# Design and Planning of Maintenance Logistics Networks

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# Abstract


**Design and Planning of Maintenance Logistics Networks**

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The system up-time and availability play an essential role in industrial, maritime, aeronautical, and health care sectors. These sectors utilize, in general, several advanced capital systems, such as gas turbines, radars, airplane engines, and MRI-scanners. Most of these technical devices contain expensive and low-demand repairable parts. The management of maintenance logistics networks deal with decisions both in the design and planning phases. In the design phase, goals such as the allocation of users to maintenance centers and spare-part provisioning are pursued. On the other hand, the planning phase deals with decisions in terms of workforce, capacity, and aggregate planning. Maintenance planning is a hard task due to several conflicting constraints, such as sporadic demand, uncertain repair and inspection time, limited capacity, and availability of resources (inventory and certified operators). Our problem of interest is mainly motivated by maintenance logistics networks in the context of gas turbine engines. The maintenance service providers to these devices are confronted with the interaction of workforce training and operations planning along with demand and repair time uncertainty, that introduce new challenges to the management of these logistics networks.

In the first part of this thesis, we devise a decision model to obtain the optimal size of workforce, training schedule, repair quantity, as well as number of repair jobs to outsource so as to minimize the cost of repair operations, spare part stock, training, outsourcing, and penalties incurred for the delayed delivery of repaired equipment over a planning horizon. Then, we evaluate the impact of integrating workforce training with operational planning decisions in maintenance facilities. Besides, we analyze the role of risk mitigation strategies such as outsourcing of repair jobs to other maintenance centers and borrowing of certified operators in the presence of demand fluctuations by formulating a two-stage stochastic programming model.

The second part of the thesis is an effort to incorporate the repair time uncertainty into the decision model developed in the first contribution. We propose a multi-stage stochastic programming model for integrated production and workforce planning under independent random repair times of faulty components. Then, we develop an approximate decomposition algorithm, based on Lagrangian relaxation approach, to efficiently solve the problem for real-size instances. This algorithm relies on decomposing the MSP model into sub-models corresponding to component scenario trees and coordinating them via a sub-gradient algorithm to obtain a high-quality feasible solution.

In the final part of this thesis, given an MLN that provides maintenance/repair services to geographically dispersed equipment users, we propose a two-stage robust optimization model for collaborative design and planning of maintenance networks under demand uncertainty. The goal of this model is to determine the optimal allocation of customers to each maintenance center along with the initial stock level of different spare parts in each facility so as to minimize the cost of late deliveries under worst-case demand scenarios. We consider component and operator sharing strategies as the recourse actions in this model to hedge against the demand surge. The proposed approach is compared with a deterministic model by the aid of Monte-Carlo simulation on several test instances inspired by a real case study.

# Preface

This thesis has been prepared in "Manuscript-based" format under the co-supervision of Dr. Masoumeh Kazemi Zanjani from the department of Mechanical, Industrial and Aerospace Engineering, Concordia University and Dr. Mustapha Nourelfath from the department of Mechanical Engineering, Laval University. This research was financially supported by Le Fonds de recherche du Québec-Nature et technologies (FRQNT) and the Natural Sciences and Engineering Research Council of Canada (NSERC). All the articles presented in this thesis were co-authored and reviewed prior to submission for publication by Dr. Masoumeh Kazemi Zanjani and Dr. Mustapha Nourelfath. The author of this thesis acted as the principal researcher and performed the mathematical models development, programming of the solution algorithms, analysis and validation of the results, along with writing the first drafts of the articles.

The first article entitled "Integrated planning of operations and on-job training in maintenance logistics networks", co-authored by Dr. Masoumeh Kazemi Zanjani and Dr. Mustapha Nouelfath was published in Journal of Reliability Engineering and System Safety in July 2020. The second article entitled "Workforce training and operations planning for maintenance centres under demand uncertainty", co-authored by Dr. Masoumeh Kazemi Zanjani and Dr. Mustapha Nourelfath was published in International Journal of Production Research in Feburary 2021. The revised version of the third article entitled "An approximate decomposition algorithm for multi-item tactical planning with independent random parameters", co-authored by Dr. Masoumeh Kazemi Zanjani and Dr. Mustapha Nourelfath was submitted to Computer and Operations Research in June 2021. The fourth article entitled "Robust collaborative maintenance logistics network design and planning", co-authored by Dr. Masoumeh Kazemi Zanjani and Dr. Mustapha Nourelfath was submitted to International Journal of Productions Economics in August 2021.

***To my beloved parents and sister***

*Hossein, Shekoufeh, Shirin*

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In addition, I am grateful to four beloved people who have meant and continue to mean so much to me. Although they are no longer of this world, their memories continue to regulate my life. My grandparents, Ahmad, Mahvash, Fatemeh, and my uncle, Dr. Shahab Daneshmandi, who passed away due to the Covid-19 disease.

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# Chapter 1

# Introduction

## 1.1 Overview

For advanced capital systems like trains, planes, medical devices, radars and antenna components, high system availability is essential. Based on the usage time and maintenance policy (preventive or corrective maintenance), these components or systems are confronted with failures and down-times. Such disruptions are generally very costly and require a responsive logistics network to replace the failed item by a functional one in a timely manner. Traditionally, the users of complex equipment own their logistics network to maintain the system availability. Such user maintenance networks are common in the military bases (Muckstadt, 2004). On the contrary, due to the technical issues, price and high variety of these equipment, the users prefer to outsource major components of service upkeep to the Original Equipment Manufacturer (OEM) or an external service provider (Basten & Van Houtum, 2014).

Maintenance logistic networks (MLN) are comprised of suppliers in the upstream, central and local maintenance facilities in the midstream and customers in the downstream echelons. Their goal is providing maintenance service to the users of the above-mentioned technical devices. They are also featured with complicating characteristics such as the large number and diversity of spare parts, extremely sporadic component failure rates (random demand), uncertain inspection and repair times, high prices of spare parts, and financially remarkable effect of spare part shortage.

The OEM or external maintenance service providers are confronted with a variety of decision-making problems in the MLN. Such decisions can be classified into strategic, tactical and operational categories. While the former encompasses the long-term decisions on the configuration of the network, inventory control policies in each echelon, and the allocation of customers to different local maintenance facilities, the latters revolve around operations management in maintenance facilities. Work-force sizing, capacity planning, production planning, procurement, and inventory planning in addition to workforce scheduling and training are few examples among others.

Motivated by the requirements and constraints of such maintenance service providers, the focus of this thesis is on the design of maintenance logistics networks and planning of operations in maintenance facilities. Integrated workforce training and operations planning in some maintenance centers is a complex problem due to the various types of constraints such as training certificate expiration and capacity constraints. On the other hand, maintenance service providers constantly deal with a high degree of uncertainty in terms of the number of failed components and inspection and repair time, which further necessitates a robust planning approach.

This thesis significantly contributes to the existing literature on maintenance logistics network management. First, this research is an attempt to incorporate training of certified operators on a periodic basic to abide by stringent safety regulations of maintenance service providers. Moreover, the integration of training schedule with repair operations is another original contribution of this thesis. Besides, this thesis contributes to the literature by incorporating the repair time uncertainty into the problem by assuming the independent repair time of components that leads to a notoriously complex MSP model as a result of merging component scenario trees. Another contribution is focused on developing an efficient decomposition algorithm to overcome the computational complexity of the proposed MSP model. The final significant contribution of this thesis relies on introducing the notion of resource sharing in MLNs by proposing a collaborative two-stage robust optimization model for maintenance service providers.

In what follows, we first provide a brief description of the studied problems. We then present the research scope and objectives. Finally, we provide the outline of this thesis.

## 1.2 Problem description

In this thesis, we confine our attention to an MLN that comprises of a central repair facility, a set of local maintenance facilities, and equipment users. The local maintenance facilities receive preventive and corrective maintenance jobs from several equipment users that are geographically dispersed and own different families of equipment that share a set of (common) components in their modular structure. The major operations in maintenance facilities encompass the preliminary inspection followed by replacing defective components with the functional ones according to a dual sourcing strategy for spare parts replenishment. More precisely, they can be replenished from the inventory of repaired parts and/or the central repair facility. While facing the random failure of equipment components and consequently random spare part demand, the service provider is also dealing with uncertain repair and procurement lead-times. All the operations in maintenance service providers are carried out by certified operators who are only competent to work on a subset of modules. Further, the validation period of such certificates is limited in many industries. For example, in the aeronautical industry, operators can be certified for specific tasks (inspection, assembly, repair, etc.) for a validation period of one or two years. Therefore, designing a framework that aggregate the training plan with procurement, production, and inventory plans can help the maintenance providers to balance their operations and workforce level to reduce their costs.

Variations in the number of failed components and the repair time of components are two major sources of uncertainty in the context of maintenance logistics network. The repair time of some complex components (e.g., rotors in gas turbines) is an uncertain parameter that depends on the skill level of operators, age, and condition of devices. Repair time randomness may delay the delivery of devices to the users, incurring high penalty costs to the maintenance facilities. Given the uncertainty in components repair times, the deterministic setting described earlier is extended into a multi-stage stochastic programming model. Due to the size of the underlying scenario tree, an approximate decomposition algorithm is necessary to efficiently solve the problem for real-size instances.

Finally, rather than focusing on a single maintenance service provider, it is worth to investigate the design and planning of a maintenance logistic network under demand uncertainty. More precisely, by assuming the formation of a grand coalition among the local maintenance facilities in the network, these centers can (temporarily) share a certain percentage of components and certified operators among each other in order to avoid delayed delivery of repaired devices in case of demand surge in some facilities. In this context, a two-stage robust decision model is proposed that seeks the optimal allocation of users to these centers along with the stock pre-positioning levels for different components in each facility as strategic decisions. The tactical planning decisions in this model, on the contrary, are defined for each demand scenario. These decisions incorporate the quantity of repair jobs scheduled in each facility along with the quantity of resources that must be exchanged among different facilities in the network in order to satisfy the demand.

## 1.3 Scope and objectives

To fill the gap in the existing literature, the main contribution of this thesis is to develop a comprehensive decision framework for the robust design and planning of maintenance logistics networks in the context of repairable spare parts. Given the problem description, the specific objectives are summarized as follows.

(1) To evaluate the impact of integrating workforce training with operations planning decisions in maintenance facilities.

(2) To devise a decision model to obtain the optimal repair workforce size, training requirements, quantity of faulty components to repair, as well as the quantity of repair jobs to outsource in maintenance centers.

(3) To analyze the role of risk mitigation strategies such as outsourcing and borrowing of certified operators in the presence of high demand volumes in these facilities.

(4) To explicitly incorporate the uncertain repair time of the components into the proposed mathematical model by the aid of multi-stage stochastic programming model.

(5) To devise an efficient solution algorithm to efficiently solve the resulting multi-stage stochastic programming model.

(6) To incorporate collaboration in terms of sharing scarce resources among different facilities in a maintenance logistics network design and planning problem.

(7) To propose a robust decision model for collaborative maintenance network design and planning under demand uncertainty.

(8) To compare the proposed robust optimization model with a deterministic approach by the aid of Monte-Carlo simulation.

## 1.4 Organization of the thesis

This manuscript is divided into five chapters, which are listed below. Chapter 2 addresses the fundamental problem of this study that is integrating workforce training decisions with operations planning in the context of maintenance logistics network. The decisions to be made are the optimal repair workforce size, training requirements, quantity of faulty components to repair, as well as the quantity of faulty spare parts to outsource so as to minimize the cost of repair operations, spare part stock, training, outsourcing, and penalties incurred for delayed delivery of repaired equipment over a one-year planning horizon. Besides, a two-stage stochastic programming framework is proposed to incorporate the demand uncertainty in this problem. In chapter 3, through modeling uncertainty in the repair time of repairable components, the deterministic model is extended to a multi-stage stochastic program. The proposed model is then solved via an approximate decomposition algorithm, based on Lagrangian relaxation approach. This algorithm relies on decomposing the MSP model into sub-models corresponding to component scenario trees and coordinating them via a sub-gradient algorithm to obtain a high-quality feasible solution. Chapter 4 presents a a two-stage robust optimization model for collaborative design and planning of maintenance networks under demand uncertainty. The proposed robust approach is compared with a deterministic model with the aid of Monte-Carlo simulation. Finally, chapter 5 provides concluding remarks as well as several avenues for future research.

# Chapter 2

# Integrated planning of operations and on-job training in maintenance logistics networks

This chapter merges the following two papers published in 2020 and 2021:

(1) Shayan Tavakoli Kafiabad, Masoumeh Kazemi Zanjani, and Mustapha Nourelfath. ”Integrated planning of operations and on-job training in maintenance logistics networks.” Reliability Engineering  System Safety 199 (2020): 106922.

(2) Shayan Tavakoli Kafiabad, Masoumeh Kazemi Zanjani, and Mustapha Nourelfath. ”Workforce training and operations planning for maintenance centres under demand uncertainty.” International Journal of Production Research (2021): 1-13.

## Abstract

Companies that provide repair & overhaul services to the users of complex technical systems are confronted with uncertain volume of demand when making tactical decisions such as workforce training and planning of repair operations over an annual planning horizon. Given the high importance of equipment availability (e.g., gas turbines) to the users (e.g, power plants), any delay in the delivery of repaired equipment caused by demand uncertainty would lead to significant penalties and loss of customer goodwill. In this chapter, first a deterministic integrated planning of operations and on-job training in maintenance logistics networks is proposed. Moreover, a two-stage stochastic programming model is proposed to obtain the optimal number of items to repair, spare part inventory, and the number of operators to train with the goal of minimising the total expected cost of maintenance operations and late delivery. Outsourcing and borrowing are introduced as corrective actions to mitigate the risk of late delivery in the presence of demand uncertainty. The numerical results highlight the effectiveness of incorporating uncertainty into the mentioned tactical planning decisions and mitigation strategies in controlling the cost.

## 2.1 Introduction

High system availability is critical for advanced technical systems such as aircraft engines and medical devices. Based on the usage time and maintenance procedure, these systems are confronted with failures and system downtime. Such disruptions are generally very costly and need a responsive logistics network to replace the failed item by a functional one promptly. On the contrary, the late delivery of such equipment to the customer can lead to increased system downtime and consequently, a significant profit loss. Traditionally, users of complex equipment own their logistics network to maintain system availability. Such user maintenance networks are common in the military bases (Muckstadt, 2004). On the contrary, due to the technical issues, price and high variety of equipment, the users prefer to outsource major components of service upkeep to the Original Equipment Manufacturer (OEM) or an external service provider (Basten & Van Houtum, 2014).

The Maintenance Logistics Network (MLN) in the context of complex equipment is typically a three-echelon supply chain. It comprises spare parts suppliers in tier one, central repair facility

and local maintenance facilities in tier two, and the equipment users in the downstream layer. Local maintenance facilities, in general, provide services to a cluster of equipment users based on geographic proximity. For instance, in the context of the gas turbine engines, power plants are the users that outsource the maintenance to a maintenance service provider (e.g., a gas turbine manufacturer maintenance division). On its turn, the service provider replenishes the inventory of spare parts and/or repairable parts from other suppliers (e.g., a gas turbine repair shop for turbines).
Maintenance facilities play a vital role in MLN. These facilities receive advanced capital equipment for preventive or corrective maintenance. Each equipment usually contains several modules, and each module consists of independent repairable components (e.g., rotor, nozzle, etc.). The major operations in the maintenance facility consist of disassembly, initial fault diagnostic, and inspection followed by replacing the failed parts with functional ones and the final assembly of modules. In the replacement phase, if the component is available in stock of the maintenance facility, it will be replenished from the internal warehouse, and the failed component is replaced by another component which is as good as new. Otherwise, the component is back-ordered until the failed part is repaired at the central repair facility. All defective parts are shipped to the central maintenance facility for repair/reconditioning. The suppliers in the upstream level are responsible for replenishment of consumable parts as well as work-kits to maintenance facilities. Figure 2.1 represents the conceptual model of a maintenance logistics network comprised of one central repair facility and one local maintenance facility.

All the tasks mentioned above are carried out by certified operators who are only competent to work on a subset of modules. Further, the validation period of such certificates is limited in many industries. For example, in the aeronautical industry, operators can be certified for specific tasks (inspection, assembly, repair, etc.) for a validation period of one or two years. This limited validation period of certificates necessitates the periodic re-training of workers on different competencies, which in many cases is an on-job training by an available certified operator. Therefore, the aggregation of the training plan with procurement, production, and inventory plans can help the maintenance providers to balance their operations and workforce level to reduce their costs.

The critical issue in maintenance facilities is the prompt delivery of repaired devices given the significantly high cost of system down-time for the users. Such facilities are also confronted

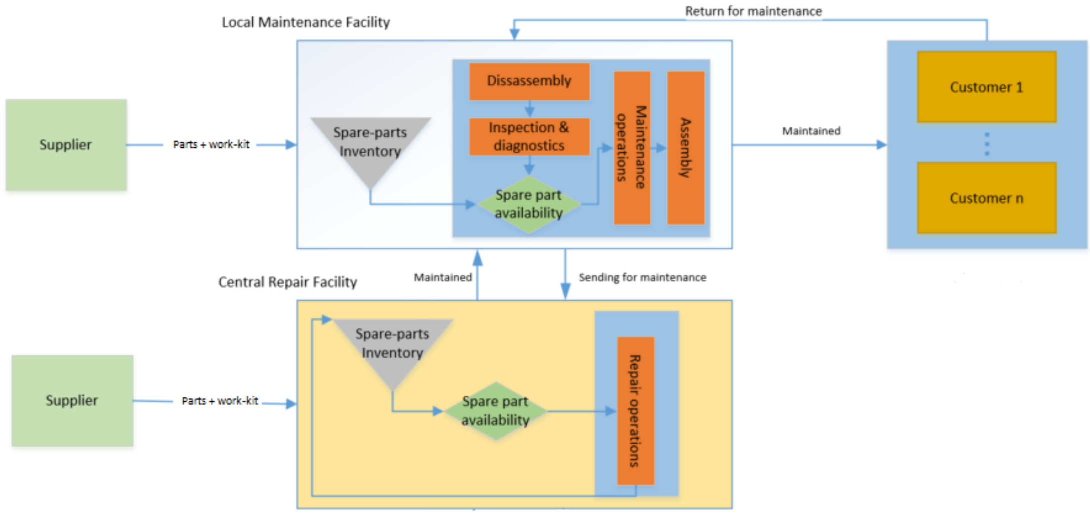


Figure 2.1: Conceptual framework of a maintenance logistics network

with demand fluctuations and considerable back-ordering (penalty) costs as a result of late delivery of devices to the users. The penalty cost is in fact a contractual term to compensate the equipment unavailability in users' site. For instance, the maintenance facilities are expected to recompense power plants for the production loss of electricity due to unavailability of turbines. In face of demand volume beyond their capacity (i.e., equipment and workforce), these facilities might exert different mitigation strategies such as outsourcing the repair operations to other facilities. Alternatively, they can also resort to temporarily borrowing available certified operators from similar facilities. This strategy is particularly justified in industries, where idle operators during low-demand periods cannot be laid-off either due to scarcity of certified operators or union rules. When adopting the mentioned mitigation strategies, the maintenance facility must seek a trade-off between the service level and financial viability of operations.

In this chapter, we propose a decision support model to maintenance service providers in order to assist them to determine the optimal level of maintenance/repair operations, inventory level of spare parts, along with the workforce training plan in each period (month) over an annual planning horizon under uncertain demand. When dealing with optimization models that incorporate uncertain parameters in the right-hand-side or coefficients of constraints, stochastic programming (Birge & Louveaux, 2011) and robust optimization (Bertsimas & Sim, 2004) approaches are among the most suitable methodologies in order to obtain an implementable solution that does not rely on foreseeing uncertain events.

In this study, we assume that the R & O contracts are signed in the beginning of planning horizon, such that each contract has a different delivery due date. Nevertheless, the service provider needs to finalize the plan for repair operations and training requirements prior to finalizing the contracts. In practice, this can be considered as a capacity plan that is usually determined based on historical demand information before the start of the next planning horizon. This step is denoted as the first-stage of the decision making process when dealing with random events such as demand (See, e.g., Birge and Louveaux (2011)). The second stage corresponds to the stage, where complete information on random demand is revealed to the decision maker, i.e., the first period of the planning horizon. In this stage, if the demand exceeds the capacity determined in the first-stage, some corrective (recourse) actions must be taken into consideration to mitigate the risk of late delivery of products. Once the maintenance contracts are finalized, the facility obtains full information on the demand for the upcoming planning horizon. This is also due to the fact that in case of PM and over-haul services, which are the main focus of this study, the majority of critical components are required to be replaced. In other words, demand has a stationary behaviour over the planning horizon once the contracts are finalized. Hence, we adopt a two-stage stochastic programming ($2SP$) approach (Birge & Louveaux, 2011) to model the integrated planning problem. The main idea behind this approach is to obtain first-stage decisions such that the expected cost of recourse actions over the scenario set plausible to random events is minimized.

The proposed $2SP$, in particular, seeks the optimal level of replacement/repair operations along with the training plan for the available workforce in the absence of accurate demand forecasts (first-stage). In the second-stage, once more information on the maintenance contracts for the upcoming planning horizon is available, the model provides a number of recourse (corrective) actions to mitigate the risk of high demand volumes. More precisely, two types of such mitigation strategies have been taken into consideration: *i*) outsourcing the maintenance of a number of received equipment to other maintenance facilities, albeit, at a higher cost compared to in-house operations cost; and *ii*) increasing the capacity of repair via temporarily borrowing certified operators from other facilities. The objective of the $2SP$ model is to minimise the expected cost of operations, training, outsourcing, borrowing, and the late delivery of equipment under a set of scenarios plausible to the uncertain demand over the planning horizon.

The contributions of this study are twofold: First, we incorporate the decisions related to workforce training plan into the aggregate spare parts provisioning and operations planning for a maintenance service provider. The novelty of this paper in terms of modeling is considering the limited age of operators' certificate in the mathematical model and integrating on-job training with operations planning. As shown by a literature review in the next section and to the best of our knowledge, there is no existing work investigating the integration of workforce training into tactical decisions in maintenance logistics networks by using mathematical programming. Second, outsourcing and borrowing strategies are proposed to mitigate the risk of late delivery in the presence of high demand volumes in the framework of a two-stage stochastic programming model. The uncertain demand is also modelled as a set of scenarios based on historical data and experts opinion. To the best of the authors' knowledge, the integrated planning of repair operations and workforce training under demand uncertainty has not been studied in the literature.

The rest of this chapter is organized as follows. In Section 2.2, the recent works related to the context of maintenance logistics networks are reviewed to highlight the paper contribution. In Section 2.3, the Deterministic integrated planning of operations and on-job training in maintenance logistics networks are discussed. Integrated planning of operations and on-job training in maintenance logistics networks under uncertain demand is proposed in Section 2.4. In Section 2.5, a case study is provided, and a sensitivity analysis is performed on different parameters of the proposed model. Finally, Section 2.6 summarizes the paper and proposes some future research directions.

## 2.2 Literature review

In this section, by analyzing the literature, different decision-making problems in the context of maintenance logistics network are reviewed. Then, some related papers on integrated maintenance with aggregate planning models are investigated. Then, recent papers related to the role of training in maintenance operations are reviewed. Finally, the motivation of this paper is clarified based on the literature.

The maintenance logistics networks are confronted with a variety of decision-making problems. Such decisions can be classified into strategic, tactical, and operational categories. While the strategic level encompasses the long-term decisions on the configuration of the network, inventory control policies, and the allocation of customers to different local maintenance facilities, the tactical and operational levels revolve around operations management in maintenance facilities. Work-force sizing, capacity planning, production planning, procurement, and inventory planning in addition to workforce scheduling and training are a few examples among others. Several studies in the literature ((Muckstadt, 2004), (Basten & Van Houtum, 2014), (Manikas, Sundarakani, & Iakimenko, 2019), (Eruguz, Tan, & van Houtum, 2017) and (Zhu, van Jaarsveld, & Dekker, 2020)) investigated repairable spare part inventory management in maintenance logistics networks (MLN), whereas others ((Candas & Kutanoglu, 2007), (Rappold & Van Roo, 2009), and (Wheatley, Gzara, & Jewkes, 2015)) investigated MLN network design problem.

Only a handful of articles investigated integrated operations planning in such value chains at tactical and operational levels. Zanjani and Nourelfath (2014) studied the integrated planning of spare parts procurement, inventory, and production in a maintenance facility by proposing a multi-stage stochastic programming model. The goal was to minimize the expected cost of procurement, production, and late delivery under an uncertain amount of maintenance demand. Weinstein and Chung (1999) investigated a model that integrates the selection of the maintenance policy with the aggregate production planning. Sensitivity analysis was conducted to test the impact factors such as maintenance activity, failure significance, maintenance activity cost, and aggregate planning policy on maintenance policy selection. Sitompul and Aghezzaf (2011) investigated an integrated hierarchical production and maintenance planning for a multi-stage production environment. They developed a hierarchical production planning that consists of an aggregate and detailed planning. Two categories of maintenance actions were investigated in their model. The preventive maintenance policy was considered in the aggregate planning model and the machine failures, which require corrective maintenance policy, discussed in the detailed planning. Fitouhi and Nourelfath (2014) developed an integrated tactical production planning with noncyclical preventive maintenance policy for multi-state systems. The model simultaneously determines the quantity of inventory, back-order, items to produce, set-up times, and preventive maintenance times. Fakher, Nourelfath, and Gendreau (2017) proposed a multi-product capacitated lot-sizing problem in an in-house maintenance network. Their model coordinated the quality production planning, maintenance scheduling, and process inspections to minimize the total cost. In the same context, Fakher, Nourelfath, and Gendreau (2018) investigated a capacitated aggregate planning to integrate production, maintenance and quality decisions with the objective of maximizing the expected profit. Rezaei-Malek, Siadat, Dantan, and Tavakkoli-Moghaddam (2019) proposed an integrated planning of part quality inspection and preventive maintenance in a multi-stage production system. Their model obtained the optimum place and time for preventive maintenance and part quality inspection while optimizing system total cost and productivity.

In (Phogat & Gupta, 2017), the authors mentioned that lack of appropriate strategic planning and the lack of training are among the main challenges of maintenance operations toward an efficient maintenance management system. They also discussed that the effective training aid to change the mindset of operators from traditional maintenance approach to the modern one and help the companies to optimize the number of operators and expand their adaptability. In the realm of maintenance logistics networks, only a few studies considered the integration of workforce capacity, training planning, and operations scheduling in a repair facility. Sleptchenko, Turan, Pokharel, and ElMekkawy (2017) investigated the joint optimization of cross-training of operators and spare parts provisioning of repairable parts in a maintenance facility that perform repair operations. Recently, Sleptchenko, Al Hanbali, and Zijm (2018) studied the integrated optimization of workforce capacity planning and spare parts provisioning in a supply chain comprising of one maintenance facility and multiple demand points. They assumed that for each failed item, an operator and the corresponding spare part should be allocated; hence, the unavailability of each leads to shortages. There are some other papers ((Norman, Tharmmaphornphilas, Needy, Bidanda, & Warner, 2002), (De Bruecker, Van den Bergh, Beliën, & Demeulemeester, 2015) and (Techawiboonwong, Yenradee, & Das, 2006)) that investigate workforce sizing and training in other context like production planning and scheduling.

The survey of the literature reveals that the majority of the studies assume cross-trained operators and an unlimited validation period for operators' certificates. As a consequence, the importance of integrating workforce training into other tactical decisions in a maintenance facility was not investigated in the literature. Furthermore, there is no existing paper that incorporates demand uncertainty into this problem; hence the efficiency of outsourcing and borrowing strategies in controlling the total maintenance cost and reduce the risk of late delivery.

## 2.3 Deterministic integrated planning of operations and on-job training in maintenance logistics networks

Consider a maintenance facility that receives advanced expensive technical devices (e.g., gas turbine engines) mainly for overhaul services over a planning horizon T indexed by $t$. Each device has several modules; each consists of independent repairable components. In this context, R is the set of components indexed by $r$. $d_{r,t}$ is the number of component $r$ scheduled to be replaced in period $t$. Given that the majority of modules/components must be replaced by functional ones in overhauled devices, the demand of components is considered as a deterministic parameter. This can be further justified by the fact that the periodicity of overhaul services for different users are clearly defined in their contract with the maintenance service provider. The devices undergo the disassembly (to their components) and inspection by the certified operators. We define inspection as any task undertaken to determine the condition of components and to find the tools, labor, materials, and equipment required to repair the components. In this phase, each component is inspected by the certified inspectors to find out whether or not the component needs replacement. In case the replacement is required, the failed component would be replaced by a functional one. $Q_{r,t}$ is the number of replaced component (ready for assembly) in period $t$. The latter is replenished from the internal inventory if the component is available in stock in the maintenance facility. $I_{r,t}$ is the number of component $r$ available in stock in period $t$ and the unit inventory holding cost and safety stock are denoted by $h_r$ and $SS_r$, respectively. Otherwise, the component would be back ordered until the failed part is repaired/reconditioned. $Q_{r,t}^{rep}$ is the number of component $r$ that should be inspected and eventually replaced in period $t$ with unit internal maintenance cost $k_r$. Without loss of generality, we only consider the cost of replacing one unit of component as the unit maintenance cost. $l_{rt}^{rep}$ is the replacement time of component $r$ in period $t$. $B_{r,t}$ is the shortage quantity of component $r$ in period $t$. It should be mentioned that the number of delayed repaired devices is calculated at the component level. In other words, the component with the highest amount of shortage (due to unavailability of spare part or the certified operators) determines the total number of devices that cannot be delivered in-time. $b$ is the unit penalty cost for late delivery of each equipment and $\theta_t$ is the auxiliary variable representing the maximum quantity of shortages among all the components.

To perform the maintenance operations, the operators need valid certificate related to specific skills. Here we assume skill set S indexed by $s$ consists of inspection and repair competencies. $\alpha_{r,s}$ is the labor consumption factor of component $r$ per skill type $s$ in period $t$. The certificates are valid for a specific duration per skill set $e_s$. Besides, after the certificate expiration date for a given task, the operator is not allowed to perform that task. In other words, the inspection/repair operations on a component will be delayed if the certified operators are not available. The total number of operators per skill type $s$ is $W_s$ and the initial number of $a$-months old certified operators per skill type $s$ in period 0 is $E_{a,s,0}$. $A_s$ is the set of possible certificate ages associated with skill $s$ indexed by $a$ ($|A_s|$=max$\{a$ in period 0$\}$+ |T|). $E_{a,s,t}$ is the number of $a$-months old certified operators per skill type $s$ in period $t$. Thus, the company needs to train the operators with unit cost $n_s$ before the expiration date to maintain a sufficient number of certified operators. $R_{a,s,t}$ is the number of $a$-months old operators of skill type $s$ who are under training in period $t$. The training is performed on the job on a one-by-one basis. In other words, there should be one certified operator available to train an operator whose certificate is expired. Further, to instruct each operator on a specific skill in a given period, there should be a demand for that skill during that period. The goal of the company is to deliver the repaired devices according to promised due dates and to avoid penalties for the late delivery. In this study, in order to investigate the impact of the inventory level of spare parts as well as the number of operators with valid certificates on the delays in the delivery of repaired devices, we envisage two outsourcing policies. More precisely, we assume that the company has the option to temporarily borrow required number of certified operators $V_{s,t}$ for a given skill type in a specific period from another maintenance facility with unit cost $q_s$ if enough certified operators are not available in that period. $G_s$ is the maximum allowable number of certified operators with skill type $s$ that can be borrowed in each period. Alternatively, the maintenance operations for a given component can be outsourced to another maintenance facility at a significantly higher cost as compared with borrowing operators from that facility. $Q^{out}_{r,t}$ is the number of component $r$ that should be outsourced with unit outsourcing cost $u_r$ to an external maintenance facility for inspection/repair in period $t$. $l^{out}_r$ and $M^{out}_{r,t}$ are the outsourcing lead time and maximum capacity of external maintenance facility to repair component $r$ respectively. Besides, the available budget for repair, training and outsourcing in each period is $L_t$, $O_t$ and $P_t$ respectively. It is worth to mention that the aforementioned outsourcing and borrowing strategies can also be considered as the mitigation actions in case the demand is higher than the actual repair capacity in the facility.

