Operating room planning with the pooling of downstream beds among specialties: A stochastic programming approach

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Abstract

Operating room planning with the pooling of downstream beds among specialties: A stochastic programming approach

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In this research, we study a stochastic operating room planning problem with the possibility of restricted pooling of downstream beds among different specialties. Here, we suppose that there is a limited number of beds that can be shared among specialties. In this problem, surgical durations and patients' length of stay are stochastic. We developed a two-stage stochastic integer programming model, where in the first-stage we decide on 1) the number of ICU and wards beds to be allocated to each specialty, and 2) the allocation of surgeries to operating rooms during the planning horizon. In the second stage, we decide on 1) how many shared beds in ICU and wards are allocated to which specialties on each day during the planning horizon, 2) the surge capacity required to satisfy downstream service to patients, and 3) the overtime incurred in each operating room during the planning horizon. The proposed model aims at minimizing the total cost including the patients' waiting cost, postpone cost, overtime and fixed cost of operating rooms, and the cost of downstream surge capacity.

We have implemented the proposed stochastic programing model in a sample average approximation framework. We have carried out extensive computational experiments to evaluate the effectiveness of several pooling policies for downstream beds and also the efficiency of the proposed sample average approximation algorithm. We have also performed an extensive sensitivity analysis of cost and the stochastic parameters to provide managerial insights. Our results demonstrated that the sharing policy among different specialties in the downstream units enhance the functionality of the system up to 19.53%. Moreover, the results indicated that the solutions obtained by proposed stochastic model outperforms the solutions from the corresponding deterministic problem by 17.43% on average.

Dedication

To Reza and Rahimeh who taught me how to dream limitlessly

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Chapter 1

Introduction and literature review

In this chapter, we cover the introduction and the literature review that are relevant to our work.

1.1 Introduction

Despite all the shocks and abnormalities, which have influenced healthcare industry in these years, this sector has successfully managed to keep its essence and structure. In any healthcare system, managers try to keep the expenses and the revenue at the minimum level. In this regard, they use mathematical models to identify the bottlenecks of their systems and remove them. Based on the existing literature, operating rooms are the most crucial part of a hospital which bears the burden of the majority of expenses and revenue. Umali and Castillo (2020) and Research and Markets (2019) state that almost two-thirds of revenue of each hospital comes from operating rooms while taking account for 40% of its expenses.

In this context, the main role of health practitioners is to allocate surgeries to operating rooms over a planning horizon such that medical resources including the available times of operating rooms are used as efficiently as possible. This planning procedure is a very difficult task because the health practitioners must take into account many other restrictive details such as the limited number of beds in ICU and wards. Efficient management of operating rooms considering the limited downstream resources is even a more complicated task because the patients' length of stay (LOS) are uncertain and therefore the health practitioners have a hazy view of the available number of beds in ICU and wards within the next few days.

In the practice, ICU and wards beds are divided between different specialties to avoid any conflict between the surgical groups. However, a fixed and inflexible allocation of downstream resources to specialties could cause inefficient use of them. For instance, a waste of recourses happens when the patients of one specialty happen to stay shorter than expected in ICU beds while patients of another specialty require to stay in ICU longer than expected. In this case, the future surgeries of the latter specialty could be cancelled due to unavailable ICU beds, while extra ICU beds are available for the other surgical group. A questions that arise in this context is whether having some shared downstream beds could improve the management of operating rooms and if it does by what margin it enhances the efficiency of the system.

In the literature, several researchers have studied the operating room management considering the limited number of beds in downstream, while addressing the stochastic essence of patients' length of stay. Min and Yih (2010) proposed a two-stage stochastic programming model for an operating room planning problem with a limited number of ICU beds. They formulated the limited number of beds as a hard constraint. Jebali and Diabat (2015, 2017) extended this work to the case of limited number of beds in wards, with and without considering emergency patients. Zhang et al. (2019) developed a similar two-stage stochastic programming model for the operating room planning problem, where patients are denied from admission to ICU when all beds are occupied. They proposed an approximate dynamic programming model as the solution approach. Also, Zhang et al. (2020) proposed a column-generation-based heuristic to solve a modified version of model proposed by Min and Yih (2010). None of these works have specifically studied the effect of downstream-bed pooling in the presence of uncertain length of stay. We refer the reader to Section 2 for a more comprehensive review of the existing works including those proposing robust optimization models.

Although there is a rich literature on stochastic operating room planning problem, to the best of our knowledge, there is no paper studying the effect of downstream-bed pooling in the presence of uncertain length of stay. The main contributions of this research are as follows:

- For the first time, we study the pooling of downstream beds in ICU and wards among different specialties in an operating room planning problem where surgical durations and patient's length of stay are stochastic. Our goal is to determine whether pooling of downstream beds results in enhancing the efficiency of the operating room planning and if so by what margin. The possibility of sharing beds in ICU and wards among specialties has not been studied in the stochastic programming models in the literature.
- We formulate the problem as a two-stage stochastic programming model where surgical durations and patients' length of stay in downstream resources are stochastic. We embed the proposed stochastic programming model in a sample average approximation framework.
- We provide extensive computational results to evaluate the improved efficiency as a result of sharing beds among specialties and provide managerial insights. We also perform a broad

sensitivity analysis to evaluate the behavior of the proposed model with different parameter settings.

The remainder of this research is structured as follows: In the rest of this Chapter, we have provided an extensive literature review. In Chapter 2, we have described the problem definition. In the same Chapter, we have proposed a two-stage stochastic model for the stochastic operating room planning problem with the possibility of downstream-bed pooling. In Chapter 3, we have provided a sample average approximation method that we have implemented to obtain statistical bounds from the proposed stochastic programming model. In Chapter 4, we have carried out an extensive computational experiments. Finally, we have concluded this research in Chapter 5 and have provided some suggestions regarding future studies.

1.2 Literature review

In operating room planning problem with downstream resource constraints, it is vital to recognize the main critical factors. Among them, uncertainty in surgical durations and patients' length of stay in downstream beds play a significant role. Handling these uncertainties by developing appropriate model is the main challenge. Regarding the randomness of these two parameters, we can categorize the papers in the literature into three categories. Some researchers have considered these parameters deterministic and proposed heuristics or mixed integer programming models to solve the problem. On the other hand, there is a stream of works that have assumed these parameters to be uncertain with known or unknown probability distributions and have developed stochastic optimization and robust optimization accordingly.

1.2.1 - Literature review of deterministic models

At first, we discuss the deterministic models in the literature for operating room planning problems with limited downstream resources. In this category, Beliën and Demeulemeester (2008) targeted human resources including nurses and operating room scheduling, simultaneously. They developed an integer programming model and two sets of solutions for two pricing problems. They used a standard dynamic programming approach and a mixed integer programming model to solve these subproblems. Blake and Donald (2002) proposed an integer programming model for the surgery planning in Toronto's Mount Sinai hospital. They performed a heuristic to assign different surgical categories to operating rooms. Their model resulted in significant improvements such as reduction in political maneuvering, reduction in conflict between department and between surgeons and better communication and cooperation between departments in terms of allocation of surgical blocks. Cardoen et al. (2007) considered an operating room scheduling problem and solved it by a branch-and-price approach. Then, they developed two pricing algorithms to solve subproblems. Their proposed dynamic programming model was resulted in higher quality solutions in a more reasonable time. By using a realistic data set, they tested their model with both exact and heuristic algorithms.

Fei et al. (2008) developed an integer programming model to formulate a surgical case assignment problem and then used Dantzig-Wolf decomposition to reformulate it. They ran a branch-and-price algorithm on the set partitioning problem that was formulated as the master problem. Fei et al. (2006) studied a weekly planning for operating theatres and developed a heuristic method to solve it. Consequently, they performed a hybrid genetic algorithm in order to solve the daily planning based on the result obtained from the first step. Guinet and Chaabane (2003) broke the overall scheduling problem into two steps. First, they assigned patients to different operating rooms and then scheduled the allocated surgeries in each room separately to allocate sufficient resources to them. They mainly focused on the first step and used an extension of Hungarian method to solve their model heuristically.

1.2.2 - Literature review of stochastic optimization models

The next category of papers in operating room planning are the ones developing stochastic optimization models. In this direction, researchers consider one or multiple sources of uncertainty to make their model more realistic. Some of the most common origins of uncertainty are stochastic surgical durations, patients' length of stay in downstream beds, and random arrival times of patients. Beliën and Demeulemeester (2007) considered the number of patients in ORs and the length of stay of patients to be uncertain. They mainly concentrated on two kinds of models: mixed integer programming approach and metaheuristic approach. They eventually stated that the best results are obtained by metaheuristic models while MIP models are more comfortable to use. In another effort, Kumar et al. (2018) developed a stochastic mixed integer programming model to manage the patient flow in master surgery scheduling, assuming the length of stay is not deterministic. Then they considered multiple planning horizons and scenarios for length of stay to evaluate their model.

Lamiri et al. (2008) concentrated on patient and OR scheduling based on their arrival and registration for being operated. They considered two groups of emergency and elective patients. They proposed a stochastic programming model at first. Then, they implemented a combination of Mont Carlo simulation and MIP to solve the model. Saadouli et al. (2014) mainly focused on surgical durations as the main source of uncertainty. First, they allocated different surgeries to different days and operating rooms. Then, they developed two MILP models such that each of them dealt with the overtime and underutilization problems. They observed that both models performed significantly great comparing to manual assignment in different instances. While some researches have targeted a single uncertain parameter, some others considered more than one. Molina-Pariente et al. (2016) developed a stochastic OR scheduling model with uncertain surgical durations, the arrival time of emergency surgeries, and surgeons' capacity. They presented a Monte Carlo method based on sample average approximation (SAA) method. They eventually, compared the results of their model with the exact model and two heuristics developed for solving the deterministic version of the model.

Compared to our work, the most relevant works in the literature are Min and Yih (2010), Jebali and Diabat (2015, 2017), Zhang et al. (2019, 2020). The problem setting in these works are more similar to ours compared to existing works in the literature. In all of these works, researchers have studied an operating room planning problem considering the limited number of beds in downstream, while addressing the stochastic essence of patients' length of stay. Min and Yih (2010) developed a two-stage stochastic programming model for allocation of patients to surgical blocks over a planning horizon, while considering a limited number of ICU beds. They supposed that the restriction on the limited number of ICU beds is a hard constraint and cannot be violated in any scenario. They evaluated their stochastic programming model in a SAA framework. Jebali and Diabat (2015) developed Min and Yih (2010)'s model by considering both ICU and wards beds. They also evaluated the improved stochastic programming model using the sample average approximation approach. Jebali and Diabat (2017) also studied an extension of the same research stream by modeling the constraint of the limited number of ICU beds as a chance constraint and used the SAA approach proposed by Luedtke and Ahmed (2008) as the solution method. Zhang et al. (2019) also proposed a similar two-stage stochastic programming model for the operating room planning problem. In their model, patients are denied from admitting to ICU when all beds are occupied. They developed an approximate dynamic programming model to solve the model.

Zhang et al. (2020) considered the same operating room planning problem and extended the stochastic programming model proposed by Min and Yih (2010). They developed a Dantzig-Wolf decomposition and offered a column-generation heuristic to solve it.

As discussed earlier in this Chapter, none of these works have considered the possibility of pooling of ICU and wards beds among different specialties where length of stay are uncertain. In this research, we focus on the improvement in the efficiency of operating room planning resulted from a restricted sharing of downstream beds among surgical groups.

1.2.3 - Literature review of robust optimization models

The third group of papers on operating room planning are the ones which developed robust optimization models. Robust optimization is a conservative approach that is independent from the probability distributions of the uncertain parameters. Shehadeh and Padman (2021) proposed a distributional robust optimization model to allocate elective surgeries to various surgical blocks. They assumed surgical duration and length of stay are uncertain, but with given information on their mean values and ranges. They reformulated the problem as an exact nonlinear mixed integer programming model and then linearized it. Shanshan et al. (2017) developed a robust chance-constrained framework to schedule surgeries over a planning horizon such that the total cost is minimized, while the restrictions in the downstream are not violated. In this work, researchers considered uncertain surgical duration, while length of stay were deterministic. Furthermore, Moosavi and Ebrahimnejad (2018) offered a multi-objective model that minimized the number of deferred patients, waiting cost, and ORs idleness, and overtime. Then, they developed its robust counterpart with uncertain surgical durations and length of stay, considering upstream and downstream units in addition to emergency demands.

Moosavi and Ebrahimnejad (2020) also have used a new two-stage heuristic algorithm that is implemented both on a deterministic multi-objective model and the model with uncertainty. The main concern in their work was determining the number of operating rooms to open and how to assign surgeries to different operating rooms. The heuristic algorithm showed some significant improvement in both cases especially in the case of uncertainty. Neyshabouri and Berg (2017) proposed a two-stage robust optimization model to allocate patients to multiple surgery blocks. They considered uncertainty for surgical durations and length of stay. In formulating, the second-stage model, they considered penalties for the violation of the ICU bed constraint. Then, they developed a column-and-row generation algorithm to solve it.

Chapter 2

Problem definition and two-stage stochastic programming model

In this chapter, first we provide the problem definition and then we represent the two-stage stochastic programming model.

2.1 Problem definition

We study an operating room planning problem where several patients are going to be allocated to operating rooms over a planning horizon. In each operating room, there is a limited regular time available for performing surgeries. The durations of surgeries are stochastic and therefore there is a possibility to have overtime at the end the available time of operating rooms. There is a fixed cost associated with opening each operating room for each day. Patients belong to different surgical specialties that do not share operating rooms. This means that patients allocated to the same operating room must belong to the same surgical specialty. Each patient has a time window that indicates the earliest and the latest days on which the surgery can be performed. If the latest day is within the planning horizon, we refer to the patient as a mandatory patient that must be operated in the planning horizon before the deadline. Otherwise, the patient is an optional patient that can be postponed to the next planning horizon for the operation.

