Gödel's Mathematical Intuition and Platonism

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Abstract

This paper has two objectives. The first is to present an interpretation of Gödel's concept of mathematical intuition and defend it against other interpretations like Charles Parsons'. The second objective is to show the necessity of realism for Gödel's mathematical intuition. The first section seeks to show what mathematical intuition is and how it works, focusing on Gödel's works and unpublished texts. Consequently, from this section, I will show that, for Gödel, the concept of mathematical intuition emerges and develops parallel to his platonistic ontological commitment. Gödel's Platonism and mathematical intuition involve not only an ontological dimension but also an epistemological dimension. In the second section, I will discuss Parsons' 1995 paper "Platonism and mathematical intuition in Kurt Gödel's thought", in which he argues for a separation between mathematical intuition and Gödel's Platonism. What I will show is that this separation is not possible in Gödel since according to the recent publications of his philosophical notebooks and works prior to 1964, mathematical intuition was already implicit and, contrary to what Parsons argues, it is not something that arises before and independently of Gödel's realism.

§ 1. Introduction

Gödel's Platonism has been quite widely ridiculed. Bertrand Russell, for instance, says, "Gödel turned out to be an unadulterated Platonist, and apparently believed that an eternal "not" was laid up in heaven, where virtuous logicians might hope to meet it hereafter" (Russell 1968, 341). Perhaps due to this negative reception of Gödel's Platonism, some commentators, such as Parsons, have tried to distance Gödel's remarks on mathematical intuition from his Platonism. While there may be no knock-down argument against such interpretations, I wish to lay out and defend an interpretation of Gödel's philosophy of mathematics where his Platonism plays a central role and is inextricably linked to the notion of intuition. Through the explanation of mathematical intuition and its necessity for Platonism presented in this paper, it is possible to reach the point of view Gödel cohesively defends in his works. This position, which describes mathematical intuition as an epistemic faculty and has a realist component, seems closer to what Gödel expected from an epistemic theory of intuition.

This paper consists of two parts: The first is an explanation of how Gödel's concept of mathematical intuition works, and the second is an analysis of Parsons' 1995 article, "Platonism and mathematical intuition in Kurt Gödel's thought". The second part of this paper aims to show that Platonism is intrinsically linked to mathematical intuition, despite Parsons' conclusion in 1995.

The first part of this paper shows the functioning of Gödel's mathematical intuition by distinguishing the two moments or processes that constitute it. Starting from the supplement to "What is Cantor's continuum problem" [1964], an analogy of mathematical intuition as a type of perception is shown. Thus, the first moment consists of mathematical intuition understood as perception. This moment, which I will call "objective," involves an independent element (the mathematical object) and a passive one (the impression it produces in us). This moment is recognized as perceiving objects that are real and independent of us, from which we obtain "data" through the impression of these objects in us. The second moment is subjective and consists of mathematical intuition understood as reason. This second moment involves the passive and the active element, where from what is given, there is a process of constructing ideas, propositions, and other rational processes. Therefore, as described, and primarily from the first objective moment, mathematical intuition implies a real ontological commitment concerning the objects of mathematics, as Gödel proposed.

The second part of this paper deals with three main arguments that Parsons presents in "Platonism and mathematical intuition in Kurt Gödel's thought". This paper describes Gödel's concept of mathematical intuition as a theory of reason rather than a theory of intuition, where, as the name implies, there would be exclusively a subjective and rational component. Parsons identifies Gödel's concept of mathematical intuition as a *de dicto* intuition, or intuition as immediate evidence given through the senses. Therefore, Parsons argues that Gödel admits an intuition that does not need an ontological commitment beyond what is given in space-time. This argument implies that this intuition limited to predicates about natural numbers (i.e., classical, finite, and intuitionistic mathematics) does not require Platonism. Consequently, the epistemic side of mathematical intuition for the axioms of set theory has a plausibility or belief comparable to the belief on a hypothetical and formalistic level or the belief of someone who simply denies set theory. Parsons thus concludes that Gödel's concept of mathematical intuition is not connected to Platonism.

In opposition to Parsons' arguments, I present texts before [1964] where Gödel adopts a realist position that uses a type of sense that perceives abstract concepts or objects as he describes mathematical intuition later on. Thus, there is evidence of a mathematical intuition linked to the reality of these objects instead of an independent development, as Parsons indicates. Consequently, mathematical intuition, at least for Gödel, is not restricted to classical, finite, or intuitionistic mathematics, but rather all objects of mathematics are of the same real nature (distinct from that of physical objects), which are perceived and understood through mathematical intuition. Regarding the credibility of mathematical intuition and its epistemic value, it is shown that, for Gödel, the act of believing in the axioms of set theory (though not only) originates from the level of plausibility and credence given to mathematical intuition as an additional sense. Thus, mathematical intuition involves Platonism insofar as it is an epistemic faculty for perceiving and understanding the objects of mathematics. Platonism is thus intrinsic to mathematical intuition.

§ 2. Mathematical Intuition

Among Gödel's works, there are several accounts of the relation between realism and mathematical intuition (see note 4) and how the latter would work. The main idea of this section is to present mathematical intuition, the way Gödel wanted it to work, and how it was present in works prior to the supplement to "What is Cantor's continuum problem" [1964]. I want to show that mathematical intuition does not result from an interpretation of the readings of Kant and Husserl but that these readings and interpretations partially respond to questions that Gödel was already raising about mathematical intuition since 1937. That is to say, Gödel has the idea of intuition as a faculty that allows us to grasp mathematical objects and concepts. He has the idea of a process for obtaining knowledge from these entities, and this faculty and process are born from his realism.

Contrary to what commentators such as Parsons and Tieszen suggest, mathematical intuition does not arise from readings on the grasping, representation, and study of objects from Kant and Husserl but from a realist thesis. It is true that phenomenology and idealism will allow Gödel to clarify his mathematical intuition. However, these contributions will never call into question the ontological commitment from which he started.

Gödel did not give an explicit definition of mathematical intuition. However, according to all that he proposed and based mainly in [1964], we can say that mathematical intuition is a type of perception that functions analogously to sensual perception and that allows us to grasp mathematical objects and concepts that exist independently of us just like objects in the physical world. Gödel's lack of clarity on the notion of mathematical intuition has led to speculation and different interpretations. Most commentators explain this intuition as a particular analogy to the sensory perception of external objects (see Tieszen 1984; 2002; 2011; Parsons 1977; 1979; 1995; Wrigley 2022). Köhler (2014) explains mathematical intuition as a sixth sense, contrary to Chihara (1990, 21), for whom an intuition, analogous to a perception of objects not contained in space-time, is a mystical or theological belief. From Gödel's perspective, though, mathematical intuition is not a faculty that provides immediate knowledge about what is perceived but helps to form ideas from the given to understand and comprehend the objects and concepts of mathematics. This comprehension and understanding of what is perceived are achieved through a process of reflection and clarification of ideas. The most discussed lines of [1964] on mathematical intuition are the following:

> It should be noted that mathematical intuition need not be conceived of as a faculty giving immediate knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we form our ideas also of those objects on the basis of something else which is immediately given. Only this something else here is not, or not primarily, the sensations. That something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combinations of sensations, e.g., the idea of object itself, whereas, on the other hand, by our thinking we cannot create any qualitatively new elements, but only reproduce and combine those that are given. Evidently the "given" underlying mathematics is closely related to the abstract elements contained in our empirical ideas. It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective, as Kant asserted. Rather they, too, may represent an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to another kind of relationship between ourselves and reality (1964, 268).

This quotation presents mathematical intuition as structurally analogous to sensible perception. In both cases, indeed, Gödel distinguishes with some care three elements. First, we have external objects, sensible or mathematical, which are independent of us. We have, secondly, what Gödel designates by the formula "the given." It indicates the result of the external object's effect on us. It is, then, as Gödel says at the end of the quotation, the presence in us of these objects. The formula used also emphasizes the passive and involuntary character of how such an effect is produced. If it is something given, it is because it did not originate in us. Finally, there are the ideas that result from our appropriation of what is given to us. We will call these three elements the independent, the passive, and the active[1].

Although Gödel is not entirely explicit, I contend that there is sufficient evidence in Gödel's writings to show he conceived the notion of intuition (mathematical or sensible) as involving three elements: the independent one, the passive one, and the active one. In such a case, we can distinguish two distinct aspects or moments of mathematical intuition. The first does not depend on us and occurs when the object -- real, external, and independent of us − produces an impression on us. This moment

[1] In this first presentation of the passage, I leave aside certain aspects I will speak about later.

goes from the independent to the passive element. The second one, on the contrary, is subjective and active. In it, from the object's effects on us, we form ideas, beliefs, convictions, propositions, etc. This second moment goes from the passive element to the active element. These two moments are not mutually exclusive. In fact, a complete presentation of mathematical intuition should include and articulate them. If the first one is emphasized, intuition will be considered just another sense and unfailingly imply admitting an ontological commitment. Some of Gödel's texts make the same emphasis. If, on the contrary, the second moment is emphasized, intuition will be seen as a rational process through which ideas are obtained and refined. This emphasis is also present in Gödel and is the one that prevails in much of the specialized criticism. Indeed, the phenomenological reading of intuition depends on it, a reading that unfortunately tends to deprive it of any realist foundation.