Based on the context and assumptions mentioned above, the company is looking for the optimal spare part inventory level, number of components to inspect/repair, number of operators to train as well as the number of temporary operators to borrow, and the number of outsourced maintenance jobs in each period in the planning horizon. The goal is to minimize the total cost consists of the repair, inventory, outsourcing of components, late delivery, training, and borrowing operators. The objective function of the problem is as follows:

$$
\begin{aligned}
Minimize \sum_{t\in T}\sum_{r\in R} k_r Q^{rep}_{r,t} + \sum_{t\in T}\sum_{r\in R} h_r I_{r,t} + \sum_{t\in T}\sum_{r\in R} u_r Q^{out}_{r,t} \\
+ b\sum_{t\in T}\theta_t + \sum_{t\in T}\sum_{s\in S}\sum_{a\in A_s} n_s R_{a,s,t} + \sum_{t\in T}\sum_{s\in S} q_s V_{s,t}
\end{aligned} \tag{2.1}
$$

The objective function 2.1 encompass three category of constraints namely operations, workforce and domain constraints.

***Operations constraints***

$$Q_{r,t} - B_{r,(t-1)} + B_{r,t} = d_{r,t} \qquad \forall t \in T, \forall r \in R \tag{2.2}$$

$$Q^{rep}_{r,(t-l^{rep}_{rt})} + Q^{out}_{r,(t-l^{out}_{r})} + I_{r,(t-1)} - I_{r,t} = Q_{r,t} \qquad \forall t \in T\backslash 1,...,l^{rep}_{rt}, \forall r \in R \tag{2.3}$$

$$\theta_t \geq B_{r,t} \qquad \forall t \in T, \forall r \in R \tag{2.4}$$

$$\sum_{r\in R} \alpha_{r,s} Q^{rep}_{r,t} \leq \sum_{a=1}^{e_s} E_{a,s,t} + V_{s,t} \qquad \forall t \in T, \forall s \in S \tag{2.5}$$

$$Q^{out}_{r,t} \leq M^{out}_{r,t} \qquad \forall t \in T, \forall r \in R \tag{2.6}$$

$$I_{r,t} \geq SS_r \qquad \forall t \in T, \forall r \in R \tag{2.7}$$

$$\sum_{r\in R} k_r Q^{rep}_{r,t} \leq L_t \qquad \forall t \in T \tag{2.8}$$

$$\sum_{r\in R} u_r Q^{out}_{r,t} \leq P_t \qquad \forall t \in T \tag{2.9}$$

$$Q^{rep}_{r,t} = 0 \qquad \forall t \in 1,...,l^{rep}_{rt}, \forall r \in R \tag{2.10}$$

$$Q^{out}_{rt} = 0 \qquad \forall t \in 1,...,l^{out}_{r},, \forall r \in R \tag{2.11}$$

Constraints (2.2)-(2.11) represent various limitations related to operations planning. Constraints (2.2) imply the balance between the number of components scheduled to be replaced, the actual number of replaced components, and the shortages in two consecutive periods. Constraints (2.3) is the inventory balance constraint for components. It ensures that the inventory level of each component in each period equals to its inventory in previous period minus the number of repaired and outsourced components while considering the inspection and repair times as well as outsourcing lead time. Constraints (2.4) calculate the maximum amount of shortages of components in each period. In other words, the component with the highest amount of shortage will identify the total number of delayed deliveries. Constraints (2.5) limits the total number of components that can be repaired in each period based on the number of available certified operators in addition to the number of temporary operators who will be borrowed from other facilities. Constraints (2.6) state the maximum capacity of other facilities in repairing outsourced components. Constraints (2.7) ensure the safety stock in each period. Constraints (2.8) and (2.9) imply the maximum available budget for repair operations and outsourcing. Constraints (2.10) and (2.11) set the initial values of repair and outsourcing decisions.

***Workforce constraints***

$$E_{(a+1),s,t} = E_{a,s,(t-1)} - R_{a,s,(t-1)} \qquad \forall t \in T, \forall s \in S, \forall a \in A_s \tag{2.12}$$

$$E_{1,s,t} = \sum_{a \in A_s \setminus 1} R_{a,s,(t-1)} \qquad \forall t \in T, \forall s \in S \tag{2.13}$$

$$\sum_{a \in A_s \setminus 1} R_{a,s,t} \leq \sum_{r \in R} Q^{rep}_{r,t} \qquad \forall t \in T, \forall s \in S \tag{2.14}$$

$$\sum_{a \in A_s \setminus 1} R_{a,s,t} \leq \sum_{a=1}^{e_s} E_{a,s,t} \qquad \forall t \in T, \forall s \in S \tag{2.15}$$

$$\sum_{a \in A_s} E_{a,s,t} + \sum_{a \in A_s \setminus 1} R_{a,s,t} = W_s \qquad \forall t \in T, \forall s \in S \tag{2.16}$$

$$V_{st} \leq G_s \qquad \forall t \in T, \forall s \in S \tag{2.17}$$

$$\sum_{s \in S} \sum_{a \in A_s} n_s R_{a,s,t} \leq O_t \qquad \forall t \in T \tag{2.18}$$

Based on the aforementioned assumptions, the certificates of the operators are valid for a specific duration. Thus, in order to perform repair operations, the company requires a workforce planning in order to maintain sufficient number of certified operators. Constraints (2.12)-(2.18) represent various limitations related to workforce training. As we need to track and update the certificate age of the operators, constraints (2.12) and (2.13) are proposed. constraints (2.12) ensure the certified operators' balance in two consecutive periods. In other words, if the company trains the operators with age $a$ in period $t$, the number of certified operators with age $a + 1$ decreases in the next period. For example, for $t = 6$ and $a = 3$, equation $E_{4,s,6} = E_{3,s,5} - R_{3,s,5}$ means that if the company trains in period 5 a number $R_{3,s,5}$ of 3-months old certified operators, this number ($R_{3,s,5}$) is subtracted from the number $E_{3,s,5}$ of certified operators with age 3 in period 5 to decrease the number$E_{4,s,6}$ of certified operators with age 4 in period 6. Constraints (2.13) state that the number of operators under training in each period is equal to the number of certified operators with age 1 in the next period. In other words, after the on-job training in period (t-1), the operators are able in period t to perform repair operations associated with each skill. Constraints (2.14) specify that the number of operators under training should be less than the number of components under inspection. In fact, under high volume demands, we want to avoid training operators who are not immediately needed. Constraints (2.15) ensure that the number of operators under training is less than the number of certified operators associated with each skill. The flow constraints (2.16) ensure the balance between the total number of employees, number of certified and uncertified operators, in addition to the number of operators under training. In fact, these constraints ensure that the total number of operators ($W_s$) is the sum of the total number of certified operators and the total number of operators under training. Constraints (2.17) state the allowable number of certified operators that the company can borrow from other maintenance providers. Constraints (2.18) specify the maximum available budget for training in each period.

***Domain constraints***

$$Q_{r,t}, Q^{rep}_{r,t}, I_{r,t}, B_{r,t}, \theta_t, E_{a,s,t}, R_{a,s,t} \in \mathbb{Z}^+ \qquad \forall t \in T, \forall s \in S, \forall a \in A_s, \forall r \in R \tag{2.19}$$

Constraints (2.19) are the domain constraints.

## 2.4 Integrated planning of operations and on-job training in maintenance logistics networks under uncertain demand

The demand (at the device level) is a random variable that can be represented as a set of scenarios with known probabilities. It is worth to mention that the demand scenarios can be either generated based on experts' opinion and or by discretizing the underlying probability distribution (e.g., Normal). Furthermore, the demand might follow a cyclic pattern over the planning horizon that can be represented as a scenario set with corresponding probabilities based on historical data.

The demand at the component level can be accordingly calculated from the Bill-of-Material corresponding to each device.

As mentioned earlier, a two-stage stochastic programming ($2SP$) approach is adopted in this study to formulate the integrated planning of repair operations and workforce training. This is due to the stationary behaviour of demand over the planning horizon. In other words, once the R & O contracts are signed in the beginning of planning horizon, the number of components that must be replaced under a PM policy or overhaul procedure can be calculated for the entire horizon depending on the due date of each contract. Hence, the decision-making process can be divided into two-stages that are distinguished in terms of the availability of information on the demand. The first-stage (present time) corresponds to the moment where the maintenance contracts for the upcoming planning horizon are not yet finalized; nevertheless, the facility needs to identify the required capacity of repair operations in addition to the on-job training schedule for the operators. The second-stage, on the contrary, represents the beginning of the planning horizon, where a more clear picture on the demand pattern (scenario) is revealed to the decision-maker. In this stage, several corrective actions are envisaged to protect the first-stage plan against random fluctuations in demand. Note that the second-stage decisions are defined for each period in the planning horizon and their value depends on the corresponding scenario. In a two-stage stochastic programming framework, the first-stage decisions need to be identified without full knowledge on the uncertainty. That is why, they are not indexed by scenarios. Contrarily, the second-stage (recourse) decisions are only determined once an actual scenario is observed, hence they are indexed by scenarios.

The first-stage tactical planning decision in a maintenance facility encompasses the number of components that should be repaired in each period in the planning horizon. The second-stage tactical planning decisions that are determined based on the demand outcome (scenario) in each period incorporate: i) the number of replaced components (ready for assembly)(this quantity is replenished from the internal inventory if the component is available in stock; Otherwise, the component would be back-ordered until the failed part is repaired/reconditioned); ii) the number of components available in stock; and iii) the shortage quantity of each component.

To perform the maintenance operations, the operators need valid certificate related to specific skills. The certificates are valid for a specific duration per skill set. Besides, after the certificate expiration date for a given task, the operator is not allowed to perform that task. In other words,

the repair operations on a component will be delayed if the certified operators are not available. Thus, the company needs to train the operators before the expiration date to maintain a sufficient number of certified operators. It is worth to mention that the company would incur a training cost per trainee that encompasses the cost of low productivity of operators during training sessions. The training is performed on the job on a one-by-one basis. In other words, there should be one certified operator available to train an operator whose certificate is expired. Further, to instruct each operator on a specific skill in a given period, there should be a demand for that skill during that period. We assume further that the duration of each training session does not exceed one period in the planning horizon. The (first-stage) training decisions correspond to the number of certified operators of different skill types and certificate ages available to work in each period, in addition to the number of operators of different skill types and certificate ages who are under training in each period in the planning horizon.
In order to minimise the delays in the delivery of repaired devices as a results of demand uncertainty, two corrective actions are envisaged. More precisely, it is assumed that the company has the option to temporarily increase the repair capacity via borrowing a limited number of certified operators for a given skill type in a specific period from another maintenance facility at a certain cost if enough certified operators are not available in that period. Hence, two additional second-stage decisions are defined as the number of operators to temporarily borrow for one period (month) along with the number of components that can be repaired in each period under each demand scenario after adding the additional temporary workforce capacity. Alternatively, the maintenance operations for a given component can be outsourced to another maintenance facility at a significantly higher cost compared to borrowing operators from that facility. The corresponding (second-stage) decision is, hence, defined as the number of components that should be outsourced to an external maintenance facility for inspection/repair in each period under each demand scenario. Without loss of generality, we are assuming that the transportation cost of repaired components by subcontractors is included in the unit outsourcing cost.

Based on the context and assumptions mentioned above, the company is looking for the optimal spare part inventory level, number of components to repair/replace, number of operators to train as well as the number of temporary operators to borrow, and the number of outsourced maintenance jobs in each period in the planning horizon. The goal is to minimise the total expected cost over all demand scenarios comprising of the replacement, repair, inventory, outsourcing, late delivery, training, and borrowing operators. The description of parameters and decision variables is provided in Tables 2.1 & 2.2.

Table 2.1: Sets, indices and parameters

| | |
|---|---|
| $T$ | The planning horizon indexed by $t$ |
| $R$ | The set of components indexed by $r$ |
| $S$ | The set of skills indexed by $s$ |
| $A_s$ | The set of possible certificate ages associated with skill $s$ indexed by $a$ (\|$A_s$\|=max$\{a$ in period 0$\}$+ \|T\|) |
| $\Omega$ | The set of all plausible scenarios indexed by $\omega$ |
| $pr(\omega)$ | The probability of scenario $\omega$ |
| $d_{r,t}(\omega)$ | The number of component $r$ scheduled to be replaced in period $t$ in scenario $\omega$ |
| $l_r^{rep}$ | The repair time of component $r$ |
| $SS_r$ | The safety stock of component $r$ |
| $M_{r,t}^{out}$ | Maximum capacity of external maintenance facility to repair component $r$ |
| $h_r$ | The unit inventory holding cost ($) of component $r$ |
| $k_r$ | The unit internal maintenance cost ($) of component $r$ |
| $c_r$ | The unit replacement cost ($) of component $r$ |
| $u_r$ | The unit outsourcing cost ($) of component $r$ |
| $b$ | The unit penalty cost ($) for late delivery of each piece of equipment |
| $n_s$ | The unit training cost ($) of operators per skill type $s$ |
| $q_s$ | The unit borrowing cost ($) of certified operators with skill type $s$ |
| $O_t$ | The available budget ($) of training in each period $t$ |
| $L_t$ | The available budget ($) for repair in each period $t$ |
| $P_t$ | The available budget ($) for outsourcing in each period $t$ |
| $\alpha_{r,s}$ | The labor consumption factor (number of required operators) for component $r$ per skill type $s$ in period $t$ |
| $e_s$ | The certificate validation period (months) of skill type $s$ |
| $G_s$ | The maximum allowable number of certified operators with skill type $s$ that can be borrowed in each period |
| $W_s$ | The total number of operators per skill type $s$ |
| $E_{a,s,0}$ | the initial number of $a$-months old certified operators per skill type $s$ in period 0 |

Table 2.2: Decision variables

| | |
|---|---|
| $Q_{r,t}(\omega)$ | The number of replaced components (ready for assembly) in period $t$ in scenario $\omega$ |
| $I_{r,t}(\omega)$ | The number of component $r$ available in stock in period $t$ in scenario $\omega$ |
| $Q_{r,t}^{rep}$ | The number of component $r$ that should be repaired in period $t$ |
| $Q_{r,t}^{temp}(\omega)$ | The number of component $r$ that can be repaired in period $t$ in scenario $\omega$ after adding additional temporary workforce capacity |
| $B_{r,t}(\omega)$ | The shortage quantity of component $r$ in period $t$ in scenario $\omega$ |
| $\theta_t(\omega)$ | The auxiliary variable representing the maximum quantity of shortages among all the components in period $t$ in scenario $\omega$ |
| $E_{a,s,t}$ | The number of $a$-months old certified operators per skill type $s$ in period $t$ |
| $R_{a,s,t}$ | The number of $a$-months old operators of skill type $s$ who are under training in period $t$ |
| $V_{s,t}(\omega)$ | The number of borrowed certified operators of skill type $s$ in period $t$ in scenario $\omega$ |

The objective function of the problem can be formulated as follows:

$$
\begin{aligned}
Minimise \sum_{t\in T}\sum_{r\in R} k_r Q^{rep}_{r,t} + \sum_{t\in T}\sum_{s\in S}\sum_{a\in A_s} n_s R_{a,s,t} + \sum_{\omega\in\Omega} p(\omega)(\sum_{t\in T}\sum_{r\in R} c_r Q_{r,t}(\omega) \\
+ \sum_{t\in T}\sum_{r\in R} h_r I_{r,t}(\omega) + \sum_{t\in T}\sum_{r\in R} k_r Q^{temp}_{r,t}(\omega) + \sum_{t\in T}\sum_{r\in R} u_r Q^{out}_{r,t}(\omega) \\
+ b\sum_{t\in T}\theta_t(\omega) + \sum_{t\in T}\sum_{s\in S} q_s V_{s,t}(\omega))
\end{aligned}
\tag{2.20}
$$

The objective function 2.20 incorporates the cost of conducting repair operations under regular capacity and training of operators (first-stage cost) along with the expected cost of replacing components, inventory, repair operations conducted by extra operators borrowed from other facilities, outsourcing, shortage, and borrowing of operators from other facilities. Three category of constraints namely operations, workforce and domain constraints are taken into consideration.

***Operations constraints***

$$Q_{r,t}(\omega) - B_{r,(t-1)}(\omega) + B_{r,t}(\omega) = d_{r,t}(\omega) \qquad \forall t \in T, \forall r \in R, \forall \omega \in \Omega \tag{2.21}$$

$$Q^{rep}_{r,(t-l^{rep}_r)} + Q^{out}_{r,t}(\omega) + Q^{temp}_{r,(t-l^{rep}_r)}(\omega) + I_{r,(t-1)}(\omega) - I_{r,t}(\omega) = Q_{r,t}(\omega) \qquad \forall t \in T\backslash 1,...,l^{rep}_{rt}, \forall r \in R, \forall \omega \in \Omega \tag{2.22}$$

$$\theta_t(\omega) \geq B_{r,t}(\omega) \qquad \forall t \in T, \forall r \in R, \forall \omega \in \Omega \tag{2.23}$$

$$\sum_{r \in R} \alpha_{r,s} Q^{rep}_{r,t} \leq \sum_{a=1}^{e_s} E_{a,s,t} \qquad \forall t \in T, \forall s \in S \tag{2.24}$$

$$\sum_{r \in R} \alpha_{r,s} Q^{temp}_{r,t}(\omega) \leq \ V_{s,t}(\omega) \qquad \forall t \in T, \forall s \in S, \forall \omega \in \Omega \tag{2.25}$$

$$Q^{out}_{r,t}(\omega) \leq M^{out}_{r,t} \qquad \forall t \in T, \forall r \in R, \forall \omega \in \Omega \tag{2.26}$$

$$I_{r,t}(\omega) \geq SS_r \qquad \forall t \in T, \forall r \in R, \forall \omega \in \Omega \tag{2.27}$$

$$\sum_{r \in R} k_r Q^{rep}_{r,t} \leq L_t \qquad \forall t \in T \tag{2.28}$$

$$\sum_{r \in R} u_r Q^{out}_{r,t}(\omega) \leq P_t \qquad \forall t \in T, \forall \omega \in \Omega \tag{2.29}$$

$$Q^{rep}_{r,t} = 0 \qquad \forall t \in 1,...,l^{rep}_{rt}, \forall r \in R \tag{2.30}$$

$$Q^{temp}_{r,t,\omega} = 0 \qquad \forall t \in 1,...,l^{rep}_{rt}, \forall r \in R, \forall \omega \in \Omega \tag{2.31}$$

Constraints (2.21)-(2.31) represent various limitations related to operations planning. Constraints (2.21) imply the balance between the number of components scheduled to be replaced, the actual number of replaced components, and the shortages in two consecutive periods. Constraints (2.22) is the inventory balance constraint for components. It ensures that the inventory level of each component in each period equals to its inventory in previous period minus the number of repaired components (by using regular and temporary workforce) and outsourced components while considering the internal repair times into consideration.It is assumed that the outsourced components are received in the same period via emergency shipment from another maintenance facility. Constraints (2.23) calculate the amount of shortage of components in each period. It should be mentioned that the number of delayed repaired devices is calculated at the component level. In other words, the component with the highest amount of shortage (due to unavailability of spare part or the certified operators) determines the total number of devices that cannot be delivered in-time. Constraints (2.24) limits the total number of components that can be repaired in each period based on the number of available certified operators. Constraints (2.25) limits the total number of components that can be repaired in each period by temporary certified operators who are borrowed from other facilities. Constraints (2.26) state the maximum capacity of other facilities in replenishing outsourced components. Constraints (2.27) ensure the safety stock in each period. Constraints (2.28) and (2.29) imply the maximum available budget for repair operations and outsourcing. Constraints (2.30) and (2.31) set the initial values of repair decisions for both regular and temporary workforce.

***Workforce constraints***

$$E_{(a+1),s,t} = E_{a,s,(t-1)} - R_{a,s,(t-1)} \qquad \forall t \in T, \forall s \in S, \forall a \in A_s \tag{2.32}$$

$$E_{1,s,t} = \sum_{a \in A_s \setminus 1} R_{a,s,(t-1)} \qquad \forall t \in T, \forall s \in S \tag{2.33}$$

$$\sum_{a \in A_s \setminus 1} R_{a,s,t} \leq \sum_{r \in R} Q^{rep}_{r,t} \qquad \forall t \in T, \forall s \in S \tag{2.34}$$

$$\sum_{a \in A_s \setminus 1} R_{a,s,t} \leq \sum_{a=1}^{e_s} E_{a,s,t} \qquad \forall t \in T, \forall s \in S \tag{2.35}$$

$$\sum_{a \in A_s} E_{a,s,t} + \sum_{a \in A_s \setminus 1} R_{a,s,t} = w_s \qquad \forall t \in T, \forall s \in S \tag{2.36}$$

$$V_{s,t}(\omega) \leq G_s \qquad \forall t \in T, \forall s \in S, \forall \omega \in \Omega \tag{2.37}$$

$$\sum_{s \in S} \sum_{a \in A_s} n_s R_{a,s,t} \leq O_t \qquad \forall t \in T \tag{2.38}$$

Constraints (2.32) track and update the certificate age of the operators and ensure the certified operators' balance in two consecutive periods. In other words, if the company trains the operators with age $a$ in period $t$, the number of certified operators with age $a + 1$ decreases in the next period. Constraints (2.33) state that the number of operators under training in each period is equal to the number of certified operators with age 1 in the next period. Given that the training on a certain skill can only be carried out only if a part that requires the same skill has been scheduled in a given period, Constraints (2.34) and (2.35) state the number of operators under training should be less than the number of available components under inspection and certified operators associated with each skill, respectively. Constraints (2.36) is the flow constraints and ensure the balance between the total number of employees, number of certified and uncertified operators, in addition to the number of operators under training. Constraints (2.37) state the allowable number of certified operators that the company can borrow from other maintenance providers. Constraints (2.38) imply the maximum available budget for training in each period.

***Domain constraints***

$$Q^{rep}_{r,t}, E_{a,s,t}, R_{a,s,t}, Q_{r,t}(\omega), I_{r,t}(\omega), B_{r,t}(\omega), \theta_t(\omega), Q^{temp}_{r,t}(\omega), Q^{out}_{r,t}(\omega) \in \mathbb{Z}^+ \quad \forall t \in T, \forall s \in S, \forall a \in A_s, \forall r \in R, \forall \omega \in \Omega \tag{2.39}$$

## 2.5 Numerical results

The computational experiments in this section are conducted on a case study in the context of gas turbine. Our objectives are to: (1) analyze the impacts of including training decisions in the operations planning model; (2) investigate the role of outsourcing and borrowing strategies on annual penalty and total costs; (3) asses the impact of fluctuations in demand, inspection and replacement time, labor consumption factor, certificate validation period, repair cost and safety stock; and (4) compare the efficiency of the plan obtained from the $2SP$ model with a deterministic plan under different stochastic settings (demand patterns). In particular, the goal is to investigate the role of outsourcing and borrowing strategies on the total cost as a response to fluctuations in demand. The key performance indicators in all tests incorporate the cost of repair, holding, outsourcing, penalty, training, borrowing, in addition to the annual total cost, the number of trainees, and the number of operators with expired certificates at the end of the planning horizon. The proposed model is solved by CPLEX 12.8 run on a computer equipped with an Intel Core i7 at 3.4 GHz and 8GB of RAM under Windows seven system. In what follows, we first present the details of the case study. Afterwards, we present the results of sensitivity analysis on the aforementioned parameters.

### 2.5.1 Case study

In the context of gas turbine engines, maintenance service providers are dealing with different engines (industrial, trent, jet engines, etc.). The main modules common in any gas turbine engines are the compressors, combustors, and turbines. The basics types of compressors are centrifugal flow and axial flow ones, and both are responsible for compression of the air before expansion in the turbines. The main components of a centrifugal flow compressor are impellers and a diffusers. Besides, the main parts of an axial flow compressor are rotor, blades, and stator vanes. The combustor is responsible for burning fuel (supplied from fuel spray nozzles) with air (supplied by the compressor) with maximum heat release and minimum loss in pressure. The main components of combustors are combustion chamber, injectors, vaporizer, and nozzles. The turbines are responsible for supplying the power to drive the compressor and accessories. The basic components of the turbine are the combustion discharge nozzles, nozzle guide vanes, discs, and blades (Soares, 2015). The maintenance operators working with these modules need valid certificates on certain skills (competencies) such as inspection and disassembly/assembly. An operator associated with a given category of skills could have certificates on all associated sub-categories. For example, the inspection skill could encompass main build inspection and conformance check, while the disassembly/assembly skill could contain calibration and balancing competencies of each module.

We investigate a maintenance facility that receives a specific kind of gas turbine engine consisting of three major modules. Also, we only consider five expensive and critical repairable components with higher failure rates in these modules. The number of repairable components scheduled to be replaced $d_{r,t}$ is changing on the interval of $(0, 5)$. The holding costs and safety stock of components of all modules are considered as $h_r = 40$ and $SS_r = 1$, respectively. 12 operators with different certificate ages are assigned to each skill type (inspection and disassembly/assembly) at the beginning of planning horizon. The planning horizon consists of 12 months and the other parameters are set as $l_r^{rep} = 1$, $l_r^{out} = 1$, $L_t = 3000$, $O_t = 3000$, $P_t = 1500$, $k_r = 150$, $u_r = 600$, $n_s = 50$, $q_s = 300$, $b = 1000$, $M_{r,t}^{out} = 20$, $e_s = 9$, $G_s = 3$, and $\alpha_{r,s} = 1$.

The above numerical values have been inspired by a real industrial case. Using CPLEX optimization software, for all the problem instances considered, CPU time is less than two seconds.

### 2.5.2 Demand scenarios

In this study, the uncertain demand is approximated as a set of subjective scenarios based on experts' opinion and historical data. More specifically, we assume that the number of devices that are received during the upcoming planning horizon are featured by a cyclic pattern. We generate all possible scenarios by considering four different stochastic settings. In setting 1, 9 scenarios have been generated by considering low, average and high demand with variations in the range of $+/-$ 15% of the average value on a 6-month cyclic pattern. More precisely, the demand for the first six months is set as low (15% less than average), average, or high (15% above average; whereas the demand over the next six month might remain the same or change. For instance, the demand for the first cycle (6-month) can be low and it can remain low or it can change to average or high in the second cycle. This leads to 9 possible demand scenarios over the planning horizon. Setting 2 is similar to the first setting except that the variation range of demand is within $+/-$ 30% of the average value. In settings 3 and 4, 27 scenarios have been generated by considering low, average, and high demand that, respectively vary between $+/-$ 15% and $+/-$ 30% of the average value on a 3-month cyclic pattern. For instance, the demand in the first, second, third, and forth cycles (3-month) can follow a pattern similar to "low, low, low, low" or "low, low, average, low", etc. Without loss of generality, we assume equal probabilities for the mentioned scenarios. Note that settings 1 and 3 are similar in terms of demand variability level ($+/-$ 15% of the average value) and distinguished in terms of seasonality pattern. In the same vein, settings 2 and 4 represent higher variability of demand over the average value albeit at different seasonality levels.

### 2.5.3 Sensitivity analysis on the deterministic model

The sensitivity analysis of changes in demand $d_{rt}$ is presented in Table 2.3. The results in this table correspond to the increase and decrease in base values by 30 % and 15%. As expected, by increasing $d_{r,t}$, the penalty, training, borrowing and total cost increase. Also, it can be remarked that for low demand, the outsourcing and borrowing strategies are not much effective. On the contrary, these strategies play an important role in reducing shortages (penalty cost) at high demand values.

Table 2.3: Sensitivity analysis on changes in demand of repairable components $d_{rt}$

| $d_{rt}$ | Repair cost ($) | Holding cost ($) | Outsourcing cost ($) | Penalty cost ($) | Training cost($) | Borrowing cost ($) | Number of trainees | Number of expected workers | Annual total cost ($) |
|---|---|---|---|---|---|---|---|---|---|
| **-%30** | 5,850 | 3,800 | 0 | 0 | 600 | 0 | 12 | 24 | 10,250 |
| **-%15** | 8,250 | 3,200 | 4,800 | 0 | 800 | 0 | 16 | 24 | 17,050 |
| **0** | 7,350 | 3,240 | 10,800 | 1,000 | 800 | 2,400 | 16 | 20 | 25,590 |
| **+%15** | 8,100 | 3,280 | 10,200 | 4,000 | 800 | 9,000 | 16 | 24 | 35,380 |
| **+%30** | 8,100 | 2,800 | 11,400 | 46,000 | 1,100 | 13,200 | 22 | 2 | 82,600 |

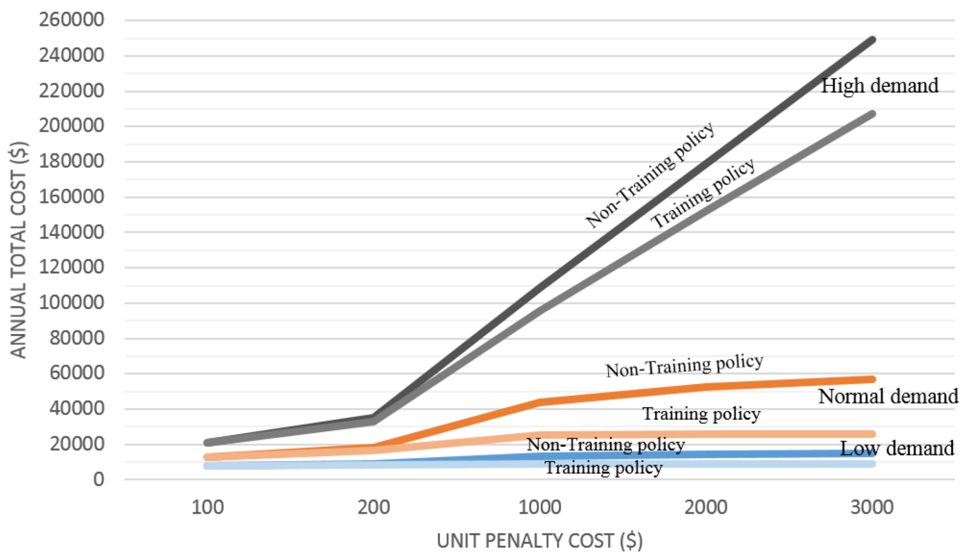


Figure 2.2: Impact of training on the total annual cost under three demand patterns

To investigate the effectiveness of training, the model was evaluated under different demand patterns: (a) low-demand (-%30), (b) normal-demand and (c) high-demand (+%30) under two policies: 1) non-training and 2) training. The trend of total annual cost and the penalty cost under the three demand patterns and the two training policies is provided in Fig. 2.2 and Fig. 2.3, respectively. The results indicate that adopting a training policy decreases the annual penalty and total costs under all three demand patterns. However, for unit penalty cost $b$ close to the training cost, the training policy is not much effective, and the cost saving is rather negligible (less than 1 percent). Besides, the outsourcing and borrowing policies are not effective for low unit penalty cost and low-demand scenarios. On the contrary, by increasing $b$, the total cost under a training policy is much lower than the non-training policy. In particular the cost saving for high-demand values and unit penalty cost is more significant.

We next conduct sensitivity analysis on the certificate validation period (months) $e_s$. When $e_s$ is small (e.g., 3 months), the system faces a large penalty cost due to the lack of certified operators.

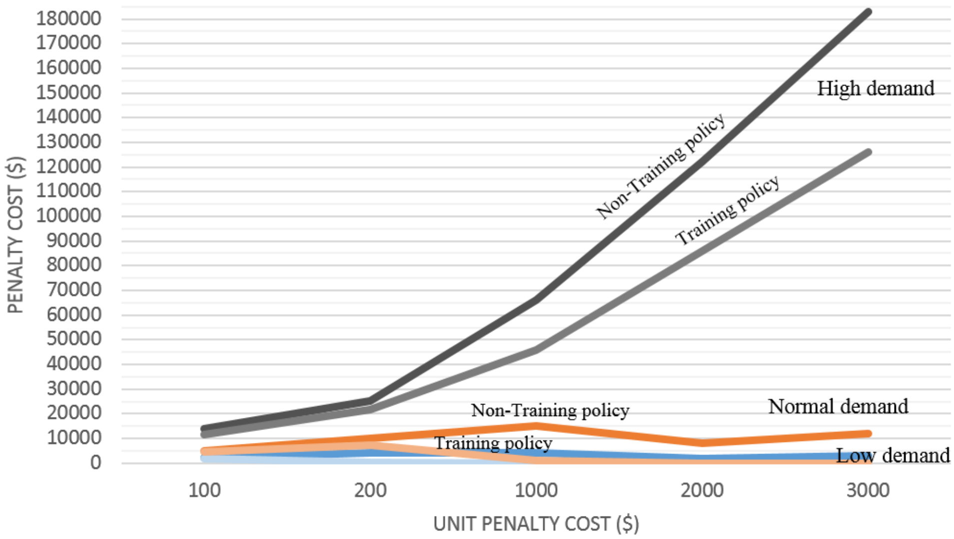


Figure 2.3: Impact of training on the penalty cost under three demand patterns

By increasing the duration of validity, the maintenance facility has more certified operators in each period. Therefore, the company can repair and store more components due to increased capacity. Thus, repair and holding costs of the system increase at higher validation periods. In the same vein, as the maintenance facility has enough capacity, the penalty cost is also reduced and as the facility is not dependent on other maintenance providers, the outsourcing and borrowing costs decrease. Further, due to increased validation period of certificates, the operators are trained less frequently; hence the training cost and the number of trainees decrease. Also, as the unit borrowing cost $n_s$ is less than unit outsourcing cost $u_r$, the borrowing cost decreases drastically, but the outsourcing cost first increases smoothly and then decreases. Fig. 2.4 represents the trade-off between repair, training, outsourcing and borrowing costs by increasing $e_s$.