After surgeries, the patients are transferred to ICU and wards consecutively and usually stay in each area for a few days. Each surgical specialty has its own number of beds in ICU and wards. For example, cardiovascular, neurology, and orthopedic may have 12, 10, and 9 beds in ICU, respectively. As in the common practice, we suppose that these beds are reserved for the corresponding specialty and will not be occupied by the patients of other specialties in any case. Besides, to study the effect of pooling for downstream beds in this research, we suppose that there is a limited number of ICU beds that are shared between specialties. Therefore, in the case that a new cardiovascular patient is operated and needs an ICU bed, but all beds of this specialty are already occupied, the patient may be allocated to one of the available shared ICU beds. If there is not any available ICU bed in the cardiology part of ICU or the shared ICU area, the patient will use a surge capacity at an extra cost, which refers either to hiring a part-time ICU nurse, or paying an existing ICU nurse overtime to take care of the patient, or transferring the patient to another hospital with available ICU beds. We consider that there is the same limited bed capacity for different specialties in wards area with the possibility of having shared beds as well. In this problem, management of downstream beds is specifically more difficult because the patients' length of stay are stochastic and therefore the number of patients occupying beds does not follow a clear trend as patients may stay longer or shorter than expected.

In the next section, we propose a two-stage stochastic programming model to formulate this problem. In the first stage, the decision maker determines 1) the number of ICU and wards beds to be allocated to each specialty, and 2) the allocation of surgeries to operating rooms during the planning horizon. Here, we suppose that there is a limited number of beds that can be considered shared between different specialties. This is necessary in practice to avoid conflict between specialties. In the second stage, the decision maker determines 1) how many shared beds in ICU and wards are allocated to which specialties on each day during the planning horizon, 2) the surge capacity required to satisfy downstream service to patients, and 3) the overtime incurred in each operating room during the planning horizon. These variables are computed based on first-stage decision on the allocation of surgeries to operating rooms and downstream beds to specialties. The objective function is to minimize the total cost including the patients' waiting cost, postpone cost, overtime and fixed cost of operating rooms, and the cost of downstream surge capacity.

2.2 Two-stage stochastic programming model

In this chapter, we developed a two-stage stochastic programming model. In the first stage, we decide on the allocation of the downstream beds to specialties, which operating rooms to open on each day, and also set the allocation of surgeries in the planning horizon. In the second stage, we determine how to allocate the shared beds in ICU and wards to different specialties on each day in the planning horizon. We also compute the incurred overtime in operating rooms and the surge capacity in ICU and wards. The first-stage objective function minimizes the patients' waiting cost, postpone cost, and fixed cost of operating rooms, while the second-stage objective function consider overtime cost of operating rooms and the cost of downstream surge capacity.

2.2.1 - First-stage model

The sets, parameters, and variables in our model are as follow.

Sets:

I	:	The set of surgeries (or patients). We have $I = I_1 \cup I_2$.
I_1	:	The set of mandatory surgeries (or patients) with latest surgery days within the planning horizon.
I_2	:	The set of optional surgeries (or patients) with latest surgery days out of the planning horizon.
S	:	The set of specialties.
D	:	The set of days in the planning horizon.
D_i	:	The set of days on which surgery i can be operated with respect to its time window.
R	:	The set of operating rooms.
R_i	:	The set of operating rooms in which surgery i can be operated in the case that operating rooms are not identical and special equipment is available in some operating rooms.
H	:	The set of downstream units. Here, we consider $H = \{1,2\}$ where 1 and 2 refer to ICU and wards, respectively. It is a general setting which can consider cases with more than two post-surgical recovery areas.

Parameters:

M_h	:	The total number of beds in downstream h.
α_h^{shared}	:	The percentage of beds in downstream h that are shared between specialties.
$\alpha_i^{waiting}$	:	The daily waiting cost of patient i.
e_i	:	The earliest day in the time window of surgery i.
s_i	:	The specialty corresponding to surgery i.
$c_{id}^{waiting}$	:	The waiting cost of surgery i if it is performed on day d. It is pre-computed by $c_{id}^{waiting} = \alpha_i^{waiting}(d - e_i)$.
$c_i^{postpone}$	:	The cost of postponing surgery i to the next planning horizon.
c^{OR}	:	The fixed cost of opening an operating room.

A_s^{min} : The minimum number of operating rooms that must be allocated to specialty s in the planning horizon.

A_s^{max} : The maximum number of operating rooms that can be allocated to specialty s in the planning horizon.

Variables:

x_{ird} : 1 if we allocate surgery i to operating room r on day d, 0 Otherwise.

x'_i : 1 if we postpone surgery i to the next planning horizon, 0 Otherwise.

y_{rd} : 1 if we open operating room r on day d, 0 Otherwise.

z_{srd} : 1 if specialty s is allocated to operating room r on day d, 0 Otherwise.

u_{sh} : The number of beds in downstream h assigned to specialty s.

Based on the given notation, we introduce the first-stage model as follows:

$$\min_{x,x',y,z,u} \sum_{i\in I}\sum_{d\in D_i}\sum_{r\in R_i} c_{id}^{Waiting} x_{ird} + \sum_{i\in I_2} c_i^{Postpone} x'_i + \sum_{r\in R}\sum_{d\in D} c^{OR} y_{rd} + Q(x,x',y,z,u) \tag{1}$$

Subject to:

$$\sum_{d\in D_i}\sum_{r\in R_i} x_{ird} = 1 \qquad i \in I_1 \tag{2}$$

$$\sum_{d\in D_i}\sum_{r\in R_i} x_{ird} + x'_i = 1 \qquad i \in I_2 \tag{3}$$

$$\sum_{s\in S} z_{srd} = y_{rd} \qquad r \in R, d \in D \tag{4}$$

$$x_{ird} \le z_{s_i rd} \qquad i \in I, r \in R_i, d \in D_i \tag{5}$$

$$A_s^{min} \le \sum_{d\in D}\sum_{r\in R} z_{srd} \le A_s^{max} \qquad s \in S \tag{6}$$

$$\sum_{s\in S} u_{sh} \le \left(1 - \alpha_h^{shared}\right) M_h \qquad h \in H \tag{7}$$

$$x_{ird} \in \{0,1\} \qquad i \in I, r \in R_i, d \in D_i \tag{8}$$

$$x'_i \in \{0,1\} \qquad i \in I_2 \tag{9}$$

$$y_{rd} \in \{0,1\} \qquad r \in R, d \in D \qquad (10)$$

$$z_{srd} \in \{0,1\} \qquad s \in S, r \in R, d \in D \qquad (11)$$

$$u_{sh} \geq 0 \qquad s \in S, h \in H \qquad (12)$$

Objective function (1) consists of four parts. The first three of them calculate patients' waiting cost, postpone cost, and the opening cost of opening operating rooms, respectively. In the last segment, we have $Q(x, x', y, z, u)$ that represents the expected second-stage cost. Constraint (2) indicates that each mandatory surgery must be allocated to one operating room on a single day in the planning horizon. Constraint (3) implies that optional surgeries are either allocated a surgical block or postponed to the next planning horizon. Constraint (4) states if an operating room is opened on a day, it must be assigned to exactly one specific specialty. Constraint (5) guarantees that surgery i can be operated in operating room r on day d only if this block is assigned to the specialty of surgery i denoted by s_i. Constraint (6) restricts the number of operating rooms that each specialty can have in the planning horizon. Constraint (7) indicates that, in each downstream, the number of non-shared beds allocated to different specialties cannot be more than the total number of available non-shared beds.

2.2.2 - Second-stage model

In the following, we present the sets, parameters, and variables used in the second-stage model.

Sets:

Ω	:	The set of stochastic scenarios.
I_s	:	The set of surgeries that belong to specialty s.

Parameters:

c_h^{bed}	:	The cost one unit of surge capacity in downstream h.
$c^{Overtime}$	:	The per-minutes cost overtime in an operating room.
A	:	The total regular available time in each operating room.
O^{Max}	:	The maximum allowed overtime in each operating room on each day.
$t_{i\omega}$	:	The duration of surgery i in scenario ω.

$l_{ih\omega}$	:	The length of stay for patient i in downstream h in scenario ω.
$\xi(\omega)$	:	The vector of uncertain parameters including surgical durations and length of stay in Scenario ω.

Variables:

$o_{rd\omega}$	:	The overtime of operating room r on day d in scenario ω.
$v_{shd\omega}$	:	The number of patients belonging to specialty s that use a surge capacity in downstream h on day d in scenario ω.
$q_{shd\omega}$	:	The number of shared beds that occupied by patients of specialty s in downstream h on day d in scenario ω.

We formulate the second-stage model as follows:

$$Q(x, x', y, z, u, \omega) = \min_{q,v,o} \sum_{s\in S}\sum_{h\in H}\sum_{d\in D} c_h^{bed} v_{shd\omega} + \sum_{d\in D}\sum_{r\in R} c^{Overtime} o_{rd\omega} \tag{13}$$

Subject to:

$$\sum_{i\in I_s}\sum_{r\in R_i}\sum_{\substack{d'\in D_i:\, d'+\sum_{h'=1}^{h-1} l_{ih'\omega} \le d \text{ and} \\ d'+\sum_{h'=1}^{h} l_{ih'\omega} > d}} x_{id'r} \le u_{sh} + q_{shd\omega} + v_{shd\omega} \qquad h\in H, d\in D, \omega\in\Omega, s\in S \tag{14}$$

$$\sum_{s\in S} q_{shd\omega} \le \alpha_h^{shared} M_h \qquad h\in H, d\in D, \omega\in\Omega \tag{15}$$

$$\sum_{i\in I: d\in D_i \text{ and } r\in R_i} t_{i\omega} x_{idr} \le A + o_{rd\omega} \qquad r\in R, d\in D, \omega\in\Omega \tag{16}$$

$$0 \le o_{rd\omega} \le o^{Max} \qquad r\in R, d\in D, \omega\in\Omega \tag{17}$$

$$v_{shd\omega} \ge 0 \qquad s\in S, h\in H, d\in D, \omega\in\Omega \tag{18}$$

$$q_{shd\omega} \ge 0 \qquad s\in S, h\in H, d\in D, \omega\in\Omega \tag{19}$$

Objective function (13) minimize the total second-stage cost including the cost associated with using the surge capacity in different downstream by patients of different specialties and the overtime cost in operating rooms. The second-stage cost $Q(x, x', y, z, u)$ in objective function (1) is calculated by $E_{\omega\in\Omega}[Q(x, x', y, z, u, \xi(\omega))]$ where $E_{\omega\in\Omega}[.]$ computes the expected value over scenarios $\omega \in \Omega$.

Constraint (14) implies the restriction on the number of available beds in different downstream. In this constraint, $\sum_{\substack{d'\in D_i:\, d'+\sum_{h'=1}^{h-1} l_{ih'\omega} \leq d \text{ and} \\ d'+\sum_{h'=1}^{h} l_{ih'\omega} > d}} x_{id'r}$ is equal to 1 if patient i, that is operated in operating room r, is in downstream h on day d. This is because $d' + \sum_{h'=1}^{h-1} l_{ih'\omega} \leq d$ indicates that the patient has left the previous downstream $(h-1)$ not later than day d and $d' + \sum_{h'=1}^{h} l_{ih'\omega} > d$ shows that he/she will leave downstream h after day d. Therefore, the left-hand side of Constraint (14) computes the total number of patients that are in downstream h on day d in scenario ω. Whenever the left-hand side of this constraint is larger than u_{sh}, the model prefers to compensate the shortage by giving positive values to $q_{shd\omega}$ and then $v_{shd\omega}$. This is because the shared ICU beds are available free of cost, while the surge beds, denoted by $v_{shd\omega}$, are penalized in the Objective function (13).

Constraint (15) shows that the total number of shared beds allocated in each downstream cannot be more than the total number of beds available for sharing. Constraint (16) declares that a limited regular time A is available in each operating room on each day. If the total surgical time on the left-hand side of Constraint (16) is more than the regular available time, then the overtime is considered by giving a positive value to $o_{rd\omega}$. Furthermore, Constraint (17) confines the allowed overtime.

Chapter 3

Solution methodology

One of the main challenge in using stochastic programming models is that solving them becomes exponentially more difficult as the number scenarios increases. Therefore, in this research, we develop a sample average approximation algorithm (SAA). This method uses Monte Carlo simulation to estimate the expected value of objective function based on a number of random independent identically distributed (i.i.d.) samples. It generates these samples iteratively and solve the extensive form of the stochastic programming model for a limited number of scenarios in each iteration separately. Then, the lower and upper bounds of the objective function is estimated using the outputs of all iterations. Here, we use $|N|$ to denote a number of randomly generated scenarios where, N = $\{\omega_1, \dots ,\omega_{|N|}\}$ is the set of scenarios. In SAA, we approximate the second-stage objective value

$$\mathbb{E}_\omega\left[\sum_{s\in S}\sum_{h\in H}\sum_{d\in D} c_h^{bed} v_{shdw} + \sum_{d\in D}\sum_{r\in R} c^{Overtime} o_{rdw}\right] \tag{20}$$

subject to (14)-(19)

by

$$\min\frac{1}{|N|}\left[\sum_{s\in S}\sum_{h\in H}\sum_{d\in D} c_h^{bed} v_{shdn} + \sum_{d\in D}\sum_{r\in R} c^{Overtime} o_{rdn}\right] \tag{21}$$

subject to (14)-(19).