These two moments are not only evident in [1964], but they can also be found in much earlier texts, such as Gödel's recently published philosophical notebooks[2] (the following quote was written between 1934 and 1941):

> The sole purpose of the construction is a “clarification” of an idea (at least insofar as the formal rules that it obeys become clear). However, the clarification does not consist in “seeing” the ideas (we are in a way “blinded” in this respect, i.e. our construction actually goes “into the void”), but we merely “complete” their existence, as we do in cases of the heteropsychological, external objects and physical theories. In spite of this, we have a “sense” for ideas (in the most primitive form, as we sense what is behind a proof, what a mathematically essential proof step is, what a decidable proposition is etc.), likewise we have a sense for the

[2] In this exact quotation below, Gödel defends that ideas are not perceived. However, his vocabulary is confusing since he suggests the need to distinguish between concept [Begriff] and idea [Ideen]. Gödel differentiates processes between primitive ideas (which, from examples given by Gödel himself, would refer to concepts) that are perceived and ideas that are constructed from the previous ones (ibid.). Rightly, Engelen, in the corresponding notes (2019, 184, n. 221, 225; 185, n. 226; 187, n. 238), refers to Leibniz. When Gödel returns to this notion of idea (ibid., 187) referring to simple (primitive) ideas that can be "seen" in Leibniz's sense, the terminology changes to a "sense" that would be developed for "Sehen des Begriffes oder der Operationsregel?" (ibid., 82).

Now, noting that these are not just annotations that Gödel could have taken from Leibniz, Gödel says in his following remark:

> Remark: That the “connection to reality” gets lost means that one no longer recognizes the corresponding concepts when using words, but a substitute [possibly the intensive occupation with mathematics has this effect, since it is necessary in mathematics to substitute the symbol for the concept as the concepts themselves are remote from perception], with which one can nevertheless correctly operate in a positivist (nominalistic) manner.

> heteropsychological and the objects of the outer world (an intuition). In all of these cases, we have a symbolic reconstruction of something invisible (in our case the ideas) (2019, 184)[3].

Gödel did not publish these philosophical notebooks. Neither did he propose a unified theory of mathematical intuition. My approach is to examine it as Gödel seems to study it, i.e., under these two moments that constitute mathematical intuition. In what follows, I will consider them individually, starting with the moment that goes from the independent to the passive element.

§ 2.1 Mathematical intuition as an additional sense

There are several works in which Gödel links mathematical intuition with perception[4]. We have just examined one in which a structural analogy is established between the two. Now, if this analogy is taken a little further, one would have to accept that mathematical intuition, as well as perception, allows us to grasp aspects of objects, not the objects themselves, which means that via perception (mathematical or sensible), we do not grasp objects completely (1947, 180). The notion of the given, which I interpreted as the immediate and involuntary effect that objects produce in us, captures this partiality and incompleteness. We will examine how we "'complete' their existence" (to use the formula I quoted at the end of the previous section (2019, 184)) in a moment. For now, I would like to point out that the explicit ontological commitment of both sense perception and mathematical intuition would suffice to neatly separate Gödel's account from Husserl's, even if the former often uses the latter to explain the process of the formation of ideas and the way we clarify them through reflection. Contrary to Husserl, both perceptual and intuitive processes presuppose the independent existence of objects for Gödel[5].

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[3] The symbolic reconstruction of something invisible can be interpreted as the Platonic form of mathematical intuition. I am not referring to the Platonic sense in which the objects of mathematics are real entities independent of our consciousness. I am speaking of how Plato refers to the invisible that is grasped as a thing in itself. Concerning things that are perceived through the senses, Plato says in the voice of Socrates (Phaedo, 79a):

> Now isn’t it true that these you could touch, see and perceive with the other senses, but that when it comes to those that stay in the same state, you could never get hold of them with anything other than the reasoning of your thought, such things being unseen and not visible?

Gödel's Platonism may not be a "Platonic Platonism" as Elsby and Buldt (2019, 382) argue; however, the handling of mathematical intuition as an epistemological faculty that induces an ontological commitment could be seen to be centered on Plato's theory of ideas. According to Wang (1990, 169), this is the case as Gödel's concepts are Plato's ideas.

[4] [2019], [1944], [1964], [*1953/9-III], [*1953/9-V], [*1961/?].

[5] da Silva (2005, 557) says:

If an object is given through the senses, we have an impression of that object in us, which in the case of a physical object, is a sensation. Gödel claims in [1964], however, that in addition to this sensation, something else is given, something that is qualitatively different from the sensible datum itself, namely, the concept of object. For example, when I see a chair, I can be aware of its particularities through the sensible data I receive from it. However, I can only recognize it as such if I acquire, at the same time, data from the concept of chair that enable me to constitute a unity[6]. This conceptual data can be interpreted in the same way as the data proper to mathematical intuition or sensory perception, i.e., as the result of an involuntary process of affectation in us of an external and independent object. However, it is unclear if, by virtue of the analogy that the passage is presenting, one must suppose in mathematical intuition an analogous integration of two qualitatively different kinds of given, corresponding (it seems necessary to suppose) to two different kinds of real objects. One would be tempted to suppose so because, as we shall see, Gödel assumes that perception is structurally analogous to mathematical intuition and even goes so far as to treat the latter as an additional sense.

A first passage illustrating this process is in "Is mathematics syntax of language? " (*1953/59-V, 359). In this draft, Gödel discusses the possibilities of realization of the syntactical program in mathematics, which consists in reducing mathematics to syntax of language. This reduction would imply that mathematical propositions have no content, thus eliminating the use of any intuition. To show the absurdity of this program, he introduces the notion of mathematical intuition and what it would hypothetically entail for it.

> [I]f we had a physical sense whose objects were of a similar regularity and similarly separated from those of the other senses, we could interpret also the propositions based on impressions of this sense to be syntactical conventions without content and associate no facts or objects

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> …Husserl clearly undermines the interpretation accepted by many analytical philosophers that, since he does not contest the independent existence of the world, there is an irreducible realist core in the phenomenological theory of perception. In fact, nothing of the sort is the case, since consciousness *of* a real world is a purely intentional experience confined to the transcendental field, it *does not* and it *cannot* support the *thesis* (already eliminated by the *epochê*) that there is a real world independent of consciousness that is given to me in perception.

This quotation refers to a passage from Husserl's *Ideas I,* § 90, and shows that the existence of objects in Husserl's phenomenology is not independent of us. Therefore, as a perception, Gödel's concept of mathematical intuition does not have a phenomenologist reading.

[6] Cf. (Hallett 2006, 122-127).

> with them or their constituents. The similarity between mathematical intuition and a physical sense is very striking.[7]

As can be seen, the whole hypothesis, absurd for Gödel, is based on the closeness between mathematical intuition and perception. A testimony of Wang, approved by Gödel, goes much further:

> There are more similarities than differences between sense perceptions and the perceptions of concepts. In fact, physical objects are perceived more indirectly than concepts. The analog of perceiving sense objects from different angles is the perception of different logically equivalent concepts. If there is nothing sharp to begin with, it is hard to understand how, in many cases, a vague concept can uniquely determine a sharp one without even the *slightest* freedom of choice. 'Trying to see (i.e. understand) a concept more clearly' is the correct way of expressing the phenomenon vaguely described as 'examining what we mean by a word.' Gödel conjectures that some physical organ is necessary to make the handling of abstract impressions (as opposed to sense impressions) possible, because we have some weakness in the handling of abstract impressions which is remedied by viewing them in comparison with or on the occasion of sense impressions. Such a sensory organ must be closely related to the neural center for language. But we simply do not know enough now, and the primitive theory on such questions at the present stage is likely to be comparable to the atomic theory as formulated by Democritus (Wang 2016, 85).[8]

[7] Gödel continues:

> It is arbitrary to consider "This is red" an immediate datum, but not so to consider the proposition expressing modus ponens or complete induction (or perhaps some simpler propositions from which the latter follows). For the difference, as far as it is relevant here, consists solely in the fact that in the first case a relationship between a concept and a particular object is perceived, while in the second case it is a relationship between concepts.