The results of sensitivity analysis on the inspection and replacement time of repairable components $l_r^{rep}$ indicate that by increasing $l_r^{rep}$, the penalty and total costs increase. Also, the holding and training costs and the number of trainees are decreasing as well. Besides, the outsourcing and borrowing costs are increasing for small to medium inspection and replacement times. Nevertheless, for $l_r^{rep}$ more than two months, these strategies are not much effective; thus, the outsourcing and borrowing costs are decreasing. Fig. 2.5 represents the trade-off between repair, training, outsourcing and borrowing costs by increasing $l_r^{rep}$.

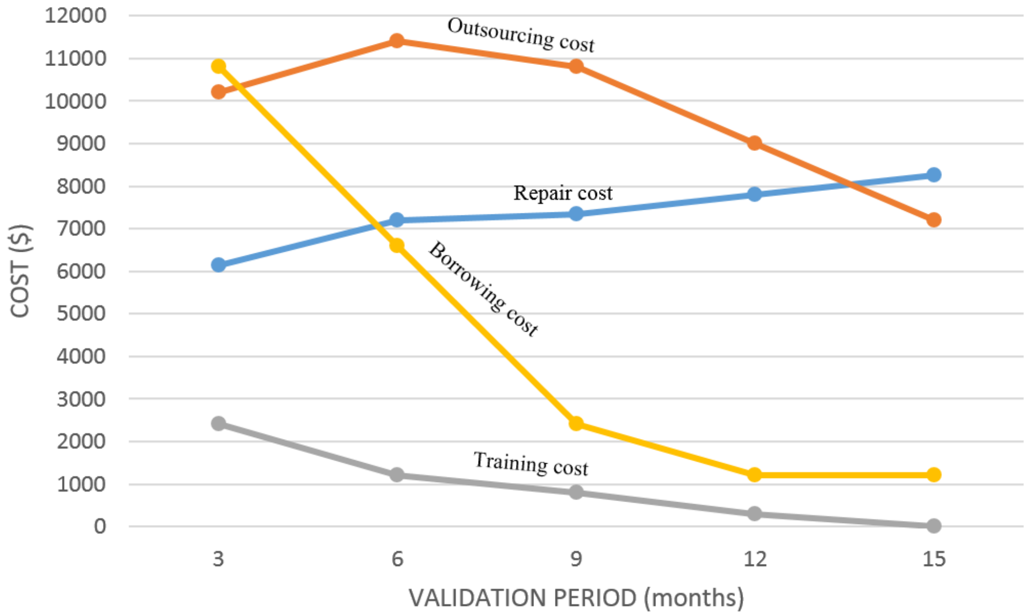


Figure 2.4: Trade-off between repair, training, outsourcing and borrowing costs by increasing $e_s$

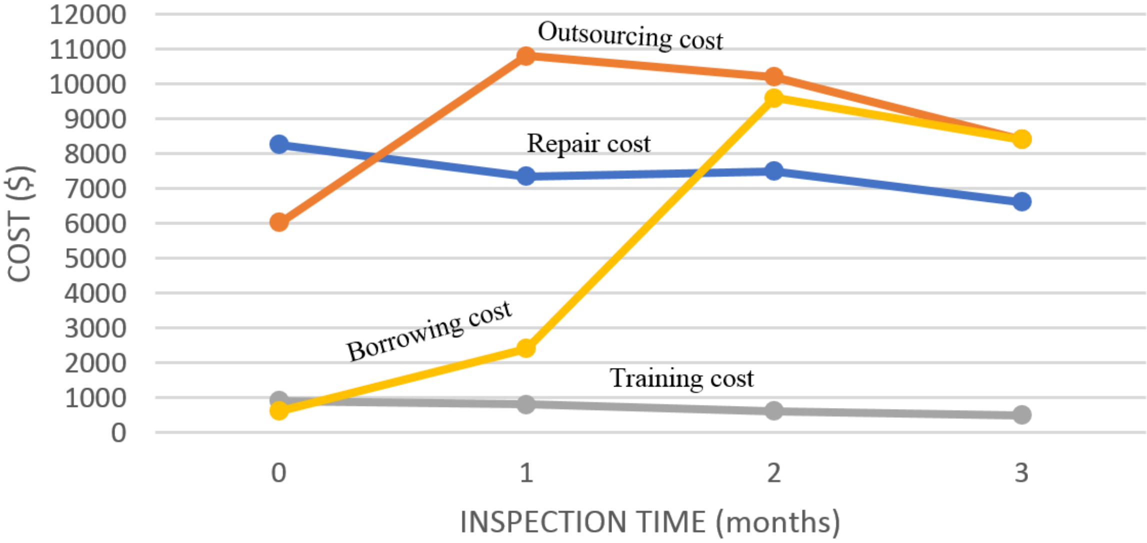


Figure 2.5: Trade-off between repair, training, outsourcing and borrowing costs by increasing $l_r^{rep}$

The sensitivity analysis on the unit outsourcing cost $u_r$ indicate that for the lower amount of $u_r$, the borrowing cost is zero. Vice versa, for the higher amount of $u_r$, the outsourcing cost is zero. Thus, there is a trade-off between borrowing and outsourcing cost. Besides, by increasing $u_r$, the company decides to repair the components internally which increases the repair cost. Furthermore, the outsourcing strategy does not have any significant impact on training and holding costs. Fig. 2.6 represents the trade-off between repair, training, outsourcing and borrowing costs by increasing $u_r$.

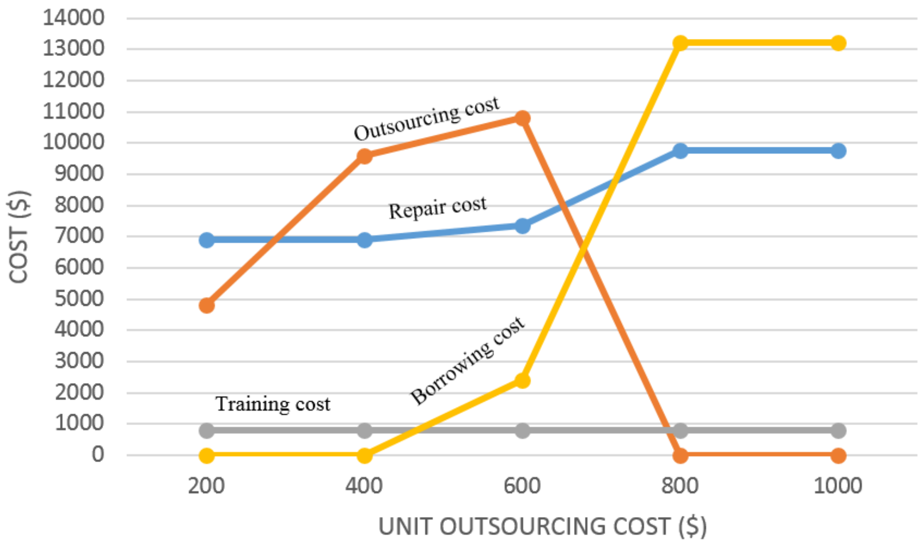


Figure 2.6: Trade-off between repair, training, outsourcing and borrowing costs by increasing $u_r$

The sensitivity analysis on changes in labor consumption factor $\alpha_{r,s}$ reveal that by increasing $\alpha_{r,s}$, the training and total costs increase. For lower values of $\alpha_{r,s}$, as the company has enough capacity, the outsourcing and borrowing strategies are not effective. However, for higher values of $\alpha_{r,s}$, outsourcing and borrowing strategies play an important role in controlling the penalty and total costs due to reduced internal capacity. Besides, by increasing $\alpha_{r,s}$, the repair operations are performed externally (outsourcing), and the repair cost decreases. Fig. 2.7 represents the trade-off between repair, training, outsourcing and borrowing costs by increasing $\alpha_{r,s}$.

With increasing the repair cost $k_r$, the company decided to outsource the repair operations and as a result, outsourcing cost increases. Besides, when $k_r$ is high, it's not beneficial for the system to borrow certified operators and therefore the borrowing cost decreases. It can be concluded that for small changes in $k_r$, the holding cost, penalty cost, training cost, number of trainees and

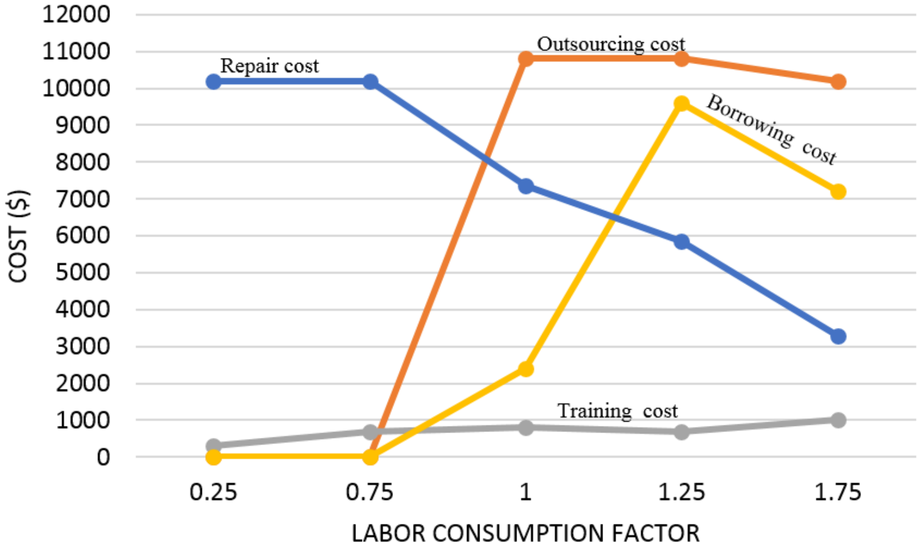


Figure 2.7: Trade-off between repair, training, outsourcing and borrowing costs by increasing $\alpha_{r,s}$

expired operators remain constant. However, by increasing $k_r$ drastically, the average penalty cost increases and the average training cost decreases. Fig. 2.8 indicates the trade-off between repair, training, outsourcing and borrowing costs with increasing $k_r$.

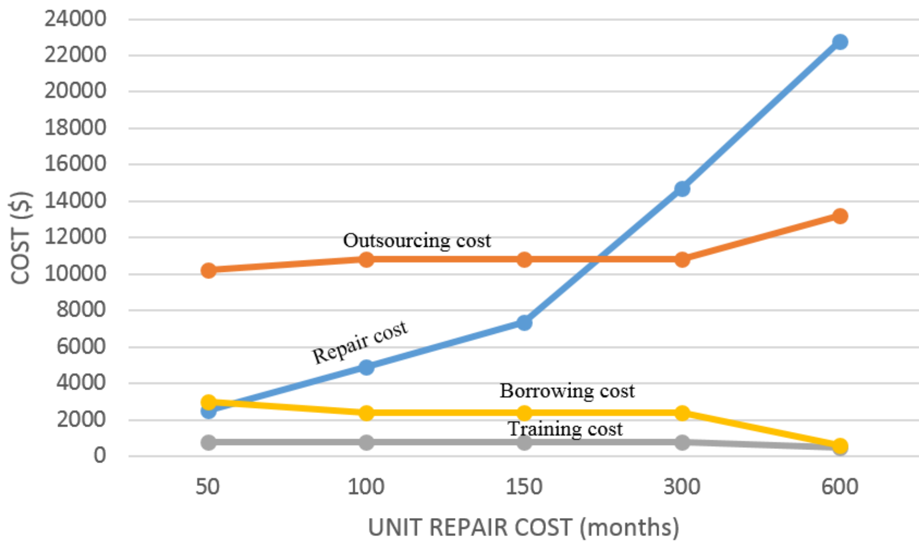


Figure 2.8: Trade-off between repair, training, outsourcing and borrowing costs with increasing $k_r$

The sensitivity analysis of changes in safety stock $SS_r$ is presented in Fig. 2.9. This sensitivity analysis reveals that increasing $SS_r$ does not have any significant impact on the training cost. However, with increasing $SS_r$, the repair, holding and borrowing costs increase. Recall from the model that the reason of shortages is related to lack of certified operators and/or lack of components. To increase the safety stock, the company has to borrow more certified operators to perform repair operations, and the borrowing cost increases. Also, we remark that when the value of $SS_r$ increases, the company perform the repair operations internally and the outsourcing cost decreases.

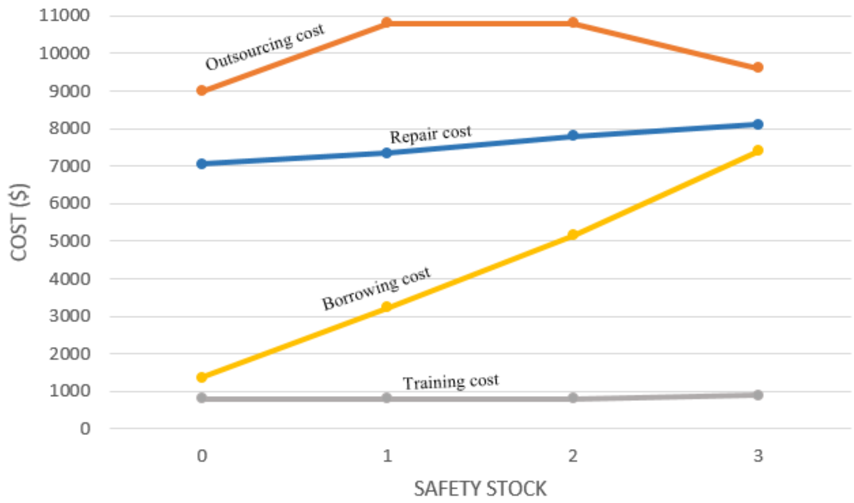


Figure 2.9: Trade-off between repair, training, outsourcing and borrowing costs with increasing $SS_r$

### 2.5.4 Value of stochastic solution

In this section, we aim to compare the performance of the plan obtained from the stochastic model (2.20)-(2.39) with the one determined by a deterministic model, where the expected value of demand is taken into consideration. It is worth noting that the deterministic model does not include the two mitigation strategies (i.e., outsourcing and borrowing). This can be justified by the fact that the average value of historical demand volumes are usually balanced with the capacity of the facility in real-life applications. In order to compare the cost-efficiency of the mentioned plans, we measure the Value of Stochastic Solution ($VSS$) for different stochastic settings introduced in section 2.5.2. Let us denote the optimal objective function value of the stochastic model as $2SP$ and the expected cost of deterministic solution as $EDS$. The latter is calculated via plugging the optimal solution of the deterministic model ($Q^{rep}_{r,t}, E_{a,s,t}, R_{a,s,t}$) as the first-stage decisions into model (2.20)-(2.39) without considering outsourcing and borrowing strategies. $VSS$ is then calculated as the difference between the objective function value of $EDS$ and $2SP$ models, as indicated in the following equation:

$$VSS(\%) = \frac{EDS - 2SP}{EDS} 100 \tag{2.40}$$

The results are summarized in Table 2.4, where the deterministic solution, $EDS$ and $2SP$ for all the four settings are compared in terms of repair, outsourcing, borrowing, penalty cost of late delivery, and total cost. Based on the total cost of $EDS$ and $2SP$, the $VSS(\%)$ is calculated in the last column. As it can be observed in the row corresponding to the deterministic model, the main portion of the total cost is associated with the cost of conducting repair operations. Penalty cost of late delivery is less significant given that the expected values of demand has been taken into consideration. This is also due to the fact that the capacity of the maintenance facility is generally enough to respond to the average value of demand. Nonetheless, according to the $EDS$ row, implementing the deterministic plan under random demand would result in significant amount of shortage of components and penalty cost of late delivery.

Similar to the deterministic model, the cost of conducting repair operations constitutes the major portion of total cost in the $2SP$ model. On the contrary, the stochastic model yields significantly lower penalty cost due to the shortage of components. This is due to adopting borrowing and outsourcing strategies in this model. Another interesting observation is that both deterministic and stochastic models used the maximum capacity of repair operations based on the results provided in “Repair Cost” column.

Table 2.4: The results of deterministic, $2SP$ and $EDS$ models

| Setting | Model | Repair cost ($) | Penalty cost ($) | Outsourcing cost ($) | Borrowing cost ($) | Total cost ($) | VSS (%) |
|---|---|---|---|---|---|---|---|
| Deterministic | | 15,900 | 1,500 | - | - | 34,120 | |
| Setting 1 | EDS | 15,916 | 15,500 | - | - | 48,681 | 31 |
| | 2SP | 16,150 | 833 | 2,533 | 400 | 37,103 | |
| Setting 2 | EDS | 16,783 | 40,167 | - | - | 74,934 | 59 |
| | 2SP | 12,766 | 3,000 | 3,866 | 2,466 | 47,077 | |
| Setting 3 | EDS | 15,916 | 15,389 | - | - | 48,529 | 31 |
| | 2SP | 16,144 | 444 | 2,933 | 377 | 36,845 | |
| Setting 4 | EDS | 16,744 | 39,056 | - | - | 73,745 | 60 |
| | 2SP | 17,016 | 2,277 | 3,888 | 2,666 | 45,948 | |

Finally, the comparison of $VSS$ values between stochastic settings 1 and 2 (and respectively 3 and 4) reveal that the demand variation has a high impact on $VSS$. In other words, the stochastic model significantly outperforms the deterministic one when increasing the demand variability. Nevertheless, the comparison between the first and third (and respectively second and forth) stochastic settings, showcases the negligible impact of demand seasonality on the $VSS$.

### 2.5.5 Managerial insights

The focus of this research is on improving the efficiency of maintenance logistics networks. For instance, given a PM contract between the user and the maintenance service provider, adopting this model guaranties the contract terms via improving the reliability and efficiency of maintenance operations. Based on the result of sensitivity analysis, the following managerial insights can be derived.

1. With the integration of training decisions with other operational decisions, the maintenance facilities can reduce the annual penalty and total costs up to 40 percent especially when the unit penalty cost is high.

2. Outsourcing and borrowing are useful strategies to mitigate the risk of shortages at a high rate of demand for the repaired components. The maintenance facilities can decrease their total costs by exerting an appropriate outsourcing and borrowing contracts with other maintenance facilities.

3. Fluctuations of demand, inspection and replacement time, and labor consumption factor have the highest impact on KPIs. As a consequence, it is recommended to hire more certified operators to hedge against high demand volumes and extended repair times.

## 2.6 Conclusion and future research

In this paper, we analyzed the impact of incorporating the training plan into other tactical planning decisions in a maintenance facility that offers various maintenance services to the users of technical devices with a modular structure. Further, our contribution is unique in the sense that the limited age of operators' certificate, the on-by-one, and on-job training aspects have not been previously investigated in the training planning problems in the literature. Our case study demonstrated the effectiveness of adopting a training policy in reducing the penalty cost of late delivery and the total cost up to 30%. This impact is expected to be more significant in other stochastic settings where the variability of demand is higher. Besides, our results revealed that adopting a deterministic decision model, where the expected volumes of demand are taken into consideration can lead to significant amounts of penalty cost as a result of late delivery of repaired devices. In particular, under demand patterns with high levels of variability, the gap between the cost of stochastic and deterministic plans can reach up to 60%. Our experimental results also revealed a trade-off between unit outsourcing and borrowing cost. More precisely, under high outsourcing costs, the maintenance facility chooses to borrow an operator to perform the repair operations internally.

It is noteworthy that implementing the proposed decision model in maintenance facilities is expected to improve the efficiency of maintenance operations. In other words, adopting an optimal training plan while scheduling maintenance jobs along with the borrowing/outsourcing strategies would increase the capacity of operations in such facilities. Under a PM contract between the user and the maintenance service provider, adopting the proposed model would guaranty the realization of contract terms as a result of increased capacity and improved efficiency. As a consequence, this would positively impact the availability and reliability of the fleet of equipment at the user's site. The proposed structure for workforce constraints by considering the limited age of operators' certificate can be exerted in other tactical planning problems such as production, service industries and health care systems, where the operations must be carried out by certified operators due to safety regulations and product standards.

In this study, we assumed a full collaboration between the maintenance facility under investigation and its competitors in terms of sharing resources (operators and facility). Whereas, in practice, the external facilities are not committed to accept outsourced repair jobs and/or share their resources unless an incentives mechanism are put in place. Designing a collaboration mechanism among such service providers is an interesting avenue of research. We also assumed that the maintenance policy is already defined. For the industrial context motivating our study, this assumption is realistic because maintenance contracts (between the user and the maintenance service provider) are designed at the strategic level (long term planning decisions), whilst we rather confine our attention in this paper to the tactical level decisions (mid-term planning).

# Chapter 3

# An approximate decomposition algorithm for multi-item tactical planning with independent random parameters

This chapter corresponds to the following journal paper:

Shayan Tavakoli Kafiabad, Masoumeh Kazemi Zanjani, and Mustapha Nourelfath. ”An approximate decomposition algorithm for multi-item tactical planning with independent random parameters”. Revision submitted to Computers and Operations Research, June 2021.

## Abstract

Integrated planning of production and workforce training is a challenging task in a multi-product setting where the demand and/or production lead-time of different items are independent random variables. With a particular focus on maintenance facilities, this study proposes a multi-stage stochastic programming (MSP) model for integrated production and workforce planning under independent random repair times of faulty components. An approximate decomposition algorithm, based on Lagrangian relaxation approach, is also developed to efficiently solve the problem for real-size instances. This algorithm relies on decomposing the MSP model into sub-models corresponding to component scenario trees and coordinating them via a sub-gradient algorithm to obtain a high-quality feasible solution. Our numerical experiments conducted on a range of problem instances demonstrate the significant value of incorporating repair time uncertainty in this problem setting and the effectiveness of the proposed solution methodology in overcoming the computational complexity.

## 3.1 Introduction

Businesses are involved with medium-term decision-making (also known as tactical planning) problems on a regular basis while confronting various sources of uncertainty. In particular, we may refer to the planning of production quantity, inventory level, workforce level, logistics, and training requirements in a multi-product setting under uncertain demand, production/procurement lead-time (LT), and resource availability. Given the multi-period structure of such problems, the above uncertain parameters are, in general, featured with a dynamic behaviour over the planning horizon; hence they are commonly modeled as a scenario tree (ST). The latter is a viable way of discretizing a dynamic stochastic process over time. It incorporates a number of stages, each including a set of uncertain outcomes, represented as nodes with a certain probability. Each path from the root node (present time) to the leaf node (e.g. last period in the planning horizon) in a ST captures the evolution of all information trajectories over time. The multi-stage stochastic programming (MSP) approach, accordingly, can be explored as a prominent approach in formulating these problems when incorporating the underlying scenario trees corresponding to uncertain parameters. This approach relies on defining the decision variables for the nodes of the ST and optimizing the expected value of the objective function over all nodes. Nevertheless, such MSP models suffer from the curse of dimensionality as the underlying scenario trees grow exponentially in size when dealing with multiple independent random processes corresponding to each item (product). More precisely, the MSP model is formulated based on a large-scale master ST as a result of merging STs corresponding to each product. This makes the MSP model computationally intractable when dealing with realistic-scale tactical planning problems that incorporate 6-12 periods (stages).

This study is motivated by integrated production and workforce planning (IPWP) in the context of maintenance facilities responsible for the repair and overhaul of complex devices such as gas turbines. Due to the technological limitations and large number of spare parts required to maintain such devices, the users prefer to outsource the maintenance jobs to an external service provider (Basten & Van Houtum, 2014). Nevertheless, such facilities are expected to abide by stringent safety regulations and delivery deadlines. In other words, as a result of quality non-conformities and late delivery of repaired devices, they would incur significant amounts of penalty. This is mainly due to the negative impact of such phenomena on users' system availability, e.g., availability of gas turbines for power generation in utility companies.

The major operations in maintenance facilities encompass the preliminary inspection of devices followed by their disassembly (into their components) and replacing the defective items with the functional ones from the inventory of repaired components. All operations in such facilities are carried out by a fleet of certified operators who are only qualified to perform a subset of inspection and repair operations. Furthermore, some industries, such as gas turbine maintenance centers require the periodic re-training of the workforce on different skills according to an on-job basis by available certified operators. In other words, they issue certificates with a limited validation period (2-3 years) after each training session. This necessitates the aggregated planning of training and production so as to balance the operations and workforce levels and ultimately avoid the delayed delivery of repaired devices. In other words, the IPWP problem aims to determine the optimal quantity of repair operations along with the training schedule in each period over a (medium term) planning horizon at the minimum cost. The above decisions must abide by two main groups of constraints, namely operations and training constraints in order to adjust the balance between repair quantities, inventory/backorder, and workforce levels while respecting specific training protocols.

It is worth to mention that the repair time of some complex components (e.g., rotors in gas turbines) is an uncertain parameter that depends on the skill level of operators, age, and condition of devices. Repair time randomness may delay the delivery of devices to the users, incurring high penalty costs to the maintenance facilities. Thus, this uncertain factor must be incorporated into tactical planning decisions in order to reduce the risk of late delivery. To the extent of the authors' knowledge, no prior work in the literature focus on integrating workforce training with production decisions in maintenance facilities by incorporating the repair time uncertainty. To fill this gap, our first contribution revolves around addressing the IPWP problem by considering the random repair time of different components involved in these modular-structured devices. In this study, components repair times are assumed to be independent random stochastic processes that can be modeled as STs. This is a realistic assumption given that the age and condition of different components in such devices (e.g., nozzles and rotors in gas turbines) are usually independent of each other. The mentioned STs are then merged together to form a master ST, which grows exponentially in size as the number of components and stages (periods) increases. The IPWP problem is accordingly formulated as a multi-stage stochastic mixed-integer programming (MS-MIP) model, which is computationally intractable for problem instances with multiple components and a planning horizon comprising of 6-12 periods.

Our second contribution is focused on developing an efficient decomposition algorithm to overcome the computational complexity of the proposed MS-MIP model for IPWP problem. This algorithm relies on decomposing the MS-MIP model, formulated based on the master ST, into component sub-models by relaxing the binding workforce capacity constraint. The sub-models are then coordinated into a feasible solution by adding the Lagrangian penalty terms in their objective function and implementing the sub-gradient algorithm. Nevertheless, component sub-problems are *per se* intractable MS-MIP models given the exponentially large number of nodes in the master ST. Hence, an approximation algorithm is proposed that relies on formulating component sub-models based on their corresponding ST rather than the master ST. Afterwards, the Lagrangian penalty terms are also approximated in each sub-model by considering the other sub-models' solutions. It is noteworthy that the proposed approximate decomposition algorithm provides a lower bound (LB) to the optimal objective function value of the MS-MIP model. An efficient repair mechanism is also developed to obtain a high-quality solution from the converged solution of this algorithm.

The proposed solution methodology in this study is an original contribution that can be applied to other multi-component, multi-period tactical planning models under uncertainty as demonstrated in a set of numerical experiments.

The rest of this paper is organized as follows. A review of recent literature is provided in Section 3.2. In Section 3.3, the problem description and the MSP model are presented. In Section 3.4, the solution methodology is provided. In section 3.5, the numerical results are presented. Finally in Section 3.6, conclusions and directions for future research are provided.

## 3.2 Literature review

In this section, a brief review of the most pertinent modeling and algorithmic approaches for addressing tactical planning under uncertainty is primarily provided. Afterwards, we confine our attention to the prevailing research on production and workforce planning under uncertainty in maintenance facilities that is the main focus of the current study.

### 3.2.1 Tactical planning under uncertainty

Multi-stage stochastic mixed-integer programming (MS-MIP) (Birge & Louveaux, 2011) has been widely applied for various tactical planning problems under uncertain parameters (e.g., supply and demand) that are featured with a dynamic behavior over the planning horizon. For instance, Kazemi Zanjani, Nourelfath, and Ait-Kadi (2010) propose an MSP model for sawmill production planning under demand uncertainty. An MSP model is developed in (Körpeoğlu, Yaman, & Aktürk, 2011) for master production planning problem in a production environment with uncertain demand, controllable processing times, and a nonlinear profit function. Guan and Philpott (2011) propose a quadratic multi-stage stochastic programming model for production planning under supply and demand uncertainty in the context of New Zealand dairy industry. A dynamic outer-approximation sampling algorithm is applied to efficiently solve the MSP model. A two-stage stochastic programming model is proposed in (Megahed & Goetschalckx, 2018) for multi-product supply chain tactical planning in the context of wind turbine industry under uncertain demand and suppliers' yields and lead times. The authors in (Alonso-Ayuso, Escudero, Guignardb,

& Weintraub, 2020) propose a novel scenario tree structure and an multi-stage stochastic mixed-integer programming model to solve a strategic and tactical planning problem in the context of forest harvesting networks under uncertain timber production. An uncapacitated multi-item multi-echelon lot-sizing problem within a remanufacturing system is investigated in (Quezada, Gicquel, Kedad-Sidhoum, & Vu, 2020) by considering random coefficients in the bill-of-material of the used products. The problem is formulated as an MSP model and a branch-and-cut algorithm is proposed to efficiently solve the model.

Generally speaking, MS-MIP models that incorporate a large number of complicating binary and/or integer variables as a result of the growth in the size of the underlying ST are computationally demanding. Among the variety of algorithms proposed in the literature for solving such models, we may refer to scenario decomposition (SD) and scenario cluster decomposition (SCD) strategies. Progressive Hedging Algorithm (PHA) (Rockafellar & Wets, 1991) is one of the scenario decomposition techniques that has been applied as a heuristic to solve MS-MIP models. The main idea behind this algorithm is to decompose the original MSP model into deterministic scenario sub-models. Afterwards, the sub-models are coordinated to an implementable solution by incorporating Lagrangian penalty terms in their objective function. Løkketangen and Woodruff (1996) propose a heuristic algorithm based on PHA and Tabu Search for solving multi-stage stochastic binary integer programming models. Despite several advantages, SD algorithms suffer from non-convergence or long CPU times for solving large-scale MS-MIP models. Watson and Woodruff (2011) propose algorithmic innovations to enhance the performance of PHA in the context of large-scale two-stage discrete optimization problems. The notion of scenario clustering has been proposed by several authors (e.g., (Escudero, Garín, & Unzueta, 2016; Kazemi Zanjani, Bajgiran, & Nourelfath, 2016)) to alleviate the shortcomings of SD algorithms. This approach relies on decomposing the original ST into smaller sub-trees. Afterwards, the MSP model is decomposed into scenario cluster sub-models by relaxing the non-anticipativity condition (NAC) in the root node of sub-trees. The latter conditions indicate that in each stage of the ST, the decision maker cannot foresee the outcomes of random events corresponding to future stages. Finally, the lack of NAC is compensated by adding Lagrangian penalty terms in the objective function of corresponding sub-models within a LR scheme. The authors in (Escudero et al., 2016) propose an SCD algorithm for solving MS-MIP model by exploring four different methods for updating

Lagrangian multipliers. Kazemi Zanjani et al. (2016) propose a hybrid SCD algorithm where an *ad-hoc* decomposition algorithm is developed to efficiently solve scenario cluster sub-models within an SCD framework. One remarkable observation in reviewing the above state-of-the-art decomposition algorithms is their limitation in solving large-scale MS-MIP models that involve millions of nodes in their underlying scenario tree. In other words, the scenario cluster sub-models would remain challenging to be solved by commercial solvers or exact decomposition algorithms when dealing with such large-scale STs.

### 3.2.2 Production and workforce planning under uncertainty in maintenance facilities

Maintenance facilities are confronted with different tactical decisions, such as the planning of workforce capacity and training/operations schedule. In the context of aircraft maintenance, De Bruecker, Beliën, Van den Bergh, and Demeulemeester (2018) propose a hierarchical planning approach incorporating three mixed-integer programming models associated with workforce scheduling, skill-mix planning, and training scheduling problems. While the first two models are concerned with constructing and optimizing the workforce schedule of the next maintenance season, the training model is only concerned with constructing a feasible training schedule during the current season. It is worth to note that the main focus of this study is on an operational-level training and personnel scheduling in the context of unlimited age of operators' certificates. The authors in (Sleptchenko et al., 2018) investigate the integrated planning of workforce capacity and spare part provisioning in a maintenance facility that serves multiple demand points. The integration of workforce training and operations planning in maintenance facilities is studied in (Tavakoli Kafiabad, Zanjani, & Nourelfath, 2020) under the assumption of deterministic demand and repair time of faulty components. The tactical planning model was solved by a commercial solver.

