PRODUCTION PLANNING AND SCHEDULING WITH TIME-RELATED PROCESSING COST CONSIDERATIONS FOR A HEAT TREATMENT SHOP

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ABSTRACT

PRODUCTION PLANNING AND SCHEDULING WITH TIME-RELATED PROCESSING COST CONSIDERATIONS FOR A HEAT TREATMENT SHOP

Zhen Yan

Due to production customization and diversification, many manufacturing companies face greater challenges to cope with uncertainties related to material supply, market demands and increased cost. The purpose of this study is to optimize the production process in a multi-product heat treatment shop to reduce production cost. We consider a multi-item multi-level production planning and scheduling problem in this research with production cost related to the waiting time of the components to be processed. A non-linear integer programming model is developed to describe the considered problem. After linearization, the model is solved to optimality using IBM® ILOG® CPLEX® Optimization Studio. We also propose a heuristic solution method to solve the considered problem for fast solutions of much larger problem sizes. Computational results of numerical examples indicate the mathematical model generates optimal results in capturing detailed problem features such as larger product variety which can be missed in using the heuristic method. The results also show that the developed mathematical model can be used to solving various production planning and scheduling problems with practical considerations such as time-related manufacturing cost functions.

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Chapter 1 INTRODUCTION

This chapter introduces the background of the research, provides a brief summary of the conducted research work and presents the outline of the thesis.

1.1 Background

Production planning and scheduling is one of the most classic topics in modern management science. It aims at allocating employees, equipment and materials to meet the targeted customer's demand in the most efficient way. It has been widely studied and applied in manufacturing industry. Automotive industry is a typical example. The automobile is regarded as the one of the greatest and most important inventions in human history which is called "the machine that changed the world". Today, automotive manufacturing technology is mature and there are numerous automotive manufacturers across the globe. We conducted a research of a heat treatment shop in a gearbox manufacturing company and applied the method of production planning and scheduling to reduce its manufacturing cost.

1.2 Gearbox and Heat Treatment

A gearbox is often called a transmission which allows a vehicle to operate efficiently through speed change. Gears and shafts are common and important components in gearboxes (shown in Figure 1.1). They connect with each other to transfer energy. Due to frequent friction, these components require a certain degree of hardness and toughness to maintain high reliability and durability in which heat treatment plays an essential role. Heat treatment is a modern processing technology whose purpose is to change the the mechanical property or combination of mechanical properties of metals or metal alloys by changing their metallographic structures. In our study, heat treatment mainly contains two parts: carburizing and post heat treatment which includes processes of press-hardening shaping, cleaning and shot peening. Component obtains a hard and high-strength but brittle layer during the carburizing and later post heat treatment improves the toughness of the layer. Thus, the component meets the processing requirements.

Figure 1.1 Gears and shafts (Ningbo Jixing Machinery Manufacturing Co., Ltd.)

1.3 Challenges and Motivations

In the 20th century, the transformation of a European invention (the automobile) into an American innovation by Henry Ford has been perhaps the greatest change in America economic history (Thomas 1969). It changed the method of production from private business to mass production. Since then, the automobile has been no longer only the luxury of the aristocracy but more commonly become a type of transportation for everyone. With the development of automotive manufacturing technology, more and more automobile models are designed to meet customer's diversified needs which is a new challenge for producers. In our study, the heat treatment shop is responsible for the whole company's heat treatment task in which different types of products are processed. Furthermore, heat treatment processing time could reach to a few hours which is much longer than general machining processing time. In order to meet customer's demands, the shop maintains a high volume of inventory. Another chain reaction is the high demand of heat treatment racks. During heat treatment, components are treated under over 1000°C temperatures (An et al. 2019) and it requires special heat-resistant and deformation-resistant racks (shown in Figure 1.2). One pair of the rack approximately costs ten thousand dollars with a half-year lifespan. Additionally, heat treatment consumes a great amount of energy and electricity bill is one of the top production costs for the shop. Thus, it is imperative to optimize the production process and reduce the manufacturing cost to improve the company's competitiveness.

Figure 1.2 Heat Treatment Rack (Eternal Bliss Casting & Forging Co., LTD)

1.4 Contributions

This thesis focuses on production planning and scheduling for a heat treatment shop and the contributions of this research work are described as follows:

- ✓ Conducting a review of recent research on topics related to production planning, scheduling and heat treatment.
- ✓ Conducting a study of production planning and scheduling for heat treatment process.
- ✓ Developing an integer non-linear mathematical model for the considered problem which includes objective function, parameters, variables and constraints for optimal solutions.
- ✓ Developing a linearization method to solve the mathematical model.
- ✓ Proposing a heuristic method for feasible and near-optimal solutions.

1.5 Outline of the Thesis

The remainder of the thesis is organized as follows: Chapter 2 categorizes and summarizes a review of some relevant literature on production planning, scheduling and heat treatment. In Chapter 3, a production planning and scheduling mathematical model and a heuristic method are presented in detail. Chapter 4 presents numerical examples with specific parameters to illustrate and compare the established model and the heuristic method with computational results. Finally, Chapter 5 concludes the study with a summary and future research directions.

Chapter 2 Literature Review

2.1 Related Literature

This section represents a review of the literature on research in the area of production planning, scheduling and heat treatment.

2.1.1 Production Planning and Scheduling

Production planning can be viewed as planning of available resources and planning of production activities to transform materials or components into final product (Pochet and Wolsey 2006). It is one of the most widely applied method in manufacturing industry management. A manufacturing company's planning has different-level content with a wide range of time scales. Long-term planning determines the structure of supply chain or the whole company. Mid-term planning is considering about decisions such as production targets. Lastly, short-term planning is carried out on a daily or hourly basis to determine when and how many assignments of tasks in each unit. At production level, short-term planning is also referred to as scheduling (Maravelias and Sung 2009).

Acevedo (2012) studied a production planning problem with raw material shelf-life considerations. "The concept of shelf-life can be defined as the time period during which a product can be stored without loss of function for which it was designed, or without loss of its usability." Due to the perishability of raw material, that when and how much raw material to order at each period can lead to different inventory costs and disposal costs. The author defined a multi-item multi-level production planning problem with material perishability considerations and proposed a mixed integer linear programming model to optimize the total cost. In the study, production is driven by customer's demands and raw material order is driven by production planning. Apart from typical production planning variables, such as the amount of production at each time period, the amount of product at inventory, the model introduced a core variable: component consumption. More specifically, component consumption contains the following attributes: component type, reception time and consumed time. At each time period, the consumed component has to not only meet the shelf-life requirement but also be within available quantity limit. The author presented different example problem instances by considering the factors of batch size and lead time to illustrate the model and analyzed and discussed the computational results as well.

Scheduling is a decision-making process on a regular basis in manufacturing and service industries. "It deals with the allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives" (Pinedo 2012).

In traditional scheduling optimization problems, it is often assumed that the problem domain remains stationary. However, most of real-life cases are dynamic, such as rush orders, order cancellation, equipment failure happening during the production process. Under this condition, dynamic scheduling is more and more studied. Baykasoğlu and Ozsoydan (2018) presented a case study of dynamic scheduling problem of parallel heat treatment furnaces. Dynamic factors considered in the study referred to unpredictable and uncontrollable events which include either changes the problem domain, such as cancellations of jobs and furnace breakdowns, or changes related problem data, such as changes in due dates and lot sizes. They proposed a multi-start and constructive research algorithm as solution to handle the dynamic factors. Finally, the proposed algorithm was embedded into ERP system which decreases energy consumption and increase of annul income for the company.

Energy consumption in industry filed accounts for a large percentage and during the past ten years, a numerous amount of literature on energy-efficient scheduling was republished. Gao, Huang, Sadollah et al (2020) conducted a review of energy-efficient scheduling in intelligent production systems. The authors analyzed and discussed 90 publications from 13 journals. They classified articles based on five indicators: shop floor, models, approaches and algorithms, objectives and aspects of energy consumption. One of the most notable conclusions is that approximately 59% of the analyzed publications employed or improved swarm intelligence and evolutionary algorithms, such as genetic algorithm and grey wolf optimizer. The reason of frequent use of swarm intelligence and evolutionary algorithms is that they have lower computational time and higher efficiency for solving large-scale scheduling and optimization problems. Another remarkable conclusion is that more and more publications are focusing on multi-objective problems which considered both energy-related objectives and traditional objectives together for obtaining better decisions. However, the analysis of the relationship between energy-related objectives and traditional objectives is limited and this can be one of the future directions to study on this topic.

Today, single factories are facing greater challenges to react appropriately to uncertain market environment, small-size and diverse demands (Bennett and Lemoine, 2014). More and more companies have changed their production framework from a single factory to a multi-factory supply chain to resist risks (Lei, D., Yuan, Y., Cai, J. & Bai D. 2020).

Maravelias and Sung (2009) stated that due to the recent trend towards production customization and diversification which leads to fluctuating demands and multi-product facilities, the existing assets have to be utilized close to their capacity. The production planning of heavily loaded units subject to complicated constraints is a challenging task because production target has to be feasible while approaching to system limits. Therefore, it is necessary to integrate production planning and scheduling to confront the challenge.

Shah (2005) proposed an approach for the integration of mid-term production planning and short-term scheduling and presented the standard lot-sizing formulation that is often used in production planning system and discussed main solution strategies developed to solve different integrated models.