This quote has been used as an example of the intuition of truth values in propositions or of intuition of concepts as demonstratives or propositions. However, one must remember the context in which Gödel is speaking. The hypothetical picture is that syntactical conventions of symbols can replace mathematical intuition. Therefore, both "This is red"— through a physical sense similar to mathematical intuition—, and the proposition expressing modus ponens are propositions without content to which no objects or empirical facts can be associated. The above quotation says nothing concerning the objects involved in the process of mathematical intuition, as Gödel posits it. It says, on the contrary, what would happen with physical senses similar to mathematical intuition if syntactical conventions could replace the latter. That is, propositions based on the impressions of concepts or objects in both senses would also be syntactical conventions.

[8] Although this book is entirely authored by Hao Wang and shows his philosophical positions that did not always coincide with those of Gödel, the quotations I present in this paper belong to chapters of the book that were additions reviewed and approved by Gödel. In the Introduction (Wang 2016, x-xi), the process to which the book was subjected can be found in detail.

That is to say, Gödel not only thinks that mathematical intuition works analogously to sensory perception, but he believes in the existence of an organ that constitutes its base and that allows us to perceive abstract objects in the same way that sensory organs allow us to perceive objects in the physical world. Now that this organ is related to language shows a capacity to transform what is given into ideas and propositions that transmit and allow the understanding of what we perceive.

The idea of a physical organ that allows us to grasp abstract impressions and that is related to that which, in some way, allows us to assimilate language is difficult to justify. However, it is not difficult to show, in the recently published philosophical notebooks, that Gödel was interested in finding a corporeal foundation for the process. I do not pretend, by any means, to solve this problem but to show tentative ideas of Gödel concerning this organ. With ideas from Kirchner and Kant, Gödel wonders whether the ear is the sense organ for the perception of abstract concepts (2019, 210). He also refers to the organ of perception as the mind (2021, 236). These two sources date from 1940, the former is from January, and the latter is from October. In this last reference, Gödel is not assuming a position in which mathematical objects are perceived or presented to the mind. On the contrary, he identifies the mind as an organ, i.e., Gödel is, in a way, materializing the mind.

Let us summarize this section. Gödel compares mathematical intuition with perceptual processes. Since, for him, the latter involves an ontological commitment, the former requires it as well. Since the former involve a merely involuntary first contact with us (the impression or the given), the latter also presupposes it. Nevertheless, this comparison also entails recognizing the limitations of both epistemic resources. What we have of the objects captured in both cases are only those impressions that correspond merely to aspects of the object that gave rise to them. To properly arrive at the knowledge of that object from what is given in both cases requires other processes. They will not, however, be arbitrary processes because they will depend, in the long run, on a referent external to us. What will end up being known is the perceptible object or the mathematical object. Wang (2016, 84-85) describes it quite well:

> ‘If we begin with a vague intuitive concept, how can we find a sharp concept to correspond to it faithfully?’ The answer Gödel gives is that the sharp concept is there all along, only we did not perceive it clearly at first. This is similar to our perception of an animal first far away and then nearby. We had not perceived the sharp concept of mechanical procedures sharply before Turing, who brought us to the right perspective. And then we do perceive clearly the sharp concept.

If, on the other hand, we were dealing with arbitrary processes that do not require any objectivity or realism, we would, as Gödel argues, eventually have clear ideas about mathematical objects since they would ultimately be our creations. However, despite the precision developed in mathematics so far, there are problems that have not been solved (*1951, 314). It would be correct to state that even if mathematical objects were our own creations, our ideas of them are neither certain nor infallible. Nevertheless, for Gödel, these ideas or propositions restrict the creator's freedom, and whatever causes this restriction is independent of the creation. Thus, even if the objects of mathematics were our creation, there would be something related to the objects that is not. Gödel mentions the syntactical program as an attempt to precise the notion of "free creation," and similar to the drafts of *1953/9, Gödel finds that for the syntactical program to be attainable, a proof of (non-partial) consistency is necessary, which no finite, formalist or constructivist system can give (*1951, 314-6).

Now, I will describe the subjective moment or process, where the passive element is related to the active element.

§ 2.2 Mathematical intuition as reason

We have just seen the first moment that characterizes mathematical intuition, recognized as the analogy with sense perception. I continue with the second moment, where mathematical intuition is characterized as a rational process. This second aspect focuses on the subjective moment of intuition and not on the analogy with perception, although the latter remains a premise. This second moment focuses on identifying intuition with reason and involves proposing, as Parsons (1979) and Tieszen (2011) have argued, a theory of reason. However, it is not the case that under this second aspect, there is no ontological commitment.

This second moment of intuition consists of a process that goes from the given to the ideas formed from it. It seems correct to assert that both the Kantian notion of synthesis[9], *epoché*, and Husserl's categorical intuition (which do not, however, presuppose any ontological commitment) answer questions about the method of handling, reflecting upon, and clarifying objects and concepts in mathematics even though the realism of these elements is a premise in Gödel. The quote on page [3] of this paper from [1964] continues as follows:

> However, the question of the objective existence of the objects of mathematical intuition (which, incidentally, is an exact replica of the question of the objective existence of the outer world) is not decisive for the problem under discussion here. The mere psychological fact of the existence of an intuition which is sufficiently clear to produce the axioms of set theory and an open series of extensions of them suffices to give meaning to the question of the truth or falsity of propositions like Cantor's continuum hypothesis. What, however, perhaps more

[9] For a correct interpretation of what elements of Kant's transcendental idealism are reflected in Gödel's philosophical work, see (Hallett 2006, § 3).

> than anything else, justifies the acceptance of this criterion of truth in set theory is the fact that continued appeals to mathematical intuition are necessary not only for obtaining unambiguous answers to the question of transfinite set theory (of the type of Goldbach's conjecture), where the meaningfulness and unambiguity of the concepts entering into them can hardly be doubted. This follows from the fact that for every axiomatic system there are infinitely many undecidable propositions of this type (1964, 268-9).

From the above quotation, it can be inferred that the axioms and propositions of set theory result from mathematical intuition. They are not, therefore, something given. The axioms and propositions of set theory and, in general, mathematics emerge after the involuntary process I explained in the previous section. By no means should we suppose that this involuntary process is superfluous since it is a necessary condition for the formulation of axioms and propositions. *Mathematical intuition* is a unique process involving two distinct and complementary moments. It is itself in charge of becoming the given and of processing it rationally.

Gödel does not pretend to know how the production and reflection of our ideas work and therefore turns to Husserl's phenomenology as "a science which claims to possess a systematic method for such a clarification of meaning" (*1961/?, 383). In the same passage, Gödel characterizes the clarification of concepts as a shift in perspective toward our use of these concepts, analogous to Husserl's acts of consciousness. In the quotation cited on page [4] (2019, 184), Gödel says that the clarification of ideas does not consist of "seeing" the ideas, but that clarification consists in a construction that completes the existence of abstract entities in us that starts from the immediately given. The comparison made with external objects and physical theories, in this case, coincides with what Gödel has in mind in [1964] (268) when, from our perceptions, we can establish physical theories that correspond to experience in the real world. Not only that, the completion of abstract entities in us coincides with [1964] (269) when Gödel says that abstract entities or elements exist in our empirical ideas.

To conclude this section on mathematical intuition, I want to insist that these two processes are by no means separate mathematical intuitions. It should be noted that for Gödel, mathematical intuition functions primarily as an additional sense and, in addition, processes the information that we obtain through it. Mathematical intuition, for Gödel, becomes the whole process, from the organ affected by real elements to the clarification of ideas we form from the given. It is tempting to leave aside the first section since the existence of an organ that allows us to perceive objects and mainly concepts does not seem a viable option in a physiological sense[10]. However, if one thinks of the eye of the soul as that which

[10] Not taking mathematical intuition seriously as an additional sense has led to the current phenomenological and Kantian models. These models, in turn, have led to the claim that Gödel was not good at philosophy or did not have a good understanding of it. I do not think Gödel's position is a lack of understanding on his part but a lack of understanding on ours. To ignore the realist

perceives the invisible in a Platonic sense (as in Plato's theory of forms) and not so much as an absurd theological sense, it is feasible that Gödel is thinking in a unified philosophical way. That is, Gödel's philosophical work does not only encompass logic and mathematics. Gödel's philosophical notebooks show a deep interest in psychology, physics, and religion, among others. Likewise, he expresses his interest in conceptual thinking, not exclusive to mathematics and logic. Thus, the restriction given to Gödel as a logician and mathematician, excluding him from any other area of philosophy, is a trivial reading.

