The Effect of Viscosity on the Rotating Waves and Polygonal Patterns within a Hollow Vortex Core

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Abstract

The question of how viscosity influences the development of instabilities within a rotating shallow layer of liquid, which gives rise to polygonal patterns, has been investigated experimentally. A phase diagram of the existence regions of these polygonal patterns is constructed in (Fr, Ta) plane, where Fr is the Froude number and Ta is the Taylor number. The results show that the effect of the viscosity on the domain of existence of the patterns depends on the initial fluid height above the disc. The results also show that the variation of viscosity does not affect the locking ratio between the rotational frequencies of the pattern to the disc; the two frequencies remain locked at approximately 1/3.

Keywords:
Swirling flow; polygonal pattern; vortex dynamics; flow instabilities

1. Introduction

When water is placed in an open stationary cylindrical container and set in rotation by a rotating disc located at the bottom of the container, gener-

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ates coherent structures. These resemble to a large extent to the ones often observed in nature. This type of flows are used as a laboratory model to study complex geophysical and astrophysical phenomena [1, 2, 3]. In general, the dynamics and stability of such fluid motion involve a solid body rotation and a shear layer flow. The shear layer flow occurs in the outer region, because of the cylindrical confining walls, while the solid body rotation evolves in the inner region.

When the layer of water, above the disc, is shallow and the disc's speed is relatively high, a circular hollow-vortex core is formed. This devoid of liquid region can undergo series of spontaneous symmetry-breaking, which have the form of rotating polygons. The existence of these shapes, reported first by Vatistas [4], became recently the subject of growing interests. Indeed, several experimental studies were conducted to understand the mechanism behind their formation and dynamics [7, 6, 5, 10, 11]. In addition, several theoretical studies have also been conducted recently with the aim to explain the mechanism behind the rotating polygonal patterns [13, 16, 8, 9]. In these studies the stability of the rotating hollow-core vortex was investigated. Assuming the base flow as potential, Tophøj *et al.* [13] identified resonance between gravity and centrifugal waves to be the cause of the polygonal patterns formation. The model by Tophøj *et al.* was found quite appropriate when the disc's speed is high. However, when the disc's speed is moderate, the central region of the rotating flow is nearly in a solid-body rotation. In this situation, the suitable swirling flow model might be of the Rankine's type, which combines a solid body rotation of the inner region with the potential flow of the outer region [8]. Using Rankine's vortex model for the base rotating flow, a new instability mechanism was proposed [9]. This mechanism involves the interaction between gravity and "Kelvin-centrifugal" waves. The rotating wave patterns were also described as traveling cnoidal waves, solutions of a Kortewege-de Vries equation [16].

The theoretical models mentioned above assumed that the fluid is inviscid; which is attractive because it allows for significant simplifications to the problem. However, the experiments indicated that the fluid viscosity could affect the phenomenon of the polygonal patterns formation and its role is intriguing. Recently, a simple experiment was conducted with liquid Nitrogen in a hot kitchen pot at temperature of 20^0C [13]. In this case, the Leidenfrost condition could be reached and a thin boiling film was formed around the solid walls. The Nitrogen liquid was brought to rotation by stirring it rapidly with a spoon. A cascade of polygons starting from 6-*gon* to 2-*gon* were observed within the hollow-core during the spin down process until the liquid Nitrogen reached its quiescent state filling the entire bottom. In the experiment the thin boiling film reduced the friction between the walls and the rotating liquid Nitrogen. Other experiments, conducted with relatively high viscosity fluids showed that viscosity could affect significantly the shape of the rotating patterns [15]. In these experiments, patterns up to 11-*gon* were observed and found able to travel in opposite direction to the disc rotation. Moreover, higher N-*gon* were observed at lower disc's speed, while the lower N-*gon* were observed at higher disc's speed. The phenomenon was also found to exhibit strong hysteresis. In fact, it was discovered that completely different patterns formed during the disc spin-up and spin-down procedure. Experiments with fluids of moderate viscosities are very few. Jansson *et al.* [14] conducted experiments with ethylene glycol (its viscosity is approximately seven times than that of water); they found that viscosity reduces the number of observable N-*gon*. They also revealed that viscosity appears to have weak influence on the rotating frequency of the polygonal pattern.