Production planning and scheduling in supply chain has become a popular research topic and the number of publications is growing rapidly. So there is a need of reviewing and summarizing on this topic. Lohmer and Lasch (2021) presented a systematic literature review on production planning and scheduling in multi-factory production networks and indicated further research potentials. The authors analyzed 128 articles and categorized all literature in a table based on shop configuration, factory type, demand, network structure and solution method. Then they distinguished the multi-factory planning problem from a single-factory problem and studied the objective and solution method for each multi-factory planning problem. Finally, they drew conclusions and explained the limitations of the research. "The bibliometric analysis revealed that academic attention has increased over the last decade, focusing on flow shops and job shops. A trend from genetic algorithms to iterated greedy algorithms, knowledge-based systems and learning algorithms for the different multi-factory problems is apparent." Furthermore, the authors reckon that "studying integrated problems, utilising adopted sophisticated meta-heuristics, examining neglected performance measures and the interconnection of multi-factory configurations with related research disciplines" is the direction for future study on production planning and scheduling of multi-factory problems.

2.1.2 Heat Treatment

Heat treatment plays an important role in many fields, such as the automotive, transport and other industries. It is always required for materials to obtain a certain level of strength, hardness, toughness and ductility. Since the first commercial application of heat treatment at the Packard Motor Car Co. to harden crankshafts used in the car company's 1937 engine, many new technologies have been developed. Jur č i (2020) briefly reviewed the development of advanced technology over the past decades, such as physical vapour deposition, which is much more environmentally friendly compared to traditional chemical vapour deposition, and many others.

In manufacturing processing, heat treatment operations often are done in separated workshops. The process cycle is broken and a manufacture cycle could be significantly prolonged (Dobrza ĕ ski 2002). In order to better understand heat treatment process, Smoljan (2007) analyzed the process of quenching, tempering and case hardening and found process planning of heat treatment has to obey the same principles that are valid for other common manufacturing processes. The author concluded that process planning of heat treatment must be based on the process performance, not on the microstructure transformations and other chemical and physical processes that appear during the heat treatment.

Carburizing is one of the most important heat treatment processes. The component is heated in a carbon-rich environment atmosphere to obtain a hard layer on its surface. Atmosphere-carburized and press-quenched gears have been widely accepted based on its mature technology and proven results. Nowadays, an alternative technology, vacuum carburizing with high pressure gas quenching has been specified. The use of variable gas pressure and flow techniques has successfully replaced oil quenching. Otto and Herring (2002) studied the difference between atmosphere and vacuum carburizing technologies and pointed out the advantages of the new generation technology. Vacuum carburizing is easily integrated, with greater automation control and has more consistent results. However, there are disadvantages at the same time: greater initial investment and higher standard of cleanliness.

Fakhurtdinov, Ryzhova, and Pakhomova (2017) studied the advantages of vacuum carburization by analyzing the processing chemical compositions. They evaluated the advantages and disadvantages of two types of atmosphere of gas carburization methods at first. Then, they compared those with vacuum carburization method and drew the conclusion that vacuum carburization method leads to lower product variations of surface carbon concentration, 1.5-2 times faster for layer formation, needlessness of controlling carbon potential of the gas medium, approximately 3.5 times higher of gas medium usage efficiency, lower electricity consumption and better environmental friendliness.

2.2 Summary

There are numerous research achievements on the topic of either production planning and scheduling or heat treatment. Inspired by reviewing the related literature, we can find that the research on production planning and scheduling of heat treatment process is limited. Additionally, with our project experience in a heat treatment shop, our team decided to conduct a research on this topic and details are presented in Chapter 3.

Chapter 3 Problem Definition and Mathematical Model

In this chapter, details of the studied problem in a heat treatment shop are presented with the development of a mathematical programming model. A heuristic method is proposed to find feasible and near-optimal solutions efficiently as well.

3.1 Problem Definition

We studied a heat treatment shop which processes different types of gears and shafts for different gearbox models. In the shop, for each gear and shaft, it will go through three states: machined, carburized and post heat treated. Machined items are ordered from other machining shops and processed into carburized ones in heat treatment shop; Carburized items are processed into post heat treated ones in heat treatment shop and finally they are delivered to other machining shops.

To better describe each state of gears and shafts, let A, B and C represent three different phases and Figure 3.1 is constructed for illustration.

Phase A: from ordering until the start of carburizing

Phase B: from carburizing until the start of post heat treatment

Phase C: from post heat treatment until the start of delivery

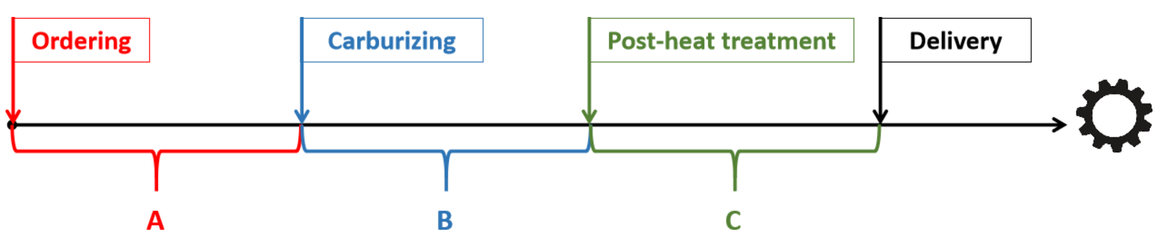

Figure 3.1 Different Phases of Gears and Shafts

The production planning and scheduling is driven by customers' demands (from machining shops after heat treatment). In this problem, we define gears and shafts as components assembled on different products (gearboxes).

The production process details are presented as follows: At first, the order request of machined component from other shops is made and there is lead time between order request and reception. When a machined component is well received, it can be either utilized directly or stored as inventory. Then, if a machined component is scheduled to be carburized, after a certain amount of production time, the machined component is converted into a carburized component which can be either processed (post heat treated) immediately or kept as on-site inventory. It is notable to mention that the time interval between a component being finished carburized and starting post heat treatment has an impact on processing cost. Then when a carburized component is scheduled to be post heat treated, it is converted to well heat treated component. It can be stored as inventory or delivered to other customers if there is a demand at that period. During the process, the value of component increases which leads to a higher inventory cost. More details of production costs considered in this study are introduced in Section 3.2.

A flow chart is constructed to illustrate the process (shown in Figure 3.2).

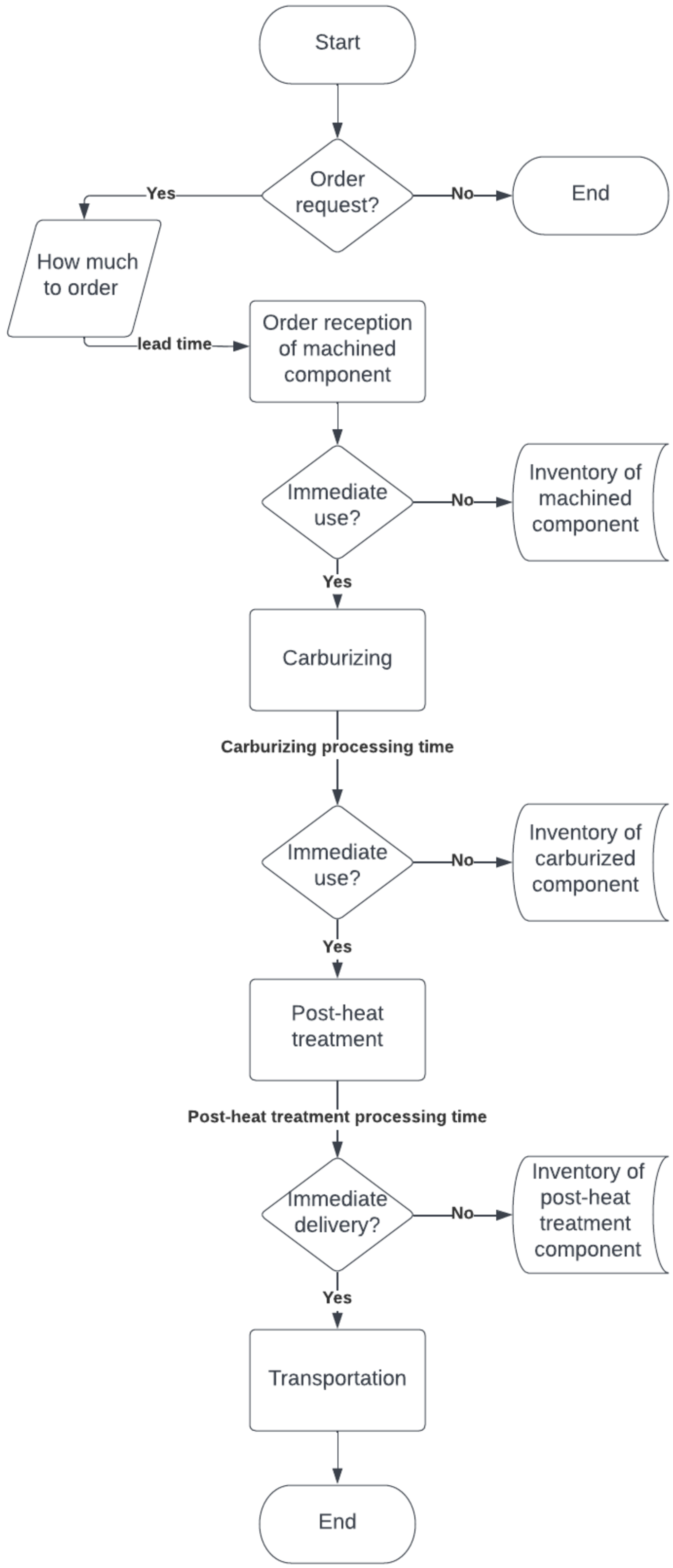

Figure 3.2 Heat Treatment Process Flow Chart

From the flow chart, we can conclude that this is a multi-item multi-level production problem. Production planning and scheduling method is applied to reduce the total manufacturing cost.