> Remark: I am apparently neither talented nor interested in combinatorial thinking (card games and chess, and poor memory). I am apparently talented and interested in conceptual thinking. I am always interested only in how it works (and not in the actual execution). Therefore, I should dedicate myself to the foundations of the sciences (and philosophy). This means: Not only the foundations of physics, biology and mathematics, but also sociology, psychology, history (world, earth, history of mankind). This means an overview of all sciences and then foundations (which is also what I am primarily interested in).
>
> I have been originally interested in explaining the phenomena of everyday life in terms of higher concepts and general regularities, hence physics (2020, 346-7).

Conceptual thinking that focuses on the foundations of mathematics is only one aspect of Gödel's interests, as seen in this quote from 1937. To say that "something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combinations of sensations, e.g., the idea of object itself" (1964, 272) implies that this kind of intuition, this other sense, is not unique to mathematics but understands and comprehends concepts present even in objects given through sense perception[11]. This position could not and is not the belief of a person without a realist ontological commitment (not only of objects of mathematics). It is clear that what Gödel calls conceptual

commitment is also to dismiss much of Gödel's work consisting of his perseverance concerning his analogy with sense perception. Even if the word or notion of intuition, as Gödel put it, is not explicitly found in some of his philosophical works, his analogy is persistently there.

[11] Through Wang (1990, 189-191), we know that Gödel's *conceptual realism* goes far beyond mathematics and even logic. However, Wang does not consider this position possible in concepts outside mathematics, such as poverty, beauty, good, etc. The impossibility expressed by Wang *suggests* a discussion with Gödel on the generalization of perception of concepts outside mathematics. On the other hand, these concepts that Wang opposes are linked to Plato's theory of forms since these concepts (beauty, the good, etc.) are mentioned in the *Symposium* (209e-212a) and *Phaedrus* (262b-266c), among others. If the discussion could have been given in these terms, the physical organ referred to by Gödel as necessary for the handling of abstract impressions would be, in terms of Plato (and Aristotle) and even Gödel (2019, 201, 205, 212; 2021, 229, 236), the mind's eye or the soul by means of which we perceive invisible things.

realism is a philosophical position that seeks knowledge from certain primitive concepts which are incompletely perceived or intuited. In addition, these intuitions are subject to change, improvement, and revision. Philosophy for Gödel must be "precise but not technical" (Wang 1990, 208), and, in epistemic and ontological terms, that was his direction. With this presentation of mathematical intuition as two processes, one perceptual and the other rational, I continue with Parsons' analysis and his arguments for the separation of Platonism from Gödel's mathematical intuition.

§ 3. Gödel and Parsons

In this section, I will propose a critical analysis of "Platonism and mathematical intuition in Kurt Gödel's thought" (Parsons 1995). Parsons defends an interpretation of Gödel's mathematical intuition in opposition to the one I presented in the previous section. The main difference lies in the importance of Platonism in developing that notion. Indeed, Parsons' purposes in his 1995 paper are, first, to explain Gödel's mathematical intuition--as an epistemic tool for knowledge of "high levels of mathematics"--so that from his point of view, it is more a theory of reason than a theory of intuition. Second, and as a consequence of the above, to show that mathematical intuition is not intrinsically linked to Gödel's Platonism (Parsons 1995, 45).

Parsons is never clear in 1995 what these two theories he opposes, that of intuition and that of reason, consist of. In "What is the iterative concept of set" (Parsons 1977, 343), he makes the distinction between two types of intuition that, according to Parsons, seem to be confused and yet are distinct. On the one hand, there is the intuition in which Parsons believes. This is a Kantian intuition that starts from sensibility and functions as a quasi-perception in which objects present themselves to the mind. On the other hand, there is the intuition which, according to Parsons, is also used in philosophy of mathematics and is understood as a *de dicto* or propositional intuition where propositions are known which, in principle, exclude the use of methods of inference and deduction, i.e., immediate evidence.

Another paper by Parsons, "Reason and intuition" (2000), allows us to understand what the author means by "reason." A reason, says Parsons, is justified by other reasons. This notion implies a relationship with the notion of argument, where there are premises and a conclusion. Premises or principles have plausibility[12] in two ways: they may be perceptual, i.e., events external to the proposition intervene, or intrinsically plausible. In the case of being intrinsically plausible, no external event is intervening. However, they are admitted as principles according to their consequences, i.e., the reason or effect for which they are held as principles. On the other hand, because of the similarity with the notion of argument,

[12] That is, they seem essentially true or are immediate evidence and are not a consequence of any other argument.

reason also implies a hierarchy in which principles give rise to sentences or judgments classified into low and high levels. Between these different levels, a relationship is established that Parsons describes as dialectical, i.e., the lower levels justify the higher ones, and the latter, in turn, validate the former (ibid., 299-302).

Suppose this is what Parsons understands in 1995 by a theory of reason. In that case, it is clear that this is a process in which internal justification dominates, except in cases where the evidence comes from external events that would justify perceptual judgments[13]. Intuitions for Parsons are considered intrinsically plausible[14] and, therefore, an instance of intuition *de dicto*[15]. This intuition is comparable to the second intuition that Parsons describes in (1977) and that we described above as immediate evidence limited to space-time. The process involved in this intuition does not require any ontological commitment since the justifying resources are essentially internal. If Gödel's mathematical intuition resembles this type of rational process, it would be more a theory of reason than one of intuition, on the one hand, and, on the other, it would not be intrinsically linked to Platonism[16].

Indeed, we may suppose that this theory of reason is Parsons' aim in Gödel:

> I think it is clear that he [Gödel] has first of all in mind what might be called rational evidence, or, more specifically, autonomous mathematical evidence··· Thus the deliverances of mathematical intuition are just those mathematical propositions and inferences that we take to be evident on reflection and do not derive from others, or justify on a posteriori grounds, or explain away by a conventionalist strategy (Parsons 1995, 59).

In both instances (2000 and 1995), when Parsons speaks of what follows from propositional intuition, he refers to propositions or principles that are assumed to be true through (mathematical) evidence or common sense. In both cases, these are processes linked to reason. Additionally, these principles are intrinsically plausible and therefore fallible and do not necessarily provide knowledge or cannot be proven to be absolutely certain.

[13] It should be noted that when Parsons refers to external evidence and perceptual judgments or evidence, he refers to perception referring to the senses. That is elements that do not transcend the sphere of space-time.

[14] This implies that in the case of mathematics, axioms in both classical mathematics and set theory are intrinsically plausible intuitions whose justification is given through the dialectic between axioms and their consequences (ibid., 304-309). This consideration is posterior and anticipates Parsons' intentions in 1995, so we will not consider it in what follows.

[15] Further on, in this same paper of 2000, Parsons talks about how an analogy between intuition and perception could be given, that is, to take intuition as applied to objects and not to propositions.

[16] Parsons (2000) will, in fact, speak of a "rational intuition" inspired by his works on Gödel's mathematical intuition.

In addition to this general reason for separating mathematical intuition and Platonism, Parsons proposes two supplementary arguments[17] (Parsons 1995, § 6), which I will present below. Before doing so, and to avoid confusion[18], I will henceforth refer to the intuition Parsons proposes in 1995 and refers to propositional intuition as "intuition" or "immediate evidence," whereas "mathematical intuition" will be the exclusive term for what Gödel shows in his works, and I explain in the first part of this paper.

Parsons' first argument shows that Gödel admitted a notion of intuition that did not require an ontological commitment. Indeed, in "Russell's mathematical logic" [1944], Gödel speaks of intuition as a fallible belief one has about what seems obvious. He describes intuition as "common sense assumptions of logic" (1944, 131). According to Parsons, that intuition is enough for classical, finite, and intuitionistic mathematics and does not require any ontological commitment. Parsons says that restricting mathematical intuition to this intuition may be limiting, but it is not incoherent (Parsons 1995, 70). There can, therefore, be mathematical intuition without Platonism.

Parsons' second argument revolves around the notion of "credence." Giving credence implies having confidence that something happens when it is presented to me. For Gödel's epistemology, it is necessary to give credence to what follows from concepts and propositions given by intuition[19], particularly the axioms of set theory or the concept of set. However, the concepts or axioms of set theory are not presented by sensible perception and are not evident for Parsons. The credence they may have comes to them, for Gödel, precisely from the independence of the reality they refer to. However, given that we are dealing with concepts and axioms that are not evident, Parsons suggests that the same credence could be given to them by a formalist or a hypothetical position in this respect. Intuition and realism are thus separable to that extent (Parsons 1995, 70-71).

I will deal with these two specific arguments and the general reason Parsons gives for separating mathematical intuition and Platonism in Gödel. I will begin with the latter and then address each argument in turn.

§ 3.1 Theory of reason and theory of intuition

[17] In addition to these two arguments, there is an observation to which I will refer later.

[18] Confusions about which intuition exactly Parsons is referring to in some of his arguments are common. I will address these confusions for the interpretation that best suits Parsons' intentions.

