Three Essays on R&D Competition with Spillovers: Theory and Experiment

Sinda Kassab

A Thesis

In the Department

of

Economics

Presented in Partial Fulfillment of the

Requirements For the Degree of

Doctor of Philosophy (Economics) at Concordia

University Montréal, Québec, Canada

May 2024

CONCORDIA UNIVERSITY
School of Graduate Studies

This is to certify that the thesis prepared

By: **Sinda Kassab**
Entitled: **Three Essays on R&D Competition with Spillovers: Theory and Experiment**

and submitted in partial fulfillment of the requirements for the degree of

Doctor Of Philosophy (Economics)

complies with the regulations of this University and meets the accepted standards with respect to originality and quality.

Signed by the Final Examining Committee:

______________________________ Chair
Dr. Ian Irvine

______________________________ External Examiner
Dr. Radovan Vadovic

______________________________ Examiner
Dr. Ming Li

______________________________ Examiner
Dr. Dipjyoti Majumdar

______________________________ Examiner
Dr. Jan Victor Dee

______________________________ Supervisor
Dr. Huan Xie

Approved by ______________________________
Dr. Christian Sigouin, Graduate Program Director
Department of Economics

May 21, 2024

____
Dr. Pascale Sicotte, Dean
Faculty of Arts and Science

Abstract

Three Essays on R&D Competition with Spillovers: Theory and Experiment

Sinda Kassab, Ph.D.
Concordia University, 2024

This thesis consists of three chapters. The first chapter reports a laboratory experiment on dynamic patent races in an indefinite horizon with complete information. In the experiment, we examine how the players react to a leader/follower or symmetric/asymmetric position as well as the distance between the initial knowledge stock and the target. Our results show that the individual average effort is highest for the players who are in a tie position, second highest for the leaders and lowest for the followers and the spillovers in the previous round significantly increase the players' investment in the current round. By comparing the first and second half of the session, we observe an overall learning effect on the pure-strategy equilibrium play, but efficiency loss remains throughout the session.

The second chapter investigates the effect of R&D subsidies on the innovating firms' quality investment choices and profits as well as social welfare in a duopoly market with product substitutability, demand spillovers and consumers' quality sensitivity. Taking the non-cooperative and cooperative scenarios into account, the optimal R&D subsidy levels are solved in a way to maximize the social welfare. Compared with no-subsidy, the firms are better off under the R&D subsidy policy. Furthermore, it is always socially beneficial to subsidize the non-cooperative regime or the cooperative agreement.

The third chapter considers a two-stage strategic R&D model in a duopoly market. In the first stage, two firms decide simultaneously whether to compete or to cooperate by choosing the level of R&D investment that might decrease the investing firm's production cost and the rival's cost through the absorptive capacity. In the second stage, after observing the R&D outcome, the two firms play the classical Cournot in order to maximize their own profits. Under the stochastic R&D technology with low or high symmetric absorptive capacity, I find that the difference between

the optimal R&D expenditures under defection and those under cooperation becomes larger as the probability of success increases. Regardless of whether the absorptive capacities of the two firms are same or different, except at the critical threshold, the R&D outcomes always align with the prisoner's dilemma situation.

Acknowledgements

I would like to start by thanking Dr. Huan Xie, who expertly and passionately guided me through my PhD. I am grateful for her guidance and unwavering support throughout my graduate studies at Concordia University. I have been extremely fortunate to have a supervisor who cared so much about my work. I would also like to thank all the Professors at the Department of Economics for sharing their knowledge and expertise. I am thankful for all their teachings and knowledge.

I would like to extend my appreciation and thank the examining committee: the external examiner, Dr. Radovan Vadovic; the examiners, Dr. Ming Li, Dr. Dipjyoti Majumdar, and Dr Jan Victor Dee and the chair Dr. Ian Irvine. I also want to thank the dedicated staff who assisted me in the department and addressed all my inquiries: Ms. Elise Melancon (the former Graduate Program Assistant), whose dedication and hard work serve as a source of inspiration; Ms. Émilie Martel, Ms. Melissa Faisal and Ms. Kelly Routly, who are always helpful and truly kind. My special thanks go to Liang Wang, Alaleh Makvandi, Nickesha Ayoade, Olha Hnatyshyn, Fatina Siblini, Khaoula Saidane and Widad Achargui.

This dissertation and all of my academic achievements are dedicated to the memory of my beloved father, Younes. His unwavering love, support, and guidance have been the driving force behind my academic pursuits. I am forever grateful for the countless hours he spent discussing ideas, challenging my thinking, and encouraging me to push the boundaries of knowledge. Though he is no longer with me, his presence continues to inspire and motivate me every day.

To my mom, words cannot describe how truly thankful I am. Thank you for encouraging me to always strive for success and reach for my dreams. Thank you for always supporting me every step of the way. Thank you for all your love and guidance every day of my life. To my husband, thank you for pushing me to be stronger.

To my daughter, Dana, to my sisters, to my brothers and to my sister-in-law, thank you for your unconditional love and support. Thank you for encouraging me in miserable moments and believing in me and my abilities when I failed to believe in myself. To my big family, thank you to those who were curious about my study. Your love and support are greatly appreciated.

Contributions of Authors

Chapter 1 is a joint work with my supervisor, Dr. Huan Xie.

Chapters 2 and 3 are not co-authored.

Table of Contents

List of Figures

List of Tables

1 Introduction

Nowadays, the R&D activities constitute a crucial part in the strategies of the innovating firms. Holding the market, supporting research as a source of new knowledge, improving quality, increasing production flexibility and carrying out R&D cooperation become the firm's main missions. Because of the presence of positive externalities, the R&D activities benefit not only the firms involved but also the society as a whole. These externalities, also called spillovers are the result of a knowledge appropriation that occurs between the firm and its rival. The "spillover" concept has gained significant importance in both theoretical and empirical literature on R&D. Using both game theoretical and experimental approaches, this dissertation contributes to three important aspects related to firms' R&D activities, specifically, the firm's strategic R&D investment behavior under the competitive environment with spillovers, the motives for non-cooperative, cooperative and opportunistic behavior in the presence of symmetric and asymmetric externalities, and the implications of the R&D subsidy policies.

Based on a dynamic game of patent races, the first chapter "Dynamic Patent Races with Absorptive Capacity: An Experimental Investigation on R&D Behavior" reports an experiment which focuses on the decision-making process during the R&D competition. In this dynamic game, each firm must decide independently and simultaneously in each period how much R&D effort to dedicate towards accumulating knowledge. The goal is to reach a predetermined critical level of experience and secure the prize. The R&D level of each firm is determined by the sum of their past efforts as well as a portion of the rival firm's efforts that spills over when both firms invest in R&D.

In our experiment, we examine the competitive behavior of the subjects within a set of sequences (races). Each sequence begins with a random matching protocol, where subjects are asked to participate in a patent race game that closely resembles the one played in previous sequences but with different initial positions. Different from previous experimental work on patent races, such as Zizzo (2002), Silipo (2005) and Breitmoser et al. (2010), our study contributes to the literature by examining the impact of absorptive capacity (termed as spillover) on the player's investment behavior in a laboratory experiment. Additionally, by considering a set of sequences, we investigate the impact of the starting position (leader vs. follower, symmetric vs. asymmetric) on the investment behavior and the race outcome as well as the influence of spillovers that were realized earlier during the race on the investment decisions in the future periods.

Our results are in line with theoretical predictions, showing that players in a tie position exhibit the highest individual average effort, followed by leaders and then followers. Additionally, there is evidence of a learning effect about the equilibrium play. Towards the end of the sessions, players' behavior becomes more consistent with the pure strategy equilibrium predictions and their payoffs become closer to the efficient outcome as the gap of the initial knowledge at the start of every sequence increases. However, there is still some efficiency loss over the course of competition. As expected, our findings show that the leaders have the highest frequency of winning the race, while followers have the lowest frequency. Furthermore, the positive spillover from the previous round leads to a significant increase in terms of the incentive to invest in the next round.

In the second chapter “Government Subsidy and R&D Quality Investment”, I investigate the impact of R&D subsidies on the decisions made by firms regarding their investment in quality improvement. The investigation takes place in a duopoly market where there is substitutability between products, demand spillover effect, and consumers’ quality sensitivity. I compare and analyze the firms' optimal decisions under both non-cooperative and cooperative settings, and I solve the R&D subsidies that maximize the social welfare. I then assess the changes in firms' profitability, quality investment, and social welfare when implementing the socially optimal subsidy policy. Additionally, I examine the impact of demand spillovers and consumers' quality sensitivity on the optimal quality investment and subsidy levels through a comparative static analysis.

