentiles from the model's estimated parameters. Two cases are considered corresponding to whether the disturbances are correlated between the banks in the sector. The presentation that follows relies on Tsay (2013).

3.A.1 MA(1) Estimation: No Cross Bank Correlation

We are considering a zero-mean K-dimensional MA(1). Among K-dimensions, a univariate time series, say series k, $\{x_{k,t}\}$ is written as

$$x_{k,t} = \epsilon_{k,t} + \theta\epsilon_{k,t-1}, \quad \epsilon_{k,0} = 0, \quad t = 1, \ldots, T, k = 1, \ldots, K, (*)$$

where $\{\epsilon_{k,t}\}$ is a series of disturbance and $\epsilon_{k,t}$ is independently and identically distributed with a scale parameter σ (denoted $\epsilon_{k,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$), $\epsilon_{k,0}$ is the initial disturbance, θ is the MA coefficient and and is required to be between -1 and 1 for a stationary and invertible MA process. $x_{k,1}, x_{k,2}, \ldots, x_{k,T}$ has a variance-covariance matrix $\sigma^2\Gamma$, where Γ is

$$\Gamma = \begin{bmatrix} 1 & \theta & 0 & \dots & 0 \\ \theta & 1+\theta^2 & \theta & & \vdots \\ 0 & \theta & 1+\theta^2 & & \vdots \\ \vdots & & & \ddots & \theta \\ 0 & \dots & \dots & \theta & 1+\theta^2 \end{bmatrix}_{T\times T}$$

Let $\epsilon_k = (\epsilon_{k,1}, \epsilon_{k,2}, \dots, \epsilon_{k,T})'$, $X_k = (x_{k,1}, x_{k,2}, \dots, x_{k,T})'$, $\epsilon_k = AX_k$, where $(\cdot)'$ denotes the transpose, A is a lower triangle matrix T by T such that its non-zero element A^{ij} equals to $(-\theta)^{i-j}, j = 1, \dots, i, i = 1, \dots, T$, and $A'A$ equals to the inverse of Γ,

$$A = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ -\theta & 1 & & 0 & 0 \\ (-\theta)^2 & -\theta & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ (-\theta)^{T-1} & (-\theta)^{T-2} & \dots & -\theta & 1 \end{bmatrix}_{T\times T.}$$

With this structure and the knowledge of the disturbance distribution, the estimation method from maximum likelihood family is a commonly used. Given a probability function of the true distribution, MLE has desirable properties: the estimates are consistent and asymptotically efficient and asymptotically normally distributed.

The density function of a single series is defined to be

$$(2\pi\sigma^2)^{-\frac{T}{2}}(det(X_k'X_k))^{-\frac{1}{2}} \exp\Big(-\frac{(AX_k)'(AX_k)}{2\sigma^2}\Big).$$

In this simple case of no correlation across series, the joint density can be written as the product of K series's,

$$(2\pi\sigma^2)^{-\frac{KT}{2}} \prod_{k=1}^{K} (det(X_k'X_k))^{-\frac{1}{2}} \exp\Big(-\frac{(AX_k)'(AX_k)}{2\sigma^2}\Big), \tag{3.7}$$

which is proportional to

$$\propto (\sigma^2)^{-\frac{KT}{2}} \prod_{k=1}^{K} \exp\Big(-\frac{(AX_k)'(AX_k)}{2\sigma^2}\Big).$$

MLE then solves

$$\min_{\theta,\sigma^2} \frac{KT}{2} log(\sigma^2) + \sum_{k=1}^{K} \frac{(AX_k)'(AX_k)}{2\sigma^2}. \tag{3.8}$$

Once the estimates of θ and σ^2 obtained, the specified common univariate MA(1) process is

$$x_t^* = \epsilon_t^* + \hat{\theta}\epsilon_{t-1}^*, \epsilon_t^* \overset{iid}{\sim} \mathcal{N}(0, \hat{\sigma}^2) \tag{3.9}$$

x_t^* in fact is the sum of two normal variate, which in turn is also a normal variate and centered at zero. We may directly use the time-invariant variance $\hat{\sigma}^2(1+\hat{\theta}^2)$ to calculate the percentile; see section 2.5.1 in Tsay (2005). Accordingly, the percentile at a probability (denoted p) is calculated as

$$\hat{\sigma}\sqrt{1+\hat{\theta}^2}\,\Phi^{-1}(p), \tag{3.10}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of cumulative probability function of $\mathcal{N}(0,1)$.