[19] What is deduced from axioms and concepts using intuition can be interpreted as the usual intuition described by Parsons or Gödel's mathematical intuition. Parsons, in any case, does not clarify this (ibid., 70).

At the beginning of this section, I tried to clarify the notions Parsons uses for his argument of separation between Gödel's mathematical intuition and Platonism. Now, I will focus on the first part of his argument: "Gödel aims at a theory of reason rather than a theory of intuition" (Parsons 1995, 45). In the first section of this paper, I showed Gödel's mathematical intuition through two processes. The first process consists of the role of mathematical intuition as an additional sense and the second as reason. Thus, to speak of mathematical intuition as a theory of reason, as Parsons does, is not particularly problematic from my perspective since mathematical intuition implies a theory of reason.

The rational process or moment I describe in the first section is the subjective part and the process of forming ideas from the given. To explain this process, the Kantian method of synthesis and Husserl's phenomenology appeal to Gödel insofar as they deal with the handling, reflection, and clarification of objects and concepts from the given. The clarification of ideas, according to Gödel, does not consist in perceiving or "seeing" ideas but in an analytical process of examination and construction of abstract entities from what is in us, i.e., from elements that we have grasped through mathematical intuition. There are primitive ideas and ideas that are formed from these. A relation is thus established between strong ideas--among these the primitive ones--and weak ones, in which the latter ideas are "definable" from the former (2019, 184).

However, this rational process is not possible without the involuntary or passive process that supposes conceiving mathematical intuition as an additional sense. Gödel, in "What is Cantor's continuum problem" [1964], makes the analogy of mathematical intuition with a type of sensual perception, and in the second version of "Is mathematics syntax of language?" (*1953/9-II, 172), speaking of the programs of Hilbert, Ramsey and especially Carnap, concludes that there are mathematical facts and objects that are as objective as physical and psychological ones, but that differ in nature. It is this type of perception that allows us to access or grasp, not immediately or entirely, these mathematical facts and objects. According to the view defended here, mathematical intuition presupposes a theory of reason but is not exhausted in it.

Parsons gives an account of the first appearances of the notions of "intuition" and "perception" in Gödel's works and classifies them as distinct as early as [1944] (Parsons 1995, 56). As I said above, the intuition that Parsons thinks to find in Gödel's early works is the notion of immediate evidence. On the other hand, this intuition is distinguished from the perception of objects in mathematics for Parsons. From intuition would follow the rules of logical inference and the axioms of classical or finite mathematics, which can be considered true with some revision and, most of all, with empirical evidence (ibid., 59-60). The perception of mathematical objects, on the other hand, is interpreted by Parsons as a mere metaphor for the incomplete knowledge or understanding that the human mind has of those objects (ibid., 57). The source of this metaphorical reading is "Some basic theorems on the foundations of mathematics and their implications" [*1951] or The Gibbs Lecture, which has been considered Gödel's most significant statement for his realism. Parsons argues that the only argument Gödel gives in this lecture for his realism is that mathematics would lack content without it. This argument is insufficient for Parsons because a perspective such as that of the intuitionists also claims that mathematics has content without marrying it to the Platonism that Gödel claims to defend (Parsons 1995, 55). Without this realism, the notion of perception becomes metaphorical (ibid., 57). This perception, with a metaphorical significance, will be called in [1964] "mathematical intuition," according to Parsons.

However, this metaphorical reading is only one (I believe remote) alternative. Parsons expresses himself so that a metaphorical interpretation of perception cannot be ruled out. The metaphorical interpretation would imply that Gödel did not have in mind a perception of concepts linked to his realism, given the insufficient argumentation of his realist position in The Gibbs Lecture. That is, like the Kantian notion of intuition in which Parsons believes, Gödel would have in mind a kind of perception where concepts are presented to the mind as they are understood. This scenario would show, if not a rejection, an independence of the perception Gödel had in mind with Platonism. However, this interpretation would not be possible if one evidences a realist position of Gödel prior to The Gibbs Lecture.

Now, Gödel's ontological commitment is clear from the 1975 Grandjean questionnaire, where he states that he was a conceptual and mathematical realist since 1925[20]. This realism is something that Parsons takes into account, but he also points out that most of these statements are after [1944]. Parsons even says that realism is treated as a hypothesis (Parsons 1995, 54, n. 20). The author highlights four texts before [1944] where Gödel's realist position can be called into question[21].

1. "The present situation in the foundations of mathematics" [*1933o]
2. "The consistency of the axiom of choice and of the generalized continuum hypothesis" [1938]
3. "[Undecidable diophantine propositions]" [*193?]
4. "Lecture at Zilsel’s" [*1938a]

Of these four texts, the most problematic concerning Platonism is [*1933o], in which Gödel states that,

> The result of the preceding discussion is that our axioms, if interpreted as meaningful statements, necessarily presuppose a kind of Platonism, which cannot satisfy any critical mind and which does not even produce the conviction that they are consistent (*1933o, 50).

[20] (Wang 1990, 20).

[21] Not only realism but the belief in the determination of the continuum problem.

The unpublished essay's title indicates the subject to be worked on, that is, the situation around 1933 of the foundations of mathematics. Gödel indeed seems to assume a formalist position, and throughout this work, he raises the problems of this position and tries to solve them. However, contrary to what Parsons and Gödel's contemporaries may think, I believe this essay is not intended to show the mathematician's personal beliefs but to give a picture of the formalist situation of mathematics in an age dominated by philosophical positions away from Platonism. Further, if Gödel is expressing his own view here, rather than what he takes to be the shared view of his audience, then we have to see the remark about him being a realist as early as 1925 to be outright mistaken. Goldfarb, in his "Introductory note to *1953/9" (1995, 324), quotes a letter from Gödel in which he says that even in 1959, there are still dogmas and prejudices regarding the objective reality of concepts and their relations in foundational issues, which prevent him from publishing his work[22]. So, it is clear that he did not want to be publicly associated with what he took to be an unpopular position. Both Dawson[23] and Köhler[24] also agree in describing Gödel as an insecure individual who did not explicitly state his beliefs contrary to those of the Vienna Circle. Later, in his 1975 letter to Grandjean, Gödel explains that his work does not correspond to the atmosphere of the early 20th century (Wang 1990, 20), and Parsons is aware of this fact (Parsons 1995, 46-47).

The problem considered in [*1933o] is to find a justification for the methods of proof used in mathematics, which formalism reduces to a finite number of axioms and rules of inference. The first obstacle to finding a valid justification for the formalists arises from the incompleteness theorem since, in every system, there is a true proposition that cannot be proved in that same system (*1933o, 48). Therefore, from the outset, there is a deficiency in the methods of proof posed by formalism. The second problem posed by Gödel centers on the justification of axioms and their meaning. This problem has three instances: the non-constructive notion of existence, the notion of class, and the axiom of choice (AC). Although Gödel does not discuss the AC, the solution to these problems arises from using what

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[22] I thank Dr. Lavers for bringing Goldfarb's remark to my attention, as well as Gödel's letter to Wang (Wang 2016, 8), where he says that it is precisely thanks to his "non-finitary mathematical reasoning" that he was able to arrive at his incompleteness results, setting aside logical biases and the finitary epistemic treatment of metamathematics.

[23] Dawson says: "As a newcomer, virtually unknown outside Vienna, he may understandably have advanced his views somewhat hesitantly, unsure what reactions to expect," and continues quoting Gödel: "Gödel speaks of the "prejudice or whatever you may call it" of logicians of the time against such transfinite concepts as that of "'objective mathematical truth,'" prejudice that Gödel took pains to circumvent by his rigorous syntactic treatment in 1931" (Dawson 1986, 198).

[24] Similarly, Köhler says: "By the 1920's already, intuition had seriously fallen into disrepute − especially within the Vienna Circle. Intuition seemed unsalvageable in the wake of the "crises" of reason or intuition: Kant thought intuition to be apodictic, whereas the intuition actually used in Mathematics seemed highly unreliable, as Bolzano (1810) emphasized long ago. ⋯ Making one drastic break with older Platonism and with the traditional Rationalism which had evolved from it, Gödel found intuition to be incontrovertibly *fallible!* ⋯ But Gödel was shy and didn't press the issue in the Vienna Circle, and his vision lay fallow." (Köhler 2014, 141).

he calls "objectionable methods," such as the law of the exclusive middle (LEM) and impredicativity, which are not methods that are admissible from a formalist position (ibid., 49). These objectionable methods give rise to Platonism. According to Gödel's analysis, the alternative solutions are not satisfactory and, therefore, the only viable solution, at that moment, is Platonism. Gödel is, in a way, assuming that Platonism is the solution to the problems of formalism and that the objectionable methods work, i.e., do not lead to an inconsistency of formal systems.