In the present paper, we reexamine the role of the working fluid viscosity in the formation and the dynamics of the polygonal patterns. We investigate the role of moderate values of viscosity on the existence of polygonal pattern in Taylor and Froude numbers space. We also examine the influence of the viscosity on the frequency locking between the wave speed, at which the polygonal patterns propagate around the hollow-core vortex, and rotational frequency of the driving disc.

2. Experimental Details and Image Processing

The experiments were conducted in a 284-mm diameter stationary cylindrical container with a rotating 280-mm diameter disc near its bottom. The shape of the vortex core was imaged from above as shown in Fig. 1.

The disc's speed was controlled using a controller and incremented slowly. Sufficient time was given to the fluid flow to stabilize between increments. The disc's rotational speed ranges from 75 to 267 rpms. Experiments with tap water and aqueous glycerol mixtures, as the working fluids, were conducted with three different initial liquid heights of 20, 30 and 40 mm above the rotating disc. Eight different aqueous glycerol mixtures were used in the experiments; the viscosities of these mixtures are 1, 2, 4, 6, 8, 11, 15 and 22 times the water's dynamic viscosity, μ, (μ=1.002 $\times 10^{-3}$ Pa.s at 20^0). The swirling flow and its instabilities were imaged from above at 30 frames per

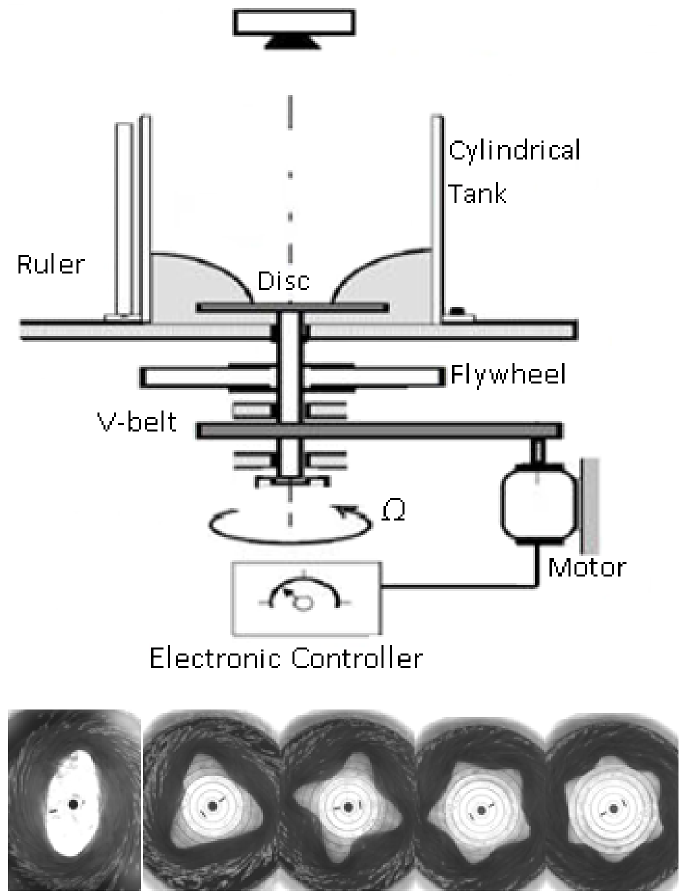

Figure 1: Schematic of the experimental setup and typical polygonal patterns following the symmetry-breaking of the circular hollow-core vortex

second, using a CMOS high-speed pco.1200hs camera. The typical polygonal patterns observed with water are shown in Fig.1.

The rotating frequency of the pattern was obtained using Fast Fourier Transform (FFT) of the time series of the radial displacement for a given point on the polygonal pattern contour, defined by its radius and its angle in polar coordinates with origin at the center of the disc; see Ait Abderrahmane *et al.* [12, 17] for further details. The patterns contours were obtained using image processing algorithm which consists of a sequence of classical image processing operations namely segmentation, noise filtration and edge detection summarized below in Fig.2.