Here, for each sample $n \in N$ constraint (14)-(19) are repeated instead of $\omega \in \Omega$.

The privilege of using SAA is that by increasing the number of samples, the obtained solution and objective value we approach the optimal solution and objective value (Kleywegt, Shapiro and Homem-de-Mello 2002). The only issue here is that using a massive number as the number of sample works against the initial intention of using SAA since it becomes a time consuming procedure. To avoid that, we solve the SAA problem $|M|$ times for a reasonable number of samples

$|N|$. Then, we calculate the average and variance of the lower and upper bound using the objective values obtained in all iterations.

The procedure of SAA can be explained as follows:

- We solve Model (21) $|M|$ times independently. In each iteration, we consider $|N|$ samples of scenarios and save the obtained first-stage solution $\hat{x}_N^m$ and the objective value $\hat{f}_N^m$ for each iteration $m \in \{1, \dots, |M|\}$. We refer to the problem that we solve in this iteration as the lower bound problem.
- We calculate the average and variance of the lower bound of the objective value using the following formulas:

$$\overline{LB} = \frac{1}{|M|} \sum_{m=1}^{|M|} \hat{f}_N^m, \tag{22}$$

$$\sigma_{LB}^2 = \frac{1}{|M|(|M|-1)} \sum_{m=1}^{|M|} (\hat{f}_N^m - \overline{LB}). \tag{23}$$

- Then, we solve Model (21) again for $|M|$ iterations for the corresponding fixed first-stage solutions $\hat{x}_N^m$ $m \in \{1, \dots, |M|\}$ obtained in the previous step. We refer to the problems solved in this step as upper bound problems. In each iteration, we consider $|P|$ samples of scenarios and save $\hat{f}_p(\hat{x}_N^m)$ that denotes the objective value of the upper bound problem in iteration m for sample p.
- Next, the average and the variance of the upper bound of the objective value is calculated by:

$$\overline{UB} = \frac{1}{|P|} \sum_{m=1}^{|M|} \hat{f}_p(\hat{x}_N^m), \tag{24}$$

$$\sigma_{UB}^2 = \frac{1}{|P|(|P|-1)} \sum_{m=1}^{|M|} (\hat{f}_p(\hat{x}_N^m) - \overline{UB}). \tag{25}$$

- We consider the minimum of average upper bound values $\overline{UB}$ obtained by (24) over different iterations as the best upper bound value and denote it by $\overline{UB}_{Best}$. Then we use $\overline{LB}$ and $\overline{UB}_{Best}$ to calculate the optimality gap by

$$Gap = \ 100\,(\overline{UB}_{Best} - \ \overline{LB})/\overline{LB}\,. \tag{26}$$

Chapter 4

Computational experiments

We used IBM ILOG CPLEX Optimization to solve the integer programming model we implemented our code in Visual Studio V12.8 in C++. The experiments were run on a computer with two AMD Rome 7502 processors, 2.50 Ghz, and a total of 64 cores. We used a single core to run each instance.

4.1 - Instance generation

In this section, we explain how we generated the instance sets. We set the number of weeks in the planning horizon to $\{2,3,4\}$. In our instances, four operating rooms available on each day. We considered 8 hours as the daily regular available time of each operating room. Also, the maximum allowed overtime for each operating room is set to three hours. Furthermore, we have considered ICU and wards two consecutive downstream units. We set the number of available ICU and wards beds to 35 and 65, respectively. For instances with two, three, and four weeks, we set the number of patients to $\{120,180,240\}$, respectively. The earliest days of time windows for surgeries are randomly generated within the planning horizon. Also, the length of time window for each patient varies from one to seven days.

In addition, each patient randomly belongs to one the seven specialties listed in Table 1. This Table also reports the average surgical time of surgeries corresponding to each specialty (Costa Jr, 2017). To generate the surgical time of patient i, we considered normal distribution $N\left(\mu_i^{duration}, \sigma_i^{duration}\right)$ where $\mu_i^{duration}$ and $\sigma_i^{duration}$ refer to the average surgical duration of the corresponding specialty and its standard deviation, respectively and set $\sigma_i^{duration} = (1/6)\mu_i^{duration}$. The coefficient of (1/6) ensures that the surgical times are generated in $\left[\mu_i^{duration} - 3\left(\frac{1}{6}\mu_i^{duration}\right), \mu_i^{duration} + 3\left(\frac{1}{6}\mu_i^{duration}\right)\right]$ with a probability of 99.73%.

Moreover, the last two columns of Table 1 reports the averages total length of stay in hospitals and its standard deviations for patients of different specialties. To randomly generate the length of stay for each patient i, we use a normal distribution $N\left(\mu_i^{LOS}, \sigma_i^{LOS}\right)$. In this distribution, $\mu_i^{duration}$ and $\sigma_i^{duration}$ respectively refer to the average length of stay of patient i and its standard deviation.

To generate a further perturbation in our instances, we supposed that the μ_i^{LOS} of different patients corresponding to the same specialty are not necessarily the same. To address this point, we set $\mu_i^{LOS} = [0.75LOS_{s_i}^{specialty}, 1.25LOS_{s_i}^{specialty}]$ where $LOS_{s_i}^{specialty}$ denotes the average of length of stay for all patients of the corresponding specialty reported under the third column of Table 1. The patients' length of stay generated from $N(\mu_i^{LOS}, \sigma_i^{LOS})$ are corresponding to all downstream units. Therefore, at the end we divide the generated length of stay $LOS_i^{patient}$ between ICU and wards using $LOS_i^{ICU} = 0.4\, LOS_i^{patient}$ and $LOS_i^{wards} = 0.6\, LOS_i^{patient}$.

There are also several cost coefficients in our model to be set. We set the fix cost of opening operating rooms and also the per-minute overtime cost to \$4,437 and \$12.37, respectively. To set the cost parameters, we consider that each patient i has an urgency level $\alpha_i^{priority}$ which are randomly picked from $\{1,2,3,4,5\}$. Then we set the daily waiting cost of patient i to $\alpha_i^{waiting} = \alpha_i^{priority} \times \$1000$. Also, the postpone cost of patients are computed by $c_i^{Postpone} = \alpha_i^{priority} \times \$15000$. We also set the cost of using one of surge capacity in wards (c_1^{bed}) to \$62.94. To compute this value, we first divided the annual salary of a wards nurse (\$54,549 as reported on www.ziprecruiter.com) by the total annual workhours (estimated by 52 weeks multiplied by 40 hours) and then divided it by 10, supposing that each nurse takes care of 10 patients on average. The obtained value represents the hourly cost of serving an extra patient in wards. Therefore, we multiplied the recent value by 24 hours and obtained \$62.94 as the daily cost of one unit of surge capacity in wards. Also, for the ICU, we use the same method with the only difference that \$54,549 was replaced by \$95,000 (reported on www.ziprecruiter.com) as the annual salary of an ICU nurse and obtained $c_2^{bed} = \$109.58$ as the daily cost of one extra surge capacity in ICU.

To generate our instance set, we considered that the number of specialties in each instance belongs to $\{1,2,3,4,5,6,7\}$. Consider all possibilities for the number of weeks in the planning horizon and the number of specialties, we have 21 combinations. For each combination, we generated 5 instances for a total of 105 instances.

Table 1-The list of specialties and details of their corresponding surgical time and length of stay (LOS)

Specialty	Surgical time (min)	Average LOS	Standard deviation of LOS
General	150.95	7.75	4.48
Neurology	135.06	7.23	5.19
Cardiovascular	189.34	5.84	3.01
Orthopedic	151.95	7.69	4.51
Urology	94	5.22	3.68
Plastic and reconstructive	157.72	6.71	4.54
Obstetrics and gynecology	79.32	5.22	2.21

4.2 - Computational results

We have separated this section of our study into four different subsections. In subsection 6.2.1, we computationally evaluate the importance of sharing beds in downstream units using our proposed stochastic programming model. In subsection 6.2.2, we tune the main parameters of our sample average approximation. Then, in subsection 6.2.3, we evaluate the performance of our model within the framework of sample average approximation. Finally, in subsection 6.2.4, we perform an extensive sensitivity analysis for cost parameters in order to measure the efficiency of our model in different situations and extract managerial insights.

4.2.1 - Sharing policy analysis

The computational experiments of this section analyze the improvement obtained by sharing beds in downstream units. We have considered three level of bed sharing in our computational analysis. In the first one, we suppose that no sharing is allowed among specialties. This means that α_h^{shared} for $h = 1,2$ is set to 0. In the second setting, we considered a "Midlevel Sharing" case where 50% of all available beds are shared among specialties. This setting refers to $\alpha_h^{shared} = 0.5$ for $h = 1,2$. In the last setting, referred to as "Full Sharing", we assume that all beds are shared among specialties without any limitation. In this case, we have $\alpha_h^{shared} = 1$ for $h = 1,2$.

To obtain the results of these three settings, we used the extensive version of our proposed stochastic programming model without SAA, to avoid confusion due to the statistical nature of SAA outputs. For each instance, we have generated 30 scenarios. We have presented the results out the above sharing policies in Table 2. In this Table, we have 18 combinations of instances with different values of the number of weeks and the number of specialties presented under the first two columns. We have ignored instances with one specialties as sharing beds is meaningless in this case. Each row of Table 2 presents the average results of five instances.

In Table 2, we have presented the results of Midlevel Sharing (setting 2) and Full Sharing (setting 3) in comparison to No Sharing (setting 1) separately. The columns of "Imp. (%)" denote the total improvement in the objective value obtained by the corresponding sharing policy (Midlevel Sharing or Full Sharing) in comparison to No Sharing policy. For instance, under the Column of Midlevel Sharing, we have $Imp.(\%) = 100\left(Obj_{No\ sharing} - Obj_{Midlevel}\right)/Obj_{No\ sharing}$, where $Obj_{No\ sharing}$ and $Obj_{Midlevel}$ denote the objective values of Midlevel Sharing and No Sharing settings, respectively. Similarly, Columns "Overtime cost Imp. (%)", "Surge cost Imp. (%)", "Waiting cost Imp. (%)", "Postpone cost Imp. (%)", and "OR cost Imp. (%)" demonstrate the contributions of each type of cost in the total improvement. This means that the sum of the recent five columns is equal to the value of "Imp. (%)".

The results of Table 2 demonstrate that the higher volume of sharing results in more improvement in the objective function. In the largest instances with seven specialties and four weeks, we observe that the values of "Imp. (%)" are 16.97% and 19.53% for Midlevel Sharing and Full Sharing, respectively. Besides, the average values of "Imp. (%)" in the last row of the Table show that Midlevel Sharing and Full Sharing lead to 11.29% and 12.38% improvement compared the No Sharing Policy. The results also show that the average improvement from Midlevel Sharing to Full Sharing is marginal around 1.09% and therefore a Midlevel Sharing could be enough to significantly improve the performance of operating room planning.

Moreover, we observe a strictly increasing trend of improvement in each set of weeks as the number of specialty rises. For example, the results of Midlevel Sharing shows that the average of total cost improvement changes from 4.56%, 4.84%, and 4.99% to 15.93%, 16.85%, and 16.97% as the number of specialties increases from 2 to 7 in instances with 2, 3, and 4 weeks, respectively. Besides, it is noteworthy that, for a fixed number of specialty, we have higher value of improvement in longer planning horizons.

Finally, the values under columns “Surge cost Imp. (%)” and “Postpone cost Imp. (%)” has the highest role in the total cost improvement. More specifically, in “Midlevel” and “Full” sharing policies, the average saving in the surging capacity are 9.18% and 9.92%, respectively. Also, the average saving obtained in postpone cost are 1.84% and 2.01% for Midlevel and Full sharing policies, respectively. For the remaining sections of this research, we run the computational experiments for the case of Midlevel Sharing policy.

Table 2- Comparison of different bed sharing policies.