At first sight, Parsons might be right: superficially, Gödel may have been speaking as a formalist. However, the way he solves the problems of formalism shows only one way out: Platonism. If Gödel proposed some solution that did not require the actual existence of the objects of mathematics, one might doubt his commitment. However, I believe that the present argument does not show a defeated formalist but a platonist disguised as a formalist giving as his only solution precisely his Platonism. If then all this is true, Gödel, in this work, is, in effect, arguing for a platonistic position without appearing to do so (in keeping with his reluctance to publicly state his own views).

Texts 2, 3, and 4, all from the 1930s, do not show a position contrary to Platonism but somewhat changes of opinion concerning the determination of the continuum hypothesis (CH) and the concept of set that he will have from "What is Cantor's continuum problem" [1947] onwards. For Parsons, it is essential to highlight Gödel's commitment to problems of proof theory. However, I do not find it necessary to discuss a mathematician's particular thinking and ability to solve a problem like CH. This attitude is part of the mathematical activity, and it is not wrong, nor contradictory, to think at some point that a problem is unsolvable and then to have the intuition that it could be.

The evidence provided by Parsons is not decisive regarding Gödel's lack of a Platonistic attitude before [1944]. Furthermore, in his philosophical notebook (2019), dating from 1934 to 1941, we find passages that also suggest a commitment to Platonism. These passages refer to the realm of concepts as real, external, and independent of us (2019, 204, 210, 211, 220). Likewise, Gödel says there are no "inexact concepts" in the same way that there are no "false facts" (ibid., 204). Therefore, we cannot think of concepts as "special" objects--as Parsons calls them--which present themselves to the mind (Parsons 1995, 62).

Parsons also suggests that his metaphorical reading holds insofar as there is not enough justification for Platonism prior to [*1951] (Parsons 1995, 58). However, this suggestion is not persuasive enough because it is not necessary to justify Platonism to believe in it. In fact, a good part of our beliefs are entirely lacking in justification and do not thereby cease to be beliefs. If this is the case, we can suppose without problem that the perception of concepts Gödel spoke about in [1944] can be interpreted as a passive grasping of external objects, as I defended in the first part of this paper. Moreover, under these conditions, we can also suppose that the "theory of reason," which implies, for me too, mathematical intuition is necessarily accompanied by a passive moment, inseparable from a realist commitment. Evidencing that Gödel's goal does not seem to be a theory of reason as Parsons proposes, I continue with the two arguments that conclude the separation of mathematical intuition from Platonism.

§ 3.2 Parsons' first argument

Let us turn now to Parsons' first argument for the separation. It holds as we saw that Gödel admits an intuition for finite, classical, and intuitionistic mathematics, determined solely by predicates on natural numbers, involving neither set theory nor abstract concepts. Gödel at no time rejects an intuition in finite terms or that concerns spatio-temporal elements. So, this intuition does seem to be conditioned to finite or classical elements. It may refer to what Gödel in "On a hitherto unutilized extension of the finitary standpoint" [1958] or the Dialectica paper calls *Anschauung* or, later, in a translation of this same paper in [1972] (never published), calls "concrete mathematical intuition" (1972, 274). Another reference to this intuition may be what Gödel calls (*1953/9-II, 194, n. 12, 13) "natural mathematical intuition." In any case, one cannot deny the existence of this intuition given the precision and accuracy of the proofs at this level of mathematics for constructivist or finitists positions (*1953/9-II, 194; *1953/9-III, 338, n.12). Since such mathematics does not seem to require any Platonism, Gödel seems to accept a (mathematical) intuition without any ontological commitment.

Parsons never clearly explains what he means by "intuition" in his argument. However, it seems reasonable to suppose that it is the immediate evidence. His argument, therefore, involves attributing to Gödel, contrary to what we have just seen, a theory of intuition that he never tries to link with the theory of reason that he has been defending throughout his article. His argument, however, raises a problem that deserves our full attention: the relation between this intuition applicable to finite mathematics, which Gödel accepts, and mathematical intuition, which applies to set theory and entails for the mathematician a clear ontological commitment. Gödel's works, in this respect, assume, as far as I understand, two incompatible answers. In both answers, however, Gödel ends up bringing Platonism into play. Both answers, therefore, conflict with Parsons' interpretation.

According to the first answer, the finitary intuition and the intuition of set theory would be two completely different epistemic elements. In the Dialectica paper [1958], Gödel, through a system **T**, carries out results on the consistency of first-order intuitionistic arithmetic (HA) and the relative consistency for Peano arithmetic (PA). However, he does so by employing abstract objects as mental constructs that would be admitted by both finitism and intuitionism. According to Troelstra (1990, 218-221), Gödel intended to contribute to Hilbert's program and to justify notions for classical and finitary mathematics from intuition *(Anschauung)* or concrete mathematical intuition. This paper does not explicitly mention the possibility of applying this concrete intuition to set theory. However, he does make the observation that the elements that lie in the propositions must not be 'intuitive' in the way Hilbert posits, i.e., objects constructed by means of combinations of spatio-temporal elements (1958, 245; 1972, 274).

Thus understood, concrete intuition is distinct from the mathematical intuition applicable to set theory. If these intuitions are distinct, Parsons' argument would not be conclusive simply because concrete intuition tells us nothing, in principle, about mathematical intuition. The fact that Gödel accepts other epistemic resources that have nothing to do with Platonism does not exclude his acceptance of a resource, mathematical intuition, that does imply it. Now, both in [1958] and in [1972], the limitations of the notion of "accessible," of the concept of "demonstrability" for transfinite induction and of ideal abstractions are considered (1958, 243, n. 3; 1972, 272, n. c). Likewise, the problem of impredicativity of the concept of function and what Gödel defines as "reductive proof" (1972, 275, n. h) arises[25]. This non-eliminable impredicativity was the subject of conversation with Bernays in a realistic tone (Feferman 2003, 295, 301). As it is well known, impredicativity involves grasping the whole, which implies Platonism (*1933o, 49-50; 1944, 128). If the two intuitions are clearly distinguished, one would have to say that the finite one is insufficient, while the one applied to set theory resolves these insufficiencies. Gödel's acceptance of this finite intuition would therefore be only circumstantial. Parsons would appeal to a finite-valued result to construct his argument, which, in any case, would already have the other problems I pointed out.

According to the second answer that can be traced in Gödel, the intuition for finite mathematics would be the same intuition applicable to set theory. The Gibbs Lecture [*1951] aims to present the implications of the incompleteness theorems. Gödel first focuses on the inexhaustibility or incompleteness of mathematics for any philosophical position, with the restriction that mathematics has content[26]. Incompleteness manifests itself in the application of the axiomatic method to the body of

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[25] This impredicativity issue in **T** is highlighted by Troelstra (1990, 233, 235) in his "Introductory note to 1958 and 1972". However, one should also take into account Troelstra's note (ibid., 236, n. k), where he quotes Gödel, saying in 1974 that there is a way of arriving at an intuitionistic interpretation of the system **T** where the concept of proof or implication does not give rise to any circularity. This quote appears in Gödel's correspondence (Feferman 2003, 210-211), but whether the letter was sent is unknown.

[26] This is also an observation of Parsons. He adds that the argument that mathematics has content is only one aspect of Gödel's realism, which he nevertheless shares with constructivists. Thus, for Parsons, this argument is insufficient to argue for a realist position (Parsons 1995, 55). But mathematics without content, for Gödel, is not an attitude that can be consistently sustained (*1951, 311). Another example would be his drafts of *1953/9 regarding the claim that mathematics has content, in the case of the syntactical program.

mathematical propositions that are stated absolutely and without additional hypotheses[27]. Axioms for Gödel must be non-arbitrary propositions, mathematically correct, and evident without proof. At the same time, they are necessary for "mathematics proper," i.e., mathematics together with formal logic. This necessity is widely recognized among philosophical positions concerning the foundations of mathematics. The problem lies in the extension of "mathematics proper" as defined by Gödel (*1951, 305)[28].

The natural starting point for Gödel is set theory, and its axiomatization, for all mathematics, reduces to it. The extension of mathematics proper consists in axiomatizing the concept of set, which gives rise to the iteration of finite sets of integers. Compared to other philosophical positions on the foundations of mathematics, the result is that this iteration process never ends, giving rise to the incompleteness of mathematics[29] (*1951, 306). Gödel affirms the need to assume axioms or rules of inference that are non-arbitrary, correct, evident, and without proof. For this purpose, he admits as necessary also to assume and axiomatize gradually the concept of set (ibid., 305), from which it is inferred that the proposition that would express the concept of set is non-arbitrary, mathematically correct, and evident without proof. Contrary to [1958], the concept of set is not a mental construct; but also, contrary to Parsons, it would be evident[30].