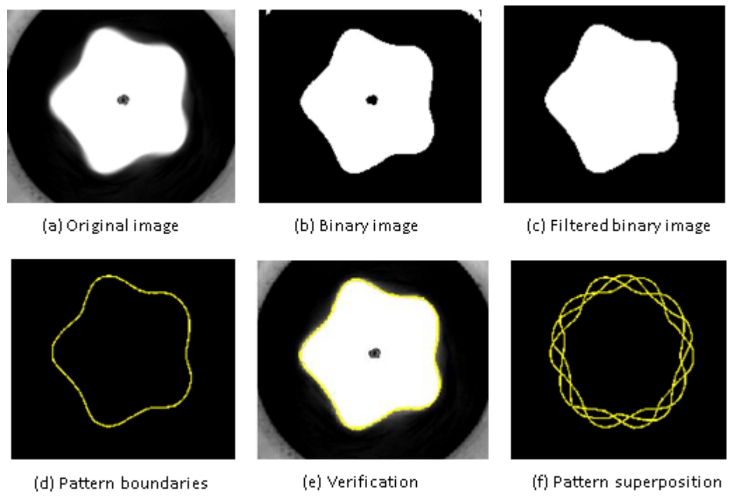

Figure 2: Summary of the image processing technique

3. Results and Analysis

3.1. Phase diagram

The experiments indicate that the pattern phenomenon involves the pattern wavenumber, N, the pattern rotational speed, ω, the fluid kinematic viscosity, ν, the initial water height above the disc, h, the disc's rotational speed, Ω, and the disc's and tank radius, R and R_t. The two latter parameters are constant in our experiment. Based on the Buckingham π theorem, the variables of the problem can be grouped in three dimensionless numbers, namely Taylor ($Ta = \frac{h^4\Omega^2}{\nu^2}$), Froude ($Fr = \frac{R\Omega}{\sqrt{gh}}$) and aspect ratio (h/R). g stands for the gravity. The Taylor number characterizes the importance of centrifugal force or inertial force due to the rotation of a fluid about a vertical axis, relative to viscous forces. The Froude number describes the significance of the centrifugal force relative to gravitational force. The aspect ratio characterizes the shallow water conditions. The goal of our experiments is to investigate the role of a moderate fluid viscosity on the formation and the dynamics of the patterns. Hence, we varied the working fluid viscosity, the disc's speed and the initial fluid height above the disc. The two latter parameters were varied in the ranges where all polygonal patterns were observed. In contrast, with others experiments [14, 10], the aspect ratio in our experiments is limited to three values only (0.14, 0.21, 0.28). These values ensure the shallow water condition and the observation of all possible polygonal patterns (up to hexagon).

A parametric study of the effects of the viscosity on patterns formation and their stability has been carried out. Fig. 3 shows the phase diagram in (Fr, Ta) plane, when the initial fluid height is h= 20 mm. With water we observed triangular, square, pentagonal and hexagonal patterns. Increasing the viscosity of the water, by adding gradually a controlled amount of glycerol, resulted on the gradual shrinking of the interval of the Froude number within which the patterns occur. Increasing the fluid viscosity triggers the transitions between unstable modes at low Froude numbers or low disc's speeds.