Compared to the previous theoretical studies on R&D policies and welfare implications (Leahy and Neary, 1997; Lahiri and Ono, 1999; Lee et al., 2017; Chen and Lee, 2023), I investigate the government's R&D subsidy based on quality levels, taking into account the demand spillover effects and the consumers' quality sensitivity under the non-cooperative, the partial cooperative, and the full cooperative scenarios.

The main results demonstrate that, in the absence of subsidies, cooperation in both quality and quantity generally leads to the highest level of quality investment compared to non-cooperative and partially cooperative scenarios. Analyzing the profit rankings under each regime, it is evident that when there is no government support and when the demand spillover differs from the critical threshold, firms have a greater incentive to cooperate in both the R&D and production stages, as this yields the highest profit. In addition to that, the results show that the consumers' quality sensitivity has solely an impact on the optimal subsidy under non-cooperation, and its influence on the level of quality investment is similar when comparing the non-cooperative and the partially cooperative scenarios. However, this effect differs when comparing the partial and full cooperative scenarios. For small demand (large) spillovers, the optimal R&D subsidy under full competition is lower (higher) than that under partial cooperation. From a social welfare point of view, the full cooperative scenario is socially less effective compared to the non-cooperative and partial cooperative scenarios.

The third chapter "R&D Investment under Symmetric and Asymmetric Absorptive Capacity" examines a two-stage R&D model in a duopoly market. In the first stage, two firms make a simultaneous decision about whether to compete or cooperate by investing in R&D to reduce the cost of production in the second stage. The model considers two possible outcomes of R&D: maintaining the original cost or reducing it with a certain probability. Additionally, there may be technological spillovers and absorptive capacity, which can benefit the other firm by diminishing its cost. After knowing the R&D outcome, each firm makes a simultaneous decision about the quantity to produce, similar to the traditional Cournot model.

Most of the theoretical investigations on R&D investment have focused on symmetric technological externalities, where the external effects are identical between firms. These studies, including D'Aspremont and Jacquemin (1988), Henriques (1990), Qiu (1997), Symeonidis (2003), and El Ouardighi, Shnaiderman, and Pasin (2014), have provided valuable insights into the decision-making process for R&D investments. However, few works have been done on the asymmetric externalities. Although previous studies have explored the spillover effect, none of them have investigated the inclusion of the opportunistic behavior in a Cournot duopoly market under the stochastic and deterministic environment with symmetric and asymmetric R&D absorptive capacity.

By considering the deterministic and the stochastic R&D technologies, as well as the potential existence of opportunistic behavior, I examine how the firm's optimal R&D investment decisions change when there is a change in terms of the symmetric or asymmetric absorptive capacity and whether the gap between the different optimal R&D expenditures is affected by the probability of success. In addition to that, I investigate whether the firms encounter a prisoner's dilemma scenario for their R&D decisions. Under the symmetric absorptive capacity, the main results show that as the likelihood of success rises, the difference between the firm’s optimal R&D expenditure in the case of defection and those in the case of cooperation widens with low absorptive capacity and narrows for those with high absorptive capacity. Additionally, firms consistently have a motivation to defect from the R&D cooperative agreement. Interestingly, other than the critical threshold, all scenarios are in line with the prisoner's dilemma situation, regardless of whether the absorptive capacity is symmetric or asymmetric.

2 Dynamic Patent Races with Absorptive Capacity: An Experimental Investigation on R&D Behavior

2.1 Introduction

Innovation, as a strategic factor of growth, has a great role in the evolution of the industrial leadership. It is one of the main factors in the technological competition that can decisively improve the firm's competitiveness. Many companies, which have been ongoing for many years, are competing in a growing market to discover an invention for the aim of obtaining the patent. As an example, Apple and Microsoft are racing to bring a hand gesture system to notebooks, vehicles, home, appliance, computers and so on.

In general, strategic interactions occur between two or more competitors who spend costly resources in a patent race in order to increase the probability of winning. As the winner gets a legal monopoly power over the technological innovation by the patent system and the loser gets nothing, a firm extracts a very high profit by winning the patent race when it sells the technological invention in a market in which rivals cannot copy it or replicate it. The competitive advantage that a firm gets is related to its ability to use, assimilate, and absorb external information toward achieving organizational goal.

In this paper, we explore experimentally the R&D investment behaviors of the firms by using a dynamic game of patent races, which follows the model similar to Fudenberg et al. (1983) and Halmenchlager (2006). We focus on the strategic decision-making during the course of the competition process. Specifically, in each period, each firm has to decide, simultaneously and independently, how much R&D effort to devote to the accumulation of knowledge in order to reach a fixed critical level of experience and obtain a prize. The effective R&D level of each firm represents the sum of its own efforts in the past periods and a fraction of the rival's efforts that spills over when both firms make investments in the R&D process.

Our experiment consists of 8 experimental sessions. In each session, subjects were asked to play ten sequences (races). Given that the timing of reaching the critical level of knowledge is unknown, every sequence was characterized by an indefinite number of periods for prize acquisition. In each sequence, we randomly and anonymously paired the subjects and varied the initial knowledge position of the players, which can be symmetric or asymmetric. We also considered different distances between the initial position and the target, which we denote as low treatment (with a long distance to the target) and high treatment (with a short distance to the target).

A real-life example of these treatments can be seen in the field of technology development where companies like Intel and AMD are racing to develop AI and high-performance computing technologies and are competing in a tight race, each with advanced technology and significant R&D investments. This has positioned them for significant competitive achievements in the near term. In contrast, Toyota and Tesla are engaged in a significant R&D competition in the electric vehicle market and face a longer-term challenge due to the need for substantial upfront investment and long-term commitment. As a result, the competition between these two companies will continue to shape the development of AI and EV technologies.[1]

To examine the order effect between the low and high treatments, among the 8 experimental sessions, we had 4 sessions starting with low treatment for the first five sequences, followed by another five sequences in the high treatment, and another 4 sessions with the reverse order. To avoid trivial results, following Zizzo (2002), we impose the rule in the experiment that firms cannot make zero effort for three successive periods in the race. Otherwise, the competition is ended and no one wins the prize.

Using this experimental design, our paper studies the evolution of competition in a set of dynamic patent races in which the type of the firms (leaders or followers or tied firms) varies at the beginning of the race. We are interested in the following questions: 1) How does the starting position (leader vs. follower, symmetric vs. asymmetric) affect the race outcome and the investment behavior? 2) What factors influence equilibrium play and efficiency level? 3) Does the existence of spillovers affect future investment behavior?

---

[1] Information collected from the following links:
https://www.hivelr.com/2023/02/nvidia-nvda-porters-five-forces-industry-and-competition-analysis/
https://www.intc.com/news-events/press-releases/detail/1672/intel-reports-fourth-quarter-and-full-year-2023-financial
https://simplywall.st/narratives/intense-competition-will-force-amd-to-sacrifice-margins-in-favor-of-randd-spend
https://www.autoblog.com/2023/02/28/toyota-rethinking-long-term-ev-strategy-ceo-koji-sato/
https://markets.businessinsider.com/news/stocks/why-toyota-may-have-the-best-strategy-in-the-ev-race-1033078343

Overall, consistent with the theoretical predictions, our results show that the individual average effort is highest for the players who are in a tie position, second highest for the leaders and lowest for the followers. By examining the first and second halves of the sessions, we find an overall learning effect in the investment behavior. The players' behavior is significantly more consistent with pure strategy equilibrium predictions in the last five sequences regardless of whether the treatment is from low to high or high to low. As predicted, the frequency of winning the race is highest (lowest) for the leaders (followers).

In the low to high treatment, we find that, if players start with a tie or leader position, the frequency of winning the race is significantly higher in the high treatment compared to the low treatment while in the high to low treatment, the results show that there is no significant difference in the frequency of winning between the low and high treatment conditional on each starting position.

Finally, we find that, as the initial knowledge gap at the start of each sequence increases, the subjects' payoffs are closer to the efficient outcome (that can be achieved when both players in race invest at low cost up to the end of the race), but the efficiency loss still incurs throughout the session. By considering all the sequences, we find that the positive spillover from the previous round significantly increases the incentive to invest in the next round.