The word "intuition" does not appear in [*1951]. Nevertheless, the non-arbitrariness, evidence without proof, and mathematical precision of propositions refer automatically to the intuition that Parsons is thinking of in his argument. However, in this lecture, this intuition would apply both to classical, finite, and intuitionistic mathematics and to set theory in the framework of a development that involves defending Platonism without nuance. It should be recalled that in The Gibbs Lecture, Gödel concludes that, in any way and from any position (involving the content of mathematics), the incompleteness of mathematics implies a conceptual realism or Platonism (*1951, 314).

Parsons establishes an epistemic difference between mathematical intuition and 'other' intuitions considering the mathematical extension of each one (Parsons 1995, 61). For Parsons, the intuition that the drafts of *1953/9 deal with is a propositional intuition or immediate evidence since Gödel justifies the existence of an intuition for the axioms of classical mathematics that even the Brouwerian school could not deny (*1953/9-III, n. 12). However, in the following footnote (Parsons 1995, 60, n. 34),

[27] Gödel argues that if this were not the case, hypothetical theorems would not exist (*1951, 305).

[28] A possible problem with this interpretation lies in the term "extension" since one could argue, as Parsons does, that this extension is a construction from the axioms that, for him, are considered evident. However, if this were the case, neither intuitionists nor finitists would oppose such an extension.

[29] Gödel goes on to explain the incompleteness of mathematics proper to positions that do not accept Gödel's extension (*1951, 308-311). However, this is not pertinent to this part of my argument.

[30] Evident but not directly apparent. It is presented by examining the axioms (*1951, 306, n. 4).

Parsons speaks of an equivalent passage which he calls "controversial" in the fourth version of *1953/9. The passage is as follows:

> It is clear that also classical mathematics was developed by means of an intuition (of the concepts of integer and set, of the continuum, of the meanings of the logical terms, etc.). That the intuition which is the basis of classical mathematics to a large extent is rejected as erroneous by the mathematicians which are called intuitionists to-day is irrelevant for assertion I. Note that all developments given in the sequel hold good no matter whether by "mathematics" one understands classical, intuitionistic, constructivistic, or finitary mathematics (Gödel [Unpublished 1953/9], IV Fassung, 1-22).

This passage, which proves problematic for Parsons and is undoubtedly confrontational to various fundamentalist positions of mathematics, shows that, at least for Gödel, mathematical intuition comprises all of mathematics, including those parts that Parsons seeks to distinguish in his first argument[31].

Under this view of the relations between finite intuition and intuition applied to set theory (between intuition and mathematical intuition), Parson's first argument is not very plausible as an anti-realist reading. Now, I will approach the second argument by Parsons, which emphasizes the epistemic role of credence in mathematical intuition.

§ 3.3 Parsons' Second Argument

The second argument for the separation of Platonism from mathematical intuition, consists in showing the credence given to mathematical intuition as an epistemic element of set theory in Gödel. Parsons supposes that the concepts of set theory are not sufficiently clear and that, consequently, the intuition of the axioms whose contents are these concepts do not provide knowledge or certainty, as would a Cartesian intuition. If the axioms do not provide knowledge or certainty that they are true, it is necessary to give them credence. However, the credence that Gödel would grant in this case, for Parsons, comes in many forms. Thus, the epistemic situation concerning credence is not inherent to Platonism, for just as Gödel, someone who assumes them hypothetically or in a formalistic way can give some credence to the axioms. Parsons claims that the credence that Gödel gives to intuition concerning the axioms and concepts of set theory does not require the existence of objects of mathematics (Parsons 1995, 70-71).

[31] It also shows that mathematical intuition is not, at least not exclusively, propositional, but that concepts (such as the concept of set) are perceived or intuited.

In my argument above, besides showing a realist ontological necessity in what Parsons calls the lower levels of mathematics, i.e., classical, finite, and intuitionistic mathematics, I show that mathematical intuition, as Gödel intended, is applied to all mathematics. Therefore, what Parsons refers to as the epistemic element is not unique to set theory but to all mathematics. Under this hypothesis, however, I will focus on the axioms of set theory, which are the focus of Parsons' analysis.

Parsons is correct in claiming that intuition cannot provide knowledge or absolute certainty about the axioms in a Cartesian way. Nevertheless, Gödel never espouses this Cartesian ideal; this is not what he aspires to in terms of knowledge. This position is something Gödel explicitly expresses in his conversations with Wang, namely that, as agents, we do not have absolutely certain knowledge. Moreover, according to Wang, it is for this reason that Gödel sometimes refers to his position as "objectivism" rather than "realism," for he says, "We are, ... more certain that we have objectivity than that we have found the right objects." (Wang 1990, 285). This thought of Wang is registered in his book, however, around 1940 Gödel expressed himself in the same way in his philosophical notebooks:

> Remark: One does not even know whether one knows something (or recognizes with absolute certainty), even though this is objectively determined in each case (Gödel 2019, 204).

Therefore, according to the above quotations, when Gödel speaks of the knowledge that could arise from intuition, or when he refers to it as a "source of knowledge,"[32] he is not referring to absolutely certain knowledge in the Cartesian manner. The reason why Gödel does not even aspire to this is that "even those states of affairs that we absolutely know for sure (2 + 2 = 4, my name is Kurt) consist of concepts that we do not fully understand" (ibid., 204)[33] because of the "very wide range of possible interpretations" (ibid., 204). This lack of absolutely certain knowledge does not indicate that there is no knowledge since, for Gödel, the process of cognition is given by two distinct modes: knowledge (certainty) and belief (plausibility), each gradually. However, "occasionally, it is not ascertainable whether the experience of 'certainty' is present or not" (ibid., 204). Then, if we refer exclusively to mathematics, the knowledge acquired through propositions has degrees of certainty and plausibility, but which are likewise always subject to correction, cultivation, or even change, for this is what Gödel refers to with the fallibility (of mathematical intuition). Therefore, the axioms of set theory are not obvious and much less

[32] (*1953/9-III, 340, § 16). It should be clarified that this expression of intuition as a "source of knowledge" is not found in any of the other drafts, including those not published.

[33] Gödel does not mention which would be the concepts behind a proposition such as 2 + 2 = 4. However, according to what we have seen in the previous section, behind this proposition would be the concept of number or even the concept of set since, for Gödel, the starting point for the axiomatization of mathematics is set theory. Furthermore, as we saw in the last quote of the previous section, all mathematics develops from the intuition of concepts such as set and integer, among others.

unquestionable. However, the lack of clarity of the concepts, in Gödel's point of view, is not a reason for not obtaining knowledge from them[34].

According to Gödel, if there is no absolutely certain knowledge, it is because of the incomplete way we perceive concepts that exist independently of us. For which Gödel, according to Wang, uses the "'argument from success,' which is an empirical inference from the fact that realism has been the most fruitful outlook up to now" (Wang 1990, 285). What Gödel brings out is that positivism, constructivism, formalism, or finitism all have structural problems that only realism can solve. The problems of formalism Gödel shows in [*1933o]; in [1944], Gödel affirms his non-constructivist position objecting to the Vicious Circle Principle (VCP); in the drafts of *1953/9, [*1951], and [*1961/?] he has an explicit position against positivism, and through the incompleteness theorems he proves the impossibility of the Hilbert program[35].

Now, if there is no absolute certain knowledge, the propositions, or in this case, the axioms of set theory, must have a certain degree of plausibility or credibility. It is because of this that the phrase so often quoted and interpreted by Parsons as an example of mathematical intuition as propositional (see Parsons 1995; 1979; 1977), "the axioms force themselves upon us as being true" (1964, 268), seems to have a different interpretation through what is presented in this paper[36]. Gödel does not seem to be asserting that the axioms are indeed true, but that, as the phrase indicates, they are forced on us as true, i.e., there is a strong possibility that they are true given the progress of mathematics through them, but it cannot be stated with certainty that they *are* true for, as Gödel emphasizes in [1944], the concepts are not perceived with sufficient clarity (139-140).

Gödel, in his philosophical notebooks, seems to posit an epistemic situation involving two types of perception: those that are states of affairs and those that are not (sensations and emotions). The former perceptions, i.e., those that are states of affairs, are in one-to-one correspondence with meaningful propositions. According to the goal of each state of affairs, they can be categorized as 1. the set of immediately attainable goals and 2. the set of believed states of affairs. Gödel explains the latter set:

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[34] I want to emphasize that, in the practice of mathematics, Parsons' statement is not sustainable. Advances and knowledge, even with the lack of clarity on the concept of set, give rise to knowledge. For example, set theory is extensive and present in all mathematical education.

[35] Gödel's position against formalism and finitism have been discussed in this paper. Goldfarb (1995, 325-6), in his "Introductory note to *1953/9", shows Gödel's position against constructivism and positivism.