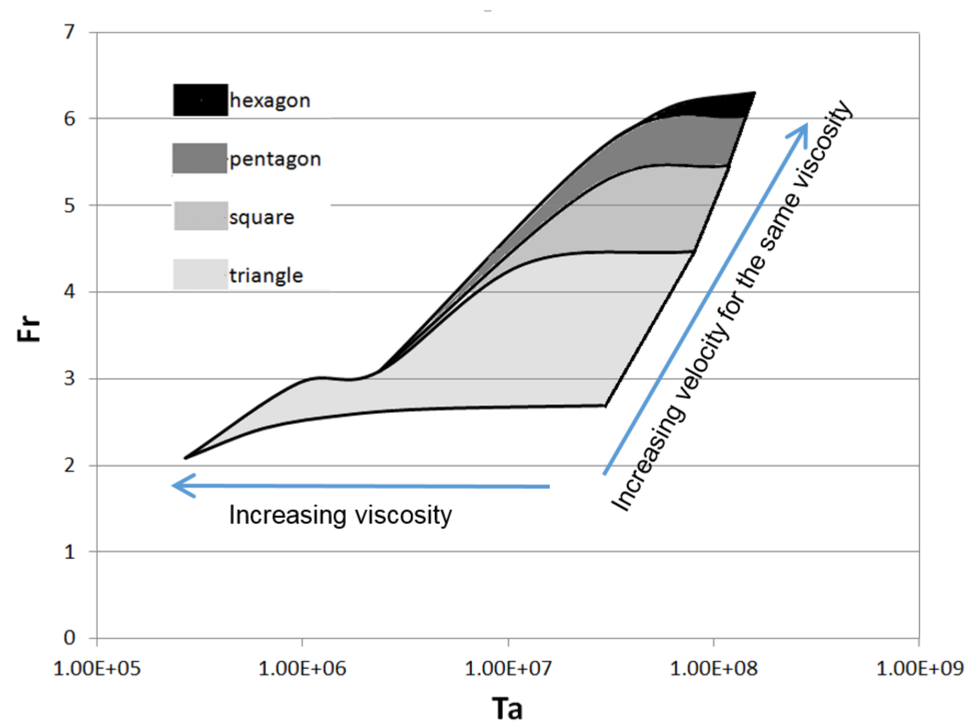

Figure 3: Phase diagram spanned by the Taylor number, Ta, and the Froude number, Fr. The initial fluid height is 20 mm.

At fluid height h= 30 mm, we observed oval, triangular, square and pentagonal patterns with water as a working fluid; see Fig.4. Similarly to the previous experiments, increasing the viscosity of the fluid resulted on the shrinking of the region of existence of the pentagonal pattern until it disappeared at relatively low viscosity. The extent of the Froude interval, within

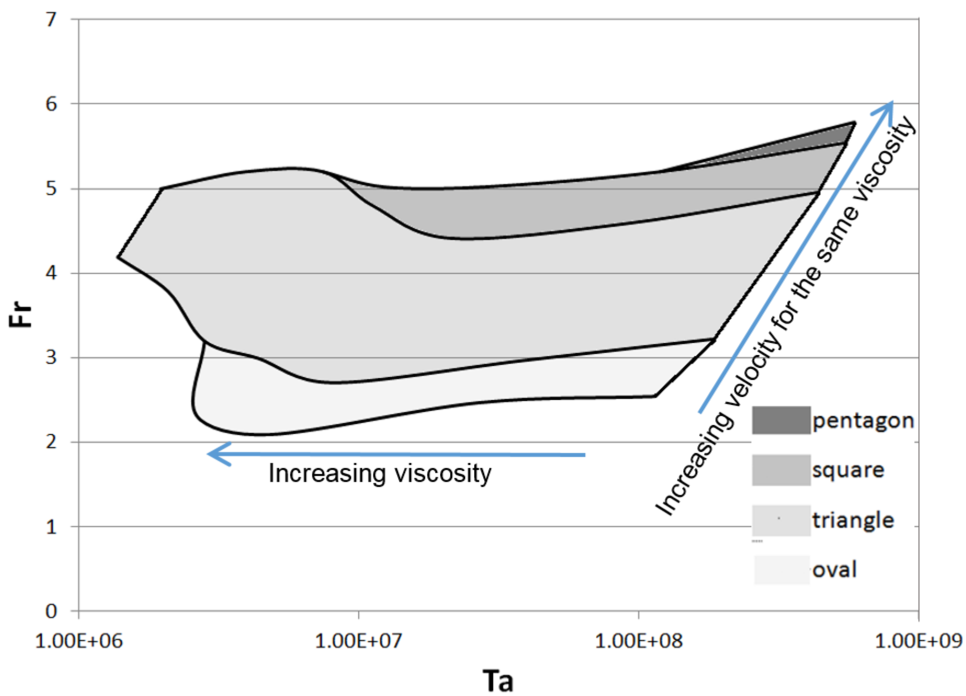

Figure 4: Phase diagram spanned by the Taylor number, Ta, and the Froude number, Fr. The initial fluid height is 30 mm.

which other patterns were observed, remained almost constant until a certain Taylor number. Bellow this threshold the extent of the intervals start to narrow mainly from their lower limits until they become null in the case of oval and square patterns. The interval of existence of the triangular pattern started to shrink at lower Taylors numbers. The triangular pattern should disappear at lower Taylors numbers, since no pattern were observed at higher viscosities, when approximately 80 % of glycerol is mixed with water. For the patterns other than the pentagon, one ca notice that bellow a certain Taylor number, the Froude numbers at which these patterns appear increase. In other words, at higher viscosities, the transitions were moved to higher Froude numbers. It is also worth highlighting that, compared with the previous results in Fig.3, the lower and upper limits of the Froude interval are decreasing gently; see Fig.4.