The authors in (Erkoyuncu, Durugbo, & Roy, 2013) categorize uncertainties confronted by maintenance service providers based on a 3-year study with 22 aeronautical experts. The results of their research indicate that the repair time uncertainty frequently arises in maintenance operations and depends on several factors, such as the inspection time, complexity of equipment, maintenance knowledge requirements, quality of components, and rate of repairability. Although extremely important in practice, the demand and repair time uncertainties have been highly neglected in the literature when proposing decision models for tactical planning in maintenance facilities. Zanjani and Nourelfath (2014) propose a multi-stage stochastic programming model for the integrated planning of spare part procurement, inventory, and production under uncertain demand in a maintenance facility. In the context of in-house maintenance, Rezaei-Malek et al. (2019) study the integrated planning of preventive maintenance and part quality inspection in a multi-stage production system under demand uncertainty. The goal is to maximize the system productivity at minimum cost.

The above review of the literature clearly indicates the paucity of research on integrating uncertainty into tactical planning decision models in the context of maintenance facilities. This is mainly due to the complexity of the resulting MS-MIP model given the multi-item, multi-stage nature of these problems in addition to the independent behavior of random parameters, such as repair time and demand corresponding to different items. Furthermore, the existing decomposition approaches that rely on a scenario clustering scheme are not efficient in solving such large-scale MS-MIP models. This study aims to fill these gaps by proposing a novel multi-stage stochastic programming model and an approximate decomposition algorithm for multi-item tactical planning by incorporating independent random parameters that evolve over time.

## 3.3 Problem description and formulation

### 3.3.1 Integrated production and workforce planning (IPWP) in maintenance facilities

Consider a maintenance facility that receives complex devices (e.g., gas turbine engines) for overhaul services. Each device is composed of different modules; each consists of several repairable components. Upon receiving the devices, they are disassembled into their components and inspected. More precisely, each component is verified by the certified operators to find out whether or not it is functional. If the component is diagnosed as defective, it would be replaced by a functional one from the internal inventory. If the maintenance facility is out of stock for a given component, the device would be backordered until the failed part is repaired/reconditioned. It is further assumed that the facility receives a fleet of aged devices for mainly overhaul services. Hence, in the majority of cases all key components must be replaced.

In order to carry out repair operations, the operators require valid certificates related to particular skill sets. In some maintenance centers, the certificates are valid for a limited period (e.g., 2-3 years) in the sense that the operators are not allowed to perform certain tasks once their certificates are expired. In this case, the inspection/repair operations might be delayed due to the unavailability of the certified operators. Therefore, the company needs to train the operators prior to the expiration date of their certificates in order to maintain a sufficient number of certified operators. For instance, in the gas turbine repair centers, the operators are certified for specific tasks (inspection, assembly, repair, etc.) for a validation period of two to four years. In fact, these facilities receive a large variety of equipment (engines) on an occasional basis. In other words, the operators rarely find the opportunity to carry out repair and inspection operations on similar product families; hence they might forget relevant technical details over time if they have not applied them over a long period of time. Nevertheless, due to the severe impact of human error in conducting inspection and repair operations on products' safety, the operators are required to repeat the training sessions on a periodic basis. In addition to the limited age of operators' certificates, in some industries, due to the complex design of products, the training is carried out on an on-job basis by certified operators. In other words, enough certified workers must be available to train operators on a certain skill set in a given period in the planning horizon. In addition, the training can be scheduled in a period only if the inspection/repair of a component that requires a similar skill set is also scheduled in that period. As a consequence, the integration of the training schedule with the production plan in these facilities provides the opportunity to balance the operations and workforce levels and ultimately minimize the late delivery of repaired devices.

The focus of this study is a context where the repair and overhaul (R & O) contracts are signed at the beginning of planning horizon, such that each contract has a different delivery due date. The critical issue in maintenance facilities is the prompt delivery of repaired devices given the significantly high cost of system downtime for the users. Such facilities are also confronted with uncertain repair time of some complex components (e.g., rotors) that increases the risk of the late delivery of the devices and considerable backordering (penalty) costs, accordingly. The penalty cost is a contractual term to compensate the equipment unavailability in users' sites. For instance, the maintenance facilities are expected to recompense power plants for the production loss of electricity due to the unavailability of turbines. In this context, we aim to develop a mathematical programming model for IPWP that determine the optimal quantity of repair operations along with the training schedule in each period over a medium term (e.g., annual) planning horizon. The objective is to minimize the total cost of operations, inventory, delays, and training by considering repair time uncertainty. Two main categories of constraints, namely operations and training constraints must be taken into consideration. While the former category mainly adjust the balance between repair quantities, inventory and backorder levels, the latter control the number of trainees and required certified operators.

### 3.3.2 Modeling the uncertain repair time

Inspired from a real gas turbine maintenance center where a fleet of aged engines are received for R & O services, it is assumed that the repair times of components involved in a complex device are independent random variables. This is a realistic assumption due to the independent life-time distribution of different components involved in the design of such modular-structured products. In the IPWP model proposed in this study, the repair time appears in the index of the repair decision variables. Hence, we need to generate discrete realizations for this parameter. To this end, we adopt a procedure to discretize the continuous distribution representing the random repair time as a set of integer outcomes with a given probability as follows:

$$Pr\{l_r^{rep} = a\} = Pr\{(a-1 < X_r \leq a) | (X_r \leq b)\} = \frac{Pr\{a-1 < X_r \leq a\}}{Pr\{X_r \leq b\}} \quad (3.1)$$

where $l_r^{rep}$ denotes a specific (discrete) realization of random repair time of component $r$; $X_r$ represents the random repair time of component $r$; $b$ is the maximum empirical threshold of repair time that can be determined based on historical data; and $a$ is an integer number in the interval of $[1, b]$. For instance, if the repair time follows an exponential distribution with the average repair time $\lambda_r$ equal to 0.83, by assuming that the maximum repair time is not longer than 2 periods, the conditional probability of having a realization of repair time equal to 1 period is calculated as follows:

$$Pr\{l_r^{rep} = 1\} = \frac{e^{-(0)\lambda_r} - e^{-\lambda_r}}{Pr\{X_r \leq 2\}} = 0.7$$

Given the multi-period nature of the IPWP problem under investigation, it is more realistic to assume that the repair time of each component follows a non-stationary (dynamic) behavior over the planning horizon (e.g., 6 periods). Hence, the uncertain repair time of each component can be modeled as a ST. An example of a 7-stage ST is depicted in Figure 3.1. In a scenario tree, each stage represents the point of time when new information on the uncertain event (repair time) is revealed to the decision-maker. In this study, we consider a period in the planning horizon as a stage, where the first stage represents current time (period zero). Each stage encompasses a set of nodes, each representing one possible outcome corresponding to the uncertain parameter (repair time). A probability is associated to each node of the ST such that the sum of probabilities of all nodes in each stage is equal to one. In the example of Figure 3.1, two outcomes are considered for each component denoted as low (e.g., 1 period) and high (e.g., 2 periods) repair times. The root node in stage 1 denotes the current state of the world (i.e., period 0 in the planning horizon). Besides, a scenario is defined as a path from the root node to each leaf node in the ST where the probability of each scenario is the product of probabilities of the nodes on this path.

Following the independency of repair times for different components, the scenario trees corresponding to each component should be merged and form a master ST (Figure 3.2). The nodes in this ST denote all possible combinations of random outcomes for all components in each stage. Due to the independency assumption, the probability of each node is thus calculated as the product of probability of corresponding outcomes (nodes) in each component ST. For instance, consider two components with independent repair times, each represented as a scenario tree similar to the one depicted in Figure 3.1. In this ST, two outcomes with equal probabilities (0.5) are considered for the repair time (high and low). Merging these STs forms a master ST similar to the one in Figure 3.2 that consists of 4 nodes in Stage 2 (period 1), 16 nodes in stage 3, 64 nodes in stage 4, 256 nodes in stage 5, 1024 nodes in stage 6 and 4096 nodes in the last stage. In this ST, nodes (2)-(5) in stage 2 have equal probabilities of 0.25 and represent, respectively, high repair time for components #1 & #2; high repair time for component #1 & low repair time for component #2; low repair time for component #1 & high repair time for component #2; and low repair time for components #1 & #2. As it can be observed in this example, the size of the master ST grows exponentially as the number of components with random repair time along with the number of periods in the planning horizon increases. This would drastically increase the computational complexity of the corresponding multi-stage stochastic IPWP model.

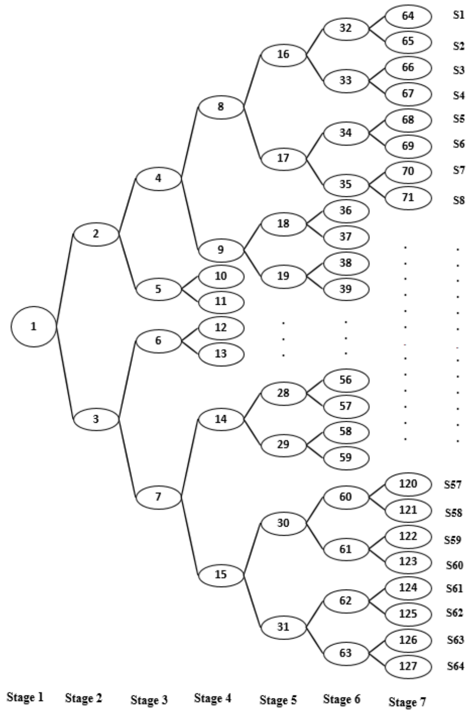


Figure 3.1: Example of a scenario tree for one component

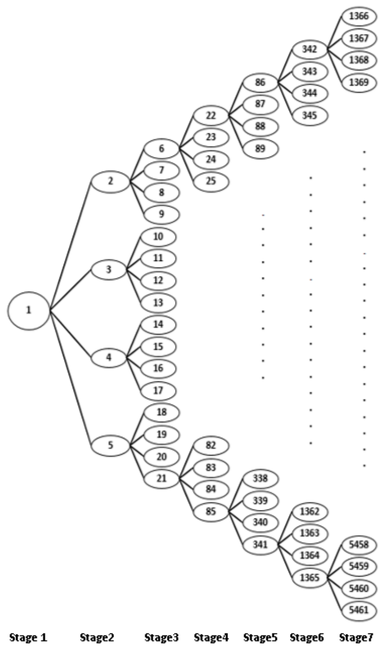


Figure 3.2: Example of master scenario tree corresponding to two components

### 3.3.3 Problem formulation

According to the ST representation of the random repair time, the IPWP problem can be formulated as a multi-stage stochastic programming (MSP) model. We assume that the stages in the master scenario tree correspond to the periods in the planning horizon. By adopting a compact formulation for the MSP model, the decision variables are defined for each node in the scenario tree. It should be mentioned that in the compact formulation, the non-anticipativity constraints (NAC) are implicitly included in the model. The description of sets, parameters, and decision variables are provided in Tables 3.1 & 3.2. The MSP model is formulated as model (3.2)-(3.14). In this model, the demand is assumed to be deterministic in the context that the equipment is received for overhaul services. As a consequence, the type of component that is required to be replaced in each device is known based on its maintenance history. In other words, according to the time required to assemble repaired/replaced components, and the requested delivery due date, the demand for the number of components to be replaced is determined ($d_{r,t}$). It is further assumed that the components of the devices go under a preliminary inspection upon disassembly where an estimation on the repair time is provided based on the condition of the component and its history of usage. Accordingly, a high or low repair time (i.e., a node index) is associated with each component.

In the IPWP problem, the repair quantity ($Q^{rep}_{r,t}$), the number of required $a$-months old certified operators ($E_{a,s,t}$) along with the number of trainees ($R_{a,s,t}$) are the here-and-now decisions that must be determined before the actual repair times for different components for the next stage are estimated; nevertheless, it is assumed that the realization of the repair time in the current stage is known to the decision-maker at the beginning of that stage. On the contrary, the number of replaced components ($Q_{r,t}$), the inventory level ($I_{r,t}$), as well as the backorder level ($B_{r,t}$) and the maximum quantity of backorder ($\theta_t$) are the wait-and-see (state) decisions that are determined once the components are gone under repair and their actual repair times are revealed. This leads to a compact formulation for the corresponding MSP model, where the NAC is implicitly applied to decision variables, indicating that the decision-maker cannot foresee the outcomes of random repair times corresponding to future stages. It is worth to note that the uncertain repair times appear in the index of $Q^{rep}_{r,(t-l^{rep}_r(n))}$. Furthermore, despite the compact representation of the MSP model, since the repair time can exceed one period (e.g., 2 periods), the decision variables are indexed both by time periods $t$ and nodes $n$.

Table 3.1: Sets, indices and parameters

| | |
|---|---|
| $N$ | Set of nodes in the (master) scenario tree, indexed by $n$ |
| $T$ | Planning horizon, indexed by $t$ |
| $R$ | Set of components, indexed by $r$ |
| $S$ | Set of skills, indexed by $s$ |
| $A_s$ | Set of certificate ages for skill $s$, indexed by $a$ ($\|A_s\|$= max $\{a$ in period 0 $\}$ + $\|$T$\|$) |
| $a(n)$ | Parent node of node $n$ in period $(t-1)$ |
| $a'_r(n)$ | Ancestor node of node $n$ in period ($t - l_r^{rep}$) for component $r$ |
| $pr(n)$ | Probability of node $n$ |
| $l_r^{rep}(n)$ | A (possible) realization of repair time of component $r$ at node $n$ |
| $d_{r,t}$ | Number of component $r$ required to be replaced in period $t$ |
| $SS_r$ | Safety stock of component $r$ |
| $h_r$ | Unit inventory holding cost of component $r$ |
| $k_r$ | Unit repair cost of component $r$ |
| $b$ | Unit penalty cost for the delayed delivery of each device |
| $n_s$ | Unit training cost per skill type $s$ |
| $O_t$ | Available training budget in each period $t$ |
| $\alpha_{r,s}$ | number of operators with skill type $s$ required for component $r$ |
| $e_s$ | Certificate age of skill type $s$ |
| $W_s$ | Total number of available operators with skill type $s$ |
| $E_{a,s,0}$ | Initial number of $a$-months old certified operators of skill type $s$ in period 0 |

Table 3.2: Decision variables

| | |
|---|---|
| $Q_{r,t}(n)$ | Number of replaced components (ready for assembly) at node $n$ in period $t$ |
| $I_{r,t}(n)$ | Inventory level of component $r$ at node $n$ in period $t$ |
| $Q_{r,t}^{rep}(n)$ | Number of repaired component $r$ at node $n$ in period $t$ |
| $B_{r,t}(n)$ | Backorder level of component $r$ at node $n$ in period $t$ |
| $\theta_t(n)$ | Maximum quantity of backorders among all components at node $n$ in period $t$ |
| $E_{a,s,t}(n)$ | Number of operators per skill type $s$ with $a$-months old certificates at node $n$ in period $t$ |
| $R_{a,s,t}(n)$ | Number of operators of skill type $s$ with $a$-months old certificates who are under training at node $n$ in period $t$ |

**Multi-stage stochastic model for the IPWP problem**

$$Minimize \sum_{n\in N} pr(n) \sum_{t\in T} \Bigg( \sum_{r\in R} (k_r Q^{rep}_{r,t}(n) + h_r I_{r,t}(n)) \\ + b\theta_t(n) + \sum_{s\in S} \sum_{a\in A_s} n_s R_{a,s,t}(n) \Bigg) \tag{3.2}$$

The objective function (3.2) minimizes the expected cost of repair, inventory, late delivery, and training over all nodes and periods in the planning horizon. The MSP model encompasses several families of constraints associated with the repair operations, workforce training, and the domain of decision variables.

***Operational constraints***

$$Q_{r,t}(n) + B_{r,t}(n) - B_{r,(t-1)}(a(n)) = d_{r,t} \qquad \forall t \in T, \forall r \in R, \forall n \in N \tag{3.3}$$

Constraints (4.2) represent the flow balance between the number of components that must be replaced ($d_{r,t}(n)$), the actual number of replaced components ($Q_{r,t}(n)$), and the quantity of back-ordered components at each node and its parent node in two consecutive periods ($B_{r,t}(n)$ and $B_{r,(t-1)}(a(n))$ ).

$$Q^{rep}_{r,(t-l^{rep}_r(n))}(a'_r(n)) + I_{r,(t-1)}(a(n)) - I_{r,t}(n) = Q_{r,t}(n) \\ \forall t \in T \backslash 1, ..., (l^{rep}_r(n)), \forall r \in R, \forall n \in N \tag{3.4}$$

The inventory balance constraints (3.4) imply that the stock level of each component at a specific node in each period ($I_{r,t}(n)$) equals to its stock level at the parent node in the previous period minus the number of repaired components by considering the random repair times. $Q_{r,t}(n)$ which represents the number of replaced components at node $n$ in period $t$ is replenished from the internal inventory if the component is available in stock.

$$\theta_t(n) \geq B_{r,t}(n) \qquad \forall t \in T, \forall r \in R, \forall n \in N \tag{3.5}$$

Constraints (3.5) calculate the maximum level of backorder among all components at a specific node in each period ($\theta_t(n)$). It is noteworthy that the (modular-structured) devices can only be delivered to customers once all of their components are replaced. Therefore, the number of delayed repaired devices is calculated at the component level. As such, the component with the highest backorder level determines the number of devices with delayed delivery.

$$\sum_{r \in R} \alpha_{r,s} Q_r^{rep}(n) \leq \sum_{a=1}^{e_s} E_{a,s}(n) \qquad \forall t \in T, \forall s \in S, \forall n \in N \quad (3.6)$$

$$I_{r,t}(n) \geq SS_r \qquad \forall t \in T, \forall r \in R, \forall n \in N \quad (3.7)$$

$$Q_{r,t}^{rep}(n) = 0 \qquad \forall t \in 1, ..., l_{rt}^{rep}(n), \forall r \in R, \forall n \in N \quad (3.8)$$

Constraints (3.6) set a limit on the level of repair operations at a specific node in each period based on the number of available certified operators over different skill sets. Constraints (3.7) ensure the safety stock at a specific node in each period. Constraints (3.8) set the initial values of repair decisions to zero in periods smaller than the repair time in each node.

***Training constraints***

$$E_{(a+1),s,t}(n) = E_{a,s,(t-1)}(a(n)) - R_{a,s,(t-1)}(a(n)) \quad \forall t \in T, \forall s \in S, \forall n \in N, \forall a \in A_s \quad (3.9)$$

$$E_{1,s,t}(n) = \sum_{a \in A_s \setminus 1} R_{a,s,(t-1)}(a(n)) \qquad \forall t \in T, \forall s \in S, \forall n \in N \quad (3.10)$$

Constraints (3.9) and (3.10) update the age of operators' certificates. Constraints (3.9), in particular, maintains the balance in terms of the number of certified operators between a node and its parent node in two consecutive periods. More specifically, training the operators with certificate age $a$ for each skill set at a given node in period $t$ would reduce the number of certified operators with age $a+1$ at the corresponding (child) node in the next period. Constraints (3.10) state the balance between the number of operators under training for each skill set at a given node in each period ($R_{a,s,t}(n)$) with the number of certified operators with age 1 at the corresponding (child) node in the next period ($E_{1,s,t}(n)$).

$$\sum_{a\in A_s\backslash 1} R_{a,s,t}(n) \le \sum_{a=1}^{e_s} E_{a,s,t}(n) \qquad \forall t\in T, \forall s\in S, \forall n\in N \quad (3.11)$$

$$\sum_{a\in A_s} E_{a,s,t}(n) + \sum_{a\in A_s\backslash 1} R_{a,s,t}(n) = W_s \qquad \forall t\in T, \forall s\in S, \forall n\in N \quad (3.12)$$

$$\sum_{s\in S}\sum_{a\in A_s} n_s R_{a,s,t}(n) \le O_t \qquad \forall t\in T, \forall n\in N \quad (3.13)$$

Constraints (3.11) limits the number of trainees to the number of certified operators available to deliver the training sessions. Constraints (3.12) guarantees the balance between the number of available operators ($W_s$), the number of certified and uncertified operators, in addition to the number of trainees. Constraints (3.13) correspond to the training budget limits available in each period in the planning horizon.

***Domain constraints***

$$Q_{r,t}(n), Q^{rep}_{r,t}(n), E_{a,s,t}(n), R_{a,s,t}(n) \in \mathbb{Z}^+ \& I_{r,t}(n), B_{r,t}(n), \theta_t(n) \in \mathbb{R}^+$$
$$\forall t\in T, \forall n\in N, \forall r\in R, \forall s\in S, \forall a\in A_s \quad (3.14)$$

## 3.4 Solution methodology

### 3.4.1 Computational complexity of the IPWP problem

The IPWP model (2)-(14) is a complex MS-MIP model with $(4\times|R||T||N|+|T||N|+|A_s||S||T||N|)$ decision variables and $(5\times|T||R||N|+4\times|T||S||N|+|T||N|+|T||S||N||A_s|)$ constraints. Although the small instances of the problem are solvable by a commercial solver, such as CPLEX, real-size instances are usually computationally intractable for the following reasons. First, by increasing the number of periods in the planning horizon, which is a realistic assumption for similar tactical planning problems, the size of the master ST grows exponentially. Besides, the independency of repair times for different components significantly elevates the size of master ST and consequently, the computational complexity of this model. For instance, by considering two/three components with independent uncertain repair times, each discretized as two outcomes, and a 12-month planning horizon, the resulting 13-stage master ST will contain approximately

$22 \times 10^6/7.7 \times 10^{10}$ nodes, respectively.

To overcome the computational complexity of this problem, two decomposition algorithms based on Lagrangian relaxation (LR) approach are proposed. LR algorithm relies on relaxing a set of complicating constraints in the original MIP model and penalizing their infeasibility by adding Lagrangian penalty terms in the objective function. It is expected then that the relaxed model will be easier to solve and/or decomposable into smaller sub-problems that can be efficiently solved by the aid of commercial solvers or *ad-hoc* algorithms Wolsey (1998). The main idea behind the decomposition algorithms proposed in this study is to decompose the MS-MIP model per component. More precisely, the proposed solution methodology can be summarized as the following steps:

1- LR based on the master scenario tree ($LRMST$): this algorithm is an LR scheme implemented by formulating component sub-models based on the master ST; hence it leads to the exact decomposition of the original MS-MIP model into component sub-models. Given that the size of the master ST could be very large for some problem instances, the resulting component sub-models could still become computationally intractable. Therefore, it is desirable to approximate these sub-models to significantly smaller models.

2- LR based on the component scenario tree ($LRCST$): this method provides an approximation to component sub-models in the $LRMST$ algorithm in order to reduce their size. The idea is to formulate these sub-models based on component scenario trees. It is noteworthy that both LR algorithms provide a lower bound (LB) to the optimal objective function of model (3.2)-(3.14). However, it is expected that the bound provided by the approximate $LRCST$ algorithm is weaker than the former algorithm due to formulating component sub-models based on their corresponding ST rather than the master ST.

3- An efficient repair mechanism is also proposed to obtain a high-quality feasible solution and upper bound (UB) based on the converged solution of $LRMST$ and $LRCST$ algorithms.

The detailed description of each step is provided in the following sections.

### 3.4.2 Lagrangian relaxation algorithm based on the master scenario tree ($LRMST$)

As mentioned earlier, the main goal of $LRMST$ is to reduce the computational complexity of the original MS-MIP model by decomposing it into $r$ component sub-models, each formulated based on the master ST. To this end, Constraints (3.6) in model (3.2)-(3.14) that bind component sub-models, must be relaxed. Afterwards, the resulting sub-models can be solved independently. Nevertheless, the obtained solution of the relaxed models might be infeasible with respect to the relaxed constraints. In other words, the number of certified operators ($E_{a,s,t}(n)$) and trainees ($R_{a,s,t}(n)$) obtained from each sub-model might violate the workforce capacity constraints (3.6). Hence, we propose an LR algorithm where possible violations of these constraints are added as the penalty terms in the objective function of each sub-model. More precisely, by introducing Lagrangian multipliers $\lambda_{s,t}(n)$, the following LR model is obtained:

$$L(\lambda_{s,t}(n)) = Minimize \sum_{n \in N} pr(n) \sum_{t \in T} \Bigg( \sum_{r \in R} (k_r Q^{rep}_{r,t}(n) + h_r I_{r,t}(n)) + b\theta_t(n) + \sum_{s \in S} \sum_{a \in A_s} n_s R_{a,s,t}(n) \Bigg) + \sum_{n \in N} \sum_{t \in T} \sum_{s \in S} \lambda_{s,t}(n) \Bigg( \sum_{r \in R} \alpha_{r,s} Q^{rep}_{r,t}(n) - \sum_{a=1}^{e_s} E_{a,s,t}(n) \Bigg) \quad (3.15)$$

Subject to:

Constraints (3.2)-(3.5) and (3.7)-(3.14).

Since we aim to decompose model (3.2)-(3.14) to component sub-models, we define $\overline{E}_{s,t}(n)$ to approximate the cumulative workforce requirement (required for all components) per each skill set and node in the master scenario trees as follows:

$$\overline{E}_{s,t}(n) = \sum_{r \in R} \alpha_{r,s} Q^{rep}_{r,t}(n) \quad (3.16)$$

It is noteworthy that the value of $\overline{E}_{s,t}(n)$ is updated in each iteration of the proposed LR algorithm based on the current value of $Q^{rep}_{r,t}(n)$ obtained from corresponding component sub-models. Besides, $\theta_t(n)$ needs to be replaced by $B_{r,t}(n)$ to track the backorders quantity per component in each sub-model. Finally, in order to improve the quality of the LB provided by the Lagrangian model, we propose to include a disaggregated version of constraints (3.6) in the component Lagrangian sub-models (Constraints (3.20)).

**Proposition 1.** The LR model $Ł(\lambda_{s,t}(n))$ can be decomposed into the following $|R|$ sub-models:

$$L_r(\lambda_{s,t}(n)) = Minimize \sum_{n\in N} pr(n) \sum_{t\in T} \Bigg( k_r Q^{rep}_{r,t}(n) + h_r I_{r,t}(n) + bB_{r,t}(n) + \sum_{s\in S}\sum_{a\in A_s} n_s R_{a,s,t}(n) \Bigg) + \sum_{n\in N}\sum_{t\in T}\sum_{s\in S} \lambda_{s,t}(n) \Bigg( \overline{E}_{s,t}(n) - \sum_{a=1}^{e_s} E_{a,s,t}(n) \Bigg) \quad (3.17)$$

Subject to:

Constraints (3.9)-(3.14) and

$$Q_{r,t}(n) - B_{r,(t-1)}(a(n)) + B_{r,t}(n) = d_{r,t} \qquad \forall t \in T, \forall n \in N \quad (3.18)$$

$$Q^{rep}_{r,(t-l^{rep}_r(n))}(a'_r(n)) + I_{r,(t-1)}(a(n)) - I_{r,t}(n) = Q_{r,t}(n) \quad \forall t \in T\backslash 1, ..., l^{rep}_r, \forall n \in N \quad (3.19)$$

$$\alpha_{r,s} Q^{rep}_{r,t}(n) \leq \sum_{a=1}^{e_s} E_{a,s,t}(n) \qquad \forall t \in T, \forall s \in S, \forall n \in N \quad (3.20)$$

$$I_{r,t}(n) \geq SS_r \qquad \forall t \in T, \forall n \in N \quad (3.21)$$

$$Q^{rep}_{r,t}(n) = 0 \qquad \forall t \in 1, ..., l^{rep}_{rt}(n) \quad (3.22)$$

**Proof.** Obvious.

Since $L_r(\lambda_{s,t}(n))$ is a nonlinear mathematical program, it can be solved by a sub-gradient algorithm (Algorithm 1). In this study, the Lagrangian sub-problems in Algorithm 1 are efficiently solved by a commercial solver (CPLEX). The stopping criterion in this algorithm is considered as a fixed number of iterations. Furthermore, Given that the convergence rate of this algorithm is dependent on the step size ($u_k$), this parameter is updated in each iteration of the algorithm.

It is worth to mention that $L_r(\lambda_{s,t}(n))$ obtained in each iteration of Algorithm 1 does not explicitly provide a LB to the optimal objective function of model (3.2)-(3.14). In fact, the LB is calculated by combining the solution of Lagrangian (component) sub-models as described in Algorithm 2. Recall from Section 3.3 that the objective function (3.2) is composed of repair, inventory, backorder, and training costs. To calculate the LB in each iteration of Algorithm 1,

**Algorithm 1** Sub-gradient algorithm ($LRMST$)

**Step 0 (Initialization):**
Assign zero to $\lambda_{s,t}^{(1)}(n)$
Assign $-\infty$ to the $LB$
Set iteration counter $k$ equal to 1
Assign initial values to $u_0$ and $p$

**While** the stopping criteria is not satisfied **do**:
**Step 1:** Solve the Lagrangian problem $L_r(\lambda_{s,t}(n))$ and update the LB (Algorithm 2)
**Step 2:** Update the step-size as $u_k$=$u_0 p^k$
**Step 3:** Update the Lagrangian multipliers as follows:

$$\lambda_{s,t}^{(k+1)}(n) = \lambda_{s,t}^{(k)}(n) + u_k \left\{ \overline{E}_{s,t}^{(k+1)}(n) - \sum_{a=1}^{e_s} E_{a,s,t}^{(k)}(n) \right\}$$
$$\overline{E}_{s,t}^{(k+1)}(n) = \sum_{r \in R} \alpha_{r,s} Q_{r,t}^{rep}(n)^{(k)}$$
$$k = k + 1$$

**End While**

the first two categories of cost are calculated by combining the repair and inventory decisions obtained from each component sub-model. Nevertheless, the backorder cost is considered as the highest backorder quantity obtained from these sub-models. In the same vein, the Lagrangian penalty cost and training cost are also calculated based on the maximum workforce and training levels ($E_{a,s,t}(n)$ and $R_{a,s,t}(n)$) obtained over all component sub-models. This is due to the fact that these decisions are not defined per component, hence it is possible that they take different values in those sub-models. By considering the maximum value over all component sub-models, the infeasibility of the relaxed constraint set (3.6) is expected to be minimized in each iteration of Algorithm 1. The proof is similar to the one provided for Proposition 2.

It should be pointed out that the component sub-models in $LRMST$ are *per se* large-scale MS-MIP models given that they are formulated based on a master ST, merging individual component scenario trees. In other words, these sub-models can be computationally intractable if the master ST grows exponentially in size due to an increase in the number of stages and components with uncertain repair time. To alleviate this complexity, an approximate LR algorithm (denoted as $LRCST$) is proposed where component sub-models are modeled based on their corresponding ST, rather than the master ST. Afterwards, the optimal value of decision variables corresponding

**Algorithm 2** Lower bound calculation algorithm ($LRMST$)

**Step 1:** Calculate the maximum Lagrangian penalty term, training and penalty costs over all component sub-models as follows:

$$SOL = \max_{r\in R}\Big\{\sum_{n\in N}\sum_{t\in T}\sum_{s\in S}\lambda_{s,t}(n)\Big(\overline{E}_{s,t}(n) - \sum_{a=1}^{e_s} E_{a,s,t}(n)\Big)\Big\}$$

$$COT = \max_{r\in R}\Big\{\sum_{n\in N}\sum_{t\in T}\sum_{s\in S}\sum_{a\in A_s} pr(n)\Big(n_s R_{a,s,t}(n)\Big)\Big\}$$

$$\theta_t(n) = \max_{r\in R, n\in N}\Big\{B_{r,t}(n)\Big\}$$

**Step 2:** Update the Lagrangian LB as follows:

$$LB = \sum_{n\in N} pr(n)\Big(\sum_{t\in T}(\sum_{r\in R} k_r Q^{rep}_{r,t}(n) + \sum_{r\in R} h_r I_{r,t}(n) + b\theta_t(n))\Big) + COT + SOL$$

to the nodes of master ST can be determined in a straightforward manner based on the values obtained in each node of component STs.