More specifically, the addressed problem considers:

- ✓ The acquisition of machined components: when and how much to order.
- ✓ Carburizing scheduling: when and how many machined components to process at each period.
- ✓ Post heat treatment scheduling: when and how many carburized components to produce at each period.
- ✓ Inventory control: how many components in different states to hold during each period.

3.2 Definitions

Some of the terms used in the thesis will be explained in this section.

3.2.1 Processing Cost

Components are packed in special heat treatment racks and treated in batches. In this problem, processing cost is the cost to process one batch of each type of components.

There are two processes: carburizing and post heat treatment. For carburizing, processing cost is constant and all the same for each batch of component. When the total processing quantity is determined, then the total carburizing processing cost is fixed. For post heat treatment, processing cost varies depending on the wait interval. The longer the interval is, the more the carburized component costs because it consumes more energy to heat up to required temperature for press-hardening shaping and takes longer for cleaning and shot peening. Thus, the processing cost of post heat treatment is time-related.

3.2.2 Idle Cost

Carburizing equipment provides required temperatures and gas atmosphere to ensure the steel's metallographic structure change in a desired way. One fact is that it takes up to more than 20 hours from operation temperature to cool down to room temperature. Furthermore, it consumes more energy to heat up from room temperature to operating temperature than to maintain the temperature within the same length of time. To ensure customer's demands are met, in addition to repairs and scheduled maintenance, the equipment keeps operating all the time.

In this problem, idle cost is the cost of maintaining each chamber of the equipment on operation at each unit of time. Regardless of others costs, consecutive production is the most energy-saving production mode.

3.2.3 Holding Cost

Holding cost is the cost to keep one batch of components in inventory at each unit of time. In operation management, the holding cost is calculated by the ratio of summation of inventory service cost, capital cost, storage space cost and inventory risk over the total value of inventory (Callarman 2020). In this problem, holding cost calculation is simplified by considering opportunity cost, employee wages and rack purchase fees. Different types of components at different states have different holding costs.

3.3 Assumptions and Notation

To structure the problem and the mathematical formulation, we make use of the following assumptions and notation.

3.3.1 Assumptions

In this thesis, the mathematical model is developed based on the following assumptions:

- ✓ The considered production planning and scheduling time horizon has multiple time periods.
- ✓ All ordering requests can be responded on time.
- ✓ Ordering lead time is greater than one time period.
- ✓ Carburizing and post heat treatment processing time is the same for all components. Compared to processing time, the difference of processing time among different components is negligible.
- ✓ Time-related processing cost is exponentially distributed
- ✓ There is no time gap among different processes
- ✓ Demand is deterministic and static
- ✓ Heat treatment process is continuous and equipment failure is not considered.
- ✓ Equipment idle cost only considers carburizing equipment idle cost.

3.3.2 Notation

Sets, parameters and variables used in the mathematical model are defined below. As defined in 3.1, the parameter or variable with symbol "a", "b", and "c" indicates the component is in phase A, phase B or phase C, respectively.

3.3.2.1 Sets

J	Set of components, $j \in J$
K	Set of products, $k \in K$
T	Set of time periods, $t \in T$

3.3.2.2 Variables

$x_{j,t}$	The amount of machined component *j* ordered in period *t*, $j \in J$, $t \in T$
$I^{a}_{j,t}$	The inventory of machined component *j* in period *t*, $j \in J$, $t \in T$
$y^{a}_{j,t}$	The amount of machined component *j* consumed in period *t*, $j \in J$, $t \in T$
$I^{b}_{j,t}$	The inventory of carburized component *j* in period *t*, $j \in J$, $t \in T$
$y^{b}_{j,p,t}$	The amount of carburized component *j* received in period *p* and consumed in period *t*, $j \in J$, $p \in T$, $t \in T$, $t \geq p$
$I^{c}_{j,t}$	The inventory of post heat treated component *j* in period *t*, $j \in J$, $t \in T$
$y^{c}_{j,t}$	The amount of post heat treated component *j* delivered in period *t*, $j \in J$, $t \in T$

3.3.2.3 Parameters

S_j	The batch size of component j, $j \in J$. In heat treatment shop, components are processed in batch which is different from machining processes.
$B_{j,k}$	The number of component j on each product k, $j \in J$, $k \in K$
LT_j	The ordering lead time of machined component j, $j \in J$
II_j^a	The initial inventory of machined component j, $j \in J$
$R_{j,t}^a$	The scheduled reception amount from previous orderings of component j in period t, $j \in J$, $t \in T$. For each component j with ordering lead time LT_j, for each time period from $t=1$ to $t=LT_j$, the component is only from previous orderings.
HC_j^a	The holding cost of machined component j, $j \in J$
LB	Carburizing processing periods
α	The number of carburizing chambers
II_j^b	The initial inventory of carburized component j, $j \in J$
$R_{j,t}^b$	The scheduled reception amount from previous processed carburized component j in period t, $j \in J$, $t \in T$
HC_j^b	The holding cost of carburized component j, $j \in J$
PB	Carburizing processing cost
IC	Carburizing equipment idle cost
β	The number of total periods of set T
λ_1	The first coefficient in time-related processing cost function, $\lambda_1 < 0$
λ_2	The second coefficient in time-related processing cost function, $\lambda_2 < 0$
λ_3	The third coefficient in time-related processing cost function, $\lambda_3 > -\lambda_1$
Q_j	The coefficient for component j in time-related processing cost function, $j \in J$
LC	Post heat treatment processing periods
II_j^c	The initial inventory of post heat treated component j, $j \in J$

$R^c_{j,t}$	The scheduled reception amount from previous processed post heat treated component j in period t, $j \in J$, $t \in T$
HC^c_j	The holding cost of post heat treated component j, $j \in J$
N	Post heat treatment equipment capacity
G_j	The post heat treatment capacity occupation of each batch of component j, $j \in J$
$D_{k,t}$	The demand of product k in period t, $k \in K$, $t \in T$

3.4 Mathematical Model

This section presents the mathematical formulation to solve the discussed problem. The formulation is an integer non-linear programming model with the objective function to minimize the total costs.

We let TC represent the total cost. The total cost is consisted of three parts: processing costs, idle costs and inventory holding costs:

- ✓ Processing costs: $\sum_{j=1}^{J}\sum_{t=1}^{T} PB\times y_{j,t}^{a} + \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t\geq p}^{T}[\lambda_1 \times e^{\lambda_2(t-p)} + \lambda_3]\times Q_j \times y_{j,p,t}^{b}$

$\sum_{j=1}^{J}\sum_{t=1}^{T} PB\times y_{j,t}^{a}$ is the processing cost for carburizing. It is assumed that the demand is deterministic and static, thus the total amount of carburized component $\sum_{j=1}^{J}\sum_{t=1}^{T} y_{j,t}^{a}$ and the carburzing cost $\sum_{j=1}^{J}\sum_{t=1}^{T} PB\times y_{j,t}^{a}$ are constant. $\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t\geq p}^{T}[\lambda_1 \times e^{\lambda_2(t-p)} + \lambda_3]\times Q_j \times y_{j,p,t}^{b}$ is the processing costs for post heat treatment, where $y_{j,p,t}^{b}$ is the amount of component *j* that have been finished carburizing in period *p* and are consumed in period *t* ($t \geq p$). In others words, there is a period interval with length of (*t-p*) between the state of being ready for post heat treatment and that of starting being post heat treated. The processing cost is associated with the length of the period interval. It is assumed that the cost is exponentially distributed. λ_1, λ_2 and λ_3 are the coefficients of the distribution function. Q_j is the coefficient for component *j* in time-related processing cost function which indicates the capacity occupation proportion of different component.

✓ Idle costs: $(\alpha \times \beta - \sum_{j=1}^{J}\sum_{t=1}^{T} y_{j,t}^{a} \times LB) \times IC$

$(\alpha \times \beta - \sum_{j=1}^{J}\sum_{t=1}^{T} y_{j,t}^{a} \times LB)$ is the summation of idle periods of all chambers. It is assumed that the production is consecutive, thus the amount of scheduled components received from previous periods (before *t=1*) approximately equals the amount of in-process component (after *t=LB*).

✓ Inventory costs: $\sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{a} \times I_{j,t}^{a} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{b} \times I_{j,t}^{b} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{c} \times I_{j,t}^{c}$

The total inventory costs are consisted by three parts: machined components, carburized components and post heat treatment components.