At h= 40 mm and water as working fluid, only oval, triangular and square patterns were observed; see Fig.5. Increasing gradually the viscosity of the

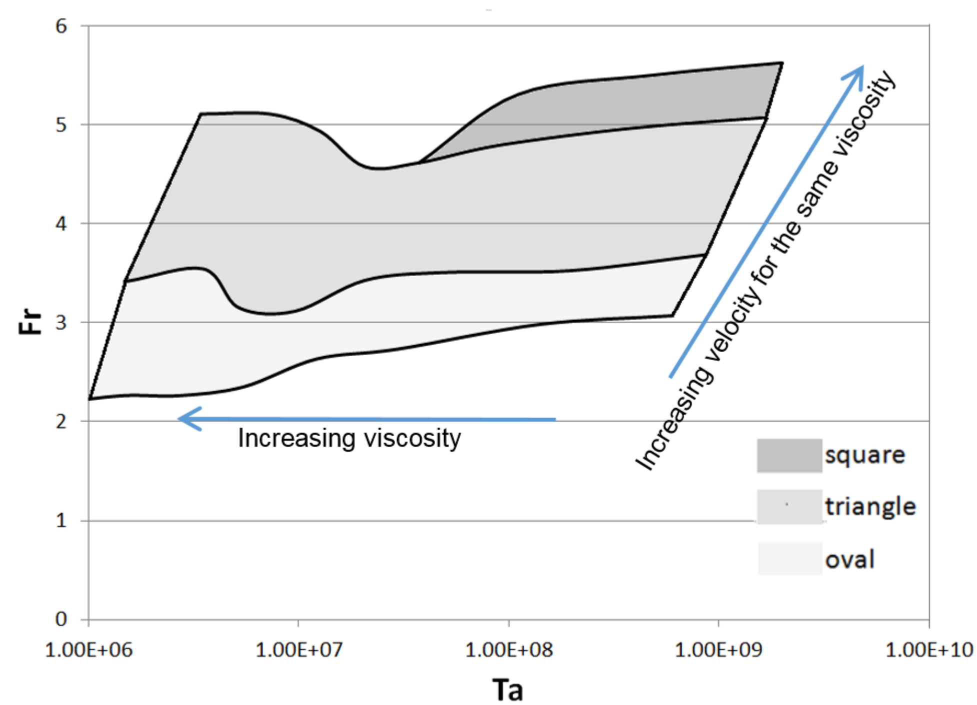

Figure 5: Phase diagram spanned by the Taylor number, Ta, and the Froude number, Fr. The initial fluid height is 40 mm.

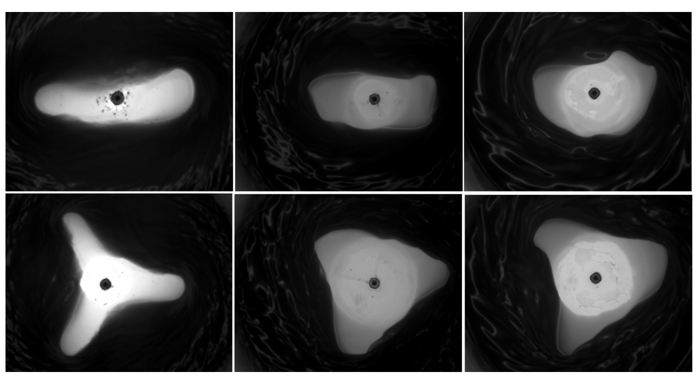

Figure 6: Evolution of oval and triangular patterns with increasing of viscosity. The snapshots correspond to three different viscosities; the viscosity is increased from left to right from 1 to 22 through 8 times μ. The increase of the viscosity tends to open up the pattern until to its quasi-circular shape.The initial fluid height is 30 mm