### 3.4.3 Lagrangian relaxation algorithm based on the component scenario tree ($LRCST$)

The prime idea behind this approximate decomposition algorithm is the relationship between the nodes of master and component STs. In fact, when decomposing the original MS-MIP model (formulated based on master ST) into component sub-models, some nodes in each stage of master ST would be indistinguishable in each sub-model. For instance, consider two component STs similar to the one depicted in Figure 3.1. In this figure, node (2)/(3) in stage two represent, respectively, high/low repair time for each component. When combining these two STs, based on the definition of nodes in master ST in section 3.3.2, it is straightforward to verify that nodes (2) & (3) in stage two of the master ST (Figure 3.2) are equivalent to node (2) in component ST when decomposing the MS-MIP model to component #1 sub-model. In fact, they both represent a high repair time for this component. Nevertheless, these nodes are not indistinguishable in the sub-model corresponding to component #2 since they represent high/low repair times for this component. In addition, nodes (4) & (5) are indistinguishable in component #1 sub-model, both representing a low repair time. In a similar fashion, nodes (2) & (4) and (3) & (5) are indistinguishable in sub-models corresponding to component #2. As a consequence, we propose to formulate component sub-models (3.17)-(3.22) based on their corresponding ST (rather than the master ST) as follows:

$$L_r(\lambda_{s,t}(n)) = Minimize \sum_{n\in N'} pr(n) \sum_{t\in T} \Bigg( k_r Q^{rep}_{r,t}(n) + h_r I_{r,t}(n) + bB_{r,t}(n) + \sum_{s\in S}\sum_{a\in A_s} n_s R_{a,s,t}(n) \Bigg) + \sum_{n\in N'}\sum_{t\in T}\sum_{s\in S} \lambda_{s,t}(n) \Bigg( \overline{E}_{s,t}(n) - \sum_{a=1}^{e_s} E_{a,s,t}(n) \Bigg) \tag{3.23}$$

Subject to:

Constraints (3.9)-(3.14) and

$$Q_{r,t}(n) - B_{r,(t-1)}(a(n)) + B_{r,t}(n) = d_{r,t} \qquad \forall t \in T, \forall n \in N' \tag{3.24}$$

$$Q^{rep}_{r,(t-l^{rep}_r(n))}(a'_r(n)) + I_{r,(t-1)}(a(n)) - I_{r,t}(n) = Q_{r,t}(n) \; \forall t \in T \backslash 1, ..., l^{rep}_r, \forall n \in N' \tag{3.25}$$

$$\alpha_{r,s} Q^{rep}_{r,t}(n) \leq \sum_{a=1}^{e_s} E_{a,s,t}(n) \qquad \forall t \in T, \forall s \in S, \forall n \in N' \tag{3.26}$$

$$I_{r,t}(n) \geq SS_r \qquad \forall t \in T, \forall n \in N' \tag{3.27}$$

$$Q^{rep}_{r,t}(n) = 0 \qquad \forall t \in 1, ..., l^{rep}_{rt}(n) \tag{3.28}$$

where $N'$ denotes the set of nodes in the component ST.

main difficulty in formulating component sub-models based on their corresponding ST is estimating the cumulative workforce requirement ($\overline{E}_{s,t}(n)$) in each sub-model. Therefore, we propose to approximate this parameter by accumulating the level of $Q^{rep}_{r,t}(n)$ obtained in each sub-model with the expected amount of repair activities obtained over all nodes in the corresponding stage in the other component sub-models. For instance, consider a three-components system where the repair time of each component is represented as the ST in Figure 3.1. The value of $\overline{E}_{s,1}(4)$ (workforce level for component #1 in node (4)), is approximated by considering $Q^{rep}_{r,1}(4)$ and the expected values of this variable in nodes (4)-(7) in component sub-models #2 and #3. This is based on the fact that when the repair time of component #1 is represented by node (4) in its corresponding ST, the repair time of the two other components could be represented by any of the possible four nodes (nodes (4)-(7)) in the STs corresponding to the two other components. Therefore, the value of $\overline{E}_{s,t}(n)$ in $LRCST$ sub-models can be approximated as follows:

$$\overline{E}_{s,t}(n) = \alpha_{r,s} Q^{rep}_{r,t}(n) + \sum_{n \in N_t} \sum_{i \in R \setminus r} pr(n) \alpha_{i,s} Q^{rep}_{i,t}(n) \tag{3.29}$$

where $N_t$ denotes the set of nodes in each stage $t$ of the component ST. The nonlinear $LRCST$ sub-models ($L_r(\lambda_{s,t}(n))$) can be solved by the aid of Algorithm 1. Afterwards, an approximate LB within the sub-gradient algorithm can be calculated by the aid of Algorithm 2 where $N$ is replaced by $N'$. Nevertheless, the exact LB can only be calculated by the aid of Algorithm 2 after converting the solution of $LRCST$ to its counterpart in the original MS-MIP model as described in the following section.

**Converting the solution of $LRCST$ to its counterpart in the original MS-MIP model**

Recall from section 3.3 that the decision variables in the original MS-MIP model are defined for the nodes of master ST. The converged value of these variables in $LRCST$ algorithm, on the contrary, are defined for the nodes of component STs ($n' \in N'$). It is quite straightforward to convert the values of $Q_{r,t}(n')$, $Q^{rep}_{r,t}(n')$, $B_{r,t}(n')$ and $I_{r,t}(n')$ to their corresponding values in the nodes of master ST. In model (3.17)-(3.22), these decisions are indexed by component $r$ and node $n$. Hence their optimal value depends on the value of demand $d_{r,t}$, which is a deterministic parameter defined per component, in addition to the repair time corresponding to each node of the master ST. Hence, the value of these decisions corresponding to the indistinguishable nodes of the master ST are identical and can be directly retrieved from the solution obtained from $LRCST$ algorithm. In other words, by assuming that a given node $n \in N$ in the master ST is the result of merging nodes $n'_r \in N'_r$ in component scenario trees $r = 1, 2, ...|R|$, and by denoting the converged solution of $LRCST$ as $\hat{Q}_{r,t}(n')$, $\hat{Q}^{rep}_{r,t}(n')$, $\hat{B}_{r,t}(n')$ and $\hat{I}_{r,t}(n')$; we obtain $Q_{r,t}(n) = \hat{Q}_{r,t}(n'_r)$, $Q^{rep}_{r,t}(n) = \hat{Q}^{rep}_{r,t}(n'_r)$, $B_{r,t}(n) = \hat{B}_{r,t}(n'_r)$ and $I_{r,t}(n) = \hat{I}_{r,t}(n'_r)$. For instance, in a two-component master ST (Figure 3.2), node (3) in period 1 (stage 2) is the result of merging nodes (2) and (3) in component 1 and component 2 STs (Figure 3.1), respectively. Hence, $Q^{rep}_{1,1}(3) = \hat{Q}^{rep}_{1,1}(2)$ and $Q^{rep}_{2,1}(3) = \hat{Q}^{rep}_{2,1}(3)$.

The training and workforce level decisions, on the contrary, could take different values in different component sub-models, although they are not indexed by components. Hence, it is less straightforward to determine their value in the nodes of master ST. The following proposition, thus, provides an approximation rule to convert the converged value of these decisions in $LRCST$ to their counterparts in the original MSP model.

**Proposition 2.** The value of workforce and training level decisions ($E_{a,s,t}(n)$ and $R_{a,s,t}(n)$) in the nodes of the master ST can be approximated as the maximum values, obtained within the $LRCST$ scheme, in corresponding nodes of component sub-models. This would guarantee the minimum amount of infeasibility in terms of workforce capacity constraints (3.6) in each node of the master ST.

**Proof.** The proof is provided in Appendix A.

According to Proposition 2, the objective function value of the converted solution provides the (largest) LB to the optimal objective value of the original MSP model (3.2)-(3.14) given that all decisions are defined for the nodes of master ST.

### 3.4.4 Upper-bound calculation heuristic

As mentioned earlier, the $LRMST$ and $LRCST$ algorithms provide an LB to the optimal objective value of MS-MIP model (3.2)-(3.14). In other words, the converged solution of these algorithms are not necessarily feasible with respect to the workforce capacity constraints (3.6). In order to repair the infeasibility and obtain a feasible solution and an UB to the optimal objective value of this model, we first propose a heuristic algorithm in the context of $LRMST$ algorithm. Let us denote the converged solution of these algorithms as ($\hat{Q}_{r,t}(n)$, $\hat{Q}^{rep}_{r,t}(n)$, $\hat{B}_{r,t}(n)$, $\hat{I}_{r,t}(n)$, $\hat{E}_{a,s,t}(n)$, $\hat{R}_{a,s,t}(n)$). The infeasibility in this solution reflects the imbalance between the number of certified operators required to carry out the repair quantity operations ($\sum_{r\in R} \alpha_{r,s}\hat{Q}^{rep}_{r,t}(n)$) and the total number of available certified operators ($\sum_{a=1}^{e_s} \hat{E}_{a,s,t}(n)$) in each node of the ST. Thus, two corrective actions are proposed to adjust either the repair quantity or number of certified operators to obtain a feasible solution in each component sub-model. More precisely, given that the cost of training is considerably lower than the penalty cost of late delivery, as the first step, the algorithm tries to increase the number of certified operators by forcing the model to train more operators. If a feasible solution exists, an UB will be obtained. Otherwise, the algorithm decreases the repair quantity in order to obtain a feasible solution (and an UB). The summary of the repair mechanism heuristic and the upper bound calculation procedure for $LRMST$ is provided in Algorithms 3 and 4.

**Algorithm 3** Repair mechanism heuristic ($LRMST$)

**Step 1:**
Calculate the amount of infeasibility in each node of component sub-models ($L_r(\lambda_{s,t}(n))$) as:

$$\widehat{\Delta E}_{s,t}(n) = \max\left\{0, \left(\overline{\hat{E}}_{s,t}(n) - \sum_{a=1}^{e_s} \hat{E}_{a,s,t}(n)\right)\right\} \qquad \forall t \in T, \forall s \in S, \forall n \in N$$

**While** $\widehat{\Delta E}_{s,t}(n) > 0$ **do**:
**Step 2:** Remove the Lagrangian term from each sub-model ($L_r(\lambda_{s,t}(n))$).
**Step 3:** Fix the values of $Q^{rep}_{r,t}(n)$ in $L_r(\lambda_{s,t}(n))$ as $\hat{Q}^{rep}_{r,t}(n)$ and calculate $\overline{\hat{E}}_{s,t}(n)$ accordingly by the aid of (16).
**Step 4:** Solve each sub-problem ($L_r(\lambda_{s,t}(n))$) after adding the following constraints:

$$\sum_{a=1}^{e_s} E_{a,s,t}(n) \geq \overline{\hat{E}}_{s,t}(n) + \widehat{\Delta E}_{s,t}(n) \qquad \forall t \in T, \forall s \in S, \forall n \in N$$

**Step 5:** If a feasible solution exists, calculate the UB by the aid of Algorithm 4;
Else go to step 6.
**Step 6:** Remove the constraints in step (4) from $L_r(\lambda_{s,t}(n))$ model and solve it by fixing the value of $Q^{rep}_{r,t}(n)$ as ($\hat{Q}^{rep}_{r,t}(n) - \widehat{\Delta E}_{s,t}(n)$); Afterwards, calculate the UB by the aid of Algorithm 4.
**End While**

As it can be observed in Algorithm 4, in order to calculate the UB, the backorder cost is considered as the highest backorder quantity obtained from these sub-models. As mentioned earlier, these decisions are not defined per component, hence it is possible that they take different values in component sub-models. Therefore, the Lagrangian penalty cost and training cost are calculated based on the maximum workforce and training levels ($E_{a,s,t}(n)$ and $R_{a,s,t}(n)$) obtained over all component sub-models.

**Proposition 3.** Considering the maximum value of workforce and training decisions over all component sub-models in calculating the UB on the optimal objective value of model (3.2)-(3.14) guarantees the feasibility of the relaxed constraint set (3.6).

**Proof.** The proof is provided in Appendix B.

**Algorithm 4** Upper bound calculation algorithm ($LRMST$)

**Step 1:** Calculate the maximum training and penalty cost over all component sub-models as follows:

$$COT = \max_{r \in R} \left\{ \sum_{n \in N} \sum_{t \in T} \sum_{s \in S} \sum_{a \in A_s} pr(n) \Big( n_s R_{a,s,t}(n) \Big) \right\}$$

$$\theta_t(n) = \max_{r \in R, n \in N} \Big\{ B_{r,t}(n) \Big\}$$

**Step 2:** Update the Upper bound UB as follows:

$$UB = \sum_{n \in N} pr(n) \Big( \sum_{t \in T} ( \sum_{r \in R} k_r Q^{rep}_{r,t}(n) + \sum_{r \in R} h_r I_{r,t}(n) + b\theta_t(n)) \Big) + COT$$

**Upper-bound calculation heuristic for the converged solution of $LRCST$ algorithm**

Recall from section 3.4.3 that the component sub-models in $LRCST$ algorithms are formulated based on component STs. The procedure to obtain an UB on the optimal objective function of model (3.2)-(3.14) is, thus, less straightforward in this context. As the first step, The repair mechanism in Algorithm 3 can be applied to the converged solution of $LRCST$ algorithm, where $n$ is replaced by $n'$ denoting the nodes of component STs. Afterwards, the repaired solution can be converted to its counterpart in the master ST by the aid of the procedure described in section 3.4.3.1. Nevertheless, this solution is not necessarily feasible due to the fact that $\overline{E}_{s,t}(n)$ in component sub-models is approximated as $\Big( \alpha_{r,s} Q^{rep}_{r,t}(n) + \sum_{n \in N_t} \sum_{i \in R \setminus r} pr(n) \alpha_{i,s} Q^{rep}_{i,t}(n) \Big)$. The left-hand-side of constraints (3.6), on the contrary, is formulated as $\sum_{r \in R} \alpha_{r,s} Q^{rep}_{r,t}(n)$. Therefore, the third step relies on verifying the feasibility of constraint (3.6) in the converted (repaired) solution of $LRCST$ algorithm. If the solution is feasible, Algorithm 4 can be applied accordingly to calculate the UB. Otherwise, the repair mechanism in Algorithm 3 is applied to this solution before calculating the UB.

## 3.5 Numerical experiments

In this section, we analyze the performance of the proposed solution methodology in terms of quality of bounds and CPU time for two classes of multi-item tactical planning problem instances. We also investigate the value of adopting a multi-stage stochastic programming approach for IPWP under repair time uncertainty rather than a deterministic approach. In what follows, we

Table 3.3: The value of parameters in the numerical experiments

| **Parameter** | $h_r$ | $SS_r$ | $\alpha_{r,s}$ | $b$ | $k_r$ | $O_t$ | $n_s$ | $e_s$ |
|---|---|---|---|---|---|---|---|---|
| Value | 40 | 1 | 1 | 1,000 | 150 | 3,000 | 50 | 24 |

first provide the details of different problem instances, followed by the results of parameter tuning for different algorithms. Afterwards, the detailed analysis of our numerical experiments are provided. All algorithms are implemented in C++ programming language using Concert Technology with IBM-ILOG CPLEX 12.10 on an Intel Core i7 3.4 GHz with 8GB of RAM.

### 3.5.1 Experimental design

Our numerical experiments are designed in the context of a maintenance facility, similar to the one described in Section 3.3.1, that receives a specific type of gas turbine engine. Among the large number of components involved in the structure of this type of equipment, we confine our attention to five expensive and critical repairable components. Some of the parameters associated with this experimental example are summarized in Table 3.3. 11 operators with two skill sets (repair and disassembly/assembly) are considered with the certificate validation period of 24 months. Besides, the initial certificate age of operators vary between 16 to 22 periods (months).

Within this context, different test instances have been generated in two classes of problems, namely IPWP, described in section 3.3, and a generic multi-item production planning (MIPP) problem where the operators' certificates have an unlimited validation period. It is worth to mention that the MIPP problem is recurrent in a wide range of industries with less stringent product safety standards. In other words, the certified operators are not required to repeat the training sessions on a regular basis. The mathematical formulation of MIPP problem is provided in Appendix C. Given the unlimited validation period of certificates, the workforce level and training decisions are not incorporated in this model. Hence, when implementing the $LRMST$ and $LRCST$ algorithms, all terms associated with these variables are not taken into consideration in Algorithms 1-4. In particular, Algorithm 1 must be adapted as described in Appendix D. Furthermore, in component Lagarngian sub-models (3.17)-(3.22) and (3.23)-(3.28), the right-hand-side of constraint sets (3.20) and (3.26) must be replaced by $W_s$.

In both classes of problem instances, two demand profiles ($d_{r,t}$) are taken into consideration, namely averages demand, determined based on the historical data available in the maintenance

Table 3.4: Size of IPWP problem instances

| Number of stages | Setting 1 | | | Setting 2 | | |
|---|---|---|---|---|---|---|
| | # of nodes in the master scenario tree | # of constraints | # of integer variables | # of nodes in the master scenario tree | # of constraints | # of integer variables |
| **5-stage** | 341 | 17,096 | 23,529 | 4,681 | 248,158 | 341,713 |
| **7-stage** | 5,461 | 305,866 | 420,497 | 299,593 | $>14.98\times10^{6}$ | $>23.96\times10^{6}$ |
| **13-stage** | $>22\times10^{6}$ | $>1.25\times10^{9}$ | $>1.79\times10^{9}$ | $>7.68\times10^{10}$ | $>3.84\times10^{12}$ | $>5.37\times10^{12}$ |

facility, as well as high demand by increasing the average value by 20%. We further analyze each class under two settings that differ in terms of the number of components with uncertain repair time. In setting 1, only two components (out of five) are featured with random repair times, both modeled as an ST similar to Figure 3.1. In this setting, the repair time can take two possible outcomes for each component. In particular, $l_1^{rep} = 1$ with probability of 0.7; $l_1^{rep} = 2$ with probability of 0.3; $l_2^{rep} = 1$ with probability of 0.8; and $l_2^{rep} = 2$ with probability of 0.2. Deterministic repair times are considered for components 3-5 with the value equal to 1 period. In setting 2, 3 components have uncertain repair times where $l_3^{rep} = 1$ with probability of 0.8 and $l_3^{rep} = 2$ with probability of 0.2. Furthermore, 3 different planning horizons are considered for both settings, including 4, 6, and 12 periods (months) that lead, receptively, to a 5-stage, 7-stage, and 13-stage scenario trees. To illustrate the size of problem instances, the number of nodes in the corresponding master ST in addition to the number of constraints and integer variables for each instance in the first class of problems (IPWP) are presented in Table 3.4.

### 3.5.2 Parameter tunning

The quality of the LB, along with the CPU time of the $LRMST$ and $LRCST$ algorithms are dependent on the value of parameters involved within the sub-gradient scheme. In particular, the maximum number of iterations ($K$), and step-size parameters ($u_0$ and $p$) are the influencing factors on the aforementioned metrics. Therefore, we aim to analyze the impact of these factors by the aid of designed experiments and $ANOVA$ on the response variable, i.e., the LB obtained from $LRMST$ and $LRCST$. The goal is to set the levels to these parameters such that the largest LB is obtained. To this end, the experiments are run on two medium-size problem instances (i.e., 5-stage and 7-stage in setting 2) by considering three levels for each parameter. Nine experiments are thus conducted over each test instance. The main effect plots of the experiments on the 5-stage test instance are provided in Figures 3.3 and 3.4. Based on this analysis, for both $LRMST$ and $LRCST$ algorithms, we set $p = 0.9$ and $K = 50$. It should be mentioned that by increasing the number of iterations from $K = 50$ to $K = 100$, the changes in the value of LBs are negligible. Moreover, $u_0 = 6.5$ and $u_0 = 1.2$ have been chosen for $LRMST$ and $LRCST$ algorithms, respectively. Similar results are obtained for the 7-stage test instance.

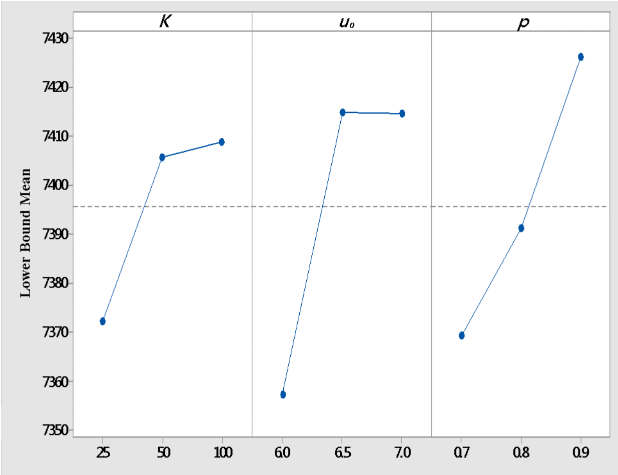


Figure 3.3: Main effect plot of parameters in $LRMST$ algorithm

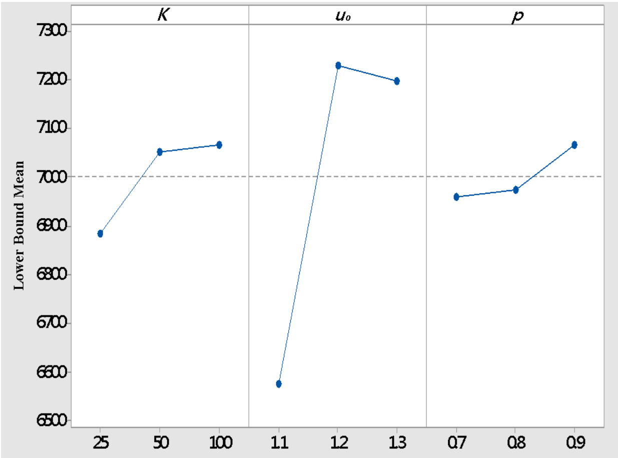


Figure 3.4: Main effect plot of parameters in $LRCST$ algorithm

### 3.5.3 Analysis of the performance of $LRMST$ and $LRCST$ algorithms

The results of implementing $LRMST$ and $LRCST$ algorithms on different classes of problem instances are provided in Tables 3.5-3.7. Table 3.5 and 3.6 present the results of the IPWP model by considering average and high demands and Table 3.7 summarizes the results of the MIPP model.

These tables have been divided into three main columns, including "CPLEX", "$LRMST$" and "$LRCST$". In "CPLEX" column, the "Best feasible solution" and "CPU time (second)" obtained by a commercial solver (CPLEX) are reported. "*" in "Best feasible solution" column indicates the optimality of the obtained solution. In some large-scale instances (e.g., setting 1, 13-stage), CPLEX is not able to find the optimal solution within a 24 hours time limit; hence, the objective value of a feasible solution with 2% optimality gap is reported. Moreover, for other instances (e.g., IPWP setting 2, 13-stage) CPLEX is not able to load the model due to the model size. Therefore, the "Best feasible solution" and "CPU time" are reported as "NA". In "$LRMST$" and "$LRCST$" columns, "LB" represents the lower bound obtained by the proposed algorithms. Besides, "Optimality Gap (%)" corresponds to the relative gap between the LB corresponding to $LRMST$/ $LRCST$ and the best feasible solution (BFS) obtained by CPLEX and is calculated as $((BFS - LB)/BFS) \times 100$). In "$LRCST$" column, "Infeasibility (%)" denotes the percentage of nodes in the component ST with infeasible solutions. "UB" represents the upper bound obtained by implementing the procedure provided in 3.4.4.1. "UB Gap (%)" denotes the relative gap between the "UB" and the BFS obtained by CPLEX and is calculates as $((UB - BFS)/BFS) \times 100)$ . Also, "UB-LB Gap (%)" measures the relative gap between "UB" and "LB" of $LRCST$ and is calculated as $((UB - LB)/UB) \times 100)$.

As it can be observed in these tables, increasing the number of components with random repair time would significantly increase the size of MS-MIP models. That is why, problem instances with a planning horizon longer than 6 months in setting 2 cannot be solved by a commercial solver. Also, in all three classes of problems, CPLEX is not able to load the model for instances that incorporate 12 periods in the planning horizon. Similar trends are observed from the results of $LRMST$ algorithm where the MS-MIP is decomposed per component sub-models, each formulated based on the master ST. Despite the high quality of the LB obtained by this algorithm (with less than 1% optimality gap on average), it is not able to solve realistic-size problem instances (7-stage and 13-stage) given that CPLEX is not able to load the corresponding component sub-models. $LRCST$ algorithm, on the contrary, can efficiently solve all problem instances. In other words, formulating the component sub-models based on component STs significantly reduces the size of these models, thus the computational time of $LRCST$ is much lower than the $LRMST$ algorithm. Furthermore, the CPU time in the largest instances (13-stage) does not exceed 7h which

Table 3.5: The results of the IPWP model with average demand

| Setting | Number of stages | CPLEX | | $LRMST$ | | | $LRCST$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Best feasible solution | CPU time (s) | LB | CPU time (s) | Optimality Gap(%) | LB | CPU time (s) | Optimality Gap(%) | Infeasibility (%) | UB | UB Gap (%) | UB-LB Gap (%) |
| Setting 1 | 5-stage | 7,509* | 12 | 7,471 | 107 | 0.51 | 7,445 | 57 | 0.86 | 2.58 | 7,546 | 0.49 | 1.33 |
| | 7-stage | 11,498 | >24h | 11,399 | 3,452 | 0.86 | 11,361 | 132 | 1.19 | 4.56 | 11,519 | 0.18 | 1.37 |
| | 13-stage | NA | NA | NA | NA | NA | 20,470 | 21,317 | NA | 7.28 | 23,319 | NA | 12.21 |
| Setting 2 | 5-stage | 7,884* | 69,060 | 7,841 | 3,478 | 0.53 | 7,814 | 68 | 0.88 | 3.22 | 7,991 | 1.35 | 2.21 |
| | 7-stage | NA | NA | NA | NA | NA | 11,776 | 124 | NA | 5.67 | 12,112 | NA | 2.77 |
| | 13-stage | NA | NA | NA | NA | NA | 21,143 | 22,647 | NA | 9.02 | 25,182 | NA | 16.03 |

is reasonable given that the tactical planning models under investigation are not required to be solved frequently. This algorithm provides high-quality solutions with small optimality gaps in a reasonable time for small and medium-size instances. In particular, the UB Gap (%) for 5-stage and 7-stage instances (setting 1) is less than 0.5 %. This clearly indicates that the proposed repair mechanism in Algorithms 3 and 4 does not significantly increase the objective function value of the solution obtained by $LRCST$ algorithm. Nonetheless, the quality of LB deteriorates by increasing the number of components with uncertain repair time and the length of the planning horizon. In particular, as the number of stages increases (e.g., 13-stage instances), the number of nodes with infeasible solutions is increased. Consequently, the gap between the repaired solution (UB) from the LB in these instances is higher than the one in 5-stage and 7-stage ones.

It is also worth to mention that in all problem instances, the proposed repair mechanism on the converged solution of $LRCST$ algorithm (Algorithm 3) is effective enough to avoid infeasibility in the converted solution of this algorithm to its counterpart corresponding to the master scenario tree. More precisely, in all the numerical experiments, the infeasibility in terms of workforce capacity constraints is resolved by reducing the quantity of repair operations ($Q^{rep}_{r,t}(n)$) according to Step 6 of Algorithm 3. This would minimize the chance of infeasibility in the converted solution of $LRCST$ algorithm given that the latter do not involve any approximation with regard to repair decisions.

Based on the results provided in Table 3.6, by increasing the demand in IPWP instances, larger LB (the total cost) values are obtained. This result clearly indicates that that capacity of the maintenance facility is not sufficient to satisfy the increased demand within the delivery due dates, which leads to a higher backorder cost. The MIPP instances (Table 3.7), on the contrary, provide the lowest cost given the unlimited validation period of operators certificates which leads to an increased capacity in the maintenance facility.

Table 3.6: The results of the IPWP model with high demand

| Setting | Number of stages | CPLEX | | *LRMST* | | | *LRCST* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Best feasible solution** | **CPU time (s)** | **LB** | **CPU time (s)** | **Optimality Gap(%)** | **LB** | **CPU time (s)** | **Optimality Gap(%)** | **Infeasibility (%)** | **UB** | **UB Gap (%)** | **UB-LB Gap (%)** |
| **Setting 1** | **5-stage** | 13,189* | 18 | 13,073 | 123 | 0.87 | 13,049 | 65 | 1.06 | 3,87 | 13,359 | 1.28 | 2.32 |
| | **7-stage** | 21,268 | >24h | 21,028 | 4,726 | 1.12 | 20,958 | 184 | 1.45 | 5,66 | 21,712 | 2.08 | 3.47 |
| | **13-stage** | NA | NA | NA | NA | NA | 36,217 | 23,178 | NA | 8,31 | 40,982 | NA | 11.62 |
| **Setting 2** | **5-stage** | 18,275* | 216 | 18,049 | 4,409 | 1,23 | 17,693 | 87 | 1.7 | 5,80 | 18,720 | 2.43 | 5.48 |
| | **7-stage** | NA | NA | NA | NA | NA | 24,218 | 213 | NA | 5,77 | 25,579 | NA | 5.32 |
| | **13-stage** | NA | NA | NA | NA | NA | 39,534 | 24,162 | NA | 9,15 | 46,912 | NA | 15.72 |

Table 3.7: The results of the MIPP model

| Setting | Number of stages | CPLEX | | *LRMST* | | | *LRCST* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Best feasible solution** | **CPU time (s)** | **LB** | **CPU time (s)** | **Optimality Gap(%)** | **LB** | **CPU time (s)** | **Optimality Gap(%)** | **Infeasibility (%)** | **UB** | **UB Gap (%)** | **UB-LB Gap (%)** |
| **Setting 1** | **5-stage** | 6,142* | 3 | 6,134 | 122 | 0.13 | 6,128 | 45 | 0.21 | 1.93 | 6,185 | 0.7 | 0.92 |
| | **7-stage** | 10,431* | 25,350 | 10,398 | 3,121 | 0.31 | 10,390 | 95 | 0.39 | 3.62 | 10,599 | 1.61 | 1.97 |
| | **13-stage** | NA | NA | NA | NA | NA | 17,968 | 18,326 | NA | 6.21 | 19,083 | NA | 5.84 |
| **Setting 2** | **5-stage** | 6,578* | 4,825 | 6,568 | 1,512 | 0.15 | 6,562 | 35 | 0.24 | 3.22 | 6,649 | 1.07 | 1.30 |
| | **7-stage** | NA | NA | NA | NA | NA | 11,776 | 115 | NA | 4.72 | 11,985 | NA | 1.74 |
| | **13-stage** | NA | NA | NA | NA | NA | 19,390 | 21,912 | NA | 6.9 | 20,927 | NA | 7.34 |

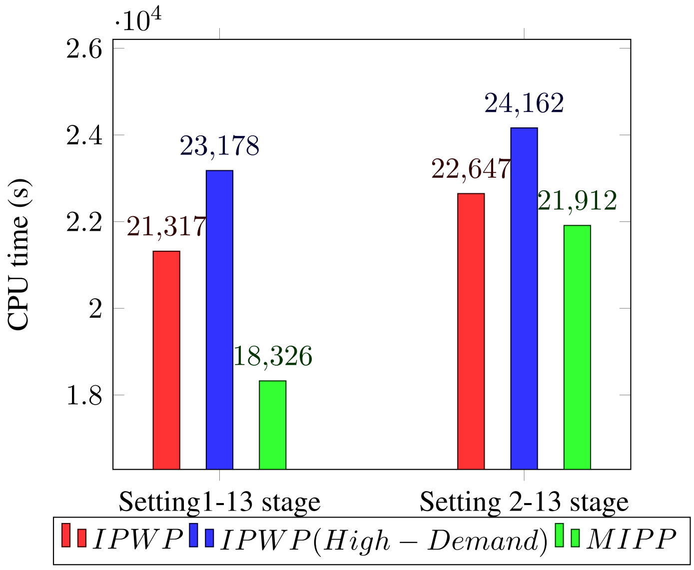


Figure 3.5: $LRCST$ CPU time comparison for different models

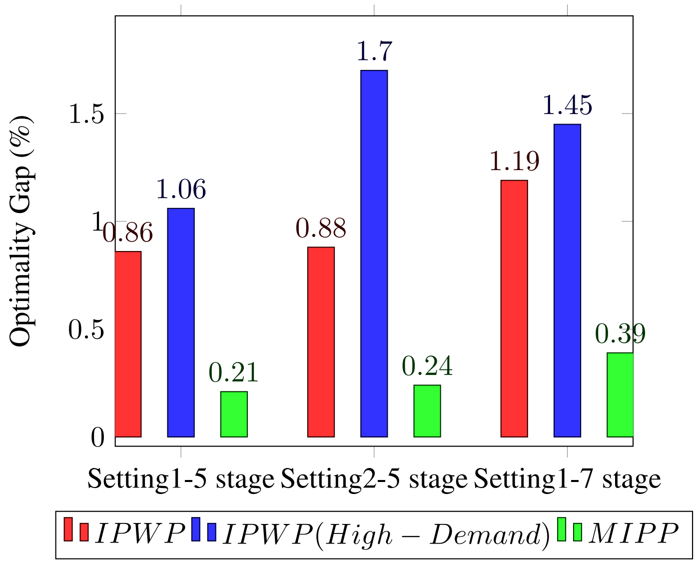


Figure 3.6: $LRCST$ Optimality Gap(%) comparison

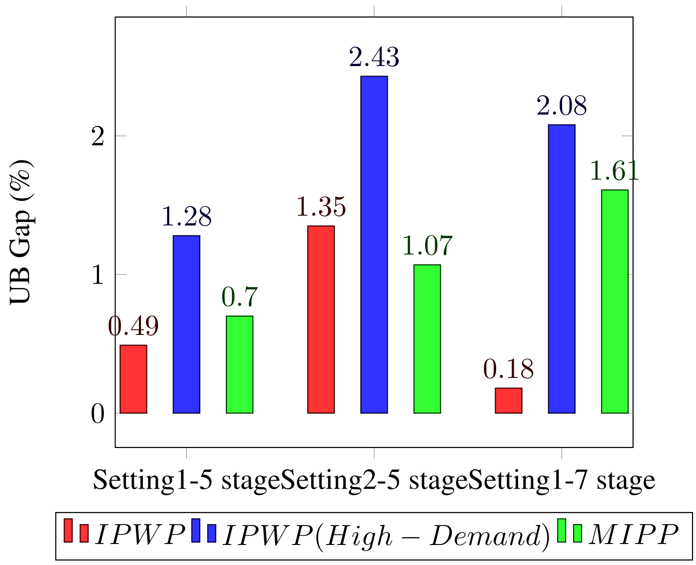


Figure 3.7: $LRCST$ UB Gap(%) comparison

The comparison between different classes of problem instances in terms of CPU time, Optimality Gap (%), and UB Gap(%) of $LRCST$ algorithm is provided in Figures 3.5-3.7. As it can be observed in Figure 3.5, the MIPP test instances are the least demanding ones in terms of CPU time. This is due the reduced number of decision variables (i.e., the absence of workforce and training variables) in these instances. Contrarily, the IPWP instances with high demand are the most difficult instances to solve by $LRCST$ algorithm. Furthermore, the CPU time required to run this algorithm for solving the problem instances in setting 2 is higher than the ones in the first setting due to the increased number of component sub-models in these instances. The comparison between the $LRCST$ Optimality Gap (%) of 5 and 7-stage problem instances (Figure 3.6) indicates that the performance of this algorithm (in terms of the quality of LB) slightly deteriorates by increasing the demand. As expected, the smallest optimality gaps are observed in MIPP instances. This result can be clearly justified based on the fact that Lagrangian component sub-models in Algorithm 5 (Appendix B) does not include workforce level decision variables. Therefore, when applying the sub-gradient scheme within the $LRCST$ algorithm, the Lagrangian penalty terms are more effective in minimizing the infeasibility in terms of workforce capacity constraints. Finally, the quality of the optimality gap slightly deteriorates by increasing the number of components with uncertain repair time (setting 2). Similar trends can be observed when comparing different classes of problem instances in terms of UB Gap (%) in $LRCST$ algorithm (Figure 3.7). In other words, for the same reason described above, the smallest percentage of infeasibility is obtained in the converged solution of $LRCST$ algorithm in MIPP test instances resulting in smaller values for the UB and consequently lower UB Gap (%) values. Finally, by comparing the values of LB and UB for 13-stage instances in Tables 3.5 and 3.7, it can be observed that the number of nodes with infeasible solutions significantly declines in MIPP instances; therefore, the UB Gap (%) in these instances is 50% lower than the IPWP ones. These results clearly indicate that $LRCST$ algorithm is more efficient in solving generic stochastic multi-item tactical planning problems where the workforce capacity does not change over the planning horizon.