Data Info.		Midlevel Sharing						Full Sharing					
No. of Weeks	No. of Spec.	Imp. (%)	Overtime cost Imp. (%)	Surge cost Imp. (%)	Waiting cost Imp. (%)	Postpone cost Imp. (%)	OR cost Imp. (%)	Imp. (%)	Overtime cost Imp. (%)	Surge cost Imp. (%)	Waiting cost Imp. (%)	Postpone cost Imp. (%)	OR cost Imp. (%)
2	2	4.56	0.06	5.61	0.42	-1.79	0.27	4.75	0.09	5.62	0.88	-2.01	0.16
	3	6.42	-0.14	7.29	-0.44	-0.32	0.03	6.69	-0.13	6.94	-0.04	0.00	-0.07
	4	8.34	-0.01	9.75	0.88	-2.25	-0.03	8.62	0.08	9.54	0.79	-1.82	0.03
	5	12.69	0.06	11.75	1.35	-0.78	0.31	13.82	0.07	12.49	1.64	-0.86	0.48
	6	13.62	0.00	10.43	0.66	2.32	0.22	15.44	0.02	13.53	1.08	0.52	0.28
	7	15.93	0.02	13.65	0.60	1.78	-0.11	17.79	-0.02	14.49	0.56	2.90	-0.14
	Average	10.26	0.00	9.74	0.58	-0.17	0.12	11.18	0.02	10.44	0.82	-0.21	0.12
3	2	4.84	-0.01	5.16	-0.07	-0.06	-0.18	4.90	0.00	7.15	0.40	-2.53	-0.12
	3	8.64	-0.21	8.21	0.54	0.14	-0.04	8.91	-0.18	6.37	0.21	2.60	-0.08
	4	12.12	-0.03	11.49	0.44	0.21	0.02	12.62	-0.06	13.94	0.99	-2.33	0.08
	5	13.21	-0.02	10.37	0.80	1.97	0.08	14.35	0.00	11.44	1.13	1.74	0.05
	6	14.34	-0.04	11.73	0.48	2.06	0.11	16.08	-0.07	12.72	0.36	2.99	0.08
	7	16.85	-0.02	11.03	-0.22	6.05	0.00	19.42	-0.04	13.03	0.70	5.65	0.08
	Average	11.66	-0.05	9.66	0.33	1.73	0.00	12.71	-0.06	10.77	0.63	1.35	0.01
4	2	4.99	-0.03	3.58	-0.07	1.66	-0.16	5.13	-0.04	3.51	-0.05	1.82	-0.11
	3	9.58	-0.10	5.62	-0.41	4.58	-0.11	9.98	-0.07	7.07	-0.19	3.21	-0.04
	4	11.43	0.00	8.87	0.83	1.67	0.06	12.42	-0.20	8.72	0.51	3.19	0.19
	5	13.82	-0.03	8.38	-0.67	5.96	0.18	15.36	-0.04	8.63	-0.49	7.14	0.12
	6	14.90	-0.02	11.04	-0.11	3.93	0.05	16.99	-0.02	10.82	-0.19	6.35	0.04
	7	16.97	-0.01	11.28	-0.27	6.05	-0.08	19.53	-0.09	12.48	-0.36	7.59	-0.08
	Average	11.95	-0.03	8.13	-0.12	3.97	-0.01	13.24	-0.08	8.54	-0.13	4.88	0.02
Total Average		11.29	-0.03	9.18	0.26	1.84	0.03	12.38	-0.04	9.92	0.44	2.01	0.05

4.2.2 - Parameter tuning of sample average approximation method

In this section, we carry out some computational experiments to tune of the parameters of the sample average approximation algorithm. These parameters include the number of iteration ($|M|$), the number of sample in the lower bound problem ($|N|$), the number of samples in the upper bound problem ($|P|$). To do so, we perform some computational analysis on one of the instances with three weeks of planning horizon and seven specialties. The reason for choosing one of the instances of this combination is that our primary computational results showed that these instances are not too computationally demanding and their medium size let them mimic the computational behavior of other larger and smaller instances. Then, we first ran the model for this instance for different combinations of number of $|N| \in \{5,10,20,30,40,50,60,70\}$ and iterations $|M| \in \{5,10,15,20,25,30,40,50\}$.

We compared the combinations based on three main features: total solution time, gap, and relative standard deviation. In Figure 1, we study the trend of the optimality gap for different values of $|N|$ and $|M|$. This figure shows that for almost all the values of $|N|$, the amount of gap decreases as the number of $|M|$ increases. Although, this decrease is more drastic for larger values of $|N|$. It is worth mentioning that the fluctuations of the gap value considerably fall after the value of $|N|$ crosses 30.

In Figure 2, we observe the behavior of Relative Standard Deviation (RSD) under different circumstances. RSD is a feature that measures the significance of the deviation in the lower bound with respective to its average value and is calculated by $RSD = SD_{LB}/LB$. In this formula, LB and SD_{LB} denote the estimated lower bound and its standard deviation. Figure 2 shows that, after crossing $|N| = 20$, the value of RSD is decreasing gradually as the number of iterations grows. Finally, in Figure 3, we observe that with growth in number of iterations, the solution time of the model increases.

Based on the above analysis, we decide to set $|N| = 30$ and $|M| = 25$ because for this parameter setting SAA leads to stable result with a small gap and RSD, while the solution time is reasonable.

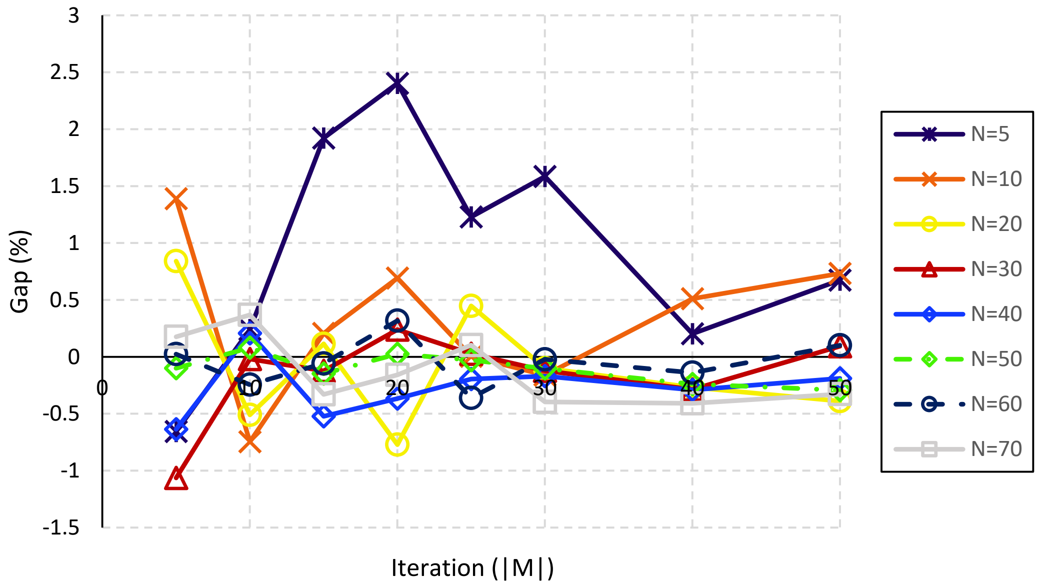

Figure 1 - Optimality gap (%) for different combinations of $|M|$ and $|N|$

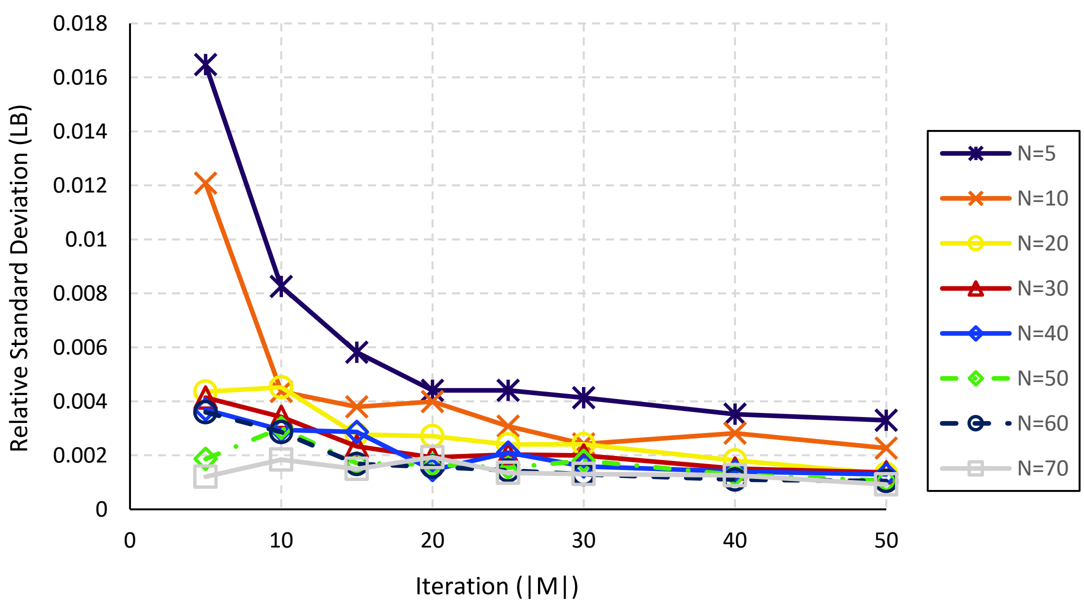

Figure 2 - The RSD of LB for different combinations of $|M|$ and $|N|$

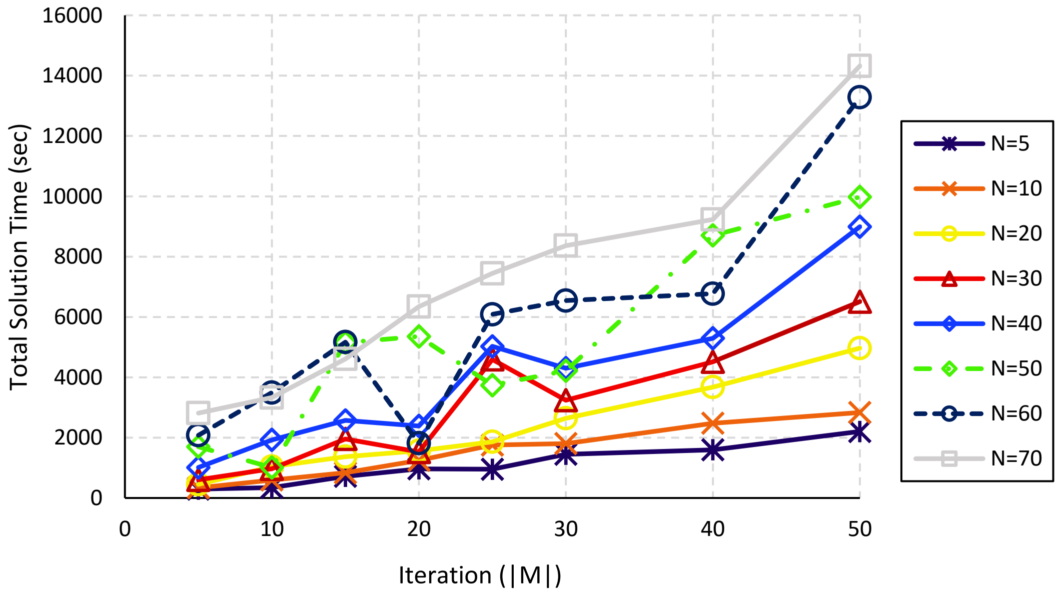

Figure 3 - Solution times (sec) for different combinations of $|M|$ and $|N|$

Next, we have to tune number of sample in the upper bound problem, that is denoted by $|P|$. For obtaining the appropriate value of $|P|$, first we fix $|N| = 30$ and $|M| = 25$ and then run the selected instance for different values of $|P|$. Figure 4 depicts RSD and solution time of the upper bound problem for different values of $|P|$. Based on Figure 4, we choose $|P| = 6000$ as it results in a very low value of RSD with a reasonable solution time.

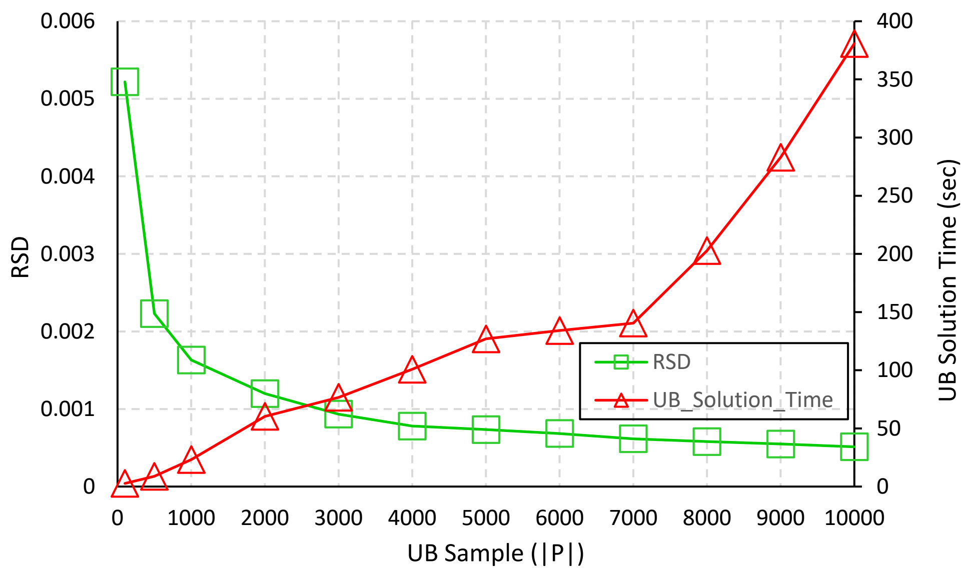

Figure 4 - The RSD and solution time of upper bound problem (sec) for different values of $|P|$

4.2.3 - Performance of SAA algorithm

In this section, we evaluate the efficiency of the proposed stochastic programming model embedded in the SAA framework using the tuned parameters. In Table 3, we have provided the results of instances with up seven specialties and four week of planning horizon.

In this table, each row presents the average of five instances for a fixed number of weeks and specialties. Column "No. of Weeks" and "No. of Spec." show number of weeks in a planning horizon and the number of specialties, respectively. Column "No. of Ite." shows the number of iterations repeated by the SAA algorithm. Column "Time (Sec)" indicates the total solution time of SAA algorithm. Columns "LB Time (%)" and "UB Time (%)" stand for the percentages of total solution time spent to solve the lower bound and upper bound problems, respectively. Columns "LB" and "SD_{LB}" indicate the average and standard deviation of lower bound. Similarly, columns "UB" and "SD_{UB}" represent the average and standard deviation of upper bound problem, respectively. The next five columns show the contribution of different costs components in the obtained objective function. Columns "Overtime cost (%)", "Surge capacity cost (%)", "Waiting cost (%)", "Postpone cost (%)", and "OR cost (%)" represent the operating rooms' overtime cost, the total surge cost in downstream units, the patients' waiting cost, the cost of postponing surgeries, and the fixed cost of opening operating rooms, respectively. Under column "Gap (%)", we have provided the optimality gap calculated by formula $100(UB - LB)/LB$. Finally, “VSS (%)” gives the value of stochastic solution, that is computed by $100(UB_{EVP} - UB)/UB_{EVP}$. Here, UB_{EVP} is the objective value of the solution obtained by solving the Expected Value Problem (EVP) and then evaluated by the scenarios of the stochastic problem. Expected Value Problem (EVP) is the deterministic version of our stochastic programming model where all random parameters are replaced by their corresponding average values from different scenarios. To obtain the amount of UB_{EVP}, we need to run the deterministic EVP for an average scenario. Then, we evaluate the obtained first-stage solution using the same stochastic scenarios. Generally, “VSS” determines how better the stochastic programming model works comparing to its deterministic EVP in terms of the objective value.