The total cost is expressed as follows:

$$
\begin{aligned}
TC = &\sum_{j=1}^{J}\sum_{t=1}^{T} PB \times y_{j,t}^{a} + \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t\geq p}^{T} [\lambda_1 \times e^{\lambda_2 (t-p)} + \lambda_3] \times Q_j \times y_{j,p,t}^{b} \\
&+ (\alpha \times \beta - \sum_{j=1}^{J}\sum_{t=1}^{T} y_{j,t}^{a} \times LB) \times IC \\
&+ \sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{a} \times I_{j,t}^{a} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{b} \times I_{j,t}^{b} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{c} \times I_{j,t}^{c}
\end{aligned}
$$

The goal of this model is to minimize the total cost: $\min\{TC\}$. Remove all constant terms and we get:

Objective function:

$$
\begin{aligned}
\min\{&\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t\geq p}^{T} [\lambda_1 \times e^{\lambda_2 (t-p)} + \lambda_3] \times Q_j \times PB \times y_{j,p,t}^{b} \\
+ &\sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{a} \times I_{j,t}^{a} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{b} \times I_{j,t}^{b} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC_{j}^{c} \times I_{j,t}^{c}\}
\end{aligned}
\qquad (3.1)
$$

Subject to:

Machined component amount balance:

$$R^a_{j,t} + II^a_j = y^a_{j,t} + I^a_{j,t} \quad \forall j \in J, \forall t \in T, t = 1 \tag{3.2}$$

$$R^a_{j,t} + I^a_{j,t-1} = y^a_{j,t} + I^a_{j,t} \quad \forall j \in J, \forall t \in T, 1 < t \le LT_j \tag{3.3}$$

$$x_{j,t} + I^a_{j,t-1} = y^a_{j,t} + I^a_{j,t} \quad \forall j \in J, \forall t \in T, t > LT_j \tag{3.4}$$

Constraint (3.2) states the machined component amount equation at the end of the first period t=1. Because LT_j is greater than 1, the ordering requested at t=1 will be received at t=1+LT_j. The origin of component j at t=1 is either from previous ordering request that is received at t=1 or the initial inventory. It is either consumed or stored as inventory. Constraint (3.3) states the machined component amount equation when the time period is from t=2 to t=LT_j. The only difference with Constraint (3.2) is that the origin is either from previous orderings or last period's inventory. Constraint (3.4) states the machined component amount equation when time period is greater than the component's ordering lead time. The origin of component at t comes from either the requested orders at (t-LT_j) or last period's inventory. These three constraints ensure the amount of machined component is balanced.

Carburizing capacity:

$$\sum_{j=1}^{J}\sum_{p=1}^{t} y^a_{j,p} + \sum_{j=1}^{J}\sum_{p=t+1}^{LB} R^b_{j,p} \le \alpha \quad \forall t \in T, t < LB, p \in T \tag{3.5}$$

$$\sum_{j=1}^{J}\sum_{p=t-LB+1}^{t} y^a_{j,p} \le \alpha \quad \forall t \in T, t \ge LB, p \in T \tag{3.6}$$

Constraint (3.5) and (3.6) ensure the total processing amount at each period is within carburizing equipment's capacity α. Based on the assumption that the processing time is the same for all kinds of component, the logic behind these constraints is that the total processing amount of LB consecutive periods is no greater than α. When $t < LB$, at each period, there are two groups of components are under processing. The first group is those starting being carburized from the first period to the current period. The second group is those starting being carburized before the first period, whose end time is after the current period. When $t \geq LB$, at each period, under-processing components are those starting being carburized from the (t-LB+1)th period to the current period.

Carburized component amount balance:

$$y^{b}_{j,p,t} = 0 \quad \forall j \in J, \forall p \in T, \forall t \in T, t < p \tag{3.7}$$

$$R^{b}_{j,t} + II^{b}_{j} = \sum_{p=1}^{t} y^{b}_{j,p,t} + I^{b}_{j,t} \quad \forall j \in J, \forall t \in T, t \geq p, t = 1 \tag{3.8}$$

$$R^{b}_{j,t} + I^{b}_{j,t-1} = \sum_{p=1}^{t} y^{b}_{j,p,t} + I^{b}_{j,t} \quad \forall j \in J, \forall t \in T, t \geq p, 1 < t \leq LB \tag{3.9}$$

$$y^{a}_{j,t-LB} + I^{b}_{j,t-1} = \sum_{p=1}^{t} y^{b}_{j,p,t} + I^{b}_{j,t} \quad \forall j \in J, \forall t \in T, t \geq p, t > LB \tag{3.10}$$

Constraint (3.7) indicates when $t < p$, $y^{b}_{j,p,t}$ is always 0. $\sum_{p=1}^{t} y^{b}_{j,p,t}$ is the total consumption amount of carburized component j in period t. Constraint (3.8), (3.9) and (3.10) ensure the carburized component is balanced in amount. In constraint (3.8), when t=1, the only possible positive value for $y^{b}_{j,p,t}$ is when p=1. Thus, $\sum_{p=1}^{t} y^{b}_{j,p,t} = y^{b}_{j,1,1}$. The origin of carburized component j is either from previous scheduled reception or the initial inventory. It is either consumed or stored as inventory. Constraint (3.9) states the carburized component amount equation when time period is from t=2 to t=LB. The only difference with Constraint (3.8) is that the origin is either from previous scheduled reception or last period's inventory. Constraint (3.10) states the carburized component amount equation when time period is greater than LB. The only difference with Constraint (3.9) is that the origin is either from previous processing or last period's inventory.

Relationship between input and output of carburized components:

$$R_{j,p}^{b} \geq \sum_{t=1}^{T} y_{j,p,t}^{b} \qquad \forall j \in J, \forall p \in T, p \leq LB, t \geq p \tag{3.11}$$

$$y_{j,p-LB}^{a} \geq \sum_{t=1}^{T} y_{j,p,t}^{b} \qquad \forall j \in J, \forall p \in T, p > LB, t \geq p \tag{3.12}$$

Constraint (3.11) and (3.12) guarantee that the consumption amount is within available amount limit. More specifically, $\sum_{t=1}^{T} y_{j,p,t}^{b}$ is the total consumption amount of carburized component j that are received at period p. It is less or equal to the available amount. When $p \leq LB$, available component is from previous schedules' reception before the first period. When $p > LB$, available component is from (p-LB)$^{\text{th}}$ period's processing.

Post heat treated component amount balance:

$$R_{j,t}^{c} + II_{j}^{c} = y_{j,t}^{c} + I_{j,t}^{c} \qquad \forall j \in J, \forall t \in T, t = 1 \tag{3.13}$$

$$R_{j,t}^{c} + I_{j,t-1}^{c} = y_{j,t}^{c} + I_{j,t}^{c} \qquad \forall j \in J, \forall t \in T, 1 < t \leq LC \tag{3.14}$$

$$\sum_{p=LC+1}^{t} y_{j,p-LC,t-LC}^{b} + I_{j,t-1}^{c} = y_{j,t}^{c} + I_{j,t}^{c} \qquad \forall j \in J, \forall t \in T, t \geq p, t > LC \tag{3.15}$$

Constraint (3.13), (3.14) and (3.15) ensure the amount of post heat treated component is in balance. Constraint (3.13) states the scenario when t=1. The origin of post heat treated component j is either from previous scheduled reception or the initial inventory. It is either delivered or kept as inventory. Constraint (3.14) states the scenario when time period is from t=2 to t=LC. The only difference with Constraint (3.13) is that the origin is either from previous scheduled reception or last period's inventory. Constraint (3.15) states the scenario when time period is greater than LC. The only difference with Constraint (3.14) is that the origin is either from previous processing or last period's inventory.

Post heat treatment equipment capacity:

$$\sum_{j=1}^{J}\sum_{p=1}^{t} y^{b}_{j,p,t} \times G_j \le N \qquad \forall t \in T, t \ge p \tag{3.16}$$

Constraint (3.16) ensures at each period, the total occupation of post heat treatment equipment is within its capacity *N*.

Demand fulfillment:

$$\sum_{l=1}^{t} y^{c}_{j,l} \times S_j \ge \sum_{l=1}^{t}\sum_{j=1}^{J} B_{j,k} \times D_{k,l} \qquad \forall j \in J, \forall t \in T \tag{3.17}$$

$$(y^{c}_{j,t} + 1) \times S_j > B_{j,k} \times D_{k,l} \qquad \forall j \in J, \forall k \in K, \forall t \in T \tag{3.18}$$

During heat treatment, components are packed in special heat treatment racks. In other words, they are processed in batch. In machining shops, components are processed by item. The parameter of batch size S_j connects heat treatment shop's processing and machining shop's demand together. Constraint (3.17) and (3.18) ensure machining shop's demand is satisfied at each period.

Non-negative integers:

$$x_{j,t}, y^{a}_{j,t}, y^{b}_{j,p,t}, y^{c}_{j,t}, I^{a}_{j,t}, I^{b}_{j,t}, I^{c}_{j,t} \in Z^{*} \qquad \forall j \in J, \forall p \in T, \forall t \in T \tag{3.19}$$

3.4.1 Linearization

In objective function, there is one non-linear term which causes complexity in computation, thus it is necessary to linearize it before solving the model. In term $\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t \ge p}^{T} [\lambda_1 \times e^{\lambda_2 (t-p)} + \lambda_3] \times Q_j \times y^{b}_{j,p,t}$, $[\lambda_1 \times e^{\lambda_2 (t-p)} + \lambda_3]$ is a coefficient which is determined by the wait interval (*t*-*p*), Q_j is a parameter and $y^{b}_{j,p,t}$ is a variable. The key point is to simplify $[\lambda_1 \times e^{\lambda_2 (t-p)} + \lambda_3]$.