Note: In the estimation procedure used in the empirical applications in Chapter 4, the beginning point may vary across the banks (different in T); in which case the A matrix is adjusted accordingly.

3.A.2 MA(1) Estimation: Fixed Cross Bank Correlation

In this specification considered in the previous section,Suppose that the K-dimensional disturbances $\epsilon_t = (\epsilon_{1,t}, \ldots, \epsilon_{K,t})'$ for $t = 1, \ldots, T$ and the disturbances are distributed as $\epsilon_t \sim \mathcal{N}(0, \Sigma_\epsilon)$,

$$\Sigma_\epsilon = \sigma^2 \begin{bmatrix} 1 & \rho & \rho & \ldots & \rho \\ \rho & 1 & \rho & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \rho \\ \rho & \ldots & \rho & \rho & 1 \end{bmatrix}_{K \times K.}$$

Let $\{x_t\}$ be a zero-mean K-dimensional MA(1) process; $x_t = (x_{1,t}, \ldots, x_{K,t})'$ for $t = 1, \ldots, T$, with $\epsilon_0 = 0_K$.

$$x_t = \epsilon_t + B\epsilon_{t-1};\ B = \theta I_K;$$

$$x_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{K,t} \end{bmatrix}_{K \times 1} = \underbrace{\begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & 1 \end{bmatrix}_{K \times K}}_{I_K} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \vdots \\ \epsilon_{K,t} \end{bmatrix}_{K \times 1} + \underbrace{\begin{bmatrix} \theta & 0 & \ldots & 0 \\ 0 & \theta & \ldots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & \theta \end{bmatrix}_{K \times K}}_{B} \begin{bmatrix} \epsilon_{1,t-1} \\ \epsilon_{2,t-1} \\ \vdots \\ \epsilon_{K,t-1} \end{bmatrix}_{K \times 1.}$$

x_t has the variance-covariance matrix,

$$\Sigma_\epsilon + \theta^2 \Sigma_\epsilon.$$

Let $X = (x_1, x_2, \ldots, x_T)' = (x_{1,1}, \ldots, x_{K,1}, x_{1,2}, \ldots, x_{K,2}, \ldots, x_{1,T}, \ldots, x_{K,T})'$;

$$X = \bar{B}\epsilon;$$

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_T \end{bmatrix}_{KT\times 1}}_{X} = \underbrace{\begin{bmatrix} I_K & 0_K & \dots & 0_K \\ B & I_K & \dots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0_K & \dots & B & I_K \end{bmatrix}_{KT\times KT}}_{\bar{B}} \underbrace{\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_T \end{bmatrix}_{KT\times 1,}}_{\epsilon}$$

where $\epsilon = (\epsilon_1, \epsilon_2, \dots, \epsilon_T)' = (\epsilon_{1,1}, \dots, \epsilon_{K,1}, \epsilon_{1,2}, \dots, \epsilon_{K,2}, \dots, \epsilon_{1,T}, \dots, \epsilon_{K,T})'$, $\epsilon \sim \mathcal{N}(0, I_T \otimes \Sigma_\epsilon)$, $\otimes$ denotes Kronecker multiplication.

Assuming $\epsilon_t \sim \mathcal{N}(0, \Sigma_\epsilon)$ yields a likelihood proportional to

$$\propto det(\bar{B}(I_T \otimes \Sigma_\epsilon)\bar{B}')^{-\frac{1}{2}} \exp\Big(-\frac{1}{2} X'(\bar{B}(I_T \otimes \Sigma_\epsilon)\bar{B}')^{-1} X\Big);$$

see equation 3.61 in Tsay (2013).