Table 3 shows that the average optimality gap of all instances is -0.04. The negativity of this value is because the lower and upper bounds are statistical estimators. However, its small absolute value demonstrate the model is working properly and finds near-optimal solutions. Moreover, in the last row of the Table, we observe that the average VSS is 17.43%, which shows the solution obtained from our proposed stochastic programming model is significantly better than the solution that one would obtain by solving EVP problem. Therefore, we can conclude that considering the uncertainty of random parameters is a critical factor in operating room planning with the possibility of pooling for downstream beds. It is also noteworthy the VSS increase as the number of specialties increases. In the largest instances with four weeks, VSS raises from 26.70% to 81.99% as the number of specialties increases from 4 to 7. The results under columns “LB Time (%)” and “UB Time (%)” also show that the lower bound problem is responsible for major part of the computational time.

Table 3- Computational results of the SAA algorithm.

Data Info.		Sample Average Approximation														
No. of Weeks	No. of Spec.	No. of Ite.	Time (Sec)	LB Time (%)	UB Time (%)	LB	SD_{LB}	UB	SD_{UB}	Gap (%)	VSS (%)	Overtime cost (%)	Surge capacity cost (%)	Waiting cost (%)	Postpone cost (%)	OR cost (%)
2	1	25	554	30.0	70.0	37319	97	37294	33	-0.06	1.71	0.37	57.81	6.38	12.47	22.96
	2	25	852	62.3	37.7	38494	97	38440	34	-0.14	3.71	0.08	57.96	9.00	9.76	23.20
	3	25	34038	98.6	1.4	39959	83	39993	32	0.09	5.16	0.72	52.37	10.80	11.25	24.85
	4	24.8	32498	98.5	1.5	39530	86	39485	31	-0.12	7.51	0.43	51.37	12.84	10.64	24.72
	5	25	46162	99.0	1.0	41826	92	41759	31	-0.16	8.50	0.35	55.11	13.43	8.26	22.84
	6	23.2	44572	99.1	0.9	43320	85	43345	31	0.07	9.90	0.26	52.52	14.53	9.34	23.34
	7	25	17805	98.4	1.6	40956	75	40953	29	0.00	10.02	0.16	51.51	16.19	7.33	24.81
Average		25	25212	84	16	40200	88	40181	31	-0.05	6.64	0.34	54.09	11.88	9.86	23.82
3	1	25	2380	23.7	76.3	61728	153	61735	47	0.01	1.70	0.33	55.09	5.51	18.95	20.12
	2	25	2253	49.8	50.2	67989	146	67872	49	-0.17	3.78	0.05	58.10	6.91	15.91	19.02
	3	21.6	54109	98.1	1.9	68451	127	68487	45	0.06	10.86	0.61	55.05	8.69	15.11	20.54
	4	23.6	61443	97.6	2.4	69761	129	69804	46	0.06	11.30	0.51	53.77	8.32	16.55	20.85
	5	25	19033	95.1	4.9	68105	115	68088	43	-0.03	11.72	0.25	54.90	9.75	13.66	21.44
	6	25	37081	97.4	2.6	71893	124	71909	44	0.02	19.48	0.25	52.03	10.94	15.85	20.92
	7	25	17567	96.6	3.4	69732	116	69601	42	-0.19	25.74	0.18	53.54	12.49	12.50	21.29
Average		24	27695	80	20	68237	130	68214	45	-0.03	12.08	0.31	54.64	8.94	15.50	20.60
4	1	25	2625	20.4	79.6	88940	155	89034	57	0.11	1.58	0.28	50.51	4.18	27.29	17.74
	2	25	2567	43.0	57.0	98904	187	98854	62	-0.05	5.50	0.05	55.91	4.69	22.15	17.19
	3	25	36284	93.6	6.4	93645	138	93534	55	-0.12	11.47	0.51	53.61	7.27	18.44	20.16
	4	25	45490	96.7	3.3	95393	166	95352	55	-0.04	26.70	0.43	51.07	9.52	18.88	20.10
	5	25	17512	91.9	8.1	99644	170	99543	55	-0.10	39.98	0.20	53.20	7.96	19.74	18.90
	6	24.4	60674	97.0	3.0	103410	148	103435	54	0.03	67.66	0.21	52.44	10.82	17.40	19.13
	7	25	16103	89.6	10.4	98570	149	98549	53	-0.02	81.99	0.13	53.62	11.45	14.46	20.35
Average		25	25893	76	24	96929	159	96900	56	-0.03	33.56	0.26	52.91	7.98	19.77	19.08
Total Average		25	26267	80	20	68456	126	68432	44	-0.04	17.43	0.30	53.88	9.60	15.05	21.17

Beside the above analysis, we depicted the insightful Figures 5, 6, 7, and 8 to illustrate how the specialties use the shared beds and surging capacities in ICU and wards. These figures help us better understand the dynamics of resource allocation in downstream by our proposed model to various specialties.

We have drawn these figures using the average results on the instances with four weeks of planning horizon and seven specialties. Figure 5 shows the surge capacity in ICU used by different specialties. The sinusoidal trend of required surge ICU beds is a very fascinating event; As we can see the cumulative used surge capacity increases during the workdays of the week and then decrease during the weekends. This makes sense because we supposed that our operating room planning problem is related to elective surgeries that are performed during the workdays. It is also very interesting to see that the model forces the specialties to use the surge capacity around the same level even though there is no explicit constraint in this regard in this proposed model.

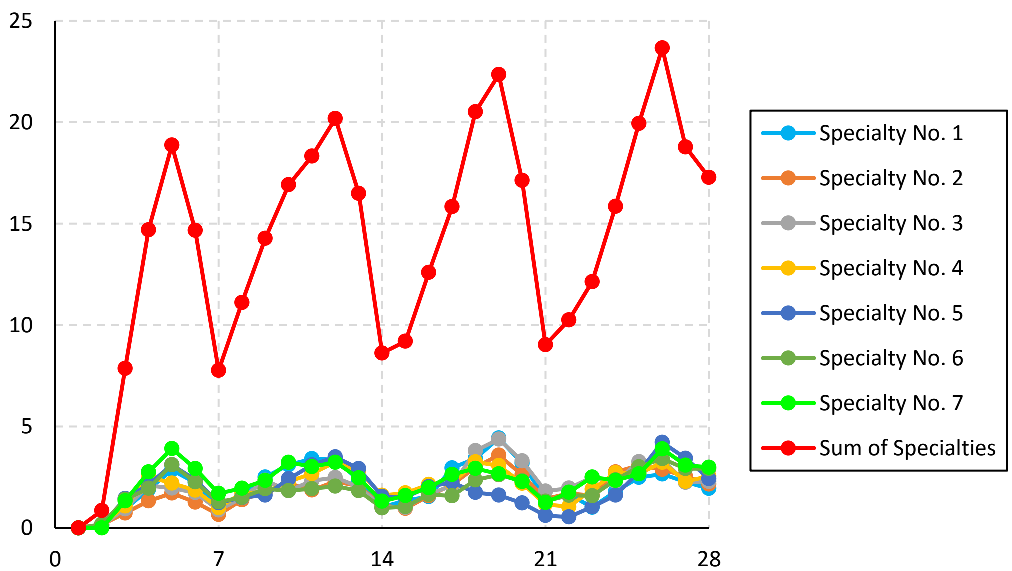

Figure 5 - Surge capacity used in downstream 1 in the planning horizon

Figure 6 depicts the number of shared ICU beds that specialties use during the planning horizon. As we can see in this figure, all shared ICU bed become occupied very quickly at the beginning of the planning horizon after three days. It is also very interesting the all specialties use fairly the same level of shared ICU beds in the planning horizon.

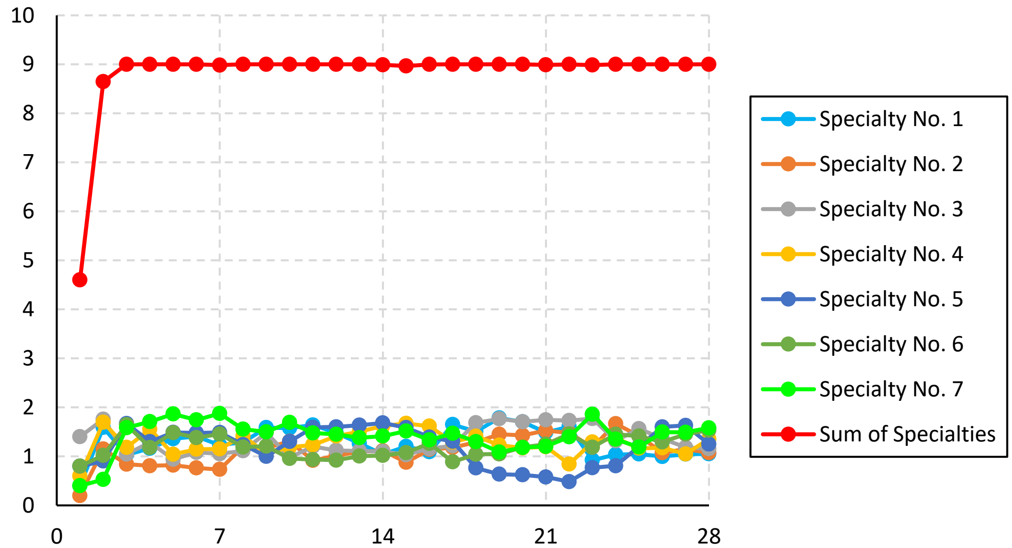

Figure 6 - The number of shared beds in downstream 1 occupied by patients of different specialties

Figure 7 shows the surge capacity used by different specialties in the wards area during the planning horizon. Compared to Figure 5, the surge capacity in wards increase more slowly and also has less fluctuation. This is because we have more beds in wards and therefore the uncertainty in patients' length of stay in this downstream can be better mitigated using the available resources.

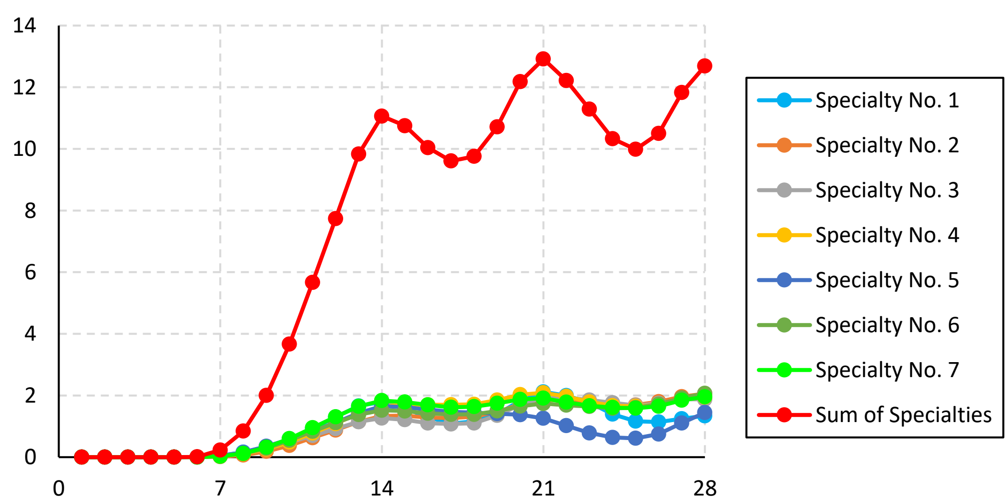

Figure 7 - Surge capacity used in downstream 2 in the planning horizon

Figure 8 shows the number of shared beds in wards used by various specialties. We can observe that the cumulative usage increases fairly slowly at the beginning until it reaches the maximum available capacity. Again, we observe that all specialties are fairly using the shared beds during the planning horizon.