In general, when $\lambda_1 < 0$, $\lambda_2 < 0$ and $\lambda_3 > -\lambda_1$, the image of function $z = [\lambda_1 \times e^{\lambda_2 v} + \lambda_3]$ is constructed in Figure 3.3.

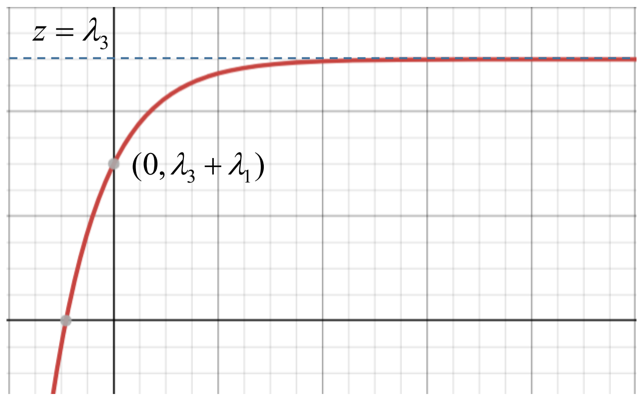

Figure 3.3 Function of $z = [\lambda_1 \times e^{\lambda_2 v} + \lambda_3]$

The limit of function *z* exists which equals λ_3 and we can get a point $(0, \lambda_3 + \lambda_1)$ from the figure as well.

In this model, $p \in T, t \in T$ and $t \ge p$. Let $v = t - p$ and we can get $v \in \{0,1,...(\beta - 1)\}$. Let $z_0, z_1, ... z_{\beta-1}$ represent the value of function *z* when *v* takes value of $0,1,...(\beta - 1)$ respectively. Then $\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t \ge p}^{T}[\lambda_1 \times e^{\lambda_2 (t-p)} + \lambda_3] \times Q_j \times y_{j,p,t}^b$ can be expressed as $\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p}^{T} z_0 \times Q_j \times y_{j,p,t}^b + \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+1}^{T} z_1 \times Q_j \times y_{j,p,t}^b + ... + \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+\beta-1}^{T} z_{\beta-1} \times Q_j \times y_{j,p,t}^b$.

To simplify computation, we assume when the curve's slope is less than 0.05, the difference between the function value and function limit can be ignored.

For example, when *m* satisfies: $\left.\frac{dz}{dv}\right|_{v=m} < 0.05$ and $\left.\frac{dz}{dv}\right|_{v=m-1} \ge 0.05$,we can get $v_0, v_1, ..., v_{m-1} < \lambda_3$ and $v_m, v_{m+1}, ..., v_{\beta-1} = \lambda_3$. Under this condition,

$$\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p}^{T} z_0 \times Q_j \times y^b_{j,p,t} + \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+1}^{T} z_1 \times Q_j \times y^b_{j,p,t} + \ldots + \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+\beta-1}^{T} z_{\beta-1} \times Q_j \times y^b_{j,p,t}$$

can be expressed as:

$$\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p}^{T} z_0 \times Q_j \times y^b_{j,p,t} + \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+1}^{T} z_1 \times Q_j \times y^b_{j,p,t} + \ldots \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+m-1}^{T} z_{m-1} \times Q_j \times y^b_{j,p,t}$$
$$+ \sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t\geq p+m}^{T} \lambda_3 \times Q_j \times y^b_{j,p,t} \quad .$$

3.4.2 Model Formulation Summary

Objective function:

$$\min\{\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t\geq p}^{T}[\lambda_1 \times e^{\lambda_2(t-p)} + \lambda_3] \times Q_j \times PB \times y^b_{j,p,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC^a_j \times I^a_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC^b_j \times I^b_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} HC^c_j \times I^c_{j,t}\} \tag{3.1}$$

Subject to:

$$R^a_{j,t} + II^a_j = y^a_{j,t} + I^a_{j,t} \quad \forall j \in J, \forall t \in T, t = 1 \tag{3.2}$$

$$R^a_{j,t} + I^a_{j,t-1} = y^a_{j,t} + I^a_{j,t} \quad \forall j \in J, \forall t \in T, 1 < t \leq LT_j \tag{3.3}$$

$$x_{j,t} + I^a_{j,t-1} = y^a_{j,t} + I^a_{j,t} \quad \forall j \in J, \forall t \in T, t > LT_j \tag{3.4}$$

$$\sum_{j=1}^{J}\sum_{p=1}^{t} y^a_{j,p} + \sum_{j=1}^{J}\sum_{p=t+1}^{LB} R^b_{j,p} \leq \alpha \quad \forall t \in T, t < LB, p \in T \tag{3.5}$$

$$\sum_{j=1}^{J}\sum_{p=t-LB+1}^{t} y^a_{j,p} \leq \alpha \quad \forall t \in T, t \geq LB, p \in T \tag{3.6}$$

$$y^b_{j,p,t} = 0 \quad \forall j \in J, \forall p \in T, \forall t \in T, t < p \tag{3.7}$$

$$R^b_{j,t} + II^b_j = \sum_{p=1}^{t} y^b_{j,p,t} + I^b_{j,t} \quad \forall j \in J, \forall t \in T, t \geq p, t = 1 \tag{3.8}$$

$$R^b_{j,t} + I^b_{j,t-1} = \sum_{p=1}^{t} y^b_{j,p,t} + I^b_{j,t} \qquad \forall j \in J, \forall t \in T, t \geq p, 1 < t \leq LB \tag{3.9}$$

$$y^a_{j,t-LB} + I^b_{j,t-1} = \sum_{p=1}^{t} y^b_{j,p,t} + I^b_{j,t} \qquad \forall j \in J, \forall t \in T, t \geq p, t > LB \tag{3.10}$$

$$R^b_{j,p} \geq \sum_{t=1}^{T} y^b_{j,p,t} \qquad \forall j \in J, \forall p \in T, p \leq LB, t \geq p \tag{3.11}$$

$$y^a_{j,p-LB} \geq \sum_{t=1}^{T} y^b_{j,p,t} \qquad \forall j \in J, \forall p \in T, p > LB, t \geq p \tag{3.12}$$

$$R^c_{j,t} + II^c_j = y^c_{j,t} + I^c_{j,t} \qquad \forall j \in J, \forall t \in T, t = 1 \tag{3.13}$$

$$R^c_{j,t} + I^c_{j,t-1} = y^c_{j,t} + I^c_{j,t} \qquad \forall j \in J, \forall t \in T, 1 < t \leq LC \tag{3.14}$$

$$\sum_{p=LC+1}^{t} y^b_{j,p-LC,t-LC} + I^c_{j,t-1} = y^c_{j,t} + I^c_{j,t} \qquad \forall j \in J, \forall t \in T, t \geq p, t > LC \tag{3.15}$$

$$\sum_{j=1}^{J} \sum_{p=1}^{t} y^b_{j,p,t} \times G_j \leq N \qquad \forall t \in T, t \geq p \tag{3.16}$$

$$\sum_{l=1}^{t} y^c_{j,l} \times S_j \geq \sum_{l=1}^{t} \sum_{j=1}^{J} B_{j,k} \times D_{k,l} \qquad \forall j \in J, \forall t \in T \tag{3.17}$$

$$(y^c_{j,t} + 1) \times S_j > B_{j,k} \times D_{k,l} \qquad \forall j \in J, \forall k \in K, \forall t \in T \tag{3.18}$$

$$x_{j,t}, y^a_{j,t}, y^b_{j,p,t}, y^c_{j,t}, I^a_{j,t}, I^b_{j,t}, I^c_{j,t} \in Z^* \qquad \forall j \in J, \forall p \in T, \forall t \in T \tag{3.19}$$

3.5 Solution Method

The mathematical programming model will be solved by using IBM® ILOG® CPLEX® Optimization Studio (Version 20.1.0). Refer to Appendix IBM® ILOG® CPLEX® OPL Model Source File to view the model formulations written in CPLEX format.

We also propose a heuristic method which provides efficient and feasible solutions. The heuristic method is introduced in detail as follows.

3.5.1 Heuristic Method

The heuristic method contains 5 steps:

Step 1. Determine a production cycle. In heuristic method, current cycle's production is for next cycle's demand.

Step 2. Determine the production mode of the next cycle and the one after the next. According to component variety and demand level, we classify four production modes: low-variety low-demand, low-variety high-demand, high-variety low-demand and high-variety high-demand. And we use LL, LH, HL and HH to represent them respectively. The standard of low or high variety is determined by if the number of component type is below or no less than a given certain number. The standard of low or high demand is determined by if the level of utilization of carburizing equipment, which can be calculated by Formula 3.20, is below or no less than a given certain number.

$$\frac{\sum_{j=1}^{J}\left\lceil \frac{\sum_{k=1}^{K}\sum_{t=1}^{T} D_{k,t} B_{j,k}}{S_j} \right\rceil \times LB}{\alpha \times \beta} \tag{3.20}$$

Step 3. Determine next cycle's inventory level. Based on the production mode of the cycle after the next, inventory level is classified in three categories: Low, medium and high. LL corresponds to low inventory level, LH and HL correspond to medium inventory level and HH corresponds to high inventory level.

Step 4. Determine current cycle's production task. The amount of each component to be processed at current cycle equals the next cycle's demand and required inventory amount minus current inventory amount. Assume that the demand and production mode don't fluctuate dramatically so that production task is always within production capacity.

Step 5. Implement the production tasks by following FIFO principle.