Given the fact that $det(\bar{B}) = 1$, the above function may be written as

$$\propto det(\Sigma_\epsilon)^{-\frac{T}{2}} \exp\Big(-\frac{1}{2} X' \bar{B}'^{-1} (I_T \otimes \Sigma_\epsilon^{-1}) \bar{B}^{-1} X\Big),$$

and

$$\bar{B}^{-1} = \begin{bmatrix} I_K & 0_K & \dots & 0_K & 0_K \\ -B & I_K & & 0_K & 0_K \\ (-B)^2 & -B & \ddots & 0_K & 0_K \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ (-B)^{T-1} & (-B)^{T-2} & \dots & -B & I_K \end{bmatrix}_{KT\times KT}$$

MLE solves

$$\min_{\theta, \sigma^2, \rho} \frac{T}{2} log(det(\Sigma_\epsilon)) + \frac{1}{2} X' \bar{B}'^{-1} (I_T \otimes \Sigma_\epsilon^{-1}) \bar{B}^{-1} X. \tag{3.11}$$

Each bank has the time invariant variance given by $\sigma^2(1+\theta^2)$. We can use the same percentile function as in the previous section:

$$\hat{\sigma}\sqrt{1+\hat{\theta}^2}\,\Phi^{-1}(p).$$

Chapter 4

Estimation of ECAP for Business Risk in Two Banking Sectors

The estimation procedures presented in the previous Chapter are applied to the two business loss data sets constructed in Chapter 2 to obtain sectoral VaR measures for different confidence levels. These are then applied to the different banks in each sector. As this approach to a formal calculation of capital that should be held by a bank against potential extreme fluctuations in business income is entirely new, no comparisons can be drawn with previous work. We do offer some comparisons with several bank's holding of what is termed buffer capital. The second section of this Chapter offers an overview of the contributions of the thesis.

4.1 Economic Capital (ECAP) for Business Income Risk Based on VaR Sectoral Estimation

The business loss data set for each sector comprises annual business income returns for each bank that has been characterized as:

$$\frac{\mathrm{RPL}_t^k}{\mathrm{IEA}_t^k} - \mathbb{E}\Big(\frac{\mathrm{RPL}^k}{\mathrm{IEA}^k}\Big), \tag{4.1}$$

where RPL_t^k is the RPL (*Residual Profit & Loss*) of bank k in a period t, IEA_t^k is the bank's corresponding IEA (average *Interest Earning Assets*).

The VaR estimates for structural representation of this data using data from 1994 to

2015 are obtained via ML estimates of the parameters θ, σ^2 (and where cross correlation is assumed ρ) using equation 3.10. These are in percentage loss returns and correspond to a VaR estimate for the entire sector. To obtain a dollar ECAP figure for a particular bank in a particular year, we simply multiply this loss estimate by the Interest Earning Assets for the bank for that year.

The VaR estimation results for income fluctuations in the US retail banking sector (respectively, the Canadian sector results) are given in Table 4.1 (Table 4.2). Based on these estimates, the ECAP figures for 2015 are given in Table 4.1 (Table 4.2) for the different banks in the sector.

We first observe that the multivariate structural estimates are in keeping with the informal data analysis that was presented at the end of Chapter 2. The estimate of the common MA(1) parameter is lower for the Canadian sector parameter than for the US sector. The contemporaneous correlation parameter is small in both sectors. We also observe that the percentile estimates are higher using the SPA approach for the pooled data than for the structural approach: the VaR 95% percentile is 43% higher for the Canadian sector and 28% higher for the US sector.

The specific ECAP numbers for each bank in their respective sectors can be found in Table 4.1 and Table 4.1. At this point it is disappointing to indicate that the results cannot be readily compared against some other benchmark values, since the formal analysis presented in the thesis is entirely novel. But we can offer the following simple comparisons of the ECAP numbers for four banks in the Canadian sector for the year 2015 (at 95% VaR) with numbers published by the bank for that year under the heading *buffer capital* (C$m):

Royal Bank	6592-ECAP	14350 -buffer
Bank of Nova Scotia	4417-ECAP	12652-buffer
Banque Nationale	1482-ECAP	984-buffer
CIBC	3016-ECAP	1287-buffer

The results for the Royal Bank and the Bank of Nova Scotia do make some sense, as the ECAP provision for potential fluctuations in business income should be an important (but not entire) part of the overall economic capital provisioning. The results for other two banks are more difficult to appreciate. We are reluctant to suggest that these banks are not sufficiently prudent in their economic capital provisioning. It is more likely that the reported number address other concerns that are not fully elaborated in the annual income statement and do not address others. Here we can only suggest that OSFI impose some uniformity across banks in reporting numbers under this particular category.