The remainder of the paper is organized as follows: Section 2.2 discusses the related literature. Section 2.3 and Section 2.4 present the theoretical model and the experimental design, respectively. Section 2.5 provides the empirical results and Section 2.6 concludes. In the appendix, we present the paths of choices that achieve the equilibrium outcome, the minimum and the maximum payoffs for the two firms given the parameters of the races that we used in the experiment.

2.2 Related literature

To better understand the forces that drive technological innovations, early models were provided by different economists. For instance, a R&D race model with knowledge accumulation was introduced by Doraszelski (2003), in which R&D efforts lead to a building up of knowledge stock and have an impact on hazard rate of a successful technological innovation. Given that the knowledge accumulation can occur across the time, the R&D investment of past periods has strategic implications since it might help the firm to make the technological discovery and therefore win the race (Doraszelski, 2003). In other word, the current and the cumulative R&D spendings are related to the probability of the firm's success.

An important strategic point is emphasized by different models in which the firm has to anticipate the R&D efforts of its rivals in order to gain a competitive advantage and to develop its abilities to identify, assimilate and exploit external knowledge that can bring promising innovations (Cohen and Levinthal, 1990; Halmenschlager, 2006). These abilities have been largely considered as an important factor for the innovating firms since Cohen and Levinthal (1989) who emphasized that a firm can gain external information from other firms if it invests in its ability to absorb technologicial externalities.

Halmenschlager (2006) introduced the absorptive capacity into a dynamic patent race model with memory of past actions based on the work of Fudenberg et al. (1983). She found that when the two firms are close in tems of their accumulated knowledge, the absorptive capacity allows the chance of catching up when the leader has a less aggressive response to the rival's reaction. When the follower doesn't conduct its R&D activity, the leader will not have the incentive to increase his effort since the rival is inactive and is far behind (Halmenschlager, 2006). In case of a tie, the firms gain the R&D effort from each other via the absorptive capacity and the pace of innovation increases.

A few experimental studies were conducted on patent races. Without considering the technological spillovers, Zizzo (2002) tested experimentally the patent race model of Harris and Vickers (1987) by taking into consideration two kinds of uncertainty: technological uncertainty (since the output and the speed in the technological development are uncertain) and dynamic uncertainty (the level of investment in R&D can change as the race continues, which depends on the firm's relative position compared to its competitors and compared to the end of the race). Different from the predictions of Harris and Vickers (1987), Zizzo (2002) found that the leaders did not appear to invest more than the followers.

Sbriglia and Hey (1994) conducted a number of experiments in multi-stage R&D competition in order to examine the effect of the learning process during the innovation race by considering the subjects' ability in solving a specific problem in a competitive environment. Their results show that the losers have, in general, spending patterns similar to the winner, but their investment strategy is either less aggressive or they are less lucky.

Silipo (2005) tested experimentally the theoretical model of a multi-stage race of Fudenberg et al.

(1983) and added the possibility of making cooperative R&D investment. His model is characterized by two stages in each period. In the first stage, firms make a sequential decision on whether to cooperate or to proceed alone up to the end of the race. The first mover chooses to make a cooperative R&D or to go by itself and the second mover chooses to accept or to refuse the cooperation proposal. In the second stage, the firms choose the R&D investment, cooperatively or non-cooperatively depending on the outcome of the first stage. If they fail to reach a cooperative agreement, they choose simultaneously and independently the investment level in the current round. The results of Silipo (2005) showed that subjects with same initial position cooperate at the beginning of the race but break this cooperative relationship as they approach the finishing line for the aim of getting the prize. Subjects with an asymmetric starting position (0,1) cooperate if the cost savings make the project profitable, while those with a position of (0,2) compete and never cooperate.

Breitmoser et al. (2010) performed an experimental study on perpetual patent races (that have the characteristic of a dynamic indefinite time horizon with uncertainty and multiple prizes), based on the original theoretical papers by Aoki (1991), Hörner (2004) and Harris and Vickers (1987). The main purpose of Breitmoser et al.'s paper is to verify in a controlled laboratory experiment whether the context-sensitivity of Markov perfect equilibria strategies will hold with real players. A number of parameters were considered in their paper, such as the discount rate, the cost and the probability of progress for high versus low research effort, and the benefits from staying ahead during the race. Their main experimental results indicate that the subjects' behavior was less context-sensitive than theory predicted.

In order to examine the impact of patents on R&D, Diduch (2010) provided a classroom experiment that allows students to determine their research effort in response to two incentive structures: the incentive for excessive research effort and the incentive for free riding. The results show that a subject can gain significant profit by being an 'imitator' rather than an innovator and that each team of students has a strong incentive to free ride on the hoped-for contributions of other teams.

Brüggemann et Meub (2017) examine experimentally how cooperation among innovators evolves with and without contest schemes. Their experimental study focuses on the impacts of two types of innovation contests on subjects' innovativeness, by comparing contests with a prize for the cumulative innovativeness and contests with a prize for the best innovation. Their results show that both contest conditions decrease the willingness to cooperate between subjects compared to a benchmark condition without an innovation contest. Interestingly, while both contests have similar impacts, they found that the most sophisticated innovation is significantly more valuable when there is a prize for the best innovation.

Aghion et al. (2018) report an experiment on the effects of competition on step-by step innovation in a dynamic investment environment with different time horizons. They found that an increase in competition increases significantly the R&D investments by neck-and-neck firm; however, it decreases the R&D investments by laggard firms.

In comparison to the work of Zizzo (2002), Silipo (2005), Breitmoser et al. (2010), Brüggemann et Meub (2017) and Aghion et al. (2018), our experimental design involves a set of sequences (races) to examine the subjects' competitive behavior. Specifically, our design employs a random matching procedure in each new sequence, where participants are asked to engage in a patent race game similar to the previous sequence but with different initial positions. We experimentally test the theoretical patent race model of Fudenberg et al. (1983) and Halmenschlager (2006), with the consideration of the absorptive capacity, which has not been explored in the previous experimental studies. In addition to that, we focus on examining the effect of the initial position on the firm's investment decisions from several aspects, when the firm's type alters from leader to follower (or vice versa) and from symmetric to asymmetric position, as well as when the distance between the initial position to the goal of knowledge accumulation varies.

2.3 Model

We first provide a brief description of the theoretical model that our experimental design is based on. We consider a patent race game consisting of two firms that are competing for the new technology's discovery. Once this new technology is available, it is going to be patented. The first developer of such technology wins the race. In each period t, the two firms i and j make a simultaneous choice of the R&D effort, denoted as $x_i(t)$ and $x_j(t)$ respectively, among three different levels: no R&D, low R&D and high R&D which are specified by 0, 1, and 2, respectively. The cost associated is respectively denoted as 0, c_1, and c_2.

Following Halmenschlager (2006), we assume that $c_2 > 3c_1$, which implies a diminishing return of the R&D process. We also assume that $\frac{Y}{2} > [\frac{(N+2)}{3}]c_2$, where Y and N denote the prize of the race and the critical level of knowledge respectively. This assumption guarantees a positive payoff if the two firms maintain the high effort level in each period up to the end of the race and share the prize.

We consider absorptive capacity in the model, that is, the firms may benefit from spillovers of the other firm's R&D investment. The spillovers, $\varphi(x_i, x_j)$, is determined by the effort level of both firm i and firm j in each period and is independent of the R&D investment in the previous periods. Following Cohen and Levinthal (1989), we assume if a firm does not make any R&D effort in a period, then it does not have such benefit. Specifically, in each period the firms gain one extra point if both firms invest at level 1 or 2, and no extra point otherwise (i.e., $\varphi(x_i, x_j) = \varphi(x_j, x_i) = 1$ if and only if $x_i, x_j > 0$).

The experience (or knowledge accumulation) for firm i at the beginning of period t, denoted by $\omega_i(t)$, is the summation of firm i's R&D investment and gained spillovers in the past periods plus the initial experience (initial knowledge stock). Correspondingly, the knowledge level which is remaining for firm i to attain the finishing line is denoted by $k_i(t) = N - \omega_i(t)$. By competing with firm j, firm i is considered the leader in period t if $k_i(t) < k_j(t)$ while the firms are considered neck and neck when $k_i(t) = k_j(t)$. We focus on the subgame perfect Markovian Nash equilibrium, in which the firms use the remaining stock of knowledge to accumulate $k_i(t)$ as the state variable in period t. Appendix A contains the detailed formulas for the dynamics of the knowledge stock and payoff function.