### 3.5.4 The value of stochastic solution

In this section, the production and workforce training plan obtained from the proposed MS-MIP IPWP model is compared with the one obtained from a deterministic model where the average repair times are taken into consideration. More specifically, the value of adopting a multi-stage stochastic programming is measured by calculating the value of stochastic solution ($VSS$). Let us denote the optimal objective function value of the stochastic model as $MSP$ and the expected cost of the deterministic solution as $EDS$. The latter is calculated by fixing the value of $Q^{rep}_{r,t}$, $E_{a,s,t}$, and $R_{a,s,t}$ in model (3.2)-(3.14) as their optimal value obtained from solving the deterministic model. Afterwards, $EDS$ is calculated accordingly as the objective function of model (3.2)-(3.14). $VSS$ is then calculated as the difference between $EDS$ and $MSP$, as indicated in the following

Table 3.8: Value of stochastic solution in 7-stage and 13-stage IPWP models

| Setting | | | Expected repair cost ($) | Expected training cost ($) | Expected backorder cost ($) | Total expected cost ($) | $VSS$ (%) |
|---|---|---|---|---|---|---|---|
| **Setting 1** | **7-stage** | $EDS$ | 6,600 | 200 | 7,299 | 15,789 | 27.17 |
| | | $MSP$ | 7,150 | 300 | 2,298 | 11,498 | |
| | **13-stage** | $EDS$ | 15,600 | 300 | 10,163 | 29,163 | 29.80 |
| | | $MSP$ | 16,227 | 500 | 624 | 20,470 | |
| **Setting 2** | **7-stage** | $EDS$ | 6,600 | 200 | 8,538 | 17,028 | 30.84 |
| | | $MSP$ | 7,206 | 300 | 2,552 | 11,776 | |
| | **13-stage** | $EDS$ | 15,600 | 399 | 12,951 | 31,951 | 33.82 |
| | | $MSP$ | 16,485 | 500 | 1,043 | 21,143 | |

equation:

$$VSS(\%) = \frac{EDS - MSP}{EDS} 100 \tag{3.30}$$

The results for the 7-stage and 13-stage IPWP models are presented in Table 8 where the $EDS$ and $MSP$ in the two problem settings are broken down into the expected repair, training, backorder, and total costs. $VSS(\%)$ is calculated in the last column based on the $EDS$ and $MSP$ values in terms of the total expected cost. As it can be observed in this table, the multi-stage stochastic programming approach significantly outperforms the deterministic one based on $VSS(\%)$ results. Furthermore, the expected backorder cost seems to be the major contributing factor to the large value of $EDS$ in all problem instances. In other words, the results indicate that implementing the plan obtained from a deterministic model leads to a high backorder cost that represents the penalty cost of late delivery of repaired devices. This is due to ignoring the possibility of extended repair times and consequently the shortage of repaired components and/or certified operators. In fact, the main advantage of the stochastic model is the flexibility to update the production (repair) quantities and training levels at every stage (period) in the planning horizon as a response to different repair time outcomes. The deterministic model, on the contrary, provides a fixed production plan without considering the upcoming changes in repair time uncertainty. That is why, the repair quantities and training levels in the stochastic model are higher than the deterministic one.

## 3.6 Conclusion

With a particular focus on maintenance facilities, we proposed two multi-stage stochastic programming models for two multi-item tactical planning problems that are featured with uncertain parameters with independent probability distributions. More specifically, we investigated two classes of problems that differ in terms of the validation period of operators' training certificates.

The first category incorporates IPWP problems, which are among the most complex tactical planning problems due to the limited validation period of operators' certificates. Hence, the workforce level and training decisions must be explicitly considered in these problems given that the certificates of operators expire over time. The second category, on the contrary, is more generic and is applicable in the context of technical devices that are required to comply with less stringent safety standards. As a consequence, once the operators are trained and certified, their certificates will be valid for an unlimited duration. In both classes of problems, the repair times of some components were assumed to be random variables, each represented as an independent scenario tree over the planning horizon. Incorporating the repair time uncertainty into an integrated workforce and production planning problem in maintenance facilities is a novel contribution that has not been previously investigated in the literature. The assumption of independence among random parameters significantly increases the size of corresponding master ST and the MS-MIP models. As another original contribution, an approximate decomposition algorithm was proposed to efficiently solve the real-size instances in both categories of problems that contain 12 periods in the planning horizon. The proposed algorithm relies on decomposing the MS-MIP models into component sub-models within an LR scheme where the sub-models are formulated based on their corresponding ST. A heuristic algorithm was also proposed to repair the infeasibility of the obtained solution within the LR framework that provides a high-quality upper bound to the optimal objective value of the original MS-MIP model.

Our numerical experiments conducted on several test instances clearly showcased the capability of the proposed approximation algorithm to efficiently solve real-size tactical planning problems. The results are in particular very promising in the generic tactical planning problems featured with a fixed amount of workforce capacity. Our results also demonstrated the importance of incorporating uncertain repair times into the tactical planning under investigation. Adopting a deterministic approach, in particular, would lead to 30% increase on average in terms of the total cost that is mainly due to the shortage of certified operators and repaired components.

The current study can be extended in terms of modeling aspects, algorithmic implementation, and application in other manufacturing sectors. From the modeling point of view, some mitigation (recourse) actions can be incorporated into the multi-stage stochastic model so as to compensate the risk of extended repair times. Partial outsourcing of repair operations to other collaborating facilities and/or temporarily borrowing available certified operators from those facilities could be considered as two strategies to hedge against the shortage of components and workforce in the presence of extended repair times. Nevertheless, such strategies heavily rely on the existence of a coalition among a set of maintenance facilities. From an algorithmic perspective, the proposed decomposition algorithm is amenable to parallel computing; hence it can be applied to efficiently solve problem instances with a large number of components with random repair times. Finally, the proposed model and solution methodology can be extended in the context of other manufacturing environments that are confronted with the random demand of products following independent probability distributions. In the same vein, they can be applied to tactical planning models that include binary set-up decisions. In this case, the component sub-models can be solved by efficient *ad-hoc* algorithms rather than a commercial solver.

# Appendix A

*Proof of proposition 2*

Recall from the component Lagrangian sub-model (3.15) that the amount of infeasibility with regard to the workforce capacity constraint (3.6) is penalized in the objective function by:

$$\sum_{n\in N}\sum_{t\in T}\sum_{s\in S}\lambda_{s,t}(n)\Big(\overline{E}_{s,t}(n)-\sum_{a=1}^{e_s}E_{a,s,t}(n)\Big)$$

Given that the workforce level ($E_{a,s,t}(n)$) decisions are not indexed per component, they might take different values at the optimal solution in different component sub-models within the $LRCST$ scheme. According to proposition 2, the value of this decision in each node of master ST ($n \in N$) is set as $\max\limits_{r\in R, n'_r\in N'_n} \hat{E}_{a,s,t}(n'_r)$, where $N'_n$ denotes the set of nodes in the $|R|$ component scenario trees that constitute node $n$ in the master ST and $\hat{E}_{a,s,t}(n'_r)$ denotes the converged value of the workforce level decisions in $LRCST$ component sub-models. It can be easily verified that $\forall r \in R|\hat{E}_{a,s,t}(n'_r) < \max\limits_{r\in R, n'_r\in N'_n} \hat{E}_{a,s,t}(n'_r)$, $\Big(\overline{E}_{s,t}(n)-\sum_{a=1}^{e_s}\hat{E}_{a,s,t}(n'_r)\Big) > \Big(\overline{E}_{s,t}(n) - \max\limits_{r\in R, n'_r\in N'_n}\sum_{a=1}^{e_s}\hat{E}_{a,s,t}(n'_r)\Big)$. Therefore, this proposition provides an approximation rule for $E_{a,s,t}(n)$ that results the smallest amount of infeasibility (if any) in terms of workforce capacity constraints.

Finally, in order to guarantee the feasibility of workforce training constraints (3.12)-(3.13), the same rule applies for converting the training decisions in the master ST. In other words, $R_{a,s,t}(n) = \max_{r\in R, n'_r \in N'_n} \hat{R}_{a,s,t}(n'_r)$, where $\hat{R}_{a,s,t}(n'_r)$ denote the converged value of training decisions in $LRCST$ component sub-models.

# Appendix B

*Proof of proposition 3*

Let us denote the repaired solution of $LRMST$ algorithm as $\hat{Q}_{r,t}(n)$, $\hat{Q}^{rep}_{r,t}(n)$, $\hat{B}_{r,t}(n)$, $\hat{I}_{r,t}(n)$, $\hat{E}_{a,s,t}(n)$, $\hat{R}_{a,s,t}(n)$. We also denote the nodes of master ST in each component sub-model $r$ as $n_r$. It can be easily verified that:

$\forall r \in R | \hat{E}_{a,s,t}(n_r) < \max_{r\in R} \hat{E}_{a,s,t}(n_r), \sum_{r\in R} \alpha_{r,s} Q^{rep}_{r,t}(n)) \leq \sum_{a=1}^{e_s} \hat{E}_{a,s,t}(n_r) \leq \max_{r\in R} \sum_{a=1}^{e_s} \hat{E}_{a,s,t}(n_r)$.

Therefore, this proposition provides an approximate value for $E_{a,s,t}(n)$ that guarantees the feasibility of workforce capacity constraints.

# Appendix C

*Mathematical formulation for multi-item production planning (MIPP) problem*

$$Minimize \sum_{n\in N} pr(n) \sum_{t\in T} \left( \sum_{r\in R} (k_r Q^{rep}_{r,t}(n) + h_r I_{r,t}(n)) + b\theta_t(n) \right) \tag{3.31}$$

***Operations constraints***

$$Q_{r,t}(n) - B_{r,(t-1)}(a(n)) + B_{r,t}(n) = d_{r,t} \qquad \forall t \in T, \forall r \in R, \forall n \in N \tag{3.32}$$

$$Q^{rep}_{r,(t-l^{rep}_r(n))}(a'_r(n)) + I_{r,(t-1)}(a(n)) - I_{r,t}(n) = Q_{r,t}(n) \qquad \forall t \in T \backslash 1, ..., (l^{rep}_r(n)), \forall r \in R, \forall n \in N \tag{3.33}$$

$$\theta_t(n) \geq B_{r,t}(n) \qquad \forall t \in T, \forall r \in R, \forall n \in N \tag{3.34}$$

$$\sum_{r\in R} \alpha_{r,s} Q^{rep}_r(n) \leq W_s \qquad \forall t \in T, \forall s \in S, \forall n \in N \tag{3.35}$$

$$I_{r,t}(n) \geq SS_r \qquad \forall t \in T, \forall r \in R, \forall n \in N \tag{3.36}$$

$$Q^{rep}_{r,t}(n) = 0 \qquad \forall t \in 1, ..., l^{rep}_{rt}(n), \forall r \in R, \forall n \in N \tag{3.37}$$

***Domain constraints***

$$Q_{r,t}(n), Q^{rep}_{r,t}(n), I_{r,t}(n), B_{r,t}(n), \theta_t(n) \in Z_0^+ \qquad \forall t \in T, \forall n \in N, \forall r \in R, \forall s \in S \quad (3.38)$$

# Appendix D

**Algorithm 5** Sub-gradient algorithm (LRMST) for MIPP problem

**Step 0 (Initialization):**
Assign zero to $\lambda^{(1)}_{s,t}(n)$
Assign $-\infty$ to the $LB$
Set iteration counter $k$ equal to 1
Assign initial values to $u_0$ and $p$

---

**While** the stopping criteria is not satisfied **do**:
**Step 1:** Solve the Lagrangian problem $L_r(\lambda_{s,t}(n))$ and update the LB (Algorithm 2)
**Step 2:** Update the step-size as $u_k$=$u_0p^k$
**Step 3:** Update the Lagrangian multipliers as follows:

$$\lambda^{(k+1)}_{s,t}(n) = \lambda^{(k)}_{s,t}(n) + u_k\left\{\overline{E}^{(k+1)}_{s,t}(n) - W_s\right\}$$

$$\overline{E}^{(k+1)}_{s,t}(n) = \sum_{r\in R} \alpha_{r,s} Q^{rep}_{r,t}(n)^{(k)}$$

$$k = k + 1$$

**End While**

# Chapter 4

# Robust collaborative maintenance logistics network design and planning

This chapter corresponds to the following journal paper:

Shayan Tavakoli Kafiabad, Masoumeh Kazemi Zanjani, and Mustapha Nourelfath. ”Robust collaborative maintenance logistics network design and planning.” Revision submitted to International Journal of Production Economics, September 2021.

## Abstract

Maintenance service providers to advanced technical devices are confronted with uncertain demand, high cost of components, and the need for certified operators. Collaboration in terms of sharing scarce resources among different facilities in a maintenance logistics network is expected to reduce the delays in the delivery of repaired devices. This study proposes a two-stage robust optimization model for collaborative design and planning of maintenance networks under demand uncertainty. The goal of this model is to determine the optimal allocation of customers to each maintenance center along with the initial stock level of different components in each facility so as to minimize the cost of late deliveries under worst-case demand scenarios. Component and operator sharing strategies are proposed as the recourse actions in this model to hedge against the demand surge. The proposed approach is compared with a deterministic model by the aid of Monte-Carlo simulation on several test instances inspired by a real case study. Our numerical experiments demonstrate the significance of adopting the proposed collaborative mechanisms among maintenance facilities in terms of cost reduction, especially when the demand fluctuation is relatively high.

## 4.1 introduction

For many advanced systems, such as gas turbines and medical equipment, high system availability is crucial. Maintenance companies are typically responsible for the repair and overhaul of these technical devices on the basis of a strategic contract with equipment users. However, these companies are confronted with the occasional surge in demand due to the sporadic failure rates of components involved in the structure of these systems. Consequently, they might experience the shortage of expensive resources, such as components, and certified operators required to perform repair operations. To ensure resource availability and reduce the risk of delays invoked by shortages, maintenance centers can resort to various strategies, such as pre-positioning more components in their internal warehouses and collaborating with other centers in the maintenance logistics network in terms of sharing components and operators according to a predetermined sharing level. Besides, estimating the probability distribution of the demand is a challenge for these centers mainly due to the unavailability of historical data on equipment failures along with accurate information on the equipment's usage pattern.

In response to the above challenges, this paper provides a robust collaborative decision model for the simultaneous strategic design and tactical planning of a maintenance logistic network comprising of a central repair facility along with a set of local repair facilities (LRFs) and equipment users. More specifically, we assume the formation of a grand coalition among the LRFs to (temporarily) share a certain percentage of components and certified operators among each other to avoid delayed delivery of repaired devices in case of demand surge in some facilities. The demand uncertainty is modeled as an interval by considering a budget of uncertainty, without assuming a specific probability distribution. In this context, a two-stage robust optimization model is proposed that seeks the optimal allocation of users to LRFs along with the stock pre-positioning levels for different components in each facility as strategic (here-and-now) decisions. The tactical planning decisions in this model, on the contrary, are wait-and-see and defined for each demand scenario. These decisions incorporate the quantity of repair jobs scheduled in each LRF along with the quantity of resources that must be exchanged among different facilities in the network in order to satisfy the demand. In fact, this model aims to determine the optimal design decisions in addition to the anticipation of tactical decisions in the presence of different demand scenarios. The objective of this model is to minimize the cost of establishing the network along with the cost of repair operations, resource exchange, and shortages under worst-case demand scenarios within a given uncertainty set.

The proposed collaborative decision model significantly contributes to the literature on maintenance logistics network management. Our first contribution revolves around incorporating a simultaneous component and operator sharing mechanism among stakeholders as a risk mitigation action into the integrated design and planning of such logistics networks in a multi-period, multi-component setting. To the extent of the authors' knowledge, no prior work in the literature considers resource sharing strategies in this problem. Our second contribution is focused on adopting a two-stage robust optimization (RO) approach Zeng and Zhao (2013) to incorporate demand uncertainty into the above-mentioned problem. Besides, we develop a Monte-Carlo simulation platform to compare the results of the two-stage RO model with a deterministic model in a realistic environment. Finally, we provide managerial insights by running sensitivity analysis experiments on a large set of problem instances inspired by a real case study in the gas turbine industry. Our goal is to investigate the impact of the proposed resource sharing mechanism on the total cost of the network and, in particular, on the shortage cost under different sharing levels agreed among the stakeholders.

The remainder of this paper is divided into the following sections. To highlight the paper contribution, Section 4.2 provides a review of recent literature on maintenance logistics network design and planning. Section 4.3 presents the problem description and formulation as a deterministic mixed-integer programming model. Incorporating the demand uncertainty into the problem under investigation is elaborated in Section 4.4. The Monte-Carlo simulation platform is provided in Section 4.5. Section 4.6 presents the numerical results. Finally, in Section 4.7, conclusions and future research directions are provided.

## 4.2 Literature review

This section primarily provides a brief review of the most relevant research on the design and planning of maintenance logistics networks. Afterwards, we narrow our focus to the prevailing research on collaborative planning in supply chains that is closely related to the current study.

### 4.2.1 Maintenance logistics network design and planning

Only a handful of papers investigate the design of maintenance networks for repairable parts. The authors in Rappold and Van Roo (2009) develop a network design model for repairable spare parts to determine the optimal facility location, user allocation, and capacity investment decisions simultaneously. Their model considers a capacitated central warehouse at the upstream level, maintenance facilities at tier two, and local warehouses at the downstream level. The authors in (Wu, Hsu, & Huang, 2011) propose an integrated model for the selection of network configuration, suppliers, and transportation modes with the aim of maintaining an average target availability while minimizing the total cost. The availability of the device in each local maintenance facility is ensured by installing sufficient resources, i.e., operators and spare parts inventory.

Another category of articles in the literature deal with the allocation of users' demand to the existing maintenance facilities and setting inventory levels of repairable parts in these facilities. Kutanoglu and Lohiya (2008) consider a network consisting of a central warehouse with infinite capacity, multiple local warehouses, and equipment users. They propose a mathematical inventory-allocation model by considering a continuous review policy, time-based service level and different transportation modes (slow, medium, fast). In the context of a utility company, Van den Berg, van der Heijden, and Schuur (2016) study a maintenance network consisting of a supplier, a central warehouse, and some advanced and basic local warehouses. Motivated by the importance of the availability of spare parts in the network, they develop a two-echelon service-parts allocation model by considering time window constraints.

Somarin, Chen, Asian, and Wang (2017) consider a network that consists of one central maintenance facility with a single repair server at the upstream level and multiple local maintenance facilities at the downstream level. They propose an allocation mechanism to find the optimal initial spare part levels at local maintenance facilities and the best reallocation strategy for the ready-to-use spare parts that have been repaired at the central maintenance facility.

In the realm of maintenance supply chains, few studies deal with integrated tactical planning problems in local repair facilities. Zanjani and Nourelfath (2014) propose a multi-stage stochastic programming model for operations planning in the context of multi-component repairable devices with the goal of minimizing the expected cost of procurement, inventory, and late delivery under uncertain demand scenarios. Sleptchenko et al. (2017) investigate the joint optimization of cross-training of operators and spare parts provisioning of repairable parts in a single echelon supply chain consisting of multiple local warehouses and one maintenance facility. The integration of operations and workforce planning in maintenance facilities is studied in (Tavakoli Kafiabad et al., 2020) under the assumption of deterministic demand for faulty components. In another work (Tavakoli Kafiabad, Zanjani, & Nourelfath, 2021), the impact of demand uncertainty on integrated operations and workforce scheduling in similar facilities is investigated.

### 4.2.2 Collaborative supply chain planning

In the realm of inventory management in maintenance logistics networks, there are several studies that investigate resource sharing mechanisms such as inventory pooling, lateral transshipment, and demand rationing strategies. The authors in (Wong, Cattrysse, & Van Oudheusden, 2005) propose a mathematical model to approximate various performance measures in a single-product, multi-facility, inventory management system where complete pooling of parts is permitted among the facilities. They consider non-zero lateral transshipment time as well as the delayed lateral transshipment and formulate the problem as a multi-dimensional Markovian problem and solve it using a two-stage solution method. The authors in (Tiemessen, Fleischmann, van Houtum, van Nunen, & Pratsini, 2013) develop a framework for dynamic demand fulfillment in the maintenance logistics network of advanced technical system that deals with users with different service contracts. The demand of users can be fulfilled from local facilities via a regular delivery, or from an external source with ample capacity via an emergency shipment. They propose a dynamic demand allocation rule that belongs to the class of one-step look-ahead policies and develop an iterative algorithm to approximate the expected total cost over an infinite planning horizon. A comprehensive literature review of inventory models with a lateral transshipment strategy can be found in (Paterson, Kiesmüller, Teunter, & Glazebrook, 2011). Collaborative planning in supply chains has been studied in several contexts in the literature, including humanitarian, healthcare and transportation networks. In the context of humanitarian networks, Doodman, Shokr, Bozorgi-Amiri, and Jolai (2019) propose a collaborative bi-objective two-stage stochastic programming model. This model determines the optimal pre-positioning of relief items in the warehouses along with the distribution of relief items after observation of demand scenarios. In particular, the authors propose a lateral transshipment strategy among the local warehouses to reduce the relief shortages. The objective is to maximize the fairness and minimize the total cost of the network. In another study, Mehrotra, Rahimian, Barah, Luo, and Schantz (2020) propose a collaborative two-stage stochastic optimization model for allocating and sharing life-saving resources in the case of COVID-19 pandemic among hospitals under random demand. They also assume a safety threshold parameter that captures the risk-aversion level of each hospital in sharing excess inventory with other hospitals in the network. Guajardo and Rönnqvist (2015) develop a mixed integer linear programming model to integrate coalition structure and cost allocation problems in the context of forest transportation networks and inventory management of spare parts. The authors report 5-15% cost saving as a result of collaboration in the forest transportation network and around 20% saving in inventory costs in the spare parts inventory management case.

The above survey of literature clearly indicates the paucity of research on developing a collaborative framework for the design and tactical planning in maintenance networks. This is mainly due to the fact that all the current papers in the literature either focus merely on the design of such networks or optimize the operations from the viewpoint of a single maintenance facility. Furthermore, the adoption of a resource sharing policy in terms of operators and components to hedge against surge of demand has never been investigated in the literature. It is also worth to mention that none of the above-mentioned studies focus on tactical planning in a maintenance logistics network; they rather study this problem from the perspective of a single maintenance facility. To fill the above-mentioned research gaps, the present paper develops a collaborative framework for the strategic design and tactical planning in maintenance networks. The proposed model adopts a resource sharing policy in terms of operators and components to hedge against surge of demand. It incorporates such resource sharing mechanisms among stakeholders as a risk mitigation action into the integrated design and planning of such logistics networks. To take into account demand uncertainty, we adopt a two-stage robust optimization approach. Furthermore, we develop a Monte-Carlo simulation and we provide managerial insights by running analysis experiments on a large set of problem instances inspired by a real case study in the gas turbine industry. The resulting proposed approach investigates the impact of resource sharing mechanism on the total cost of the network and, in particular, on the shortage cost under different sharing levels.

## 4.3 Problem description

### 4.3.1 Collaborative maintenance logistics network design and planning

Consider a maintenance logistics network consisting of a central repair facility (CRF), multiple local repair facilities (LRF) and a set of equipment users. The users in this network utilize advanced technical devices (e.g., gas turbines) comprising several modules and repairable components that are subject to random failures. Besides, the users are geographically dispersed and, based on a long-term contract, they outsource their system upkeep and overhaul services to an LRF. Therefore, each LRF is responsible for satisfying the demand of a cluster of users. Upon receiving the failed pieces of equipment, the LRF is responsible for the disassembly, inspection, and replacement of defective components. Besides, all such faulty components are repaired and recovered in the CRF. Figure 4.1 provides a conceptual framework for this network.

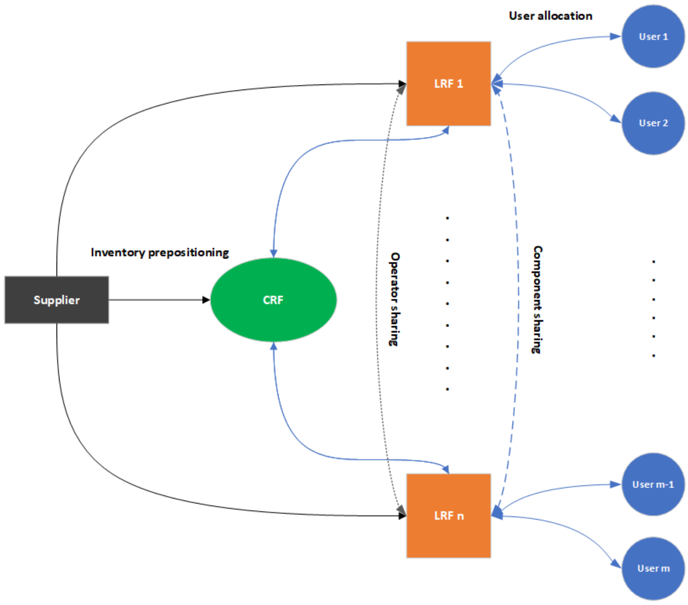


Figure 4.1: The conceptual framework of a maintenance logistics network

The collaborative design and planning problem, proposed in this study, incorporates two phases. In the design phase, the optimal allocation of users to LRFs as well as the inventory pre-positioning decisions are crucial. We further assume the demand of each user is only satisfied by one LRF. Besides, the user allocation cost is an increasing function of the distance between the user and the LRF along with the average demand of the user. The adequate initial inventory level of components in the CRF and LRFs will assure the responsiveness of the network in the tactical phase. Thus, at the beginning of the planning horizon, the CRF and LRFs are required to procure components from suppliers and store them in their internal warehouses with respect to their procurement budget and estimated demand.

The planning phase, in contrast, revolves around anticipating the optimal level of operations as a response to the demand of maintenance services for the devices that are being received by the LRFs from the assigned users over a planning horizon. We further confine our attention to a fleet of aged devices, where the majority of components must be replaced. In this phase, the repairable components would be replenished from the internal inventory of each LRF if they are available in stock. Otherwise, they would be back-ordered until the failed components are repaired in the CRF. The latter receives all failed components and carries out repair/recovery operations on them with variable repair times. Afterwards, the recovered components are shipped back to corresponding LRFs. Both the CRF and LRFs have a limited capacity for replacement and repair operations in each period of the planning horizon that is a function of the number of available operators and budget.

Given that the network is confronted with demand uncertainty in the tactical phase, the LRFs exert two risk mitigation strategies, namely component and operator sharing. In other words, it is assumed that a grand coalition has been established among the LRFs, such that each player shares a predetermined percentage of their affiliated operators and components available in stock with others. In the same vein, each LRF has the right to keep a certain level of reserved operators and components as safety stock to maintain a minimum service level. In this context, the shared operators and components are sent immediately to satisfy the unmet demand in the same period by considering the operator's relocation as well as component transportation costs.

Based on the assumptions mentioned above, we formulate the collaborative design and planning problem as a mixed integer programming model. The description of parameters and decision variables is provided in Tables 4.1 & 4.2. The operating cost drivers incorporate: (1) user allocation cost to LRFs; (2) inventory pre-positioning cost at CRF and LRFs; (3) inventory holding cost at CRF and LRFs; (4) repair and replacement costs; (5) transportation cost; (6) shortages cost; (7) Operator sharing cost; and (8) Component sharing cost. The objective of the model is to minimize the total cost of the system over the planning horizon.