4.2 Conclusions

In the evolution of bank regulation over the last thirty years, the Value-at-Risk (VaR) measure been a key metric in determining the amount of regulatory capital a bank must hold to deal prudently with its exposure to market, credit and operational risk. The original focus was on the bank's credit risk but regulatory capital was extended to cover first market then operation risks. The VaR concept was framed to be a statistical notion associated with the distribution of potential losses that could arise in a particular risk category. In applying the VaR approach the distribution is viewed as having an objective character, either derived from models with parameters calibrated to the bank's experience (as in credit risk), or based on empirical distributions with a long history of the bank's experience (market risk), or based on sparse history requiring some form of data pooling to populate the distribution (operational risk). In the three cases, the development of the distributions had an historical character.

The security supposedly provided by VaR was certainly challenged by the financial crisis in 2008. The risk analysis in place at the time appeared to be too narrowly focused, as other issues (particularly liquidity risk) came to the fore. As well, a risk analysis that focused on a sole bank outside its competitive network seemed too narrow. Other regulatory tools have been developed in recent years as a consequence of adopting a broader perspective.

But VaR remains an appropriate diagnostic instrument. The determination of what losses could occur in a dire situation is a sensible preoccupation and imposes discipline on a bank's internal analysis of the riskiness of its activities.

This thesis has maintained the VaR objective, but extends the traditional analysis along two dimensions. First, we have analyzed a notion of business risk associated with fluctuations in a bank's business income that are not tied to specific market, credit or operational events. Rather the fluctuations that we analyze are more the consequences of ongoing strategic decisions. Second, we have attempted to operationalize a sectoral approach where the losses potentially faced by a particular bank are those that are shared by its competitors. More particularly, the thesis makes three important contributions:

I. From the banks' income statements, we have constructed two major data sets covering the recent tumultuous history in banking results: one covers eighteen banks in the US retail banking sector. The data set adjusts for losses from market, credit and operational (in the sense of Basel) fluctuations and smooths one-off accounting charges to arrive at a core notion of business income. Income in this sense is then relativized to the bank's interest bearing assets to allow temporal and inter-bank comparisons.

   The second data set is constructed in the same manner for the entire Canadian banking sector over the same period. Here we faced the challenge arising from the implementation of new accounting standards in mid-sample.

II. Towards undertaking a more comprehensive sectoral analysis, we adopted two different approaches. The first is structural in which the time series character of the bank's income has been preserved in a multivariate setting with a particular econometric specification to reflect the sectoral character. The second approach pools all the data within the sector as the basis for the VaR analysis.

III. Estimation on the structural approach has used standard maximum likelihood techniques. The third contribution of the thesis has been to apply the *saddlepoint approximation* (SPA) approach to the pooled data. To contrast differences between these two approaches, we have undertaken a comprehensive Monte Carlo simulation study in the third Chapter. We find that the SPA performs more than adequately in this context.

Since such an analysis has not previously undertaken, we cannot suggest that our approach is superior to earlier results. We do offer some comparisons with the reporting of buffer capital that appears in several Canadian bank statements in 2015. However, these must be viewed as entirely tentative; the bank statements give very little information as to how the buffer is calculated or what formal objectives the buffer is intended to meet. In this regard, the thesis makes two important advances:

- The VaR analysis enables a bank to calculate a formal VaR measure of an important component of the buffer capital it must hold to operate with fiduciary prudence in its current competitive environment;
- It offers a simple analytic approach to the VaR computation via SPA.