The following proposition from Halmenschlager (2006) describes the equilibrium behavior in the subgame perfect Markovian equilibrium which uses the remaining knowledge stock $(k_i(t), k_j(t))$ as the state variable.

Proposition 1:

The unique subgame perfect Markovian equilibrium strategy is characterized as follows (where x_1 is the R&D effort of the leader and x_2 is the R&D effort of the follower):

Case 1: $(x_1, x_2) = (1,0)$ if $k_2 \geq k_1 + 2$. When the knowledge gap between the follower and the leader is greater than 2, the leader makes a low R&D effort up to the end of the race while the follower gives up the competition.

Case 2: (x_1, x_2) = (2,2) if $k_1 = k_2$. The firms choose a high effort level when they are neck and neck.

Case 3: x_1= [1, 2] and x_2= [0, 2] if $k_2 = k_1 + 1$. If the leader is one unit ahead the follower, a mixed strategy is chosen by the leader between a high and a low effort level while the follower randomizes between no effort and a high effort. In equilibrium, the latter catches up the former with a positive probability.

The proofs of proposition 1 are provided by Halmenschlager (2006).

Three possible outcomes may arise for firm i at the end of the race: 1) Firm i is the unique winner if it reaches the finishing line with a higher knowledge level than its rival. 2) Both firms win the race and share the patent value, if they reach or pass the finishing line with the same level of knowledge. 3) Firm i is the loser if it does not attain the finishing line while the other firm does $\{ k_i(t+1) > 0 \text{ and } k_j(t+1) \leq 0\}$ or if it reaches the line with a lower level of knowledge than its rival $\{k_i(t+1) > k_j(t+1) \text{ and } k_i(t+1) \leq 0\}$. In any case, the firm needs to pay the accumulated cost incurred during the R&D procedure. The firm gains the entire prize if he is the unique winner and shares the prize with the other firm equally if both firms are the winners.

In the sections below, both numerically and experimentally, we will focus on the pure strategy equilibrium behaviour (Case 1 and Case 2 in Proposition 1), since the prediction on the R&D investment in each period in these two cases is precise and the equilibrium path can be determined by the initial knowledge level. However, in Case 3, the mixed strategy and the dynamic feature of the game can lead to many different equilibrium paths given the same initial knowledge level, which makes it difficult to serve as a theoretical benchmark for the experimental data. Here we provide two numerical examples, one with symmetric initial knowledge levels (Case 2) and the other with asymmetric initial knowledge level (Case 1). We calculate the investment in each period and the dynamics of the knowledge stocks according to Proposition 1.

Example 1: Equilibrium path for the case $(\omega_1(1), \omega_2(1)) = (0,0)$, N=16

Table 2.1: Profile for Example 1

t	1	2	3	4	5	6	7
$x_1(t)$	2	2	2	2	2	2	
$x_2(t)$	2	2	2	2	2	2	
$\omega_1(t)$	0	3	6	9	12	15	18
$k_1(t) = N - \omega_1(t)$	16	13	10	7	4	1	-2
$\omega_2(t)$	0	3	6	9	12	15	18
$k_2(t) = N - \omega_2(t)$	16	13	10	7	4	1	-2
$\varphi(x_1(t), x_2(t))$	1	1	1	1	1	1	
Cost for every player	c_2	c_2	c_2	c_2	c_2	c_2	

Example 2: Equilibrium path for the case $(\omega_1(1), \omega_2(1)) = (2{,}0)$, N=16

Table 2.2: Profile for Example 2

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$x_1(t)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
$x_2(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
$\omega_1(t)$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$k_1(t) = N - \omega_1(t)$	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
$\omega_2(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_2(t) = N - \omega_2(t)$	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
$\varphi(x_1(t), x_2(t))$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Cost for player 1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	

2.4 Experimental design, hypotheses, and procedure

2.4.1 Experimental design

Based on the model, the experiment is featured by R&D races with an indefinite number of periods. In each period, the subjects' decisions of investment in R&D are simultaneous. In order to examine whether players' behavior in the experiment is consistent with the Subgame Perfect Markovian equilibrium as stated in Proposition 1, we use a within-subject design and vary the level of the initial stock of knowledge (low or high) and the distance between the initial asymmetric positions of the two firms, with a knowledge gap of 2 versus 4. Specifically, we design the races with symmetric low initial position at $(0, 0)$, symmetric high initial position at $(8, 8)$, asymmetric low initial position at $(0, 2)$ and $(0, 4)$, and asymmetric high initial position at $(8, 10)$ and $(8, 12)$.

By Proposition 1, symmetric and asymmetric initial positions lead to different levels of competition and different lengths of race. Specifically, in the symmetric case, the competition is more severe and both firms make a high effort, and correspondingly results in a shorter race. However, in the asymmetric case with a knowledge gap of at least 2, the competition is less severe and the race is much longer given that one firm will give up the race and the leading firm will only need to make a low effort.

The predictions in Proposition 1 only depend on the difference of knowledge level of the two firms in each period, k_1 compared to k_2, but the investment of the two firms in each period does not depend on how far it is from the finishing line. However, when the initial position is at a low level, we may observe a behavioral difference of the firms, since it is more difficult for the firms to figure out the equilibrium path when the race is expected to be longer.

In each session, there was a total of 10 sequences (races) ordered as in Table 2.3. The same asymmetric initial gap was always played twice in every treatment, since we switched the initial position of each subject across the two races. For instance, if the subject in race 2 has an initial position of 2, then he will have an initial position of 0 in race 3. Meanwhile, when each sequence started, subjects were randomly re-matched to avoid possible reputation effect. This way, subjects can experience both in the initial role as a leader and as a follower. As well, subjects' earnings can be more balanced and less affected by their initial roles. Based on the equilibrium path, the 10 races will take about 80 periods in total. In our program, each period takes about 1 minute.

We conducted 8 experimental sessions. For the first set of 4 sessions, we started by the low initial positions (Low treatment) for the first 5 races then we moved to the high initial positions (High treatment) for the remaining 5 races. We denoted these sessions as L-H treatment. In the other set of four experimental sessions, we reversed the order of the Low and High treatments and denoted them as H-L treatment.

Table 2.3: The set of sequences with their associated initial positions

Race		1	2	3	4	5	6	7	8	9	10
treatment	L-H	(0,0)	(2,0)	(0,2)	(4,0)	(0,4)	(8,8)	(10,8)	(8,10)	(12,8)	(8,12)
	H-L	(8,8)	(10,8)	(8,10)	(12,8)	(8,12)	(0,0)	(2,0)	(0,2)	(4,0)	(0,4)

Based on the model in Section 2.3, two assumptions, $\frac{Y}{2} > [\frac{(N+2)}{3}]c_2$ and $c_2 > 3c_1$, need to be satisfied for the uniqueness of the subgame perfect Markovian equilibrium. We chose the critical level of knowledge N=16, the prize Y=20, and the cost c_0 = 0 ECU, c_1 = 0.2 ECU, c_2= 0.8 ECU to satisfy these assumptions for all the races.

Based on the choice of these parameter values and given each set of initial experiences, we calculate the different possible payoffs as summarized in Table 2.4. In Appendix B, we show the corresponding paths of choices for the two firms and the details on how we calculate each payoff. Generally, the maximum payoff is achieved in a path where both players in a race invest at the low cost until the end of the race (in the asymmetric case) or close to the end of the race (in the symmetric case) so the spillover is mostly generated. Interestingly, the group efficient outcome can be achieved with different paths.[2] Regarding the group equilibrium outcome, which represents the total expected payoff of the two matched players, it ranges from 10.4 to 19.2 and these values depend on the equilibrium path and on the gap between the starting position and the target.