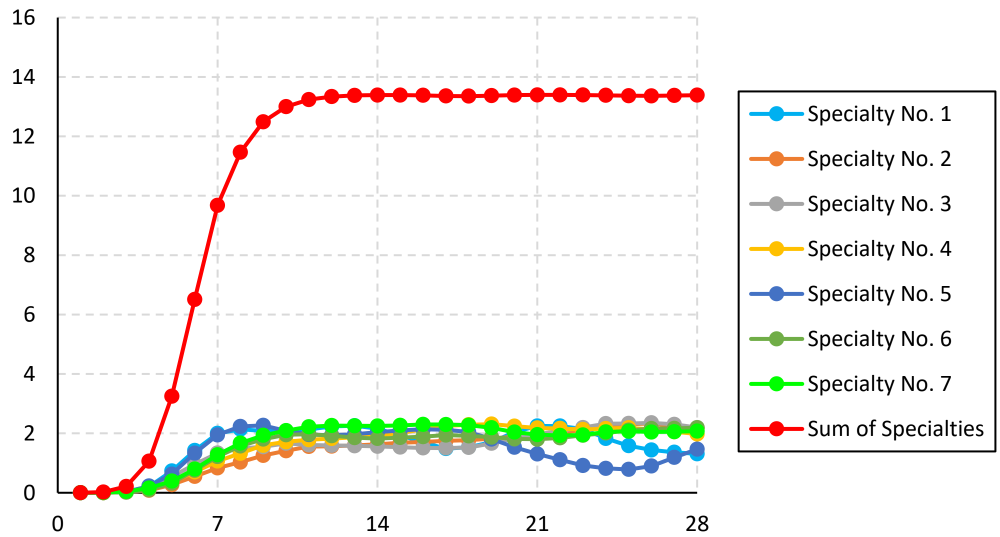

Figure 8 - The number of shared beds in downstream 2 occupied by patients of different specialties

4.2.4 - Sensitivity analysis

In this section, we perform some sensitivity analysis on the costs and uncertain parameters of the proposed model to observe the behavior of the obtained results and provide managerial insights. In Table 4, we have summarized the results obtained by performing sensitivity analysis on costs parameter independently. In this table, Column "Parameter" shows the name of the cost parameter that we have analyzed. Under Column "Value", we have different settings for the value of the cost parameters. In our sensitivity analysis, we have modified the values different cost parameters by $c_{id}^{Waiting} \coloneqq \alpha^{Waiting} c_{id}^{Waiting}$, $c^{OR} = \alpha^{OR} c^{OR}$, $c_h^{bed} \coloneqq \alpha^{Surge} c_h^{bed}$, $c_i^{Postpone} = \alpha^{Psotpone} c_i^{Postpone}$, and $c^{Overtime} = \alpha^{Overtime} c^{Overtime}$. The next five columns show the contribution of each cost component in the total cost. "Waiting Time" indicator shows the total number of days that all the patients waited before they were operated. "No. of Postpone" indicator specifies the number of patients who have been postponed to the next planning horizon. Under "No. of ORs", we have the total number of operating rooms that the model has decided to open in the planning horizon.

Table 4- Sensitivity analysis on cost coefficients.

Parameter	Value	Cost components					Indicators			
		Waiting cost (%)	Postpone cost (%)	OR cost (%)	Overtime cost (%)	Surge capacity cost (%)	Waiting time (Day)	No. of postpone	No. of ORs	Overtime (min)
$\alpha^{Waiting}$	1	12.49	12.50	21.29	0.18	53.54	79	12	66.8	10
	3	21.41	18.38	17.86	0.11	42.23	44	18	67.0	8
	5	23.75	26.58	15.68	0.09	33.90	31	25	66.2	7
	7	22.10	35.33	14.01	0.07	28.50	23	31	63.8	6
	10	16.98	47.48	12.74	0.06	22.74	11	40	62.2	5
α^{OR}	1	12.49	12.50	21.29	0.18	53.54	79	12	66.8	10
	3	11.24	12.21	40.59	0.14	35.82	90	16	59.2	11
	5	10.45	12.88	49.69	0.11	26.87	97	20	54.8	11
	7	10.11	14.92	54.56	0.11	20.29	105	25	51.2	13
	10	9.30	16.63	58.89	0.11	15.07	116	32	47.4	16
α^{Surge}	1	12.49	12.50	21.29	0.18	53.54	79	12	66.8	10
	3	12.08	38.40	11.70	0.05	37.77	105	45	62.4	5
	5	10.82	53.43	9.31	0.03	26.42	97	62	59.0	3
	7	10.64	60.83	8.11	0.02	20.40	93	72	55.4	2
	10	9.99	67.27	7.66	0.01	15.07	86	80	54.8	1
$\alpha^{Psotpone}$	0.5	9.45	29.03	22.73	0.13	38.66	44	37	62.2	6
	0.75	12.23	16.93	21.96	0.15	48.72	71	19	65.8	8
	1	12.49	12.50	21.29	0.18	53.54	79	12	66.8	10
	1.25	13.17	5.28	21.51	0.17	59.87	93	4	68.8	10
	1.5	14.24	1.99	21.95	0.15	61.66	103	1	69.8	7
$\alpha^{Overtime}$	1	12.49	12.50	21.29	0.18	53.54	79	12	66.8	10
	3	11.82	12.70	21.51	0.43	53.54	75	12	67.6	8
	5	12.20	13.31	21.28	0.72	52.49	78	12	67.0	8
	7	12.36	13.07	21.36	0.70	52.52	77	12	67.4	6
	10	12.69	12.80	21.20	1.05	52.26	82	12	67.2	6

The first parameter that we analyze is $\alpha^{Waiting}$. Figure 9-a illustrates the contribution of different cost components in the total cost for different values of $\alpha^{Waiting} \in \{1,3,5,7,10\}$. The interesting point in Figure 9-1 is that the waiting cost has a concave shape. This is because by the initial increase in the value of $\alpha^{Waiting}$ the total waiting cost increases as expected. However, at the same time, the model postpones a higher number of patients to avoid excessive waiting cost. Therefore, the increase in the total postpone cost results in decrease of the contribution of the total waiting cost in the total cost. Also, with increase of $\alpha^{Waiting}$, the OR cost and surge capacity cost decrease due to the growth in the number of postponed surgeries. Perhaps, tracking the amount of overtime cost in all the Figures seems a little irrelevant due to its insignificancy, but knowing the trend of its change could be useful. In this case, rising the value of $\alpha^{Waiting}$ reduces the portion of overtime cost gradually.

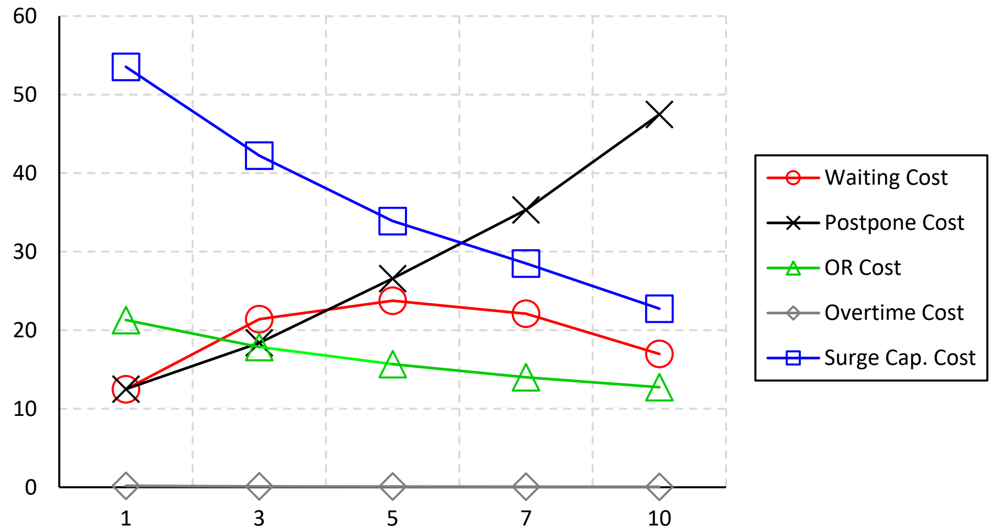

Figure 9-a - The distribution of cost components for different values of $\alpha^{\mathrm{Waiting}}$

In Figure 9-b, we depict trends for numbers of important indicators in our model for different values of $\alpha^{Waiting}$. We can observe that increase in $\alpha^{Waiting}$ reduces the total number of days that patients. This event occurs due to the higher number of postponed surgeries. Also, the number of operating rooms and overtime decrease a little in this process.

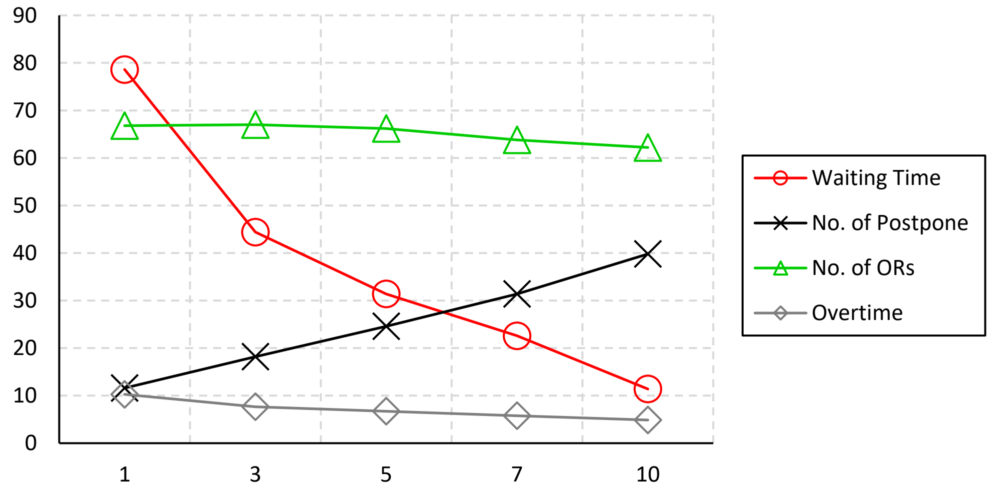

Figure 9-b - The values of cost indicators for different values of $\alpha^{Waiting}$

The next parameter we analyze is α^{OR} that is used to analyze the sensitivity of the model to the fixed opening cost of operating rooms. Figure 10-a and Figure 10-b shows that increase in α^{OR} makes the model postpone more patients while the total opening cost of ORs increases. With fewer patients in the system, we have fewer surge capacity used in downstream. In Figure 10-b, we observe that the number of waiting days increase strictly. However, Figure 10-a shows that the contribution of the waiting cost in the total cost drops as α^{OR} increases. This is justified by consider the fact that the total OR cost increases excessively with increase in α^{OR}. Also, Figure 10-b shows that the amount of overtime raises because of reduction in the number of operating rooms.

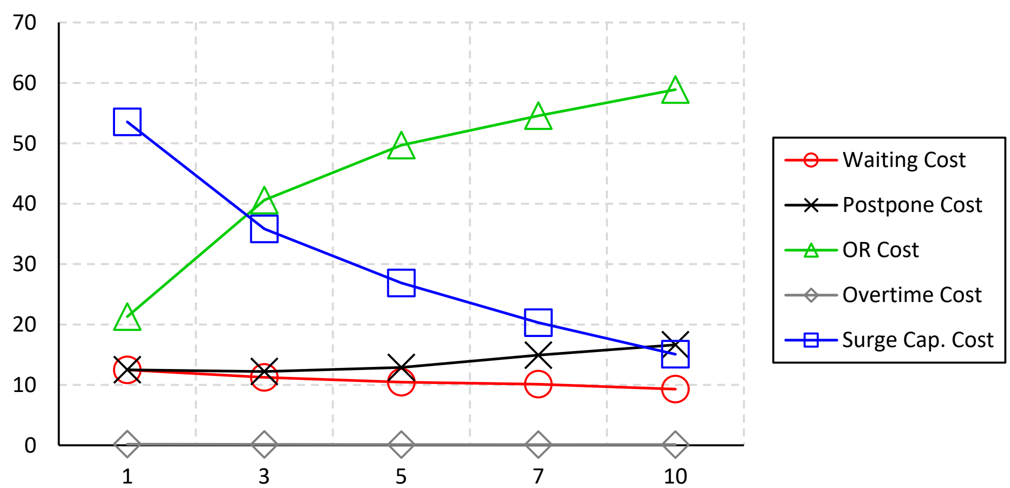

Figure 10-a - The distribution of cost components for different values of α^{OR}

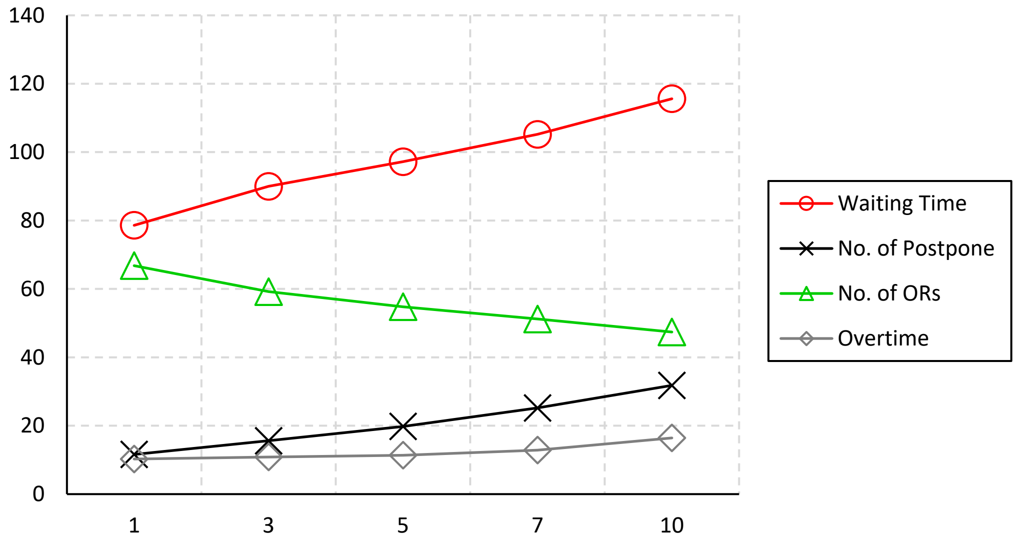

Figure 10-b - The values of cost indicators for different values of α^{OR}

The next parameter to analyze is α^{Surge} that stands for the relative importance of the cost of surge capacity in downstream compared to other cost parameters. Figure 11-a and 11-b demonstrate that increasing α^{Surge} results in a significant raise in the number of postponed patients and its corresponding cost. As depicted in Figure 11-b, more postponed surgeries result in opening few operating rooms and therefore we have less total operating room cost in Figure 11-a. With fewer scheduled surgeries, one may expect fewer number of waiting days. However, the recent indicator does not show a strict increase or decrease in Figure 11-b. This is because while the number of postponed surgeries increase, the model opens fewer operating rooms, and therefore the waiting time of patients increases from $\alpha^{Surge} = 1$ to $\alpha^{Surge} = 3$ due to fewer available operating room.