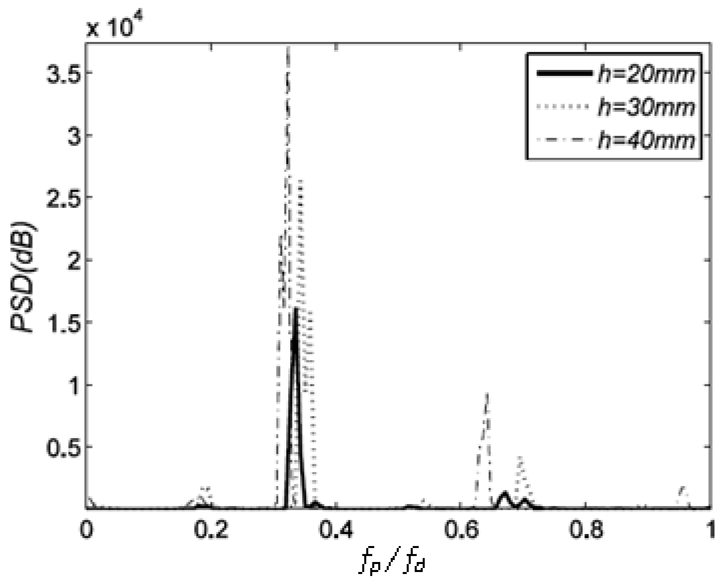

Figure 7: Power spectrum of the rotating triangular pattern for three different water initial fluid height h. The fluid is water only μ=1.002 $\times$ 10^{-3} Pa.s. The disc's speed were 161 rpms for h=20mm, 134.5 rpms for h=30mm and 175 rpms for h=40mm.

water the interval of existence of the square pattern shrunk to zero. However, the triangular and the oval patterns persisted for relatively high viscosities. The region of existence of these two patterns should shrink to zero at higher viscosities, as indicated above no clear polygonal pattern was observed when 80 % of glycerol was mixed with water.

The effect of increasing the fluid viscosity on a given pattern is shown in Fig. 6. One can easily notice that the contours of the polygonal patterns get distorted. When the fluid viscosity was increased, the shear at the disc's surface should have increased, which resulted in larger hollow-core vortex. This enlargement reduces the area of the annular inner fluid region, which is in solid body rotation. This squeeze of the annular inner region, which hosts the polygonal patterns, limits the propagation of higher wavenumber modes. This might explain why the highest modes have a narrower existence region and why they are the ones that disappear first when the viscosity is increased.

The influence of the viscosity on the polygonal patterns that follows the hollow-core vortex instability seems to depend on the initial working fluid

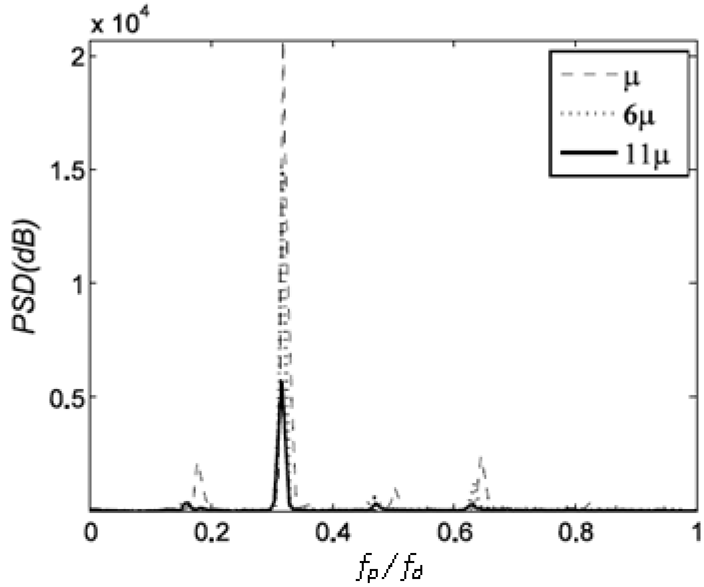

Figure 8: Power spectrum of the rotating triangular pattern observed at an initial fluid height of $h = 40$ mm for three different viscosities. The disc's speed were 175 rpms for $1 \times \mu$, 163 rpms for $6 \times \mu$ and 150 rpms for $11 \times \mu$. (μ indicates water's viscosity)

height. At lower height the viscosity lowers the disc's speed at which the hollow-core vortex undergoes instability and unstable modes bifurcate to its subsequent one. It also reduces the parameter domain of the unstable modes. At intermediate fluid height the increase of fluid viscosity shrinks the domain of existence of the highest unstable mode. However, for lower modes, the viscosity started to influence noticeably the instable modes only bellow a certain Taylor number. At high viscosities the transitions to higher modes occurs at high Froude numbers or high disc's speed. At higher fluid height, the viscosity shrunk the domain of existence of the highest unstable mode but seems to slightly enlarge the domain of the lower unstable modes.