Table 2.4: The different types of payoffs with their associated treatment

IE & Payoff / Treatment	Initial Experience (IE)	Individual Equilibrium payoff	Group Equilibrium payoff	Individual Maximum payoff	Individual Minimum payoff	Group Efficient payoff
Low	(0,0)	(5.2, 5.2)	10.4	18.2	-4.2	16.8
Low	(2,0)	(17.2, 0)	17.2	18.6	-4.0	17.2
Low	(4,0)	(17.6, 0)	17.6	18.8	-3.2	17.6
High	(8,8)	(7.6, 7.6)	15.2	19.0	-1.8	18.4
High	(10,8)	(18.8, 0)	18.8	19.4	-1.6	18.8
High	(12,8)	(19.2, 0)	19.2	19.6	-1.6	19.2

Given the convex feature of the cost function, we notice that the symmetric initial positions such

[2] In this paper, we define a group as a pair of players who compete in a race.

as (0, 0) and (8, 8) will lead to an efficiency loss in equilibrium, i.e., the group equilibrium payoff is less than the group efficient payoff. For instance, in Example 1 in Section 2.3, the efficient outcome can be reached if one firm invests each period by 1 point but the other firm gives up at the beginning of the race, which leads to the group efficient payoff $Y - 16c_1 = 20 - 16 * 0.2 = 16.8$. Intuitively, the efficiency loss in races with a symmetric position comes from the excess competition between the two firms, both of whom invest at the high level. In contrast, if the race has asymmetric initial positions, such as Example 2 in Section 2.3, in equilibrium the follower will give up from the beginning of the race, so the equilibrium outcome will coincide with the efficient outcome.

2.4.2 Hypotheses

Based on the experimental design, the comparison between the low and high treatment and between the symmetric and asymmetric initial experiences identifies the following hypotheses that are going to be verified through the statistical estimation:

Race Outcomes

Hypothesis 1: (*Starting position effect as leader vs. tie vs. follower)*

The probability for the subject to win a race is highest (lowest) when he starts as a leader (follower).

Hypothesis 2: (*Low initial knowledge versus high initial knowledge effect)*

Fixing the subject's starting position and the knowledge gap at the beginning of the race, the frequency of winning a race has no significant difference between the low and high treatment.

Investment Behavior

Hypothesis 3: *(Effort level based on the position of the round)*

In each round within the race, the subject's effort level is highest in a tie position, second highest as a leader, and lowest as a follower.

Hypothesis 4: *(Effort level between low and high treatments)*

Fixing the position of each round, the subject's effort levels in the low and high treatments are not significantly different.

Hypothesis 5: *(Spillover effect)*
The spillovers in the previous round have significant effect on the players' investment behavior in the current round.

Efficiency Loss
Hypothesis 6: *(Symmetric vs. asymmetric starting position of the race)*
In each treatment, the efficiency loss in a race with a symmetric starting position is significantly higher than with an asymmetric starting position.

Hypothesis 7: *(Efficiency loss between low and high treatments)*
In the symmetric initial position, the efficiency loss is higher in the low treatment compared to the high treatment while in the asymmetric initial position, there is no significant difference between the low and high treatment.

2.4.3 Experimental procedure

In every session, there were 10 sequences as discussed in Table 2.3. In every sequence, subjects were randomly and anonymously matched with each other in order to compete in the R&D race. Within the sequence, they remained matched with the same opponent. Subjects are explicitly informed of the matching protocol. Before moving to the next sequence, every pair had to wait until the other pairs finished their races.

In each round of a sequence, the matched players decided whether to invest 0, 1, or 2 points simultaneously, which cost respectively c_0, c_1 and c_2. Prior to proceeding to the next round, every player had to wait until his/her matched player completed his task. In each round, before the player made his decision, he would be able to observe the choice of his matched player in all the previous rounds, as well as his accumulated points and his matched opponent's accumulated points. In order to avoid the race lasting forever when no one invests, we impose a stopping rule that, if two players within a group choose zero for three consecutive rounds, the competition is terminated and the prize is lost.

If one player in the pair invested 0 point, neither of the two players in the match would receive the extra point of the spillover. Only when both players within a group invested 1 or 2 points, they would receive the point of the spillover for the round.

When one player or both players in a pair reach or pass a level of accumulation of 16 points, a sequence ends. Otherwise, the sequence will continue into next round, in which the total points and the total costs accumulated in all previous rounds of the same sequence will carry over into the next round. The outcome from every sequence will not be transferred to the next sequence given that everything will be reset all over again at the start of a new sequence.

In every sequence, the prize of 20 ECU was given to the player who first reaches or passes the level of accumulation of 16 points. If the player is the winner, he will get 20 ECU less the costs he bared to accumulate 16 points. If the two subjects accumulate 16 points in a same round, they will share the prize and each gets 10 ECU, less the costs they bared to accumulate 16 points. If the matched player accumulates 16 points first, the game ends and the other player gets no prize from the game but he still needs to pay the costs that have occurred.

At the end of the session, subjects were given 10$ for showing up on time and finishing the experiment and they were also paid their total earnings from all the sequences. The total payoffs from these sequences were converted into dollars at a known and fixed rate of 1 ECU = CAD$0.25. The experiment took place at CIRANO. It was programmed and conducted with the z-Tree software. Participants were recruited from the regular subject pool maintained by the lab in CIRANO. Each session lasted approximately 2 hours, including the first 45 minutes during which the experimenter read the instructions, solicited questions from the participants and asked them to complete a quiz to test their understanding of the key concepts covered in the experiment. The remaining time of the session (75 minutes) was devoted to playing the games. At the end, the participants were paid privately and left the laboratory one at a time.

2.5 Experimental results

In this section, we present our empirical results and compare them to the theoretical predictions. We first provide the race outcomes, then we discuss the subjects' behavior, the proportion of consistency with the equilibrium path and the efficiency loss.

2.5.1 Race outcomes

We first present the summary of the races in each session and examine the race outcomes in relation with the initial position of the players (being a leader, a follower, or in the symmetric position) in the low and high treatment. As shown in the following table, a total of 94 subjects participated in 8 experimental sessions. Out of 470 races, 50 races ended with 2 winners, 415 races ended with 1 winner and 5 races ended with no winner. Subjects earned on average, $29.48 including the $10 show-up fee.

Table 2.5: Session Summary

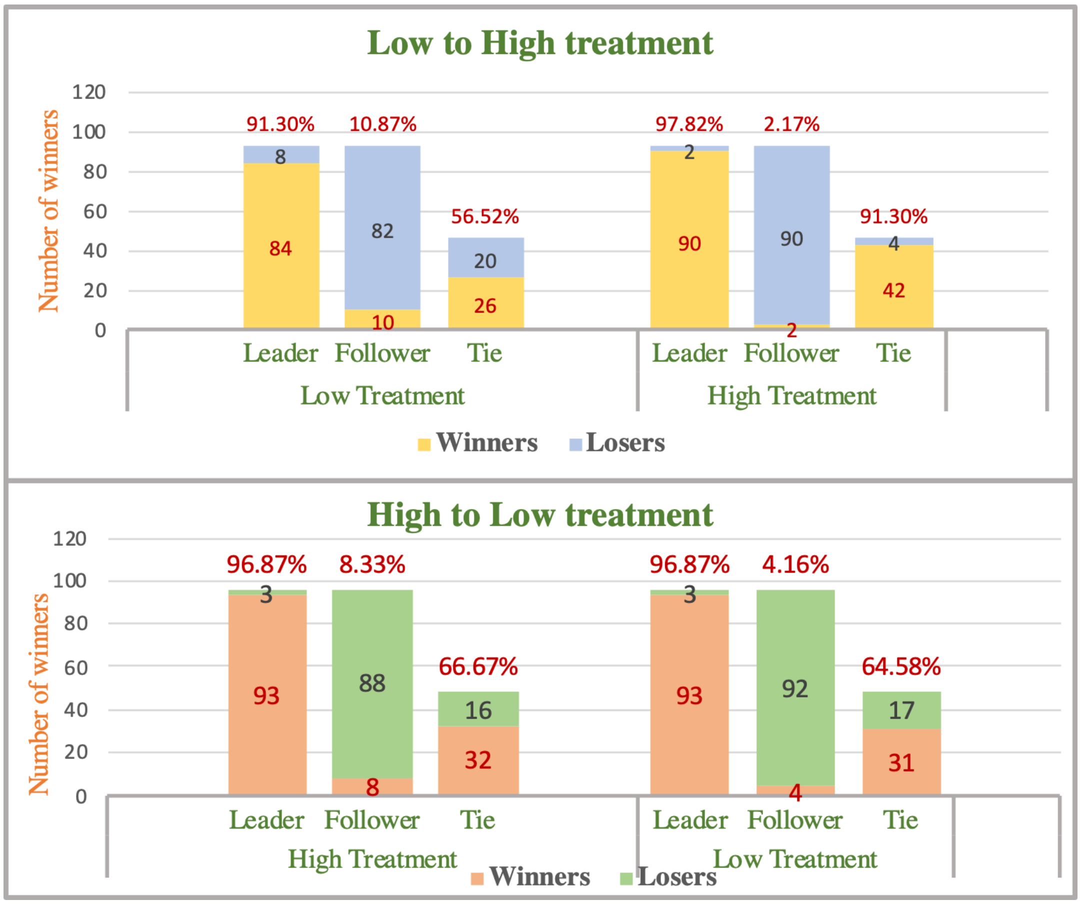
Session	Treatment	No. of Subjects	No. of Races with 2 winners	No. of Races with 1 winner	No. of Races with no winner	Total number of races
1	L-H	10	6	41	3	50
2	L-H	14	10	60	0	70
3	L-H	10	5	45	0	50
4	L-H	12	6	54	0	60
5	H-L	12	7	52	1	60
6	H-L	14	8	62	0	70
7	H-L	10	5	44	1	50
8	H-L	12	3	57	0	60
Total	N/A	94	50	415	5	470