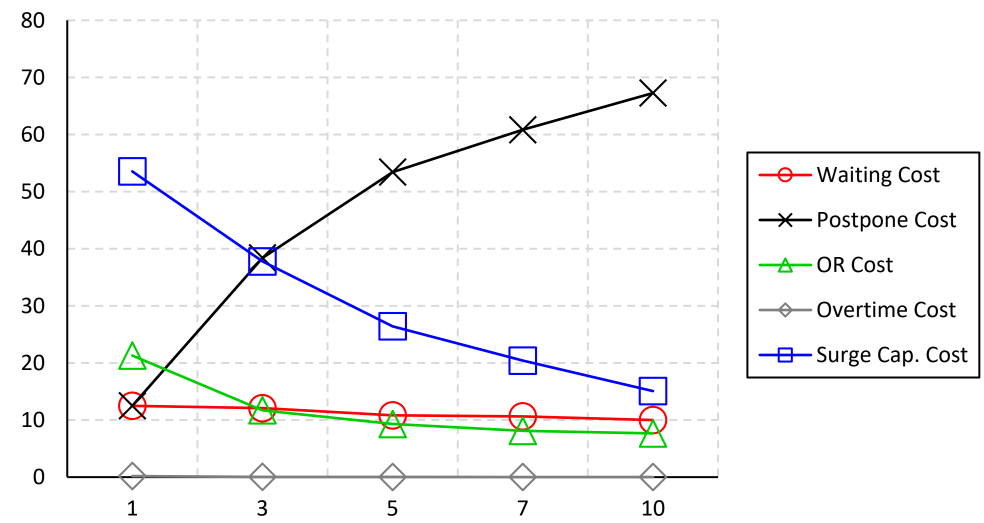

Figure 11-a - The distribution of cost components for different values of α^{Surge}

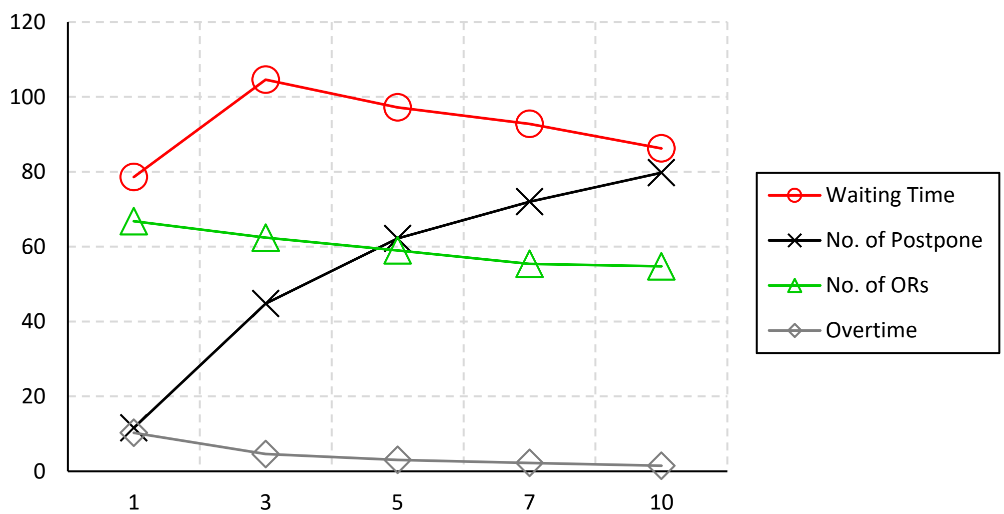

Figure 11-b - The values of cost indicators for different values of α^{Surge}

We have also analyzed the sensitivity of the model to $\alpha^{Postpone}$. As Table 4 shows, the values of $\alpha^{Postpone}$ are set to $\{0.5,0.75,1,1.25,1.5\}$ and not $\{1,3,5,7,10\}$ as in other parameter analysis. This is because we realized the values of $\alpha^{Postpone} = \{1,3,5,7,10\}$ does not provide a meaningful analysis since the model does not postpones any surgery for $\alpha^{Postpone} = 3$. This trend obviously continued for higher values of $\alpha^{Postpone}$. Therefore, we used $\alpha^{Postpone} \in \{0.5,0.75,1,1.25,1.5\}$.

In Figure 12-a, increasing $\alpha^{Psotpone}$ makes the model to postpone fewer surgeries as expected. This reduction in number of postpones results in more scheduled patients and therefore more waiting cost and surge capacity cost. Also, Figure 12-b shows that the number of opened operating rooms gradually increases. However, as in Figure 12-a, the contribution of the OR cost in the total cost is almost constant due to the significant increase in the surge capacity cost.

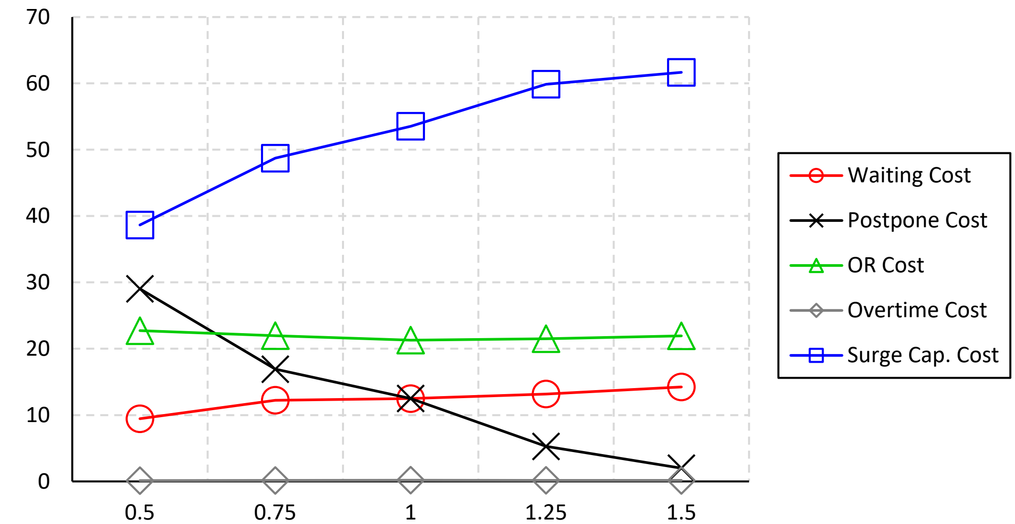

Figure 12-a - The distribution of cost components for different values of $\alpha^{Postpone}$

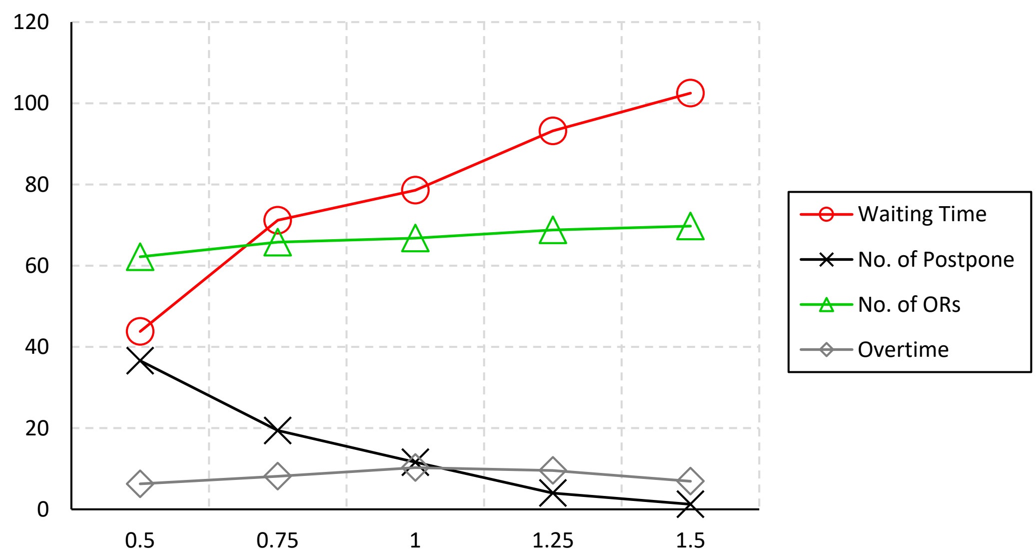

Figure 12-b - The values of cost indicators for different values of $\alpha^{Postpone}$

The last parameter in our analysis is $\alpha^{Overtime}$. In Figure 13-a and Figure 13-b, there are not significant changes in cost components and the indicators for different values of $\alpha^{Overtime}$. The only noteworthy point is that for higher volume of $\alpha^{Overtime}$ we have less amount of overtime.

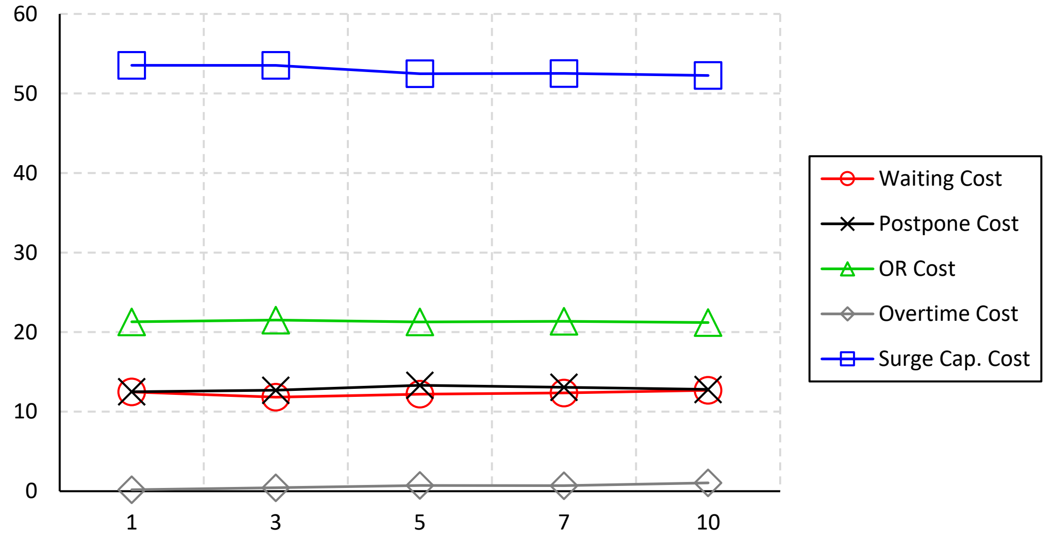

Figure 13-a - The distribution of cost components for different values of $\alpha^{Overtime}$

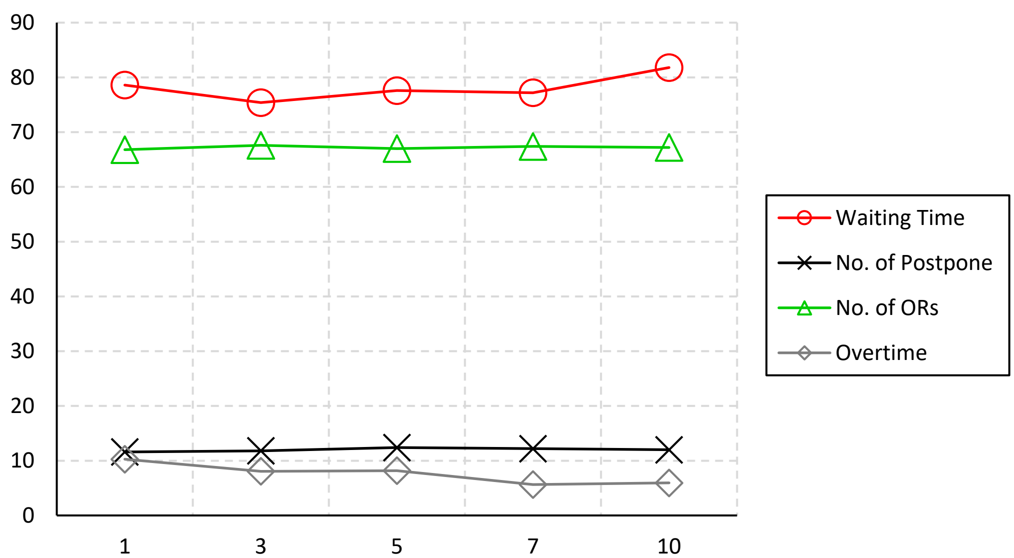

Figure 13-b - The values of cost indicators for different values of $\alpha^{Overtime}$

After scrutinizing in analysis of different cost parameters, it is now the time to perform another type of sensitivity analysis. In this part, we observe the behavior of the model in terms of cost distributions and their actual numbers when we modify the uncertain parameters we considered in our model.