Table 4.1 VaR Estimates: US Sector

Panel A: ML Estimates					
	No. of Series (K)	**No. of years (T_k)**	$\hat{\theta}$	$\hat{\sigma}^2$	$\hat{\rho}$
MLE(no cross correlation)	18	18-22	0.48	0.17	
MLE*(fixed cross correlation)	18	18	0.48	0.16	0.11
Panel B: Estimated VaR (%)					
	MLE	**MLE***	**SPA**		
$VaR_{90\%}$	0.59	0.61	0.67		
$VaR_{95\%}$	0.76	0.78	0.97		
$VaR_{99\%}$	1.08	1.11	1.57		

$\hat{\theta}$ denotes the estimate of the coefficient parameter in a Gaussian MA(1) process, $\hat{\sigma}^2$ denotes for the variance of the disturbance distribution in the process, and $\hat{\rho}$ denotes the estimate of the fixed cross-correlation among banks. MLE and MLE* respectively refer to the case of no cross-correlation and the case of fixed correlation, see Appendix to Chapter 3.

Table 4.2 VaR Estimates: Canadian Sector

Panel A: ML Estimates

	No. of Series (K)	**No. of years (T_k)**	$\hat{\theta}$	$\hat{\sigma}^2$	$\hat{\rho}$
MLE(non-correlated)	8	14-22	0.28	0.10	
MLE*(correlated)	7	17	0.36	0.08	-0.04

Panel B: Estimated VaR (%)

	MLE	**MLE***	**SPA**
$VaR_{90\%}$	0.41	0.37	0.50
$VaR_{95\%}$	0.53	0.47	0.76
$VaR_{99\%}$	0.75	0.67	1.29

See notes for Table 4.1

Table 4.3 Estimated Economic Capital (ECAP) for Business Risk: US Banks

Banks	**IEA_{2015}** \$ m	**$ECAP_{90\%}$** \$ m	**$ECAP_{95\%}$** \$ m	**$ECAP_{99\%}$** \$ m
FITB	123584	834	1205	1946
ASBC	24571	166	240	387
BBT	172673	1165	1683	2719
BOKF	27722	187	270	437
CMA	65129	440	635	1026
FHN	23456	158	229	369
HBAN	63023	425	614	993
KEY	82519	557	804	1300
MTB	91187	615	889	1436
NYCB	43622	294	425	687
PBCT	33228	224	324	523
PNC	308825	2084	3011	4864
RF	107871	728	1052	1699
STT	220456	1488	2149	3472
STI	168813	1139	1646	2659
SNV	25992	175	253	409
USB	367445	2480	3582	5787
ZION	54374	367	530	856

$ECAP_{90\%}$, $ECAP_{95\%}$ and $ECAP_{99\%}$ are respectively calculated by the product of IEA_{2015} and the corresponding VaR in Table 4.1.

Table 4.4 Estimated Economic Capital (ECAP) for Business Risk: Canadian Banks

Banks	$\mathbf{IEA_{2015}}$ C$ m	$\mathbf{ECAP_{90\%}}$ C$ m	$\mathbf{ECAP_{95\%}}$ C$ m	$\mathbf{ECAP_{99\%}}$ C$ m
DEJ	170922	859	1303	2209
BNS	579471	2911	4417	7489
BMO	766000	3848	5839	9899
CM	395616	1987	3016	5113
LB	31248	157	238	404
NA	194419	977	1482	2513
RY	864752	4344	6592	11175
TD	913804	4590	6966	11809

$ECAP_{90\%}$, $ECAP_{95\%}$ and $ECAP_{99\%}$ are respectively calculated by the product of IEA_{2015} and the corresponding VaR in Table 4.2.

References

Klaus Böcker. Modelling and Measuring Business Risk. *Pillar II in the New Basel Accord-The Challenge of Economic Capital*, pages 179–200, 2008.

James G Booth and Andrew TA Wood. An Example in Which the Lugannani-Rice Saddlepoint Formula Fails. *Statistics & Probability Letters*, 23(1):53–61, 1995.

Simon A Broda and Marc S Paolella. Saddlepoint Approximations: A Review and Some New Applications. In *Handbook of Computational Statistics*, pages 953–983. Springer, 2012.

Ronald W Butler. *Saddlepoint Approximations with Applications*, volume 22. Cambridge University Press, 2007.

Bryan Campbell and Eric Stam. Modeling Business Risk at Fifth Third Bank. *Working Paper, CIRANO Financial Group.*, 2012.

Mohamed Chaffai and Michel Dietsch. Modelling and Measuring Business Risk and the Resiliency of Retail Banks. *Journal of Financial Stability*, 16:173–182, 2015.