In next Chapter, different example instances with assumed hypothetical values of parameter are applied to demonstrate the mathematical model and heuristic method and the results are analyzed and discussed in depth.

Chapter 4 Numerical Examples, Results and Analysis

In this chapter, we present 8 numerical example instances to illustrate the mathematical model and heuristic method developed in Chapter 3. Computational results are compared and analyzed to demonstrate the effectiveness and efficiency of the mathematical model and the heuristic method.

4.1 Example Problem Instances

We present four groups of instances to compare the heuristic method and the mathematical model. The features of different problem instances are presented in Table 4.1.

Table 4.1 Different Problem Instances by Considerations

Instances	Low Variety	High Variety	Low Demand	High Demand	Heuristic Method	Mathematical Model
LL1	√		√		√	
LL2	√		√			√
LH1	√			√	√	
LH2	√			√		√
HL1		√	√		√	
HL2		√	√			√
HH1		√		√	√	
HH2		√		√		√

We define, if the number of component type is equal or greater than 6, the instance is defined to be a high-variety instance, otherwise it is a low-variety one. In our example problems, there are 2 types of products and 5 types of components in low-variety instances and there are 3 types of products and 7 types of components in high-variety instances.

4.1.1 Bill of Materials and Component Batch Size

The information of bill of materials of each component for each problem instance is shown in Table 4.2.

Table 4.2 Bill of Materials for Each Problem Instance

Instances	Product Types	Component Types						
		1	2	3	4	5	6	7
LL1, LL2, LH1, LH2	1	1	0	2	0	2	-	-
	2	0	1	0	2	2	-	-
HL1, HL2, HH1, HH2	1	1	0	0	2	0	0	2
	2	0	1	0	0	2	0	2
	3	0	0	1	0	0	2	2

In our problem, the product is counted by item and the unit of component is in batch. The information of batch size of each component for each problem instance is shown in Table 4.3.

Table 4.3 Component Batch Size for Each Problem Instance

Instances	Component Batch Size						
	1	2	3	4	5	6	7
LL1, LL2, LH1, LH2	12	12	16	16	20	-	-
HL1, HL2, HH1, HH2	12	12	12	16	16	16	20

4.1.2 Demands

Regardless of initial inventory and scheduled reception from previous periods, when the carburizing equipment utilization ratio is equal or greater than 0.7, we define the instance to be a high-demand instance, otherwise it is a low-demand one.

Assume a low-variety problem instance where $D_{1,4} = D_{2,8} = 10$, $D_{1,12} = D_{2,16} = 15$, $D_{1,24} = D_{2,24} = 20$ and the values of the rest $D_{k,t}$ are all 0. Let $T = 1..24$ (or $\beta = 24$), $\alpha = 8$ and $LB = 5$. According to formula (3.20):

$$\frac{\sum_{j=1}^{J}\left\lceil \frac{\sum_{k=1}^{K}\sum_{t=1}^{T} D_{k,t}B_{j,k}}{S_j} \right\rceil \times LB}{\alpha \times \beta} = \frac{\left(\left\lceil \frac{45\times1}{12} \right\rceil + \left\lceil \frac{45\times1}{12} \right\rceil + \left\lceil \frac{45\times2}{16} \right\rceil + \left\lceil \frac{45\times2}{16} \right\rceil + \left\lceil \frac{90\times2}{20} \right\rceil\right)\times 5}{8\times24} = 0.76 ,$$

which is higher than 0.7, so this is a low-variety high-demand instance.

The information of product demand for each problem instance is presented in Table 4.4. The carburizing equipment utilization ratio is 0.76 and 0.60 for both high-demand instances and low-demand instances respectively.

Table 4.4 Demand at Each Period for Each Problem Instance

Periods	Low Variety				High Variety					
	Low Demand		High Demand		Low Demand			High Demand		
	1	2	1	2	1	2	3	1	2	3
4	10		10		5			5		
8		10		10		10			10	
12	10		15				10			15
16		10		15						
20						10			20	
24	15	15	20	20	15		20	20		20

Note that there are demands only at the period of 4, 8, 12, 16 20 and 24. The demands of the rest periods are all 0.

4.1.3 Scheduling for Mathematical-Model Applied Instances

For mathematical-model applied instances, the demand of each product is required to be satisfied at each period. More specifically, the available amount of each component at each period mentioned above should be equal to or more than the corresponding demand. The demands of each component at each period for problem Instance LH2, calculated by formula (3.17) and (3.18), are presented in Table 4.5 for demonstration.

Table 4.5 Demands of Each Component at Each Period for Problem Instance LH2

Periods	Product Types		Component Types(in batch size)				
	1	2	1	2	3	4	5
4	10		1		2		1
8		10		1		2	1
12	15		2		2		2
16		15		2		2	1
20							
24	20	20	1	1	2	2	4

4.1.4 Production Planning for Heuristic-Method Applied Instances

For heuristic-method applied instances, production tasks are implemented by following FIFO principle. To make sure that customer's demand is satisfied at each period, there exists an inventory corresponding to different production modes. The information of component inventory is shown in Table 4.6.

Table 4.6 Component Safety Inventory for Each Heuristic-Method Applied Instances

Inventory Level	Instances	Component Inventory (in Batch Size)						
		1	2	3	4	5	6	7
Low	LL1	0	1	0	2	1	-	-
Medium	LH1	1	1	2	2	2	-	-
	HL1	0	1	1	0	2	2	2
High	HH1	1	1	1	2	2	2	3

We determine the production cycle with a length of 24 time periods and assume that the production mode won't change for the following two cycles. Under this condition, the amount of production is equal to customer's demand of the cycle.

4.1.5 Costs

For the parameters of post heat treatment processing cost, we have:

$\lambda_1 = -5$, $\lambda_2 = -1$, $\lambda_3 = -25$, $PB = 30$ and $IC = 4$

The rest parameters are listed in Table 4.7.

Table 4.7 Parameters of Different Cost for Each Example Instance

Categories	Parameters	Instances	Components						
			1	2	3	4	5	6	7
Processing Cost	Q_j	Low-variety	1	1	0.9	0.9	0.8	-	-
		High-variety	1	1	1	0.9	0.9	0.9	0.8
Holding Cost	HC_j^a	Low-variety	1.2	1.2	1.4	1.4	1.2	-	-
		High-variety	1.2	1.2	1.2	1.4	1.4	1.4	1.2
	HC_j^b	Low-variety	1.4	1.4	1.6	1.6	1.4	-	-
		High-variety	1.4	1.4	1.4	1.6	1.6	1.6	1.4
	HC_j^c	Low-variety	1.6	1.6	1.8	1.8	1.6	-	-
		High-variety	1.6	1.6	1.6	1.8	1.8	1.8	1.6

In mathematical model, the post heat treatment processing cost is expressed as follows: $\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t\geq p}^{T}[(-5)\times e^{-(t-p)}+25]\times Q_j \times y_{j,p,t}^{b}$. The image of function $z=-5e^{-v}+25(v\geq 0)$ is shown in figure 4.1.

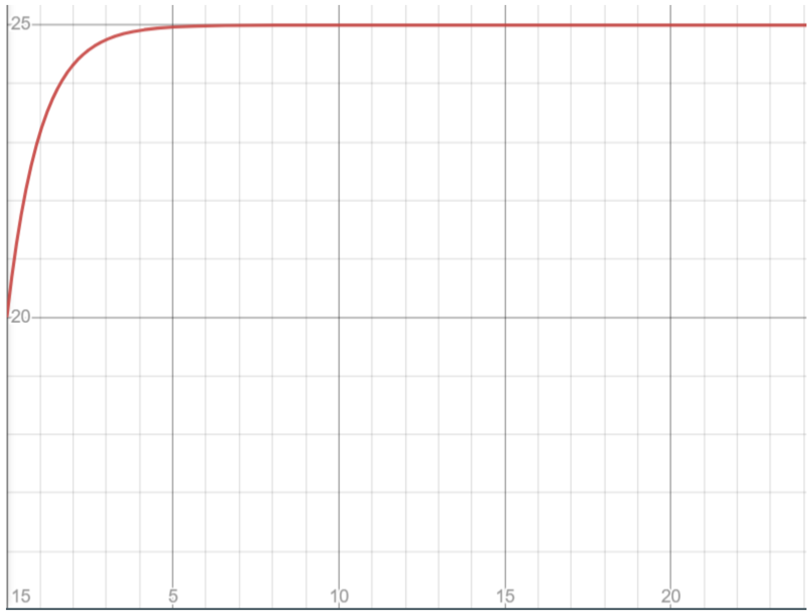

Figure 4.1 Function of $z=-5e^{-v}+25$

We implement the developed linearization method as follows:

The limit of z is 25 and we can compute that $\left.\frac{dz}{dv}\right|_{v=4}=0.09$ and $\left.\frac{dz}{dv}\right|_{v=5}=0.03$. Thus, when v is equal or greater than 5, z all takes the value of 25. Furthermore, we can find these points: (0, 20), (1, 23.2), (2, 24.3), (3, 24.8) and (4, 24.9) from the above figure.