Next, we intend to analyze the sensitivity of the model to the patients' length of stay and their surgical durations. To do so, in the revised stochastic programming model, we set $t_{i\omega} \coloneqq \alpha^{Duration} t_{i\omega}$ and $l_{ih\omega} \coloneqq \alpha^{LOS} l_{ih\omega}$. Here, $\alpha^{Duration}$ and α^{LOS} are the control parameters that we use for sensitivity analysis. Also, as explained in Chapter 2, $t_{i\omega}$ and $l_{ih\omega}$ denote the patients' surgical durations and length of stay, respectively. In our sensitivity analysis, we set $\alpha^{Duration}, \alpha^{LOS} \in \{0.5, 0.75, 1, 1.25, 1.5\}$. We have presented the results of both experiments in Table 5.

Table 5- Sensitivity analysis on the coefficients of uncertain parameters.

Parameter	Value	Waiting cost (%)	Postpone cost (%)	OR cost (%)	Overtime cost (%)	Surge capacity cost (%)	Waiting time (day)	No. of postpone	No. of ORs	Overtime (min)
α^{DOS}	0.5	12.35	13.40	21.41	0.00	52.85	78	12	67.0	0
	0.75	12.81	11.64	21.30	0.00	54.25	82	11	66.8	0
	1	12.49	12.50	21.29	0.18	53.54	79	12	66.8	10
	1.25	14.50	14.08	21.05	3.27	47.09	96	14	70.8	158
	1.5	17.78	18.44	20.95	4.62	38.21	121	20	77.6	307
α^{LOS}	0.5	31.47	0.52	54.40	0.43	13.18	92	0	70.6	10
	0.75	20.82	3.55	36.26	0.31	39.06	85	2	69.0	11
	1	12.49	12.50	21.29	0.18	53.54	79	12	66.8	10
	1.25	10.94	14.93	18.56	0.13	55.44	77	16	66.4	8
	1.5	10.23	15.55	15.35	0.11	58.76	87	19	65.4	8

In Figure 14-a and Figure 14-b, we observe the effects of change in the surgical durations. As expected, the model gradually increases the number of postpones surgeries and therefore the surge capacity cost decreases. Also, the overtime cost increases significantly due to the increase in surgical times. Moreover, due the limited available times in operating rooms, the model schedules the patients less than before and therefore the waiting time and its cost increase.

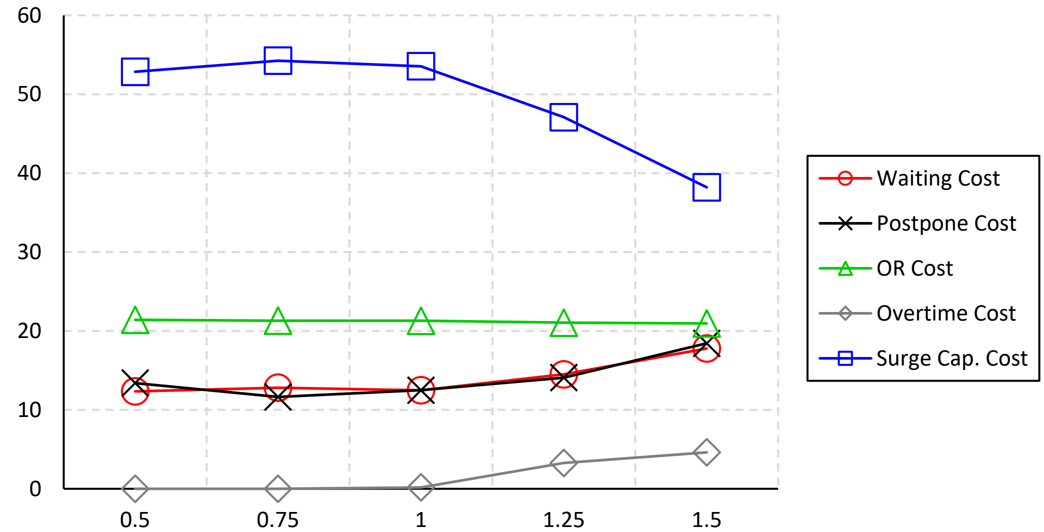

Figure 14-a - The distribution of cost components for different values of α^{DOS}

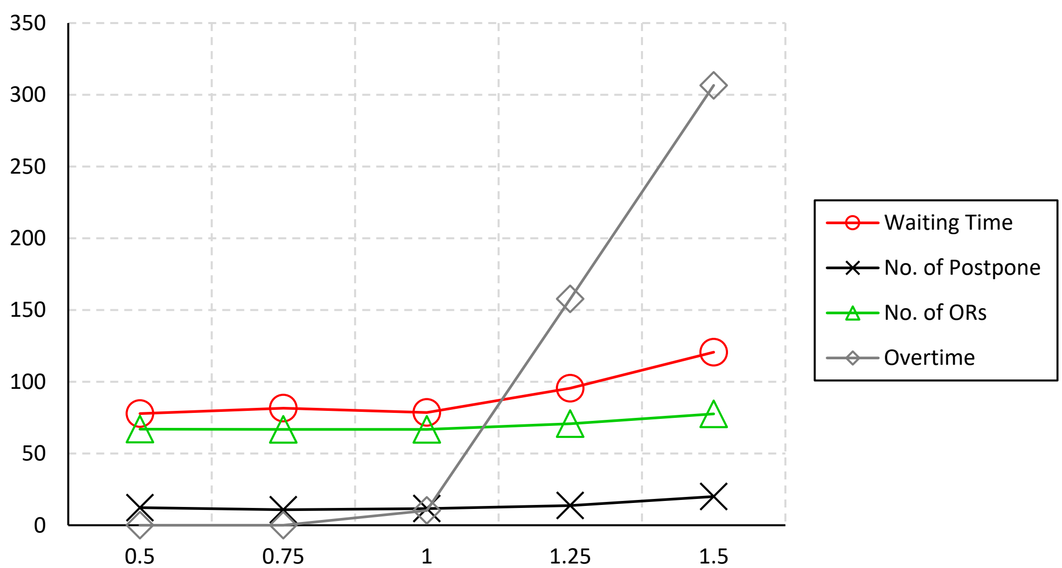

Figure 14-b - The values of cost indicators for different values of α^{DOS}

Figure 15-a and 15-b visualize the behavior of the system as the patients' length of stay increase. In a glance, with longer hospitalization time, the surge capacity cost increases significantly as expected. Also, the model tends to postpone more patients to compromise. This action makes the system less crowded which simultaneously decreases the number of waiting days. An exception in this regard is the case of $\alpha^{LOS} = 1.5$ in Figure 15-b for which number of waiting days has

increased. This is because there is a trade-off between the number of opened operating rooms and the number of waiting days; Moving from $\alpha^{LOS} = 1.25$ to $\alpha^{LOS} = 1.5$ has been caused fewer number of opened operating rooms and therefore the patients' waiting time has increases.

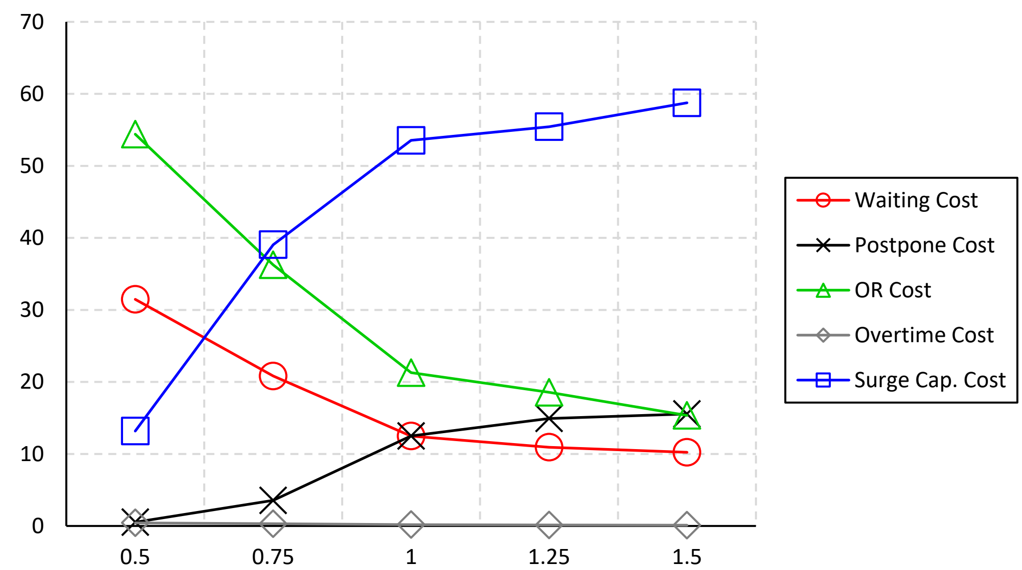

Figure 15-a - The distribution of cost components for different values of α^{LOS}

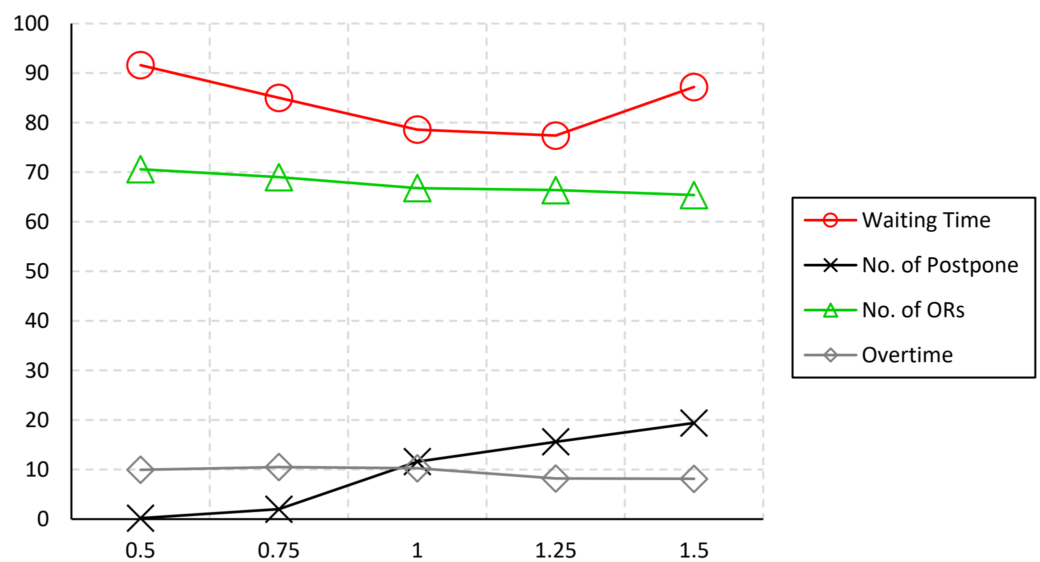

Figure 15-b - The values of cost indicators for different values of α^{LOS}

Chapter 5

Conclusion and future work

In this thesis, we studied an operating room planning problem with the goal of evaluating the effect of pooling the downstream beds among specialties. In our problem, a limited number of beds are available for sharing in ICU and wards. The main sources of uncertainty is the randomness of surgical times and length of stay. We proposed a two-stage stochastic integer programming model and embedded it in a sample average approximation algorithm. In the first stage of our model, the decision maker decides on the allocation of the non-shared beds among specialties, and the allocation of surgeries to operating rooms. Then, in the second stage, after the realization of uncertain parameters, he/she must decide on the allocation shared ICU and wards beds to specialties during the planning horizon, how many surge capacity beds are required for each specialty in downstream, and also compute the overtime incurred in operating rooms. Our model intends to minimize the total cost including patients' waiting cost and postpone cost, fixed cost of opening operating rooms, the cost of surge capacity in downstream, and overtime cost.

We performed extensive computational results with three goals: 1) evaluating the effect of different pooling policies for beds in downstream on the performance of the surgery planning, 2) evaluating the efficiency of the proposed sample average approximation algorithm, and 3) evaluating the behavior and the stability of the proposed model for a large set of instances with various cost and uncertainty parameters.

In the first set of our computational experiments, we compared the results of three pooling policies, namely, No Sharing, Midlevel Sharing, and Full Sharing. In these three strategies, we allow the sharing of 0%, 50%, and 100% of the available beds. Our computational results indicated that the two latter policies lead to 11.29% and 12.38% improvement compared No Sharing policy on average, respectively. Also, in some large instances, these improvements were respectively as high as 19.53% and 16.97%.

In the second part of our computational experiments, we tuned the parameters of our SAA algorithm by depicting the values of optimality gaps, RSD, and solution time for different values of the number of scenarios in lower and upper bound problems and also number of iterations in SAA.

In the third set of instances, we focused on Midlevel policy and evaluated the efficiency of the proposed SAA algorithm with the tuned parameters. Our results indicated that the proposed approach finds near-optimal solutions with an absolute optimality gap of 0.04%. Also, the average value of value of stochastic solution (VSS) is 17.43%. This value shows that the proposed stochastic programming approach outperforms its corresponding deterministic model by 17.43%. Moreover, VSS varies between 26.70% and 81.99% in instances with four weeks of planning horizon and 4 to 7 specialties. We also provided some insightful figure on the dynamic of shared-beds surge-capacity allocation to specialties. It was very interesting to observe the sinusoidal trend of ICU bed usage and also the fair allocation of shared beds to specialties.

In the last part of computational experiments, we performed a sensitivity analysis on various cost and uncertain parameters. We justified the behavior of cost components and system's indicator in different settings.

In this research, we focused on downstream pooling in an operating room planning problem. Future research can focus on an integrated surgery planning and scheduling with open scheduling strategies where surgeons are allowed to perform surgeries in multiple operating rooms. Moreover, the concept of pooling can be studied for upstream resources as well as post anesthesia care unit. It is also interesting to study our problem in a robust context and focus on developing a two-stage robust optimization model.

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