Table 4.1: Sets, indices and parameters

| | |
|---|---|
| $J$ | Set of LRFs indexed by $j$ |
| $I$ | Set of users indexed by $i$ |
| $R$ | Set of components, indexed by $r$ |
| $u_r$ | Unit procurement cost of component $r$ |
| $k_r$ | Unit repair cost of component $r$ |
| $h_r$ | Unit holding cost of component $r$ |
| $f_{ij}$ | Fixed allocation cost of user $i$ to LRF $j$ |
| $v_j$ | Unit transportation cost from CRF to LRF $j$ |
| $l_{jk}$ | Unit component sharing cost from LRF $j$ to LRF $k$ |
| $r_{jk}$ | Unit operator sharing cost from LRF $j$ to LRF $k$ |
| $\pi_r$ | Unit penalty cost for the shortage of component $r$ |
| $n^c$ | Available budget of CRF to repair components |
| $m^c$ | Available budget of CRF to procure components |
| $m^l_j$ | Available budget of LRF $j$ to procure components |
| $g_r$ | Repair time of component $r$ |
| $e_j$ | Number of operators affiliated with LRF $j$ |
| $\beta_r$ | Resource consumption factor (operator) to inspect/replace component $r$ |
| $d_{rit}$ | Number of components $r$ of user $i$ scheduled to be sent to LRFs for repair in period $t$ |
| $\tau_j$ | Percentage of initial inventory level of components in LRF $j$ that the LRF is willing to share with other LRFs |
| $\lambda_j$ | Percentage of reserved inventory that LRF $j$ keeps as the safety stock in internal warehouse in each period |
| $\eta_j$ | Percentage of affiliated operators of LRF $j$ that can be shared with other LRFs |
| $\zeta_j$ | Percentage of reserved operators in LRF $j$ in each period |

Table 4.2: Decision variables

| | |
|---|---|
| $D_{rjt}$ | Number of component $r$ scheduled to be replaced in LRF $j$ in period $t$ |
| $X_{ij}$ | 1 if user $i$ is assigned to LRF $j$, 0 otherwise. |
| $Y_{jkt}$ | Number of operators assigned to LRF $k$ by the LRF $j$ at the beginning of period $t$ |
| $Q^c_r$ | Quantity of component $r$ prepositioned in the CRF |
| $Q^l_{rj}$ | Quantity of component $r$ prepositioned in LRF $j$ |
| $I^c_{rt}$ | Inventory level of component $r$ in CRF in period $t$ |
| $B^l_{rjt}$ | Shortage quantity of component $r$ in LRF $j$ in period $t$ |
| $B^c_{rt}$ | Shortage quantity of component $r$ in CRF $j$ in period $t$ |
| $I^l_{rjt}$ | Inventory level of component $r$ in LRF $j$ in period $t$ |
| $Z^{cl}_{rjt}$ | Quantity of component $r$ sent from the CRF to LRF $j$ in period $t$ |
| $Z^{lat}_{rjkt}$ | Quantity of component $r$ sent from LRF $j$ to LRF $k$ in period $t$ |
| $R_{rjt}$ | Quantity of ready to assembly component $r$ in LRF $j$ in period $t$ |
| $Q^{rep}_{rt}$ | Number of component $r$ that should be repaired in the CRF in period $t$ |
| $U_{jt}$ | Number of available operators in LRF $j$ in period $t$ including the affiliated and shared operators |
| $V_{rjt}$ | 1 if LRF $j$ decides to send components to other LRFs for component $r$ in period $t$, 0 otherwise |
| $W_{jt}$ | 1 if LRF $j$ decides to share operators with other LRFs in period $t$, 0 otherwise |

### 4.3.2 Mathematical formulation

According to the problem description and assumptions mentioned above, the objective function of the model can be formulated as follows:

$$\begin{aligned} \min \quad & \sum_{i\in I}\sum_{j\in J} f_{ij}X_{ij} + \sum_{r\in R} u_r Q^c_r + \sum_{r\in R}\sum_{j\in J} u_r Q^l_{rj} + \sum_{r\in R}\sum_{t\in T} k_r Q^{rep}_{rt} \\ & + \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} h_r(I^c_{rt} + I^l_{rjt}) + \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} v_j Z^{cl}_{rjt} + \sum_{r\in R}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T} l_{jk} Z^{lat}_{rjkt} \\ & + \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} \pi_r(B^c_{rt} + B^l_{rjt}) + \sum_{j\in J}\sum_{k\in K}\sum_{t\in T} r_{jk} Y_{jkt} \end{aligned} \tag{4.1}$$

Objective function (4.1) minimizes the total cost of user allocation to LRFs, and procurement of the repairable components in the strategic phase; in addition to the costs corresponding to the tactical phase, such as the cost of repair, inventory, transportation between the CRF and LRFs, shortages, as well as component and operator sharing among LRFs. Two main categories of constraints are taken into account, including design and tactical planning constraints. Constraints (4.2)-(4.5) represent a set of conditions related to network design aspects.

***Design constraints***

$$\sum_{r \in R} u_r Q_r^c \leq m^c \tag{4.2}$$

$$\sum_{r \in R} u_r Q_{rj}^l \leq m_{r,j}^l \qquad \forall j \in J \tag{4.3}$$

$$\sum_{j \in J} X_{ij} = 1 \qquad \forall i \in I \tag{4.4}$$

$$\sum_{i \in I} X_{ij} \geq 1 \qquad \forall j \in J \tag{4.5}$$

Constraints (4.2) and (4.3) represent the available procurement budget available at the CRF and LRFs. Constraints (4.4) ensure that each user is assigned to one of the LRFs. More precisely, the demand of each user is assigned to one particular LRF in the tactical phase. Constraints (4.5) state that at least one user is assigned to each LRF. Constraints (4.6)-(4.22) indicate a set of constraints related to tactical planning decisions.

***LRF flow constraints***

$$D_{rjt} = \sum_{i \in I} d_{rit} X_{ij} \qquad \forall j \in J, \forall r \in R, \forall t \in T \tag{4.6}$$

$$R_{rjt} + B^{l}_{rjt} - B^{l}_{r,j,(t-1)} = D_{rjt} \qquad \forall j \in J, \forall r \in R, \forall t \in T \tag{4.7}$$

$$Q^{l}_{rj} + Z^{cl}_{rj1} = R_{rj1} + I^{l}_{rj1} \qquad \forall j \in J, \forall r \in R \tag{4.8}$$

$$\sum_{k \in J \setminus k \neq j} Z^{lat}_{rkjt} + Z^{cl}_{rjt} + I^{l}_{r,j,(t-1)} = \sum_{k \in J \setminus k \neq j} Z^{lat}_{rjkt} + R_{rjt} + I^{l}_{rjt} \qquad \forall j \in J, \forall r \in R, \forall t \in T \setminus 1 \tag{4.9}$$

$$I^{l}_{rjt} \leq Q^{l}_{rj} \qquad \forall j \in J, \forall r \in R, \forall t \in T \tag{4.10}$$

$$\sum_{k \in J} Z^{lat}_{rjkt} \leq M V_{rjt} \qquad \forall j \in J, \forall r \in R, \forall t \in T \tag{4.11}$$

$$M(V_{rjt} - 1) \leq I^{l}_{rjt} - (1 - \tau_j) Q^{l}_{rj} - \lambda_j D_{rjt} \qquad \forall j \in J, \forall r \in R, \forall t \in T \tag{4.12}$$

$$\sum_{k \in J} Z^{lat}_{rjkt} \leq I^{l}_{rjt} - (1 - \tau_j) Q^{l}_{rj} - \lambda_j D_{rjt} + M(1 - V_{rjt}) \qquad \forall j \in J, \forall r \in R, \forall t \in T \tag{4.13}$$

Constraints (4.6) calculate the number of components scheduled to be replaced in each LRF according to the user allocation decisions. Constraints (4.7) represent the balance between the actual number of replaced components ($R_{rjt}$), the shortages in two consecutive periods ($B^{l}_{rjt}$), and the number of components scheduled to be replaced ($d_{rjt}$) in each LRF. Constraints (4.8) and (4.9) imply the flow balance constraints in each LRF. Constraints (4.8) state the balance between the number of prepositioned components ($Q^{l}_{rj}$), the number of repaired components sent from CRF to LRF ($Z^{cl}_{rjt}$), the inventory level ($I^{l}_{rjt}$) and the number of ready to assembly components ($R_{rjt}$) in the first period. Constraints (4.9) represent the balance between the shared components between LRFs ($Z^{lat}_{rkjt}$), number of components sent from CRF to LRF ($Z^{cl}_{rjt}$), inventory level and number of components that are ready to assemble in each period. Constraints (4.10) ensure that the inventory level of components in each period ($I^{l}_{rjt}$) is less than the LRF base stock inventory ($Q^{l}_{rj}$). Constraints (4.11)-(4.13) ensure the satisfaction of component sharing conditions. More precisely, in each period, the LRF reserves a portion of demand as the safety stock ($\lambda_j D_{rjt}$). Constraints (4.11), hence, imply that the number of shared components is zero if the net inventory level is negative. Constraints (4.12) state that the LRF is only allowed to share components if the difference between the inventory level ($I^{l}_{rjt}$) and the total reserved components and safety stock is positive.

When the LRF is allowed to share components, Constraints (4.13) calculate the maximum number of shared components from LRF $j$ to other LRFs.

***CRF flow constraints***

$$Q_r^c = \sum_{j \in J} Z_{rj1}^{cl} + I_{r1}^c \qquad \forall r \in R \quad (4.14)$$

$$Q_{r,(t-g_r)}^{rep} + I_{r,(t-1)}^c + B_{rt}^c - B_{r,(t-1)}^c = \sum_{j \in J} Z_{rjt}^{cl} + I_{rt}^c \qquad \forall r \in R, \forall t \in T \backslash 1 ... g_r \quad (4.15)$$

$$I_{rt}^c \leq Q_r^c \qquad \forall r \in R, \forall t \in T \quad (4.16)$$

$$\sum_{r \in R} k_r Q_{rt}^{rep} \leq n^c \qquad \forall r \in R, \forall t \in T \quad (4.17)$$

Constraints (4.14) and (4.15) represent the flow balance constraints in each CRF. Constraints (4.14) represent the balance between the number of prepositioned components in CRF ($Q_r^c$), the number of components sent from CRF to LRF ($Z_{rjt}^{cl}$) and the inventory level of components ($I_{rt}^c$) in the first period. Constraints (4.15) guarantee the balance between the number of repaired components, considering the corresponding repair time ($Q_{r(t-g_r)}^{rep}$), the inventory level and shortages of two consecutive periods, and the number of parts sent from the CRF to LRFs. Constraints (4.16) state that the inventory of components in each period is less than the CRF's base stock inventory level. Constraints (4.17) impose the repair budget of the CRF available to repair and recover failed components.

***Workforce constraints***

$$e_j - \sum_{k \in J \setminus k \neq j} Y_{jkt} + \sum_{k \in J \setminus k \neq j} Y_{kjt} = U_{jt} \qquad \forall j \in J, \forall t \in T \tag{4.18}$$

$$\sum_{r \in R} \beta_r R_{rjt} \leq U_{jt} \qquad \forall j \in J, \forall t \in T \tag{4.19}$$

$$\sum_{k \in J \setminus k \neq j} Y_{jkt} \leq M W_{jt} \qquad \forall j \in J, \forall t \in T \tag{4.20}$$

$$M(W_{jt} - 1) \leq \eta_j e_j - \zeta_j \sum_{r \in R} \beta_r d_{rjt} \qquad \forall j \in J, \forall t \in T \tag{4.21}$$

$$\sum_{k \in J \setminus k \neq j} Y_{jkt} \leq \eta_j e_j - \zeta_j \sum_{r \in R} \beta_r d_{rjt} + M(1 - W_{jt}) \qquad \forall j \in J, \forall t \in T \tag{4.22}$$

Constraints (4.18) represent the balance between the initial number of affiliated operators ($e_j$) of LRF $j$, the number of shared operators with other LRFs ($Y_{jkt}$), the number of borrowed operators from other LRFs ($Y_{kjt}$) and the total number of operators available to work ($U_{jt}$) in each period. It should be noted that the LRFs are only allowed to share the affiliated operators and not the borrowed ones. Constraints (4.19) limit the total number of components that can be processed/assembled in each period by the total number of operators who are available in the LRF. Constraints (4.20)-(4.22) represent the total number of operators that the LRF is able to share with other LRFs based on the number of affiliated operators and the number of components scheduled to be replaced. Constraints (4.20) indicate that the number of shared operators is set to zero if the number of net available operators in a period is negative. Constraints (4.21) state that the LRF is only allowed to share affiliated operators if the difference between the number of operators willing to share ($\eta_j e_j$) and the number of reserved operators ($\zeta_j \sum_{r \in R} \beta_r d_{rjt}$) is positive. It should be mentioned that the number of reserved operators is estimated as a fraction of the labor consumption factor ($\beta_r$) multiplied by the number of components scheduled to be replaced ($d_{rjt}$). When the LRF is allowed to share affiliated operators, constraints (4.22) calculate the maximum number of operators that can be shared by LRF $j$ with other LRFs.

***Domain constraints***

$$X_{ij}, W_{jt}, V_{rjt} \in \{0, 1\} \tag{4.23}$$

$$Y_{jkt}, Z^{lat}_{rkjt}, Z^{cl}_{rjt}, I^{l}_{rjt}, I^{c}_{rt}, B_{rjt}, R_{rjt}, Q^{l}_{rj}, Q^{c}_{r}, Q^{rep}_{rjt} \geq 0 \tag{4.24}$$

## 4.4 Robust collaborative maintenance logistics network design and planning under random demand

In practice, the users' demand for the maintenance of devices is highly uncertain. Besides, estimating the probability distribution of demand is not straightforward due to the unavailability of accurate information on the usage pattern of the fleet of equipment by the users. Furthermore, even if such a probability distribution could be estimated, the resulting stochastic programming model would be notoriously complex to solve due to the multi-component and multi-period structure of the problem. Alternatively, adopting a robust optimization (RO) approach (Bertsimas & Sim, 2004) that does not rely on an exact probability distribution of uncertain demand might be a better option. This is motivated by the fact that the decision-makers in this context are typically more interested in minimizing equipment unavailability (due to delayed maintenance operations) along with the system costs under some worst-case demand scenarios. Nevertheless, a classical RO approach that considers all decision variables as here-and-now decisions would be less applicable to the problem under investigation as it contains long-term (design) and mid-term (tactical) decisions. While the design decisions can be considered as here-and-now, the tactical ones can be set once the actual outcome of the demand is revealed; hence, they should be considered as wait-and-see decisions. Therefore, a two-stage robust optimization approach (e.g., (Bertsimas, Litvinov, Sun, Zhao, & Zheng, 2012), (Zeng & Zhao, 2013)) would be the best alternative in this context. More details on the proposed approach are provided as follows.

### 4.4.1 Modeling the demand uncertainty

The deterministic model (1)-(24) provides design and planning decisions based on the average demand. In this section, we aim to incorporate demand fluctuations in the process of decision making. Inspired by similar studies in the literature ((Bertsimas & Sim, 2004), (Zeng & Zhao, 2013)), the users' uncertain demand in each period ($\widetilde{d_{rit}}$) is modeled as a convex set by considering the average demand ($\overline{d_{rit}}$) and the maximum deviation over the average value ($\widehat{d_{rit}}$). However, the uncertainty set could be also represented as other sets of convex constraints that leads to a general polyhedron. It is worth noting that RO is a conservative approach that seeks an optimal solution under the worst-case scenario. When dealing with multi-dimensional random parameters, such as the demand of different customers for multiple components in different periods, the worst-case (maximum) outcome for all customers and components in all periods is an extremely rare phenomenon that provides an overly conservative solution. Therefore, a budget of uncertainty ($\Gamma$) can be introduced in the uncertainty set to control the number of such worst-case outcomes over all customers, components and periods. Furthermore, given that it is unlikely in practice to receive maximum demand from all users in the network, we consider a budget of uncertainty ($\Gamma$) to control the variation of the demand in the interval between the average and the maximum value over all users. This would also allow us to adjust the level of conservatism in the resulting RO model. The demand uncertainty set can be accordingly represented as follows:

$$\Omega = \left\{ \widetilde{d_{rit}} \in \mathbb{Z}^+ := \overline{d_{rit}} + \sigma_{r,i,t}\widehat{d_{rit}}, \quad \sigma_{r,i,t} \in \{0,1\}, \quad \sum_{i \in I} \sigma_{r,i,t} \leq \Gamma, \quad \forall r \in R, \forall t \in T \right\} \tag{4.25}$$

According to set $\Omega$, the maintenance logistic network is expected to receive the maximum quantity of demand from up to $\Gamma$ users in each period of the planning horizon. It can be easily verified that this set incorporates a large (finite) number of outcomes (scenarios) for the uncertain demand. By considering the number of users as $M$, the number of periods as $T$, and the number of components as $R$, the total number of possible scenarios can be obtained by $\frac{R \times T \times m!}{\Gamma!(M-\Gamma)!}$.

### 4.4.2 Two-stage robust optimization model

The main idea behind the two-stage robust optimization model is to determine the optimal first stage (here-and-now) decisions such that the cost of second-stage decisions under worst-case scenarios within the given uncertainty set is minimized. As mentioned earlier, the first stage decisions are assumed to be determined before the realization of random parameters, while the second-stage decisions are made under complete information in terms of the outcome of uncertainty to hedge against random parameter perturbations. Let $x$ and $y$ be the first-stage and second-stage decision variables, respectively, and $\Omega$ be the uncertainty set, the general setting of two-stage robust optimization model can be formulated as follows (Zeng & Zhao, 2013):

$$U(x, y) = \min_{x} \; c^T x + \max_{\omega \in \Omega} \min_{y \in F(x,\omega)} b^T y \tag{4.26}$$

$$s.t. \quad Ax \geq d \qquad\qquad x \in P_x \tag{4.27}$$

where $F(x, \omega) = \{y \in P_y : Gy \geq h - Ex - M\omega\}$ with $P_y \subseteq \mathbb{R}^n_+$ and $P_x \subseteq \mathbb{R}^m_+$.

Model (4.26)-(4.27) is a nonlinear program that contains a max-min term, associated with the second-stage problem, in the objective function. Two categories of approaches have been proposed in the literature to overcome the computational complexity of this class of RO models. The first approach relies on modeling the second-stage decision variables as affine functions of the uncertain parameters and solving the two-stage RO model accordingly. Some applications of this approach are provided in (Bertsimas, Brown, & Caramanis, 2011). The second approach, on the contrary, revolves around reformulating model (4.26)-(4.27) as a bi-linear programming model by discretizing the initial uncertainty set as a scenario set. Benders-dual cutting plane and column-and-constraint generation algorithms are among the most common algorithms proposed to efficiently solve the resulting bi-linear model. By assuming that the second-stage decisions are continuous, in the Benders-dual cutting plane algorithm, the dual of second-stage decisions are explored to gradually construct the value function of the first-stage decisions (e.g., (Bertsimas et al., 2012)), (Gabrel, Lacroix, Murat, & Remli, 2014)). In other words, in this approach, model (4.26)-(4.27) is decomposed as an outer minimization (master) problem that solves the first-stage problem and contains the dual of second-stage decisions, obtained from a bi-linear inner minimization problem. This algorithm relies on adding (optimality) cuts generated by an inner optimization algorithm designated to solve the second-stage (bi-linear) sub-problems, to the first-stage problem in an iterative manner until the convergence criterion is met. The column-and-constraint generation procedure ((Zeng & Zhao, 2013) (Zhao & Zeng, 2012)), on the contrary, begins by solving the master problem by considering a subset of the extreme points of the uncertainty set. After solving the master problem and obtaining the initial first-stage solutions, new extreme points are added iteratively to the initial subset after solving associated second-stage bi-linear sub-problems. It is noteworthy that the latter bi-linear models are either solved by the aid of an *ad-hoc* algorithm or linearized by the aid of *big-M* method after adding constraints corresponding to KKT conditions.

Despite being efficient for solving certain classes of two-stage RO models, the above-mentioned algorithms explore the dual of second-stage decisions in different iterations; hence, they are not applicable to model (4.1)-(4.24) that contains binary and integer second-stage variables. Therefore, we propose an approximation algorithm by considering a random subset of scenarios ($s = 1, ..., p$) in uncertainty set $\Omega$. Given that the uncertainty set $\Omega$ is composed of a finitely large number of scenarios, considering all such scenarios would drastically increase the complexity of the corresponding two-stage RO model. Therefore, we propose to approximate this model by considering a scenario subset, randomly sampled from all plausible demand scenarios, representing set $\Omega$. This is equivalent to relaxing constraints that correspond to the scenarios that are not included in the selected sample and provides a valid relaxation (and, consequently, a lower bound) to the optimal objective value of the original two-stage RO model. By considering $\omega_s$ as a possible outcome of the uncertain parameter under scenario $s$, model (4.26)-(4.27) could be approximated as a mixed-integer programming model as follows:

$$\hat{U}(x, y_s, \theta) : \min_{x} \ c^T x + \theta \tag{4.28}$$

$$s.t. \quad Ax \geq d \tag{4.29}$$

$$\theta \geq b^T y_s \qquad s = 1, ..., p \tag{4.30}$$

$$Ex + Gy_s \geq h - M\omega_s \qquad s = 1, ..., p \tag{4.31}$$

$$x \in P_x, y \in P_y \tag{4.32}$$

The non-linear max-min term in (4.26) is linearized in model (4.28)-(4.32) by defining variable $\theta$ in the objective function that captures the maximum of $b^T y_s$ over all demand scenarios based on constraints (4.30). Besides, set $F(x, \omega)$ in (4.26) is explicitly represented as constraints (4.31), where the second-stage decision variables are indexed by scenarios. Based on the above discussions, the two-stage robust counterpart of model (4.1)-(4.24) can be formulated as follow:

$$\min_{(X,Q^c,Q^l)\in P_x} = SC + \max_{s\in\Omega} \min_{(Q^{rep},I^c,I^l,Z^{cl},Z^{lat},B^c,B^l,Y,W,V)\in P_y} TC \tag{4.33}$$

$$s.t. \quad P_x = \left\{(X, Q^c, Q^l) \in \{0,1\}^{m\times n} \times \mathbb{R}^r_+ \times \mathbb{R}^r_+ : (4.2) - (4.5)\right\} \tag{4.34}$$

$$P_y = \left\{(Q^{rep}, I^c, I^l, Z^{cl}, Z^{lat}, B^c, B^l, Y, W, V) \in \mathbb{R}^{r\times n\times t}_+ \times \mathbb{R}^{r\times t}_+ \times \mathbb{R}^{r\times n\times t}_+ \times \mathbb{R}^{r\times n\times t}_+ \times \mathbb{R}^{r\times n\times n\times t}_+ \times \mathbb{R}^{r\times t}_+ \times \mathbb{R}^{r\times n\times t}_+ \times \mathbb{R}^{n\times n\times t}_+ \times \{0,1\}^{n\times t} \times \{0,1\}^{r\times n\times t} : (4.6) - (4.22)\right\} \tag{4.35}$$

In this model, the allocation of users to LRFs and inventory pre-positioning decisions are first-stage decisions, whereas the inventory, repair, transportation, shortage, and resource-sharing decisions are second-stage decisions. The objective function in equation (4.33) minimizes the strategic costs ($SC$) and the tactical costs ($TC$) under worst-case scenarios within the uncertainty set, defined in 4.4. In other words, the strategic costs in (4.1) ($SC = \sum_{i\in I}\sum_{j\in J} f_{ij}X_{ij} + \sum_{r\in R} u_r Q^c_r + \sum_{r\in R}\sum_{j\in J} u_r Q^l_{rj}$) is equivalent to the first-stage cost $c^T x$ in (4.28); whereas $TC$, that will be defined for each scenario, is equivalent to $b^T y_s$ in (4.30). Constraints (4.34) correspond to constraints (4.2)-(4.5) with respect to the domain of first-stage decision variables. Constraints (4.35) represent constraints (4.6)-(4.22) with respect to the domain of second-stage decision variables. By indexing second-stage decision variables by scenarios $s$, the expanded form of this model corresponding to a sub-set of random demand scenarios ($S$) is provided in Appendix A.

## 4.5 Monte-Carlo simulation platform

In this section, we propose a Monte Carlo simulation algorithm to compare the maintenance network's optimal decisions, obtained from deterministic and two-stage RO approaches, in a realistic environment. More specifically, we aim to compare the expected cost of solutions provided by these approaches under realistic scenarios. Recall from Section 4.4 that the deterministic model provides design and tactical decisions based on the average demand, $\overline{d_{rit}}$. In contrast, the RO model provides these decisions under worst-case demand scenarios. In the same vein, the second-stage cost (*TC*), obtained from this model, corresponds to the worst-case scenario. Our goal is thus to assess the performance of both models in terms of second-stage decisions in a more realistic context, represented as a set of scenarios randomly generated within the interval corresponding to the minimum and maximum value of the demand. To this end, the deterministic model (4.1)-(4.24) is solved for the scenario mentioned above after fixing the first-stage decisions at their optimal values obtained from the (mean-value) deterministic and RO models. This provides the actual cost of second-stage decisions for each demand scenario based on a given first-stage decision. Afterwards, the average objective function values are calculated over all scenarios in order to compare the first-stage (design) decisions proposed by deterministic and robust models. Algorithm 1 summarizes the details of the simulation procedure. In this algorithm, a uniform distribution is considered to generate random demand scenarios. This is mainly motivated by the assumption of modelling random demand as a box uncertainty set, represented by a lower and upper bound around the average demand. Nevertheless, depending on the definition of the uncertainty set, other probability distributions can be considered in this algorithm as well.

**Algorithm 6** Monte-Carlo simulation platform

**Step 1:** Solve the deterministic model (4.1)-(4.24) and obtain the first stage decision variables ($X_{ij}$, $Q^l_{rj}$, and $Q^c_r$).
**Step 2:** Solve the two-stage robust optimization model (4.33)-(4.35), formulated based on a randomly-generated scenario set of size $M$ from the set described in (4.25) and obtain the first-stage decision variables ($X_{ij}$, $Q^l_{rj}$, and $Q^c_r$).
**Step 3:** Generate $N$ random demand scenarios ($N >> M$) from a Uniform distribution in [$\overline{d_{rit}}$-$\widehat{d_{rit}}$, $\overline{d_{rit}}$+$\widehat{d_{rit}}$].
**for** $n \in \{1, ..., N\}$ **do**
  **Step 4:** Fix the the value of first-stage decision variables in model (4.1)-(4.24) to the values obtained in **Step 1** and solve this model by considering demand scenario $n$.
  **Step 5:** Fix the the value of first-stage decision variables in model (4.1)-(4.24) to the values obtained in **Step 2** and solve this model by considering demand scenario $n$.
**end**
**Step 6:** Calculate the average objective function values of the deterministic and two-stage robust optimization models obtained from **Step 4** and **Step 5**.

## 4.6 Numerical results

The computational experiments in this section are carried out on a case study inspired by the gas turbine maintenance industry. The objectives of our experiments are threefold: (1) to assess the impact of component and operator sharing strategies on the shortage and total costs in the deterministic model; (2) to conduct similar analysis in the context of the proposed two-stage robust optimization model; and (3) to compare the results of the deterministic and robust models by the aid of the proposed Monte-Carlo simulation platform. In what follows, we first provide the details of the case study. Afterwards, the detailed analysis of our numerical experiments is provided. All models are implemented in the Python programming language using the DOCPLEX package with IBM-ILOG CPLEX 12.8 on an Intel Core i7 3.4 GHz with 8GB of RAM.

### 4.6.1 Case study

Consider a maintenance network with an upstream central repair facility (CRF), five intermediate local repair facilities (LRFs), and ten users in the downstream echelon. Among all the components that are included in the pieces of equipment (gas turbines), three expensive and critical repairable items are taken into consideration. We assume that the network's LRFs are of varying sizes. More precisely, LRFs 1 and 2 represent relatively small maintenance facilities with 18 affiliated operators and a \$16,000 procurement budget. LRFs 3 and 4 are medium-sized centers with 20 affiliated operators and a \$20,000 procurement budget per facility; whereas, LRF 5 is a large maintenance facility with 24 affiliated operators and a \$24,000 procurement budget. The user allocation costs for each LRF are provided in Table 4.3. The CRF procurement and monthly repair budgets are set to \$18,000 and \$28,000, respectively. The planning horizon is comprised of 6 periods, with an average demand of 2 repairable components per user in each period. The unit holding cost ($h_r$) is 10% of the unit procurement cost ($u_r$) and the unit shortage cost ($\pi_r$) is considered as 160% of the unit procurement cost to reflect the severe consequences of late deliveries. Besides, we assume that the repair time ($g_r$) and the labor consumption factor ($\beta_r$) are equal to 1 for all the components. The values of parameters related to the components are provided in Table **??**. The parameters related to the LRFs are provided in Table 4.5. The values of $v_j$ and $l_{jk}$ in this table are functions of the distance between facilities.

Table 4.3: The user allocation cost to each LRF

| **User** | 1 | | | | | 2 | | | | | 3 | | | | | 4 | | | | | 5 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **LRF** | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| $f_{ij}$ | 20,000 | 40,000 | 40,000 | 40,000 | 40,000 | 40,000 | 20,000 | 50,000 | 40,000 | 40,000 | 40,000 | 50,000 | 20,000 | 40,000 | 40,000 | 30,000 | 40,000 | 20,000 | 40,000 | 40,000 | 50,000 | 40,000 | 40,000 | 20,000 | 40,000 |
| **User** | 6 | | | | | 7 | | | | | 8 | | | | | 9 | | | | | 10 | | | | |
| **LRF** | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| $f_{ij}$ | 40,000 | 40,000 | 40,000 | 20,000 | 40,000 | 40,000 | 40,000 | 40,000 | 40,000 | 50,000 | 40,000 | 40,000 | 40,000 | 40,000 | 20,000 | 40,000 | 40,000 | 40,000 | 40,000 | 20,000 | 40,000 | 40,000 | 40,000 | 40,000 | 20,000 |

Table 4.4: The values of parameters related to components

| **Component** | $u_r$ | $k_r$ | $h_r$ | $\pi_r$ | $g_r$ | $\beta_r$ |
|---|---|---|---|---|---|---|
| 1 | 400 | 350 | 40 | 640 | 1 | 1 |
| 2 | 600 | 400 | 60 | 960 | 1 | 1 |
| 3 | 800 | 450 | 80 | 1,280 | 1 | 1 |

Table 4.5: The values of parameters related to LRFs

| **LRF** | $v_j$ | $\tau_j$ | $\lambda_j$ | $\eta_j$ | $\zeta_j$ | $l_{jk}$ | $o_{jk}$ |
|---|---|---|---|---|---|---|---|
| 1 | 200 | 0.9 | 0.1 | 0.9 | 0.1 | 100 | 300 |
| 2 | 200 | 0.9 | 0.1 | 0.9 | 0.1 | 100 | 300 |
| 3 | 200 | 0.9 | 0.1 | 0.9 | 0.1 | 100 | 300 |
| 4 | 200 | 0.9 | 0.1 | 0.9 | 0.1 | 100 | 300 |
| 5 | 200 | 0.9 | 0.1 | 0.9 | 0.1 | 100 | 300 |

### 4.6.2 Experimental settings

In order to validate the collaborative network design model under different circumstances in terms of demand and collaboration level among stakeholders in the network, various experimental settings are considered. In particular, in addition to the base case (average demand for all users), we investigate three demand patterns, namely Cases A, B, and C, based on the uncertainty set (4.25), described in Sub-section 4.4.1. In all cases, the value of base demand is set to $\overline{d_{rit}}$=2, and the maximum demand deviation is set to $\widehat{d_{rit}}$=3. In case A, we set $\Gamma = 2$ and randomly assign the maximum demand of 5 units for each component in each period to 2 users out of 10 in each generated scenario. The demand for the remaining 8 users is considered as the base value. In the same vein, for cases B and C, we set $\Gamma = 4$ and $\Gamma = 6$ and assign the maximum demand to 4 and 6 users, respectively. We further study each demand pattern under three resource sharing levels among LRFs, namely, no-sharing, medium, and high. It is worth noting that the no-sharing strategy, particularly in terms of operators, is a benchmark scenario that represents the current practice in the majority of maintenance networks. Under the no-sharing strategy, both the $\tau_j$ and $\eta_j$ are set to zero. This represents the case where LRFs are not willing to share their resources (operators and components) with other entities in the network. When the sharing level is medium, we set $\tau_j = 0.5$ and $\eta_j = 0.5$, representing the case where LRFs are partially willing to share their resources with each other. Finally, when the sharing level is high, we set $\tau_j = 0.9$ and $\eta_j = 0.9$. In this case, the LRFs agree to share 90% of their resources to enhance the overall performance of the maintenance network.

### 4.6.3 Impact of resource sharing policies on the network cost: deterministic model

The purpose of this section is to investigate the impact of sharing policies under different demand patterns on the expected costs of the deterministic model. To this end, for each test case, 100 replications are generated and the deterministic model is solved iteratively. The average cost of the network over these replications is broken down into the strategic costs and various categories of tactical costs as reported in Table 4.6. The average Gap (%) in this table corresponds to the relative difference between the expected total cost of the network by considering sharing (at high or medium levels) and no-sharing policies.

As shown in Table 4.6, the capacity of LRFs in terms of operators and components can meet the average demand (base case). As a result, the cost of shortages, as well as component and operator sharing, are zero. However, increasing the demand from case A to case C, would significantly increase the expected repair, shortage, and component/operator sharing costs. By analyzing the strategic costs, it can be noticed that the average user allocation cost is increasing under the no-sharing strategy. In other words, the model adjusts the number of users allocated to each LRF according to their repair capacity. However, when medium and high sharing levels are considered, the user allocation cost is not changing, and the model incorporates sharing strategies to meet the additional demand. Besides, by increasing the level of demand uncertainty, the model prepositions more components to the CRF and LRFs at the beginning of the planning horizon to hedge against augmented volumes of demand at certain user sites. Furthermore, analyzing the Gap (%) column reveals that including a resource sharing policy in the network leads to a significant cost reduction when the demand uncertainty is relatively high (case B). This, in fact, is mainly attributed to the reduction of shortage cost as a result of sharing resources among LRFs in the network. Nevertheless, it can be observed that the difference between medium and high sharing levels is not remarkable under all demand patterns. This result clearly advocates that horizontal collaboration among stakeholders helps to reduce delayed deliveries (shortages) to some extent. In fact, this is an encouraging outcome given that convincing network partners to share 50% of their resources is much more realistic than establishing an almost perfect resource sharing policy, similar to the one in Case C, among these entities. Finally, the results in Table 4.6 indicate that the sharing strategies are less effective at controlling the shortage cost under extreme levels of demand uncertainty (Case C) as compared with the two other demand patterns.