Compared to the work by Jansson *et al.* [14], who considered only two values of viscosity, several viscosities were considered in this work. The viscosity was found that it influences the polygonal pattern formation. The fluid height seems to affect the way the viscosity influence the hollow-core instability and transitions between polygonal patterns. This influence should be an interesting research question that one should investigate in the future. At this time, we can conjecture that the increase of the height of the fluid in the outer region near the walls should promote the development of a meridional flow in this flow region[18]. The development of this meridional flow combined with the increase in fluid viscosity should lead to high dissipation of perturbations energy in the outer region (shear flow region). This dissipation should affect the polygonal patterns phenomenon. Indeed, it is thought that the instability mechanism leading to the formation of the patterns is due to the energy flowing from the shear layer into the inner annular solid-like body rotation region [19]. Hence, if part of this energy is dissipated within the secondary (meridional) flow in the outer shear layer, the modes or the polygones within the hollow-core region should appear more stable. This conjuncture may explain the persistence of some low wavenumber unstable mode at high fluid viscosity, when the fluid height is relatively high. The polygonal patterns appear to be more stable when the viscosity and fluid height get increased.

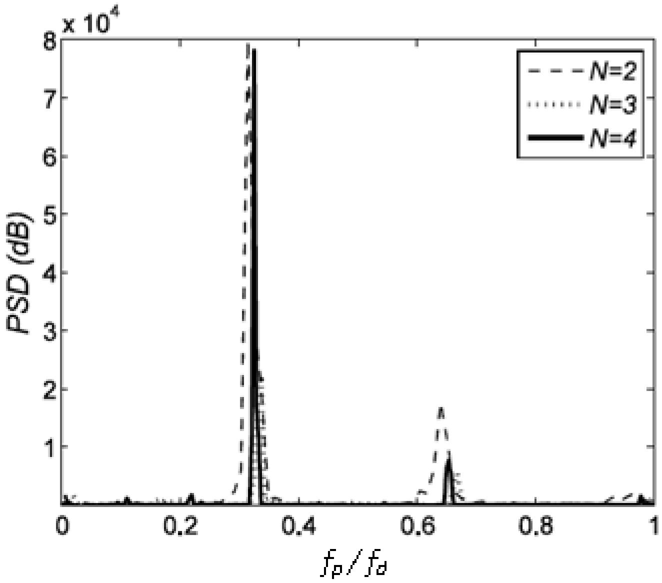

Figure 9: Power spectrum in the case of three polygonal patterns. Initial fluid height is h = 40 mm and viscosity 4μ.The disc's speed were 130 rpms for N = 2, 166 rpms for N = 3 and 227 rpms for N = 4.

3.2. Polygonal pattern frequency ratio (f_p/f_d)

Using water as the working fluid, Vatistas *et al.* [12] have found that the pattern's frequency to disc's speed ratio (f_p/f_d) is approximately constant and equals to 1/3, irrespective of the polygon and disc's speed. This observation has been confirmed for the three different heights investigated in the present study; see Fig.7, which depicts the power spectra at three different initial heights for the triangular pattern.

Similar frequency locking was observed when the viscosity of the fluid is increased. Fig.8 shows a superposition of power spectrum of the triangular pattern, obtained with three different viscosities: one, six and eleven times the water's viscosity, indicated here by μ. This figure clearly shows that the locking between frequencies of the pattern and the disc remains around 1/3 and unaffected by the fluid's viscosity. Fig.9 illustrates the power spectra of all patterns observed using aqueous glycerol mixture (four times the viscosity of water). This figure also shows that the locking holds for all the observed patterns.

4. Concluding Remarks

We investigated the effect of liquid viscosity on the formation of the polygonal patterns within the hollow-core vortex, generated by rotating a disc near the bottom of a cylindrical container under shallow layer conditions. The phase diagram in Taylor and Froude numbers plane, which delimits the regions where the various N-*gon* pattern can be observed, are displayed. The viscosity was found to reduce the domain of existence of the given pattern until its disappearance in favor of a quasi-circular shape. The experiments revealed that the initial fluid height above the disc affect the way the viscosity influence the pattern formation and their endurance. Another important observation is that viscosity had almost no effect on the frequency locking between the polygonal pattern and the disc's speed. For all viscous mixtures the polygonal pattern's rotating frequency, f_p, was found to approximately equal to 1/3 the disc's frequency, f_d. Hence, the viscosity does not affect the rotating frequency of the patterns, which confirms the results by Jansson*et al.* [14].

Acknowledgements

This work is supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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