Arun Chockalingam, Shaunak Dabadghao, and Rene Soetekouw. Strategic Risk, Banks, and Basel III: Estimating Economic Capital Requirements. *The Journal of Risk Finance*, 19(3):225–246, 2018.

Henry E Daniels. Saddlepoint Approximations in Statistics. *The Annals of Mathematical Statistics*, pages 631–650, 1954.

AC Davison and D Mastropietro. Saddlepoint Approximation for Mixture Models. *Biometrika*, 96(2):479–486, 2009.

Anthony C Davison and David V Hinkley. Saddlepoint Approximations in Resampling Methods. *Biometrika*, 75(3):417–431, 1988.

René Doff. Defining and Measuring Business Risk in an Economic-capital Framework. *The Journal of Risk Finance*, 9(4):317–333, 2008.

Matteo Fasiolo, Simon N Wood, Florian Hartig, Mark V Bravington, et al. An extended empirical saddlepoint approximation for intractable likelihoods. *Electronic Journal of Statistics*, 12(1):1544–1578, 2018.

Andrey Feuerverger and Augustine CM Wong. Computation of Value-at-Risk for Nonlinear Portfolios. *Journal of Risk*, 3:37–56, 2000.

Michael B Gordy. Saddlepoint Approximation of CreditRisk+. *Journal of Banking and Finance*, 26(7):1335–1353, 2002.

Xinzheng Huang, Cornelis W Oosterlee, and Hans van der Weide. Higher-order Saddlepoint Approximations in the Vasicek Portfolio Credit Loss Model. *Journal of Computational Finance*, 11(1):93–114, 2007.

John Hull. *Risk Management and Financial Institutions*, volume 733. John Wiley & Sons, 2012.

Chaonan Jiang, Davide La Vecchia, Elvezio Ronchetti, and Olivier Scaillet. Saddlepoint Approximations for Spatial Panel Data Models. *Swiss Finance Institute Research Paper*, (19-18), 2020.

Sojung Kim and Kyoung-Kuk Kim. Saddlepoint Methods for Conditional Expectations with Applications to Risk Management. *Bernoulli*, 23(3):1481–1517, 2017.

John E Kolassa. *Series approximation methods in statistics*, volume 88. Springer Science & Business Media, 2006.

Andrew Kuritzkes and Til Schuermann. What We Know, Don't Know and Can't Know About Bank Risks: a View from the Trenches. *Working Paper, Wharton Financial Institutions Center.*, 2008.

Andrew P Kuritzkes and Hal S Scott. Sizing Operational Risk and the Effect of Insurance:

Implications for the Basel II Capital Accord. *Capital Adequacy Beyond Basel: Banking, Securities, and Insurance*, 2005.

Robert Lugannani and Stephen Rice. Saddlepoint Approximation for the Distribution of the Sum of Independent Random Variables. *Advances in Applied Probability*, 12 (2):475–490, 1980.

Richard J Martin. Saddlepoint Methods in Portfolio Theory. *arXiv:1201.0106*, 2011.

Elvezio Ronchetti and Alan H Welsh. Empirical Saddlepoint Approximations for Multivariate M-estimators. *Journal of the Royal Statistical Society: Series B (Methodological)*, 56(2):313–326, 1994.

Gerhard Schroeck. *Risk Management and Value Creation in Financial Institutions*, volume 155. John Wiley & Sons, 2002.

Ruey S Tsay. *Analysis of Financial Time Series*, volume 543. John Wiley & Sons, 2005.

Ruey S Tsay. *Multivariate Time Series Analysis: with R and Financial Applications*. John Wiley & Sons, 2013.

Suojin Wang. General Saddlepoint Approximations in the Bootstrap. *Statistics & Probability Letters*, 13(1):61–66, 1992.

Andrew TA Wood, James G Booth, and Ronald W Butler. Saddlepoint Approximations to the CDF of Some Statistics with Nonnormal Limit Distributions. *Journal of the American Statistical Association*, 88(422):680–686, 1993.

Yuantao Zhang and Yue Kuen Kwok. Saddlepoint Approximations to Tail Expectations Under Non-Gaussian Base Distributions: Option Pricing Applications. *Journal of Applied Statistics*, 47(11):1936–1956, 2020.