Then the cost can be expressed as follows:

$$\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p}^{T}20\times Q_j\times y_{j,p,t}^{b}+\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+1}^{T}23.2\times Q_j\times y_{j,p,t}^{b}+\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+2}^{T}24.3\times Q_j\times y_{j,p,t}^{b}$$
$$+\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+3}^{T}24.8\times Q_j\times y_{j,p,t}^{b}+\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t=p+4}^{T}24.9\times Q_j\times y_{j,p,t}^{b}+\sum_{j=1}^{J}\sum_{p=1}^{T}\sum_{t\geq p+5}^{T}25\times Q_j\times y_{j,p,t}^{b}$$

4.1.6 Lead Time, Post Heat Treatment Capacity

The parameters of lead time and post heat treatment capacity occupation for each example instance is shown in Table 4.8.

Table 4.8 Parameters of Lead Time, Post Heat Treatment Capacity Occupation for Each Example Instance

Parameters	Instances	Values						
		1	2	3	4	5	6	7
LT_j	LL1, LL2, LH1, LH2	3	3	2	2	2	-	-
	HL1, HL2, HH1, HH2	3	3	3	2	2	2	2
G_j	LL1, LL2, LH1, LH2	1	1	0.8	0.8	0.6	-	-
	HL1, HL2, HH1, HH2	1	1	1	0.8	0.8	0.8	0.6

The post heat treatment capacity $N = 10$.

4.1.7 Initial Inventories and Scheduled Receptions

The total amount of initial inventory and scheduled reception from previous periods are the same for all example instances so that these factors will have negligible impact on further comparison.

For each component, all initial inventory of machined and carburized component and reception from previous periods are 0.

Table 4.9 and Table 4.10 show the parameters of initial inventories and receptions from previous processes for each post heat treated component respectively.

Table 4.9 Initial Inventory of Post Heat Treated Component for Mathematical-Model Applied Example Instances

Parameters	Instances	Initial Inventory						
		1	2	3	4	5	6	7
II_j^c	LL2, LH2	1	1	0	1	0	-	-
	HL2, HH2	1	1	0	0	0	0	1

Table 4.10 Receptions of Post Heat Treated Component from Previous Periods for Mathematical-Model Applied Example Instances

Parameter	Instances	Component	Periods		
			1	2	3~ 24
$R_{j,t}^c$	LL2, LH2	1	0	0	-
		2	1	0	-
		3	0	2	-
		4	2	0	-
		5	1	1	-
	HL2, HH2	1	0	0	-
		2	0	0	-
		3	1	0	-
		4	0	1	-
		5	2	1	-
		6	0	0	-
		7	1	1	-

4.2 Computational Results and Analysis

This section presents the computational results of each example instance mentioned in section 4.1 and the analysis of the results.

4.2.1 Computational Results

The mathematical model is solved by CPLEX and Table 4.11 is a summary of running results. LL2 and LH2 are both low-variety instances and HL2 and HH2 are both high-variety instances, hence they have same numbers of constraints, variables and non-zero integers in each pair. All models are solved with one second.

Table 4.11 Summary of CPLEX Running Results

Instances	LL2	LH2	HL2	HH2
Constraints	3132	3132	4524	4524
Variables (Integers)	3588	3588	5023	5023
Non-zero Coefficients	12058	12058	17049	17049
Objective Value	280.6	417.6	282.6	424.8
Nodes	0	0	0	0
Iterations	37	59	61	84
Avg. Time (hr:min:sec:cs)	00:00:00:36	00:00:00:36	00:00:00:26	00:00:00:36

Furthermore, we calculated all kinds of considered costs for each example instance and the results are presented in Table 4.12.

- ✓ For carburizing processing costs, we assume all instances have the same initial conditions and there is no production mode change. Thus, mathematical model and heuristic method have the same carburizing costs under the same production mode.

- ✓ For post heat treatment processing costs, mathematical model calculates the costs according to its running result on CPLEX. Heuristic method calculates the costs by multiplying processed amount and 25.
- ✓ For Inventory costs, it equals the objective value minus carburizing processing costs in mathematical models. In heuristic method, inventory cost is calculated based on Table 4.6 and Table 4.7.
- ✓ For idle costs, mathematical model and heuristic method have the same carburizing costs under the same production mode.

Table 4.12 Summary of Costs for Each Example Instance

Instances	Total Cost	Processing Cost		Inventory Cost	Idle Cost
		Carburizing	Post-heat Treatment		
LL1	1203.7	420	312.5	163.2	308
LL2	1008.6	420	250	30.6	308
LH1	1506.9	570	422.5	326.4	188
LH2	1175.6	570	338	79.6	188
HL1	1401.9	450	337.5	326.4	288
HL2	1020.6	450	270	12.6	288
HH1	1792.6	660	495	489.6	148
HH2	1232.8	660	396	28.8	148

4.2.2 Analysis

4.2.2.1 Method Comparison

Under each production mode, the total cost of mathematical model is lower than that of heuristic model (shown in Figure 4.2). The total cost decrease rates are 16%, 22%, 27% and 31% for mode LL, LH, HL and HH respectively.

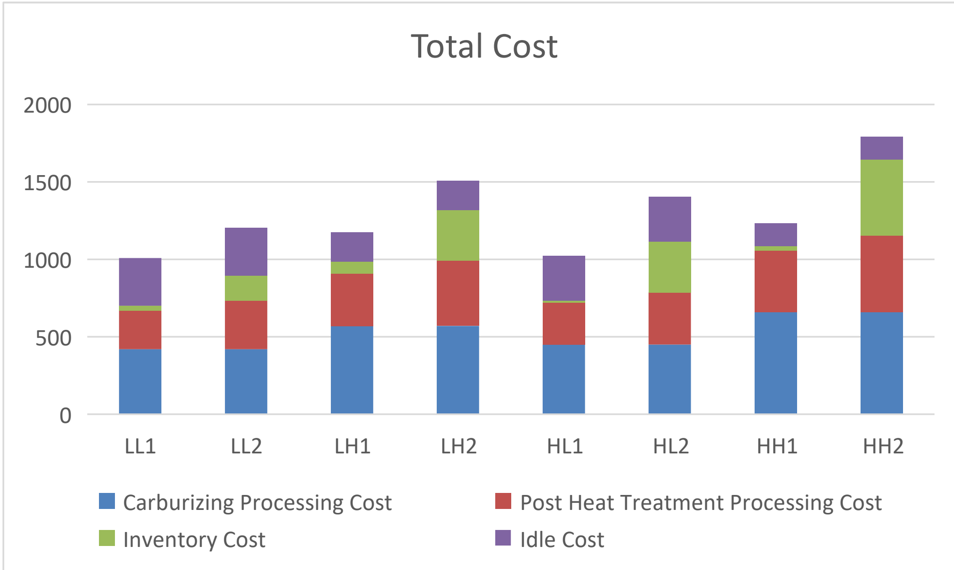

Figure 4.2 Total Cost for Each Example Instance

Furthermore, the total cost difference is consisted of two parts: post heat treatment costs and inventory costs (shown in Figure 4.3). The difference is mainly caused by inventory costs with an average percent of 76.8 among four production modes.

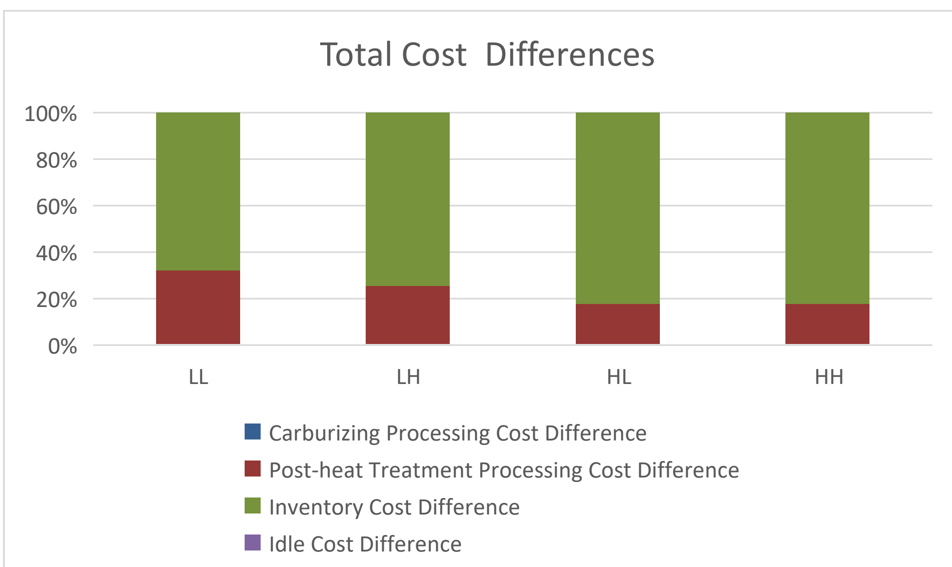

Figure 4.3 Total Cost Difference for Production Mode

In our example, all instances have the same initial condition, therefore we can use the difference of total costs between different modes for both mathematical model and heuristic method to compare these two methods. Table 4.13 and Table 4.14 show the changes of total cost under different mode-changing situations for mathematical model heuristic method respectively.

Table 4.13 Total Cost Changes for Mathematical Model under Each Production-Mode Changing Condition

From \ To	LL	LH	HL	HH
LL	0	167	12	224.2
LH	167	0	155	57.2
HL	12	155	0	212.2
HH	224.2	57.2	212.2	0

Table 4.14 Total Cost Changes for Heuristic Method under Each Production-Mode Changing Condition

From \ To	LL	LH	HL	HH
LL	0	303.2	198.2	588.9
LH	303.2	0	105	285.7
HL	198.2	105	0	390.7
HH	588.9	285.7	390.7	0

Assume that each mode-changing situation happens with equal probability. We can compute the mean and variance for two methods which is shown in Table 4.15. Mathematical model has lower mean and standard deviation.