Table 4.6: The expected costs of the deterministic model by considering different cases under 100 replications

| $d_{rit}$ | Sharing level | Expected strategic costs | | | Expected repair cost ($) | Expected shortage cost ($) | Expected component sharing cost ($) | Expected operator sharing cost ($) | Expected transportation cost ($) | Expected total cost ($) | Gap (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Expected user allocation cost ($) | Expected CRF procurement cost ($) | Expected LRF procurement cost ($) | | | | | | | |
| Base case | High | 220,000 | 0 | 60,000 | 99,000 | 0 | 0 | 0 | 48,000 | 431,800 | 0 |
| | Medium | 220,000 | 0 | 60,000 | 99,000 | 0 | 0 | 0 | 48,000 | 431,800 | 0 |
| | No-sharing | 220,000 | 0 | 60,000 | 99,000 | 0 | 0 | 0 | 48,000 | 431,800 | 0 |
| Case A | High | 220,000 | 0 | 84,208 | 122,768 | 915 | 0 | 1,848 | 59,010 | 497,887 | 0.94 |
| | Medium | 220,000 | 0 | 84,208 | 122,768 | 915 | 0 | 1,848 | 59,010 | 497,887 | 0.94 |
| | No-sharing | 220,800 | 12 | 83,776 | 122,568 | 7,354 | 0 | 0 | 58,900 | 502,628 | 0 |
| Case B | High | 220,000 | 14,360 | 93,748 | 138,898 | 16,304 | 287 | 13,050 | 69,262 | 578,894 | 6.50 |
| | Medium | 220,000 | 15,126 | 92,856 | 139,025 | 16,336 | 93 | 13,122 | 69,574 | 579,122 | 6.46 |
| | No-sharing | 239,600 | 7,9280 | 94,460 | 139,010 | 59,302 | 0 | 0 | 67,560 | 619,138 | 0 |
| Case C | High | 220,000 | 17,972 | 95,788 | 139,496 | 258,784 | 22 | 7,848 | 69,595 | 820,247 | 2.40 |
| | Medium | 220,200 | 17,968 | 95,762 | 139,513 | 259,840 | 4 | 7,272 | 69,600 | 820,989 | 2.31 |
| | No-sharing | 238,400 | 17,983 | 95,834 | 139,620 | 267,520 | 0 | 0 | 69,674 | 840,461 | 0 |

Table 4.7: The user-allocation decisions in the deterministic model

| LRF | User 1 | User 2 | User 3 | User 4 | User 5 | User 6 | User 7 | User 8 | User 9 | User 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| LRF 1 | ✓ | - | - | - | - | - | - | - | - | - |
| LRF 2 | - | ✓ | - | - | - | - | ✓ | - | - | - |
| LRF 3 | - | - | ✓ | ✓ | - | - | - | - | - | - |
| LRF 4 | - | - | - | | ✓ | ✓ | - | - | - | - |
| LRF 5 | - | - | - | - | - | - | - | ✓ | ✓ | ✓ |

The user allocation decisions obtained from the deterministic model are presented in Table 4.7. From this table, it can be observed that LRF 5 is in charge of satisfying the demands of three users and LRF 1 only serves one user. The expected number of received and shared components and operators among LRFs are reported in Table 4.8. As it can be observed in this table, LRF 5 that represents the largest facility in the network, receives temporary operators from other LRFs to perform repair operations more efficiently. More specifically, LRF 1 and LRF 2, representing the smallest facilities, share a high number of their affiliated operators with LRF 5 under cases B and C. This can be mainly attributed to the higher number of users allocated to LRF 5, as demonstrated in Table 4.7. In other words, under high-demand scenarios (such as Cases B and C), this LRF faces the highest surge of demand; hence, it requires extra resources to satisfy the demand. By observing LRF 3 and 4, that are medium-sized maintenance centers, it can be noticed that they share their affiliated operators when the demand is low and receive operators when the demand is high. As expected, only a few spare parts are shared among LRFs which is mainly due to the relatively small number of these (expensive) items initially prepositioned in the facilities.

Table 4.8: The expected number of shared/received components and operators among LRFs

| **Model** | **Sharing level** | **LRF 1** | | | | **LRF 2** | | | | **LRF 3** | | | | **LRF 4** | | | | **LRF 5** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Expected number of components | | Expected number of operators | | Expected number of components | | Expected number of operators | | Expected number of components | | Expected number of operators | | Expected number of components | | Expected number of operators | | Expected number of components | | Expected number of operators | |
| | | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received |
| **Case A** | **High** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 4 |
| | **Medium** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 4 |
| **Case B** | **High** | 1 | 1 | 14 | 4 | 1 | 0 | 16 | 4 | 0 | 0 | 7 | 5 | 0 | 0 | 6 | 6 | 0 | 1 | 0 | 25 |
| | **Medium** | 0 | 0 | 13 | 4 | 0 | 0 | 14 | 4 | 0 | 0 | 8 | 5 | 0 | 0 | 8 | 6 | 0 | 0 | 1 | 25 |
| **Case C** | **High** | 0 | 0 | 9 | 4 | 0 | 0 | 16 | 3 | 0 | 0 | 1 | 5 | 0 | 0 | 1 | 3 | 0 | 0 | 0 | 11 |
| | **Medium** | 0 | 0 | 9 | 3 | 0 | 0 | 14 | 2 | 0 | 0 | 1 | 5 | 0 | 0 | 1 | 4 | 0 | 0 | 0 | 10 |

### 4.6.4 Performance of the two-stage robust optimization model

The purpose of this section is to assess the performance of the two-stage robust optimization (RO) model under different experimental settings. To this end, 100 scenarios are generated for cases A, B, and C and the model is solved by setting the CPU time to 4 hours. It should be noted that the relative optimality gap reported by CPLEX in case A is zero; whereas, the reported average optimality gap under cases B and C is around 2.5%. The strategic cost (SC), tactical cost (TC), the total cost and the total gap % of the cost under no-sharing and different sharing policies are reported in Table 4.9 for both the deterministic model (base case) and the two-stage RO model under cases A, B and C. By analyzing the strategic costs obtained from the RO model, it can be noticed that the average user allocation costs are higher in cases B and C due to the surge of demand in these cases. Moreover, under all sharing levels, the CRF and LRF procurement costs along with the tactical costs are increasing by increasing the budget of uncertainty.

According to the results presented in this table, the total cost of RO model is greater than the deterministic model due to the fact that the former model obtains an optimal solution that is protected against the worst case demand scenario within any given uncertainty set. Furthermore, by comparing the results under cases A, B, and C in Tables 4.6 and 4.9, it can be concluded that the two-stage RO model leads to a higher Gap (%) than the deterministic model. In other words, the component and operator sharing strategies play a more crucial role in controlling the shortage and total cost when adopting a conservative decision model (i.e., RO model) as compared with a deterministic model that relies on average demand volumes. Nevertheless, similar trends can be observed in the two-stage RO approach in terms of the effectiveness of sharing strategies under different resource sharing levels and demand patterns. More precisely, these collaboration mechanisms are most effective under demand pattern B and a medium sharing level. Finally, the user-allocation decisions obtained from the two-stage RO model (under demand case B) are presented in Table 4.10, where a different pattern is observed as compared with the deterministic model (Table 4.7). In particular, the RO model assigns less users to LRF 5 in order to reduce the penalty cost invoked for the late delivery of repaired devices. One of the small LRFs, on the contrary, has more users as compared with the solution of the deterministic model.

Table 4.9: The results of the two-stage robust optimization model

| $d_{rit}$ | Sharing Level | SC | | | TC ($) | Total cost ($) | Total Gap (%) |
|---|---|---|---|---|---|---|---|
| | | User allocation cost ($) | CRF procurement cost ($) | LRF procurement cost ($) | | | |
| **Deterministic model** | **High** | 220,000 | 0 | 60,000 | 147,000 | 431,800 | 0 |
| | **Medium** | 220,000 | 0 | 60,000 | 147,000 | 431,800 | 0 |
| | **No-sharing** | 220,000 | 0 | 60,000 | 147,000 | 431,800 | 0 |
| **Case A** | **High** | 220,000 | 7,200 | 79,000 | 196,800 | 503,000 | 4.32 |
| | **Medium** | 220,000 | 6,600 | 78,600 | 198,820 | 504,020 | 4.12 |
| | **No-sharing** | 220,000 | 800 | 81,200 | 223,710 | 525,710 | 0 |
| **Case B** | **High** | 240,000 | 17,800 | 91,600 | 252,040 | 601,440 | 10.53 |
| | **Medium** | 240,000 | 17,000 | 93,000 | 254,540 | 604,540 | 10.07 |
| | **No-sharing** | 240,000 | 5,400 | 91,800 | 335,060 | 672,260 | 0 |
| **Case C** | **High** | 240,000 | 18,000 | 95,600 | 499,710 | 853,310 | 3.07 |
| | **Medium** | 240,000 | 18,000 | 95,800 | 505,410 | 859,210 | 2.40 |
| | **No-sharing** | 240,000 | 18,000 | 95,800 | 526,570 | 880,370 | 0 |

Table 4.10: The user-allocation decisions of two-stage robust model with considering case B

| LRF | User 1 | User 2 | User 3 | User 4 | User 5 | User 6 | User 7 | User 8 | User 9 | User 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **LRF 1** | ✓ | - | - | - | - | - | ✓ | - | - | - |
| **LRF 2** | - | ✓ | - | - | - | - | - | - | ✓ | - |
| **LRF 3** | - | - | ✓ | ✓ | - | - | - | - | - | - |
| **LRF 4** | - | - | - | | ✓ | ✓ | - | - | - | - |
| **LRF 5** | - | - | - | - | - | - | - | ✓ | - | ✓ |

### Impact of sample size on the performance of two-stage robust optimization model

Recall from section 4.4 that the two-stage RO model (4.33)-(4.35) is approximated by considering a scenario set, randomly sampled from the demand uncertainty set. Therefore, our goal is to analyze the impact of sample size on the optimal (robust) solution along with the variability of the obtained solution for different samples. To conduct these experiments, we chose Case B because it represents the most realistic circumstances in terms of demand uncertainty among the two other demand patterns. In other words, by considering four out of ten users experiencing the maximum demand surge, Case B is neither overly conservative, as Case C, nor optimistic, as Case A. To obtain the smallest sample size to reach a trade-off between solution variability and CPU time, we tested case B by considering 25, 50, and 100 scenarios. Accordingly, the results indicated no significant difference in terms of total cost under 50 and 100 scenarios. Afterwards, to test sample variability, we tested case B by considering 5 independent random samples consisting of 100 scenarios. The average sample standard deviation (STD), expected sample total cost (ETC) and the coefficient of variation (CV) of these five samples for different cost categories (i.e., strategic (SC), tactical (TC) and total cost) are reported in the Table 4.11. The "CV" is the ratio of STD to ETC and is calculates as $(STD/ETC) \times 100$). According to these results, the variability of total cost by considering 100 random scenarios with and without considering resource sharing strategies are 0.11% and 0.5%, respectively. Therefore, it can be concluded that this is an appropriate sample size providing a robust solution with a low variability.

Table 4.11: Results of the two-stage robust optimization model with 5 different samples

| **Sample** | $d_{rit}$ | **Sharing level** | **SC ($)** | **TC ($)** | **Total cost ($)** | **Gap (%)** |
|---|---|---|---|---|---|---|
| **No.1** | **Case B** | **High** | 349,400 | 252,040 | 601,440 | 10.53 |
| | | **No-sharing** | 337,200 | 335,060 | 672,260 | |
| **No.2** | **Case B** | **High** | 349,400 | 251,740 | 601,140 | 9.57 |
| | | **No-sharing** | 340,600 | 324,160 | 664,760 | |
| **No.3** | **Case B** | **High** | 350,800 | 251,720 | 602,520 | 9.33 |
| | | **No-sharing** | 338,200 | 326,360 | 664,560 | |
| **No.4** | **Case B** | **High** | 347,200 | 254,600 | 601,800 | 9.86 |
| | | **No-sharing** | 335,400 | 332,200 | 667,600 | |
| **No.5** | **Case B** | **High** | 349,400 | 253,330 | 602,730 | 10.07 |
| | | **No-sharing** | 338,200 | 332,060 | 670,260 | |
| **Metric** | | | | **ETC** | **STD** | **CV** |
| **Average of samples** | **Case B** | **High** | | 601,926 | 683.57 | 0.11 |
| | | **No-sharing** | | 667,888 | 3379.48 | 0.50 |

### 4.6.5 Comparison between the robust and deterministic models by the aid of Monte-Carlo simulation

In this section, we present the results of conducting Monte-Carlo simulation experiments, described in section 4.5, that aims to compare the performance of deterministic and two-stage robust network design models. The results are only provided for case B, where the impact of resource-sharing strategies deemed more significant. To choose the right sample size in the simulation experiments, we ran them by considering 250, 500 and 1000 scenarios within the interval of [1,5], representing the minimum and maximum amount of demand in these experiments.

The analysis revealed that there is a negligible difference between the results by considering 500 and 1000 scenarios. Thus, 500 scenarios are selected to compare the models. Table 4.12 summarizes the results of Monte-Carlo simulation experiments. Comparing the deterministic and two-stage RO models reveals that the strategic costs of the latter are approximately 6.5 percent higher than that of the former model. This difference is mainly due to the different user allocation pattern as well as higher number of components pre-positioned in the CRF in the RO model to better hedge against demand fluctuations. Figure 4.2 compares the expected tactical costs of deterministic and two-stage RO models. It can be observed that exerting resource sharing strategies as well as an adequate initial inventory level at LRFs and CRF can reduce the shortage cost in the two-stage RO model by up to 43% when compared to the deterministic model. This is a remarkable advantage of adopting a robust optimization approach given that maintaining a high service level (a low shortage cost) is a top priority in maintenance logistics networks. Besides, the expected repair, component sharing, operator sharing and transportation costs are higher in the deterministic case under medium and high sharing levels. Most notably, adopting the strategic decisions determined by the deterministic model would engage 43% more operator sharing cost to reduce the shortage cost, which is not a convenient solution in practice.

The expected number of shared/received components and operators obtained from the Monte-Carlo simulation experiments based on the strategic (design) decisions of both models are provided in Table 4.13. The results of the deterministic model show that at medium and high sharing levels, LRF 1 is only sharing components with other LRFs and LRF 2 is only receiving components from other LRFs during the planning horizon. LRFs 3, 4, and 5, on the other hand, receive and share components based on their actual demand in each period. According to the deterministic user-allocation decisions, reported in Table 4.7, 3 users are assigned to LRF 5. As a result, at medium and high sharing levels, LRF 5 receives a large number of operators to perform repair operations, the majority of whom are affiliated with LRF 1 that has only one user according to Table 4.7. Analyzing the resource sharing results in the two-stage RO model reveals that LRFs 3, 4, and 5 share their components with LRFs 1 and 2. Besides, at the medium sharing level, only LRF 5 shares the components with LRF 1. These results can be justified by looking into the user-allocation decisions reported in Table 4.11. In fact, in the two-stage RO model, fewer users are allocated to LRF 5 compared with the deterministic one. As a result, rather than receiving operators from other LRFs, LRF 5 shares affiliated operators with LRFs 1 and 2 to meet their needs in terms of operators to fulfill repair operations when facing a demand surge.

Table 4.12: The results of Monte Carlo simulation experiments

| Model | Sharing level | Strategic costs | | | Expected repair cost ($) | Expected shortages cost ($) | Expected component sharing cost ($) | Expected operator sharing cost ($) | Expected transportation cost ($) | Expected total cost ($) | Average Gap (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | User allocation cost ($) | CRF Procurement cost ($) | LRF Procurement cost ($) | | | | | | | |
| Deterministic | High | 220,000 | 16,000 | 92,400 | 129,965 | 16,367 | 835 | 8,926 | 65,596 | 565,377 | 8.29 |
| | Medium | 220,000 | 16,800 | 91,600 | 130,895 | 17,429 | 384 | 8,739 | 66,325 | 567,785 | 7.90 |
| | No-sharing | 230,000 | 8,800 | 92,000 | 130,417 | 76,557 | 0 | 0 | 64,188 | 616,472 | 0 |
| Two-stage RO | High | 240,000 | 17,800 | 91,600 | 129,851 | 9,347 | 480 | 5,305 | 66,572 | 576,172 | 3.88 |
| | Medium | 240,000 | 17,000 | 93,000 | 129,537 | 9,435 | 179 | 5,401 | 66,770 | 575,797 | 3.94 |
| | No-sharing | 240,000 | 5,400 | 91,800 | 134,668 | 49,569 | 0 | 0 | 65,717 | 599,458 | 0 |

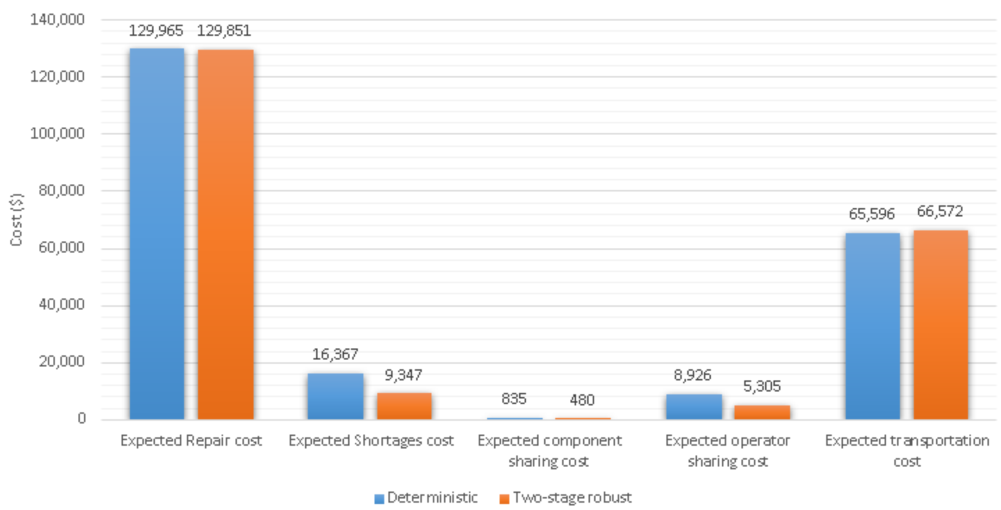


Figure 4.2: Comparison between the expected tactical costs of deterministic and two-stage RO models

Table 4.13: Monte-Carlo simulation results for resource sharing among LRFs

| **Model** | **Sharing level** | **LRF 1** | | | | **LRF 2** | | | | **LRF 3** | | | | **LRF 4** | | | | **LRF 5** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Expected number of components | | Expected number of operators | | Expected number of components | | Expected number of operators | | Expected number of components | | Expected number of operators | | Expected number of components | | Expected number of operators | | Expected number of components | | Expected number of operators | |
| | | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received | Shared | Received |
| **Deterministic** | **High** | 6 | 0 | 16 | 0 | 0 | 4 | 3 | 6 | 1 | 1 | 7 | 3 | 1 | 1 | 4 | 2 | 0 | 2 | 0 | 19 |
| | **Medium** | 2 | 0 | 16 | 0 | 0 | 2 | 2 | 6 | 1 | 1 | 6 | 3 | 1 | 0 | 4 | 2 | 1 | 1 | 0 | 18 |
| **Two-stage RO** | **High** | 0 | 2 | 1 | 6 | 0 | 2 | 1 | 6 | 1 | 0 | 4 | 2 | 2 | 0 | 6 | 2 | 1 | 0 | 5 | 0 |
| | **Medium** | 0 | 2 | 1 | 6 | 0 | 0 | 1 | 6 | 0 | 0 | 4 | 2 | 0 | 0 | 5 | 3 | 2 | 0 | 6 | 0 |

## 4.7 Conclusion and future research

In this paper, we investigated the collaborative design and planning of a maintenance logistics network, encompassing a CRF and multiple LRFs that repair and overhaul a fleet of failed technical devices. Our contribution is unique in the sense that proposing robust collaborative mechanisms among LRFs for integrated strategic and tactical planning in these networks has never been studied in the literature. To mitigate the risk of demand uncertainty, we incorporated two resource-sharing mechanisms, namely component and operator sharing strategies at different levels. To incorporate the demand uncertainty into this problem, we proposed a two-stage RO model to obtain the optimal configuration of the network along with the initial stock levels in the facilities as the first stage decisions. The second-stage decisions in this model are associated with tactical plans in terms of repair operations planning and resource sharing decisions. The objective is to minimize the cost of first-stage decisions along with the tactical costs under worst-case demand scenarios. We also provided an extensive set of numerical experiments inspired from a real case to draw managerial insights.

According to the outcomes of our numerical experiments, we first demonstrated the significance of adopting a collaborative mechanism among LRFs that relies on operator/component sharing strategies. The results indicated that incorporating these policies leads to a significant cost reduction when the demand uncertainty is relatively high. The experiments revealed that considering a moderate sharing level would, respectively, lead to 6.5 % and 10.5 % total cost reduction when adopting deterministic and robust optimization decision models. Moreover, we demonstrated the significance of incorporating demand uncertainty into the problem by the aid of Monte-Carlo simulation experiments. In particular, the results indicated that adopting an RO approach provides a higher service level (43% lower shortage cost) for maintenance service providers at the cost of pre-positioning more components at the beginning of the planning horizon and a different user allocation pattern as compared with the solution of a deterministic model. This large gap is in fact attributed to insufficient initial inventory levels assigned to each facility in the deterministic model. The RO model, in contrast, relies on the highest levels of demand according to a given budget of uncertainty; hence, it foresees more resources in these facilities.

In this study, the random demand was modeled as a box uncertainty set by considering a budget of uncertainty. Nonetheless, given that the proposed RO model is formulated by considering a scenario set, randomly selected from the uncertainty set, considering other uncertainty sets does not affect the structure of the model. It might rather affect the strategic and tactical decisions in the network. The current study can be extended in terms of modeling aspects and solution methodology. In this paper, we assumed all LRFs are in a grand coalition and share a pre-defined percentage of operators and components among each other. Besides, we assumed the LRFs have the same service level for all demand classes corresponding to different users. Nevertheless, the grand coalition is not always achievable due to varying service levels among the users, distance between LRFs, and different management policies in each maintenance facility. Thus, formulating a coalition structure problem by considering different demand classes to determine the optimal sub-coalition among network partners would be an interesting avenue of research. In the same vein, designing efficient cost-sharing mechanisms among the members to assure the stability of the coalition would be another modeling extension that is worth investigating. In terms of solution methodology, providing exact algorithms for solving the proposed two-stage robust optimization model that contains binary and integer second-stage variables is another interesting avenue of research.

# Appendix A: Two-stage robust optimization model

$$\min \quad \hat{U} = SC + \theta \tag{4.36}$$

$$s.t \quad SC = \sum_{i\in I}\sum_{j\in J} f_{ij}X_{ij} + \sum_{r\in R} u_r Q_r^c + \sum_{r\in R}\sum_{j\in J} u_r Q_{rj}^l \tag{4.37}$$

$$TC_s = \sum_{r\in R}\sum_{t\in T} k_r Q_{rts}^{rep} + \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} h_r(I_{rts}^c + I_{rjts}^l) + \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} v_j Z_{rjts}^{cl}$$
$$+ \sum_{r\in R}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T} l_{jk} Z_{rjkts}^{lat} + \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} \pi_r(B_{rts}^c + B_{rjts}^l) + \sum_{j\in J}\sum_{k\in K}\sum_{t\in T} r_{jk} Y_{jkts} \quad \forall s \in S \tag{4.38}$$

$$TC_s \leq \theta \qquad \forall s \in S \tag{4.39}$$

$$\sum_{r\in R} u_r Q_r^c \leq m^c \tag{4.40}$$

$$\sum_{r\in R} u_r Q_{rj}^l \leq m_{r,j}^l \qquad \forall j \in J \tag{4.41}$$

$$\sum_{j\in J} X_{ij} = 1 \qquad \forall i \in I \tag{4.42}$$

$$\sum_{i\in I} X_{ij} \geq 1 \qquad \forall j \in J \tag{4.43}$$

$$D_{rjts} = \sum_{i\in I} d_{rits} X_{ij} \qquad \forall j \in J, \forall r \in R, \forall t \in T, \forall s \in S \tag{4.44}$$

$$R_{rjts} + B_{rjts}^l - B_{rj(t-1)s}^l = D_{rjts} \qquad \forall j \in J, \forall r \in R, \forall t \in T, \forall s \in S \tag{4.45}$$

$$Q^l_{rj} + Z^{cl}_{rj1s} = R_{rj1s} + I^l_{rj1s} \qquad \forall j \in J, \forall r \in R, \forall s \in S \tag{4.46}$$

$$\sum_{k \in J \setminus k \neq j} Z^{lat}_{rkjts} + Z^{cl}_{rjts} + I^l_{rj(t-1)s} = \sum_{k \in J \setminus k \neq j} Z^{lat}_{rjkts} + R_{rjts} + I^l_{rjts} \qquad \forall j \in J, \forall r \in R, \forall t \in T \setminus 1, \forall s \in S \tag{4.47}$$

$$I^l_{rjts} \leq Q^l_{rj} \qquad \forall j \in J, \forall r \in R, \forall t \in T, \forall s \in S \tag{4.48}$$

$$\sum_{k \in J} Z^{lat}_{rjkts} \leq M V_{rjts} \qquad \forall j \in J, \forall r \in R, \forall t \in T, \forall s \in S \tag{4.49}$$

$$M(V_{rjts} - 1) \leq I^l_{rjts} - (1 - \tau_j) Q^l_{rjs} - \lambda_j D_{rjts} \qquad \forall j \in J, \forall r \in R, \forall t \in T, \forall s \in S \tag{4.50}$$

$$\sum_{k \in J} Z^{lat}_{rjkts} \leq I^l_{rjts} - (1 - \tau_j) Q^l_{rjs} - \lambda_j D_{rjts} + M(1 - V_{rjts}) \; \forall j \in J, \forall r \in R, \forall t \in T, \forall s \in S \tag{4.51}$$

$$Q^c_r = \sum_{j \in J} Z^{cl}_{rj1s} + I^c_{r1s} \qquad \forall r \in R, \forall s \in S \tag{4.52}$$

$$Q^{rep}_{r(t-g_r)s} + I^c_{r(t-1)s} + B^c_{rts} - B^c_{r(t-1)s} = \sum_{j \in J} Z^{cl}_{rjts} + I^c_{rts} \quad \forall r \in R, \forall t \in T \setminus 1 ... g_r, \forall s \in S \tag{4.53}$$

$$I^c_{rts} \leq Q^c_r \qquad \forall r \in R, \forall t \in T, \forall s \in S \tag{4.54}$$

$$\sum_{r \in R} k_r Q^{rep}_{rts} \leq n^c \qquad \forall r \in R, \forall t \in T, \forall s \in S \tag{4.55}$$

$$e_j - \sum_{k \in J \setminus k \neq j} Y_{jkts} + \sum_{k \in J \setminus k \neq j} Y_{kjts} = U_{jts} \qquad \forall j \in J, \forall t \in T, \forall s \in S \tag{4.56}$$

$$\sum_{r \in R} \beta_r R_{rjts} \leq U_{jts} \qquad \forall j \in J, \forall t \in T, \forall s \in S \tag{4.57}$$

$$\sum_{k \in J \setminus k \neq j} Y_{jkts} \leq M W_{jts} \qquad \forall j \in J, \forall t \in T, \forall s \in S \tag{4.58}$$

$$M(W_{jts} - 1) \leq \eta_j e_j - \zeta_j \sum_{r \in R} \beta_r D_{rjts} \qquad \forall j \in J, \forall t \in T, \forall s \in S \tag{4.59}$$

$$\sum_{k \in J \setminus k \neq j} Y_{jkts} \leq \eta_j e_j - \zeta_j \sum_{r \in R} \beta_r D_{rjts} + M(1 - W_{jts}) \qquad \forall j \in J, \forall t \in T, \forall s \in S \tag{4.60}$$

$$X_{ij}, W_{jts}, V_{rjts} \in \{0, 1\} \tag{4.61}$$

$$Y_{jkts}, Z^{lat}_{rkjts}, Z^{cl}_{rjts}, I^{l}_{rjts}, I^{c}_{rts}, B_{rjts}, R_{rjts}, Q^{l}_{rj}, Q^{c}_{r}, Q^{rep}_{rjts} \geq 0 \tag{4.62}$$

The $SC$ (4.37) contains the total cost of user allocation to LRFs, and procurement of the repairable components in the strategic phase. On the other hand, $TC_s$ (4.38) formulates the total cost of repair cost, holding cost, transportation cost between the CRF and LRFs, the shortage cost, component and operator sharing costs between LRFs in different scenarios. $\theta$ in (4.39) captures the maximum tactical cost $TC_s$ among all the realizations of demand scenarios.

# Chapter 5

# Conclusion and Future Work

## 5.1 Concluding remarks

This thesis addressed maintenance logistics network design and planning problems in the context of advanced systems distinguished by their modular structure and high cost of repairable components. It accounted for several types of problems in practice associated with design and planning of repairable components in maintenance facilities. Inspired by a real-life example in the context of gas turbine industry, we investigated the performance of risk-mitigation strategies and solution approaches on real-size instances. It was shown that the problem instances can be solved within reasonable amount of times utilizing the proposed solution schemes.

In chapter 2, we proposed a deterministic followed by a two-stage stochastic programming model for integrated workforce training and operations planning problem. In addition to the common operations planning constraints, the training constraints such as the validity period of training certificate and the required number of employees to train, the limited capacity of operators and spare parts, and the outsourcing and borrowing constraints were also taken into account. Besides, we analyzed the impact of incorporating the training plan into other tactical planning decisions as well as outsourcing and operator sharing strategies in a maintenance facility that offers various maintenance services to the users of technical devices with a modular structure.

In chapter 3, we proposed a multi-stage stochastic programming model for the integrated workforce scheduling and operations planning, which is among the most complex tactical planning problems due to the limited validation period of operators' certificates. The repair times of some components were assumed to be random variables, each represented as an independent scenario tree over the planning horizon. The repair time uncertainty was incorporated into an integrated workforce and production planning problem in maintenance facilities that has not been previously investigated in the literature. Besides, we proposed an approximate decomposition algorithm to efficiently solve the real-size instances in both categories of problems that contain 12 periods in the planning horizon. The proposed algorithm relies on decomposing the MS-MIP models into component sub-models within an Lagrangian relaxation scheme where the sub-models are formulated based on their corresponding ST. Moreover, we proposed a heuristic algorithm to repair the infeasibility of the obtained solution within the LR framework that provides a high-quality upper bound to the optimal objective value of the original MS-MIP model.

Finally, in chapter 4, we investigated the collaborative design and planning of a maintenance logistics network, encompassing a central repair center and multiple local repair facilities that repair and overhaul a fleet of failed technical devices. We proposed a collaborative mechanism among local repair facilities for integrated strategic and tactical planning in these networks. We incorporated two resource-sharing mechanisms, namely component and operator sharing strategies at different levels to mitigate the risk of demand uncertainty. Besides, we proposed a two-stage Robust optimization model to obtain the optimal configuration of the network along with the initial stock levels in the facilities as the first stage decisions. The second-stage decisions in this model were associated with tactical plans in terms of repair operations planning and resource sharing decisions. We also provided an extensive set of experiments with the aid of a Monte-Carlo simulation platform to draw managerial insights.

## 5.2 Future research directions

Immediate extensions of this thesis can revolve around the following avenues of research.

(1) Considering a variant of the proposed models accounting for uncertainty in unavailability of operators and economic parameters such as component and operator sharing costs.

(2) Designing efficient cost-sharing mechanisms among the members to assure the stability of the coalition.

(3) Developing efficient solution algorithms for the variants mentioned above would be another promising avenue of research.

(4) Given that the grand coalition is not always achievable among maintenance service providers, formulating a coalition structure problem by considering different demand classes to determine the optimal sub-coalition among network partners is another avenue of research.

(5) The proposed structure for workforce constraints by considering the limited age of operators' certificate can be exerted in other tactical planning problems such as production, service industries and health care systems, where the operations must be carried out by certified operators due to safety regulations and product standards.

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