Figure 2.1 shows the number and percentage of winners given players' initial status as a leader, follower, or in a symmetric position, in the low and high initial position respectively. We see that in both low and high treatments, the initial status has a decisive influence on the chance of winning the race. On average, being a leader in the initial position results in winning a race with about 95% frequency. The frequency of winning is always the lowest when the players start as a follower and highest when the players start as a leader. This pattern holds in both low and high initial experience, and regardless of the order of the low and high treatments.

By calculating the frequency of winning at the individual level given each position (as a leader, follower, or symmetric position), we found that the results from the two-tailed Wilcoxon signed rank tests confirm the significant difference as stated in Finding 1 ($p < 0.001$ for all the 6 pair-wise tests, Leader vs. Follower, Leader vs. Tie, Follower vs. Tie, for low treatment and high treatment respectively, 94 observations using all data in the L-H and H-L treatments).[3]

Finding 1: The frequency of winning a race is highest when players start as leaders, second highest when they start in a symmetric position, and lowest when they start as followers. This pattern holds for both low and high treatments.

Figure 2.1: Number and percentage of winners given the starting position



We also examine the order effect between the L-H and H-L treatment by conducting the two-tailed Wilcoxon signed rank tests on the individual frequency of winning between the low and high treatments given each initial position. Interestingly, when the sequence starts from low initial

[3] If we conduct the same tests separately for the L-H treatment (6 tests, 46 obs.) and H-L treatment (6 tests, 48 obs.), Finding 1 still generally holds, except for one test on the comparison between the leader position and the symmetric position in the high initial experience of the L-H treatment.

experience and moves to high initial experience (L-H treatment, Session 1 to 4), we found that the effect of the initial stock of knowledge in the low or high level interacts with the position of the players. Specifically, when the starting position is a leader (follower), the frequency of winning a race in the high treatment is significantly higher (lower) than in the low treatment ($p < 0.1$). Most importantly, when the starting position is symmetric, the frequency of winning a race in the high treatment is significantly higher than in the low treatment ($p < 0.01$). These results imply that when the initial stock of knowledge is closer to the critical level, the race outcome is converging to the equilibrium predictions. Specifically, with an asymmetric starting position, the winner of the race is more likely to be determined by the starting position in the high treatment. With a symmetric starting position, the competition in the high treatment is more severe and it is more likely both players will win the race and share the prize, which can be verified by the fact that on average 91.3% of players won the race in this case. Alternatively, we cannot exclude the possibility that subjects learn over time about the equilibrium play, since the high treatment is always in the second half of the session in the L-H treatment.

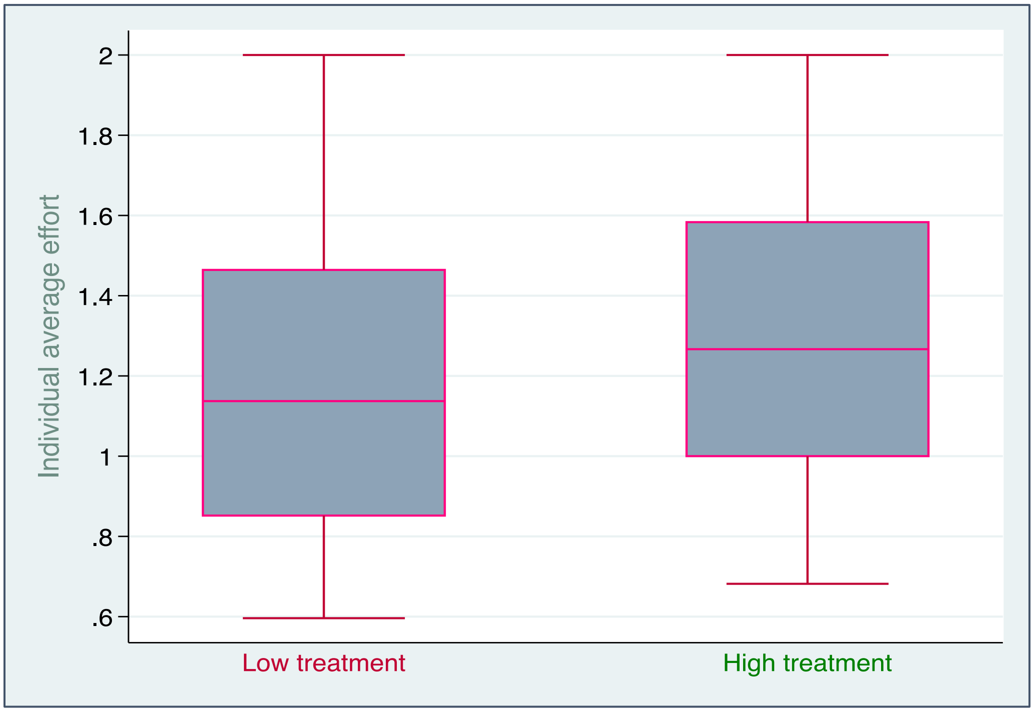
However, when the sequence starts from high initial experience and moves to low initial experience (H-L treatment, Session 5-8), we found that, conditional on each starting position, there is no significant difference in the frequency of winning between the low and high treatment, which indicates that the behavior of the players is statistically stable.

Finding 2: In the L-H treatment, the race outcome given the high initial positions is more consistent with the equilibrium predictions compared to the low initial positions. However, in the H-L treatment, the race outcome given the high initial experience is not significantly different from the low initial experience for any starting position.

2.5.2 Investment Behavior

To have a good indication of how the values of individual average effort are spread out in the data (94 observations in total), we present two boxplots in Figure 2.2. For both low and high treatment, the maximum individual average effort reaches the maximum effort level of 2. In the low treatment, the minimum value, the lower quartile, the median and the upper quartile for the individual average effort are, respectively, 0.596, 0.851, 1.137 and 1.464. In the high treatment, the corresponding minimum value and quartiles are 0.681, 1, 1.266 and 1.583, which are all higher than in the low treatment.

Figure 2.2: Box plots for the individual average effort in the low and high treatment



In order to further examine whether the investment behavior of the players depends on the position of the player in each round during the race, we present in Figure 2.3 the mean of individual average effort conditional (and unconditional) on players' position at the start of every round (as a leader, follower, or tie position) in the low and high treatment respectively.

Furthermore, we conduct the pairwise Wilcoxon signed rank tests that compare the individual average effort when players are in different positions at the start of each round (leader, follower or tie), and we report the p-values in Table 2.6. Overall, regardless of whether we consider the L-H or H-L treatment or both, the difference between the individual average effort of each position is statistically significant and consistent with Proposition 1.

Finding 3: Overall, the result from each treatment (regardless of whether it is in the L-H or H-L or both) shows that the individual average effort is the highest for the players who are in a tie position, second highest for the leaders and lowest for the followers, which supports Hypothesis 3.