Table 4.15 Statistical Analysis of Total Cost Change for Each Method

Method	Mean	Standard Deviation
Mathematical Model	103.45	93.10
Heuristic Method	233.96	195.07

4.2.2.2 Summary

The mathematical model is able to solve the defined problem with a scale of 3 types of products, 7 types of component and 24 time periods within 1 second. When time periods increase to 96, the new problem can be solved within 3 seconds. When 1 period represents 15 minutes, the time lengths with 24 periods and 96 periods are 6 hours and 24 hours respectively. The model is efficient and practical for solving short-term and mid-term scheduling problems.

In heuristic method, the production tasks are determined by next cycle's demand which contains an iterative idea. Compared to mathematical model, the total cost is higher and as product variety and customer's demands increase, the difference becomes greater. However, heuristic method is fast and easy to implement when the problem scale is large. Due to the processing cost of post heat treatment is time-related, mathematical-model-applied instances balances processing cost and inventory cost and heuristic-method-applied instances does not. Furthermore, when the production mode changes, total cost of mathematical model changes less with smaller range indicating that mathematical model is more capable of confronting variations.

Chapter 5 Conclusions and Future Research

The following are the most conclusive aspects of this research thesis. We also propose future research directions in the area.

5.1 Conclusions

Regarding the first objective for this study, we conducted a review of recent research on related topics: production planning, scheduling and heat treatment. From the review, we conclude that there is considerable amount of research on either production planning and scheduling for typical machining manufacturing or heat treatment specifying in technology innovation. However, the research on production planning and scheduling for heat treatment process is limited and therefore, a study on this topic is carried out.

We defined a multi-item multi-level production planning and scheduling problem for a heat treatment shop and developed a non-linear integer programming model to tackle the problem. The goal of the model is to minimize the total cost which is consisted of processing costs, inventory cost and idle cost. As a notable consideration worth mentioning is that one processing cost is time-related. In addition to typical variables, such as order requests and inventory cost considered in production planning problems, a core variable is defined with three attributes: the type of component, the time period when it is ready to be used and the time period it is used. It is assumed that the processing cost is exponentially distributed based on the waiting time. As a result, the multiplication of the processing cost and the variable is a non-linear term and we developed a linearization method to solve it. In addition, we proposed a heuristic method of the considered problem for efficient and near-optimal solutions. In the method, next two cycles' demands are considered to control the inventory level.

To validate the mathematical model and heuristic method, 4 groups of numerical example instances are presented under the same initial condition. When it comes to implementing the mathematical model, IBM® ILOG® CPLEX® Optimization Studio is used for optimal solutions. The analysis is conducted based on the computational results and we can conclude that as product variety and demand level go higher, mathematical method performs better in total cost optimization and heuristic method is more efficient when the problem scale is large.

The main contributions of this research are the application of production planning and scheduling in heat treatment process, the consideration of time-related processing cost and the development of a linearization method.

5.2 Future Research

There are several options to extend the research presented in this thesis. Our suggestions for future research in this area are:

- ✓ Considering dynamic demands of products, such as rush orders and order cancellations.
- ✓ Considering penalty cost. Demand does not have to be strictly satisfied at each time period and there will be penalty when delivery is delayed. Furthermore, the penalty cost could be a constant or a time-related function as well.
- ✓ Considering set-up cost. If there are more than one piece of equipment, we need to define binary variables that if each piece of equipment is operating or not. Set-up cost is considered into the total cost.
- ✓ Considering overall planning for the company. We can consider production planning and scheduling not only for the heat treatment shop but also covering machining shops from the perspective of the whole company.

These additional considerations will provide boarder applicability to the model to solve related problems.

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APPENDIX. IBM® ILOG® CPLEX® OPL MODEL SOURCE FILE

```
/*Parameters*/
int S[J]=...; //batch size
int B[J][K]=...; //bill of materials
int LT[J]=...; //lead time from order request to order reception for machined component j
int IIa[J]=...; //initial inventory of machined component j
int Ra[J][T]=...;//reception amount of machined component j in t
float HCa[J]=...;//inventory cost per batch for machined component j
int LB=...; //processing time of carburizing
int ALPHA=...; //number of carburizing chambers
int IIb=...; //initial inventory of carburized component j
int Rb[J][T]=...;//reception amount of carburized component j in t
float HCb[J]=...; //inventory cost per unit for carburized component j
float Q[J]=...; //coefficient for different component
int LC=...; //processing time,including shot peening, check quality report and production recording
int IIc[J]=...;//initial inventory of post-heat treated component j
int Rc[J][T]=...;//reception amount of post-heat treated component j in t
float HCc[J]=...; //inventory cost per unit for post-heat treated component j
float N=...; //after-carburizing treatment capacity
float G[J]=...; //after-carburizing treatment capacity occupation per unit for component j
int D[K][T]=...; //demand of product k in t

/*Variables*/
dvar int+ x[J][T]; //amount of order requests of machined component j in t
dvar int+ Ia[J][T]; //inventory of machined component j in t
dvar int+ ya[J][T]; //consumption amount of machined component j in t
dvar int+ Ib[J][T]; //inventory of carburized component
dvar int+ yb[J][T][T]; //consumption amount of carburized component j from t in p
dvar int+ Ic[J][T]; //inventory of heat-treated component
dvar int+ yc[J][T]; //delivered heat-treated component j in t
```

```
/*Objective*/
minimize    sum(j in J, t in T)HCa[j]*Ia[j][t]
            +sum(j in J, t in T)HCb[i]*Ib[j][t]
            +sum(j in J, t in T)HCc[i]*Ic[j][t]
            +sum(j in J, p in T,t in T:t==p)(20*yb[j][p][t]*Q[j])
            +sum(j in J, p in T,t in T:t==p+1)(23.2*yb[j][p][t]*Q[j])
            +sum(j in J, p in T,t in T:t==p+2)(24.3*yb[j][p][t]*Q[j])
            +sum(j in J, p in T,t in T:t==p+3)(24.8*yb[j][p][t]*Q[j])
            +sum(j in J, p in T,t in T:t==p+4)(24.9*yb[j][p][t]*Q[j])
            +sum(j in J, p in T,t in T:t>=p+5)(25*yb[j][p][t]*Q[j]);

/*Constraints*/
subject to{
//order receipt(t) + inventory(t-1) = consumption(t) + inventory(t) A
forall (j in J,t in T){
        if(t==1){
          Ra[j][t]+IIa[j]==ya[j][t]+Ia[j][t];
                }
                 if(t>1 && t<=LT[j]){
                   Ra[j][t]+Ia[j][t-1]==ya[j][t]+Ia[j][t];
                                    }
                                    if(t>LT[j]){
                                    x[j][t-LT[j]]+Ia[j][t-1]==ya[j][t]+Ia[j][t];
                                               }
                   }
//carburizing capacity
forall (t in T){
      if(t<LB){
      sum(j in J,l in T:l<=t)ya[j][l]+ sum(j in J,m in T:m>t)Rb[j][m] <= ALPHA;
              }
                 if(t>=LB){ sum(j in J, n in T:n<=t && n>=(t-LB+1))ya[j][n]<= ALPHA; }
                 }
```

```
//production receipt(t) + inventory(t-1) = consumption(t) + inventory(t) B
forall (j in J,t in T){
            if(t==1){
            Rb[j][t]+IIb==sum(p in T:p==1)yb[j][p][t]+Ib[j][t];
                        }
                         if(t>1 && t<=LB){
                         Rb[j][t]+Ib[j][t-1]==sum(p in T:p>=1 && p<=t)yb[j][p][t]+Ib[j][t];
                                                }
                                                if(t>LB){
                                          ya[j][t-LB]+Ib[j][t-1]==sum(p in T:p>=1 && p<=t)yb[j][p][t]+Ib[j][t];
                                                          }
                                    }
forall (j in J, p in T,t in T:t<p)
                yb[j][p][t]==0;
//production>=consumption
forall(j in J, t in T){
          if(t>=1 && t<=LB){
          Rb[j][t]>=sum(p in T:p>=t)yb[j][t][p];
                                    }
                                    if(t>LB){
                                    ya[j][t-LB]>=sum(p in T:p>=t)yb[j][t][p];
                                            }
                                  }
```

```
//production receipt(t) + inventory(t-1) = consumption(t) + inventory(t) C
forall(j in J,t in T){
        if(t==1){
        Rc[j][t]+IIc[j]==yc[j][t]+Ic[j][t];
                }
                if(t>1 && t<=LC){
                Rc[j][t]+Ic[j][t-1]==yc[j][t]+Ic[j][t];
                        }
                        if(t>LC){
                sum(p in T:p<=t && p>LC)yb[j][p-LC][t-LC]+Ic[j][t-1]==yc[j][t]+Ic[j][t];
                                }
                }
//after-carburizing treatment capacity
forall (t in T)
      sum(j in J,p in T:p<=t)yb[j][p][t]*G[j]<=N;
//delivery>=demands
forall (j in J, t in T)
         sum(t in T)yc[j][t]*S[j]>=sum(k in K, t in T)D[k][t]*B[j][k];
forall (j in J, k in K, t in T)
         (yc[j][t]+0.99)*S[j]>=D[j][t]*B[j][k];


}
```