[36] I do not claim that the propositional attitude of mathematical intuition is a misinterpretation in the way Parsons puts it. However, propositional intuition, derived from the perception of concepts, has more plausibility in light of this paper. Hallet (2006, 121) gives a similar analysis.

> A state of affairs is believed when it was once consciously considered to be true. The set [of believed states of affairs] also determines the set of known (= true and strongly believed states of affairs or such that can easily be deduced from them) and of unknown states of affairs (= not deducible from those that are believed) (Gödel 2020, 483).

Furthermore, a subset of the believed states of affairs is formed from those states of affairs obtained through experience or that are known (ibid., 483). Thus, credibility for Gödel is not limited to assumptions of a theory but encompasses states of affairs or perceptions that are experienced in some way or another and thus assumed to be true. This epistemic value of perception applies to set theory. Gödel expresses his preference for the position that axioms are evident truths because of his "belief" in set theory (ibid., 427).

For Gödel, "to believe" means, in addition, to act according to something, and the strength of this belief is measured in the way one acts according to this belief (Gödel 2020, 417). Then, the belief or credence given to mathematical intuition is to act according to what is given by this kind of perception. Wang says that for Gödel, intuition works by pointing to things (Wang 1990, 203). Therefore, belief arises from the decision to believe[37] in what is perceived through mathematical intuition as if it were some other kind of perception. The way Gödel put it in [1964] gives rise to this interpretation, "I don' t see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception." Hence, mathematical intuition leads us to make a decision from what we perceive concerning problems and the meaning of objects of mathematics (1964, 268).

The credibility of mathematical intuition stems from its analogy with perception and with which it shares characteristics such as fallibility. This form of credibility given to mathematical intuition, which, according to Gödel, works analogously to sensual perception, is not comparable to the position held by a formalist or someone who takes axioms hypothetically, precisely because of their analogy and their relation to reality. Gödel has been emphatic that --in terms of set theory − paradoxes are to mathematics what perceptual illusions are to the empirical sciences (*1951, 321; 1964, 268) and, therefore, says Gödel, "are frequently alleged as a disproof of Platonism, but, I think, quite unjustly." (*1951, 321). Wang puts it adequately:

[37] This decision is described here in a more sophisticated way than it would be in practice. If we are talking about any other type of perception or sense, where what is given comes from a physical organ such as sight or hearing, it is only sometimes necessary to decide whether what is perceived is true. That is, if I am cold, I do not make the decision to feel cold but am simply perceiving a change in temperature that is possibly occurring externally and independently of me. Similarly, the credibility of mathematical intuition is given by the kind of sense that Gödel sometimes refers to as reason. There are ways in which this sense is wrong, but in principle, what is given in perception, or in this case, in mathematical intuition, is shown to be true.

> G says, 'It should be noted that mathematical intuition need not be conceived of as a faculty giving an *immediate* knowledge of the objects concerned.' I take this to imply the familiar, though often deliberately overlooked (in order to discredit the appeal to intuition), fact that our intuition makes mistakes, needs cultivation, and can be corrected and extended (Wang 1990, 304).

The fallibility of mathematical intuition, as well as that of the senses, is crucial in mathematical work, for it is this fallibility that has allowed the progress of mathematics and especially of the notion of set, from Frege and Russell to Gödel's iterative notion of set that avoids the paradoxes. That is to say, the senses can deceive us in our knowledge of the external world just as mathematical intuition can deceive us in our conceptual knowledge and, specifically, in the concept of set given in mathematical knowledge. Consequently, according to the above, Parsons' alternative is not feasible since mathematical intuition as an epistemic resource needs realism for its credibility. Therefore, the difference between the credibility of a formalist or someone who rejects set theory and the credibility that Gödel grants concerning axioms cannot be dismissed, contrary to what Parsons argues.

In addition to these two arguments above, Parsons makes a final observation. This observation is that, even if mathematical intuition works the way Gödel intended, there is no justification for the existence of the objects of mathematics. That is, Gödel claims that there is a well-determined objective reality formed by concepts and theorems that are perceived and described by mathematical intuition (*1951, 320), which will allow us, in the long run, to decide problems such as CH (1964, 263-4). For Parsons, this would imply that "mathematical intuition is intuition concerning truth." Parsons' explanation is, then, valid in the sense that if there is an independent and objective reality to which we can have access through mathematical intuition, there is no way of knowing that what is perceived is necessarily the truth or falsity of problems such as CH, or, similarly, mathematical intuition does not guarantee the unfolding of concepts -as Gödel put it-- in such a way that we can arrive at the solutions required in mathematics (Parsons 1995, 71). Parsons concludes that the certainty of the intuitions of set theory can only be affirmed through the practice of mathematics, its investigation and development.

The validity of this observation is based on mathematical intuition devoid of any ontological commitment. On the one hand, we have the claim that mathematical intuition concerns truth, which, as argued, corresponds to the belief as true of what is perceived, thus mathematical intuition, in a way, does involve truth, only that these perceptions, like those corresponding to the senses, are fallible. On the other hand, Gödel's belief that concepts form a well-determined objective reality consists in the rationalistic optimism that he shared with Hilbert. That is, if there were undecidable problems for the human mind, reason would be asking questions that reason itself cannot answer. Then, reason would be "imperfect and, in some sense, even inconsistent," given the mathematical precision shown so far (Wang

2016, 324-325). Therefore, given this optimism concerning reason, there would be no reason to doubt Gödel's (and Hilbert's) belief that problems that are so far undecidable may eventually be solved.

Now, regarding a mathematical intuition that gives rise to the solutions to these problems or the unfolding of concepts, we must refer to the first part of this paper, which concerns the functioning of mathematical intuition both as perception and reason. Suppose a well-ordered reality exists where problems like CH are decidable and also that there exists a mathematical intuition that allows the impressions of the concepts of that reality in us. In that case, it is not absurd to think that this same intuition allows us to arrive at a decision for these problems in some way or at some time. Gödel's words are:

> That new mathematical intuitions leading to a decision of such problems as Cantor's continuum hypothesis are perfectly possible was pointed out⋯ (1964, 268)

Gödel points out the possibility that mathematical intuition leads us to a decision. What Parsons questions about mathematical intuition does not include this intuition as a perception of concepts, and that is why for Parsons, it is not possible that these problems can be determined by a mathematical intuition that differs from other ways of acquiring knowledge.

> ⋯ *mind, in its use, is not static, but constantly developing*, i.e., that we understand abstract terms more and more precisely as we go on using them, and that more and more abstract terms enter the sphere of our understanding (1972a, 306).

Our understanding from the given consists in formulating mathematical intuitions, even if we do not have the capacity to understand them in their totality. This incapacity is what Gödel calls the incompleteness of mathematics (Wang 2016, 324). Therefore, the epistemic factor of credence in mathematical intuition and the Platonism attached to these makes it impossible to separate them.

§ 4. Conclusion

Mathematical intuition is presented, according to Gödel, as an epistemic faculty that allows the grasping and understanding of the objects of mathematics insofar as these real objects affect us. The processes of mathematical intuition as perception and reason distinguish the functionalities that Gödel attributes to mathematical intuition in his works. The perceptual process of mathematical intuition allows us to solve questions around the notion of mathematical intuition presented both in Parsons' paper and in others where it is presumed that Gödel did not understand philosophy well. Incorporating the perceptual moment makes it possible to comprehend mathematical intuition in a complete way, of which, undoubtedly, Platonism is a part.

On the other hand, Parsons' arguments regarding the separation of Gödel's mathematical intuition from his Platonism, although valid in certain instances (e.g., mathematical intuition without any ontological commitment, without perception of independent objects, rationalistic optimism, or limited to classical, intuitionistic or finitary mathematics), do not correspond to Gödel's view of mathematical intuition. Gödel shows a realist commitment prior to what Parsons supposes, which makes it evident that the perception of concepts or mathematical intuition is a notion that captures -albeit incompletely- the objects of mathematics, including the objects of classical, finite and intuitionistic mathematics. Thus, one cannot conclude that mathematical intuition is the theory of reason that Parsons describes but that it is a theory of both intuition and reason, which has a realistic origin independent of us and by which we formalize, as much as possible, our "intuitions." Credibility as an epistemic factor is, therefore, analogous to the credibility we give to our senses in describing the reality of the physical world and is part of the process of mathematical intuition as perception. To set aside Platonism or the perceptual process of mathematical intuition is to remove a significant part of what characterizes it, which should not have been an option from the start. The perception of real objects through mathematical intuition, i.e., our ability to perceive concepts or objects independent of us, and hence the realist commitment to these objects, is intrinsically linked to mathematical intuition.

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