Figure 2.3: Mean of individual average effort given players' status in each round in the low and high treatment

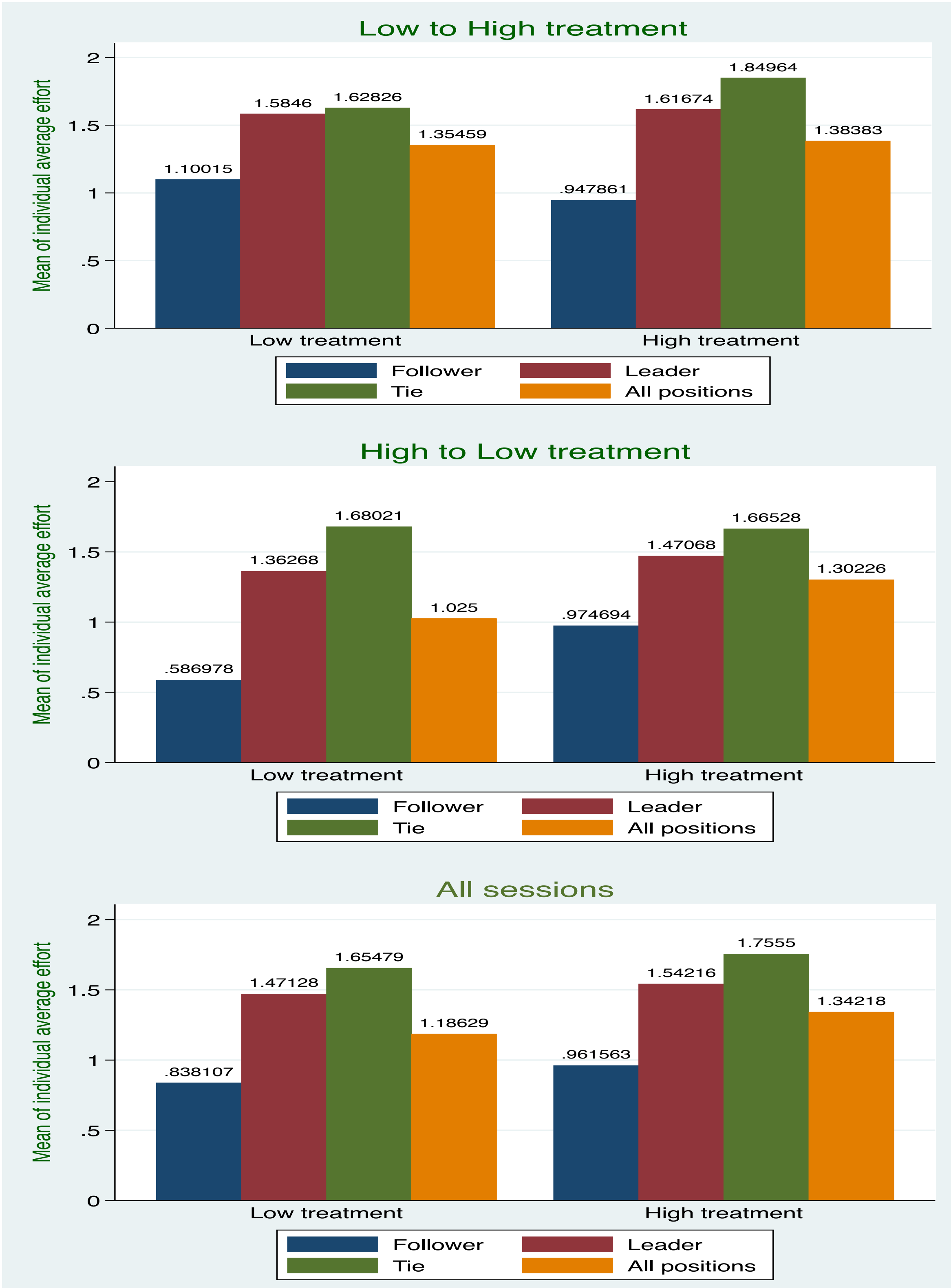


Table 2.6: P-value of signed rank tests on individual average effort (Comparison between different positions)

Treatment	Comparison	L-H treatment (Session 1 to 4)	H-L treatment (Session 5 to 8)	All sessions
Low treatment	Follower vs. Leader	0.000	0.000	0.000
	Follower vs. Tie	0.000	0.000	0.000
	Leader vs. Tie	0.566	0.000	0.001
High treatment	Follower vs. Leader	0.000	0.000	0.000
	Follower vs. Tie	0.000	0.000	0.000
	Leader vs. Tie	0.000	0.017	0.000

Table 2.7: P-value of signed rank tests on individual average effort(low vs. high treatment)

	L-H treatment	H-L treatment	All sessions
Follower	0.052 Low>High	0.000 Low<High	0.058 Low<High
Leader	0.788 Low ≈ High	0.031 Low<High	0.084 Low<High
Tie	0.002 Low<High	0.911 Low ≈ High	0.05 Low<High
All positions	0.438 Low ≈ High	0.000 Low<High	0.000 Low<High
Gap: Leader vs. Follower	0.069 Low<High	0.01 Low>High	0.587 Low ≈ High
Gap: Tie vs. Follower	0.001 Low<High	0.004 Low>High	0.968 Low ≈ High
Gap: Tie vs. Leader	0.014 Low<High	0.122 Low ≈ High	0.689 Low≈High

Table 2.7 reports the p-value of Wilcoxon signed rank tests which compare the individual average effort conditional or unconditional on players' position in each round between the low and high treatment. We find different patterns in the L-H treatment and H-L treatment.

In the L-H treatment, the results show that there is no significant difference between the low and high treatment if considering all three positions ($p = 0.438$, 46 observations using all the data in the L-H treatment). However, conditional on each position at the start of every round and in comparison with the low treatment, the high treatment shows that the individual average effort is significantly higher for the players who are in a tie position ($p = 0.002$), lower for the followers ($p = 0.052$) and not significantly different for the leaders ($p = 0.788$), which means that on average the leadership behavior is statistically stable.

In the H-L treatment, we find that there is a significant difference in terms of the individual average effort between the low and high treatment if considering all positions (p = 0.000). Furthermore, conditional on each position at the start of every round and in comparison with the high treatment, the low treatment shows that the individual average effort is significantly lower for the followers and the leaders (p = 0.000 and 0.031 respectively) and not significantly different for the players who are in a tie position (p = 0.911).

We also look at the gap in terms of individual average effort of the players who are at different starting positions (leaders vs. followers, tie vs. followers, and tie vs. leaders). As shown in Figure 2.3 and Table 2.7, we find that regardless of whether the treatment is L-H or H-L, all the gaps become significantly larger in the second half of the session except that between the leaders and the players who are in a tie position in the H-L treatment (p = 0.122). Based on the above test results, we formulate the following Finding 4.

Finding 4: Compared to the first half of the session in the L-H or H-L treatment, the last five sequences show that there is overall evidence of learning over time.

In order to further analyze the individual R&D investment behavior in a multivariate framework, we use panel data OLS regression models on the level of effort and present the results in Table 2.8. The variable "effort" is the dependent variable which is equal to 0, 1, or 2. "Know_gap" is the players' knowledge gap compared to their partner at the start of each round, which is negative (positive) when the player is a follower (leader) and 0 otherwise. "Leader" and "Follower" are dummy variables, which is equal to 1 if the players are leaders and followers, respectively, at the start of a round and is 0 otherwise. "High_treatment" is equal to 1 if it is the high treatment and 0 otherwise, and "High_low" is equal to 1 if the treatment is from high to low, 0 otherwise. We also include the interaction terms "Leader_ht" which is "Leader * high_treatment", "Follower_ht" which is "Follower*high_treatment", "High_low* high_treatment". Finally, "Spillover(t-1)" is the spillover in the previous round of the same sequence.

In all the regressions, we find knowledge gap at the start of each round has a significant effect on the effort level, but the coefficient is quite small. In the L-H treatment (Regression 1), the effort level for the players who are in a symmetric position is significantly higher in the high treatment compared to the low treatment, but such effect disappears in the H-L treatment (Regression 2).

Table 2.8: OLS regressions on individual effort

Dependent variable: **Effort**	**(1)** L-H treatment	**(2)** H-L treatment	**(3)** All sessions	**(4)** All positions	**(5)** All positions
Know_gap	0.054***	0.029***	0.040***	0.057***	0.057***
	(0.010)	(0.005)	(0.004)	(0.004)	(0.004)
Leader	-0.385***	-0.658***	-0.534***		
	(0.088)	(0.059)	(0.054)		
Leader_ht	-0.171	0.320**	0.098		
	(0.109)	(0.101)	(0.077)		
Follower	-0.429**	-1.109***	-0.788***		
	(0.137)	(0.114)	(0.094)		
Follower_ht	-0.426***	0.463***	0.039		
	(0.094)	(0.110)	(0.087)		
High_treatment	0.228***	-0.112	0.052	0.024	0.161***
	(0.066)	(0.074)	(0.052)	(0.047)	(0.032)
High_low				-0.328***	
				(0.070)	
High_low*high_treatment				0.254***	
				(0.060)	
Spillover (t-1)					
Constant	1.705***	1.816***	1.764***	1.340***	1.167***
	(0.069)	(0.046)	(0.042)	(0.054)	(0.039)
N	2130	2674	4804	4804	4804

Clustered standard errors (by subject) in parentheses
Notes: * p<0.05, ** p<0.01, *** p<0.001
Subject-period as unit of observation

These results are consistent with Figure 2.3 and Table 2.7. Regardless of whether the treatment is L-H or H-L or both (Regressions 1, 2 and 3), the effort is significantly the highest for the players who are in a symmetric position, the second highest for the leaders and the lowest for the followers. This result is more robust in the low treatment. Conditional on all sessions and all players' positions (Regression 5), we find that the effort in the high treatment is significantly higher than the effort in the low treatment ($p < 0.001$) as shown in Figure 2.3 and Table 2.7.

Conditional and unconditional on players' positions, we further examine how the investment behavior is affected under the existence of spillovers in the previous round. For that purpose, we use panel data OLS regression models on the level of effort as well and we present the results in

Table 2.9. The independent variables are defined same as in Table 2.8, except that now, we include two interaction terms "Leader_spillover" which is "Leader * Spillover(t-1)" and "Follower_spillover" which is "Follower* Spillover(t-1)".

Table 2.9: Individual effort and spillovers effect

Dependent variable **Effort**	**All Data**		**L-H treatment**		**H-L treatment**	
	(1) All positions	**(2)** All sessions	**(3)** Low treatment	**(4)** High treatment	**(5)** Low treatment	**(6)** High treatment
know_gap	0.058***	0.017***	0.060***	0.105***	0.053***	0.087***
	(0.002)	(0.004)	(0.005)	(0.012)	(0.002)	(0.009)
Spillover(t-1)	0.810***	0.329*	0.718***	0.787***	0.933***	0.594***
	(0.030)	(0.137)	(0.064)	(0.072)	(0.038)	(0.073)
leader		-0.387**				
		(0.140)				
leader_spillover		0.161				
		(0.144)				
follower		-1.150***				
		(0.136)				
follower_spillover		0.667***				
		(0.146)				
Constant	0.745***	1.511***	0.855***	0.813***	0.633***	0.890***
	(0.033)	(0.133)	(0.070)	(0.069)	(0.033)	(0.066)
N	3864	3864	1262	408	1744	450

Clustered standard errors (by subject) in parentheses
Notes: * p<0.05, ** p<0.01, *** p<0.001
Subject-period as unit of observation

Overall, conditional on all positions (Regression 1) and regardless of whether the player is a leader or a follower or in a symmetric position (Regression 2), we find that there is a positive and a significant relationship between the spillovers in the previous period and the effort in the current period, indicating that getting an external technological benefit constitutes an incentive for the players to provide more investment.

In the L-H treatment (Regressions 3 and 4), the results show that the effect of these spillovers on the R&D investment is significantly larger in the high treatment compared to the low treatment while the opposite is true for the H-L treatment (Regressions 5 and 6).

Finding 5:

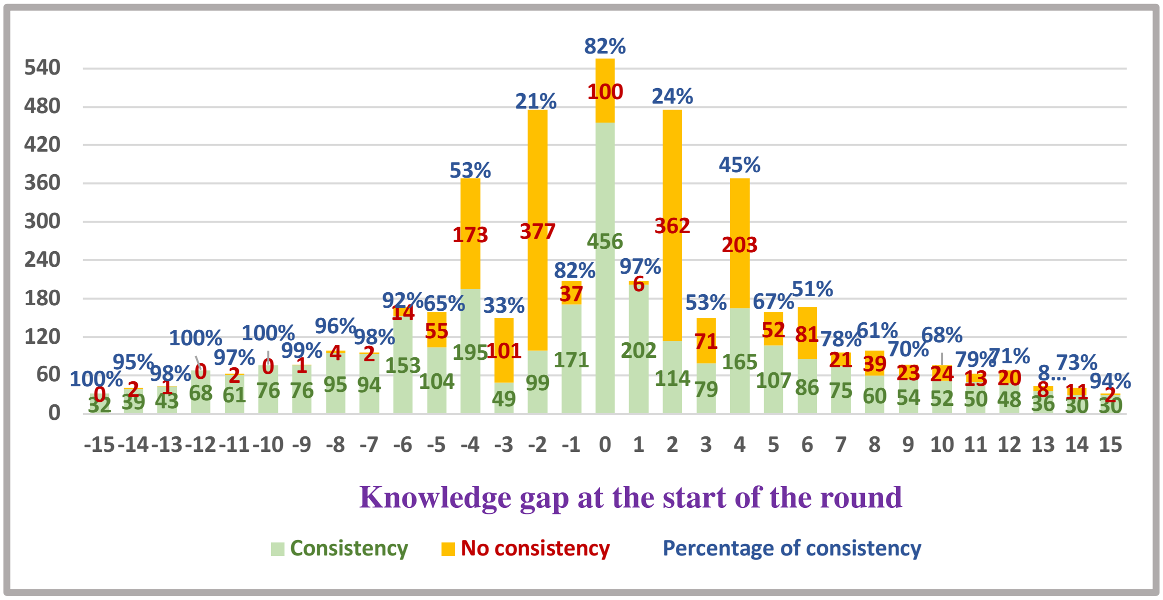
Regardless of whether the treatment is L-H or H-L or both, the spillovers in the previous round significantly increase the players' investment in the current round.

2.5.3 Equilibrium Consistency

In this subsection, we examine whether and to which extent the investment behavior in our experimental data is consistent with the equilibrium predictions. For the investment choice of each player in each round of a sequence, we create a dummy variable that takes the value of 0 or 1, where 1 indicates that the investment behavior, conditional on the knowledge gap at the beginning of that round, is consistent with the predictions in Proposition 1. Figure 2.4 presents the frequency of this dummy variable being equal to 1 or 0 given each knowledge gap at the start of the round. We see that the knowledge gap, in our data, ranges from -15 to 15. The consistency is high when the knowledge gap is large in absolute value ($k \leq -5$ or $k \geq 5$), and it is higher for followers (($k \leq -5$, with an average frequency of 94%) than for leaders ($k \geq 5$, with an average frequency of 72%) in this range. There is also high consistency with the equilibrium prediction when the players are in a symmetric position ($k = 0$) where both firms should make a high effort level based on the predictions. When the absolute value of the knowledge gap is equal to 1, the behavior of the leaders is more consistent with the mixed strategy equilibrium prediction than that of the followers, indicating that the followers not only randomize between zero and a high effort as theory predicts but also invest at a low effort. Finally, we notice that the consistency level is relatively low when the knowledge gap ranges from -4 to -2 and from 2 to 4.

To better understand the investment behavior when the knowledge gap in the range $2 \leq |k| \leq 4$, we calculate the frequency of each effort level at each knowledge gap in this range, as shown in Table 2.10. The equilibrium predicts that when the knowledge gap is greater than or equal to 2 in absolute value, the follower should give up the competition while the leader should invest at a low effort up to the end of the race. Table 2.10 shows that when the knowledge gap is equal to -2 or 2, the highest effort is chosen more than 60% of time, implying too much competition going on compared to the equilibrium prediction. The followers were trying to catch up with the competition while the leaders were trying to secure the leading position.

Figure 2.4: Frequency of the investment behavior consistent with the equilibrium predictions conditional on knowledge gap



When the knowledge gap moves to -3 and -4 for the followers and to 3 and 4 for the leaders, we observe the frequency of the high effort reduces significantly, on average by more than 20%. The followers switched to choosing zero effort more frequently, and correspondingly, the leaders increased the frequency of the low effort. This trend indicates a convergence to the equilibrium behavior.

Table 2.10: Frequency for each effort level conditional on knowledge gap

Knowledge gap at the start of each round	-4	-3	-2	2	3	4
Effort =0	53%	33%	21%	3%	3%	4%
Effort =1	16%	21%	18%	24%	53%	45%
Effort =2	31%	47%	61%	73%	44%	51%

Next, we further examine the consistency at the individual level. We calculate the individual frequency of consistency by dividing the total number of rounds of investment that is consistent with the pure strategy equilibrium predictions (PSEP) by the total number of rounds of decisions each player made. In Figure 2.5, we show the mean of individual frequency of consistency conditional (and unconditional) on players' position at the start of every round (as a leader, follower, or tie position) in the low and high treatment respectively.

Figure 2.5: Mean of individual frequency of consistency

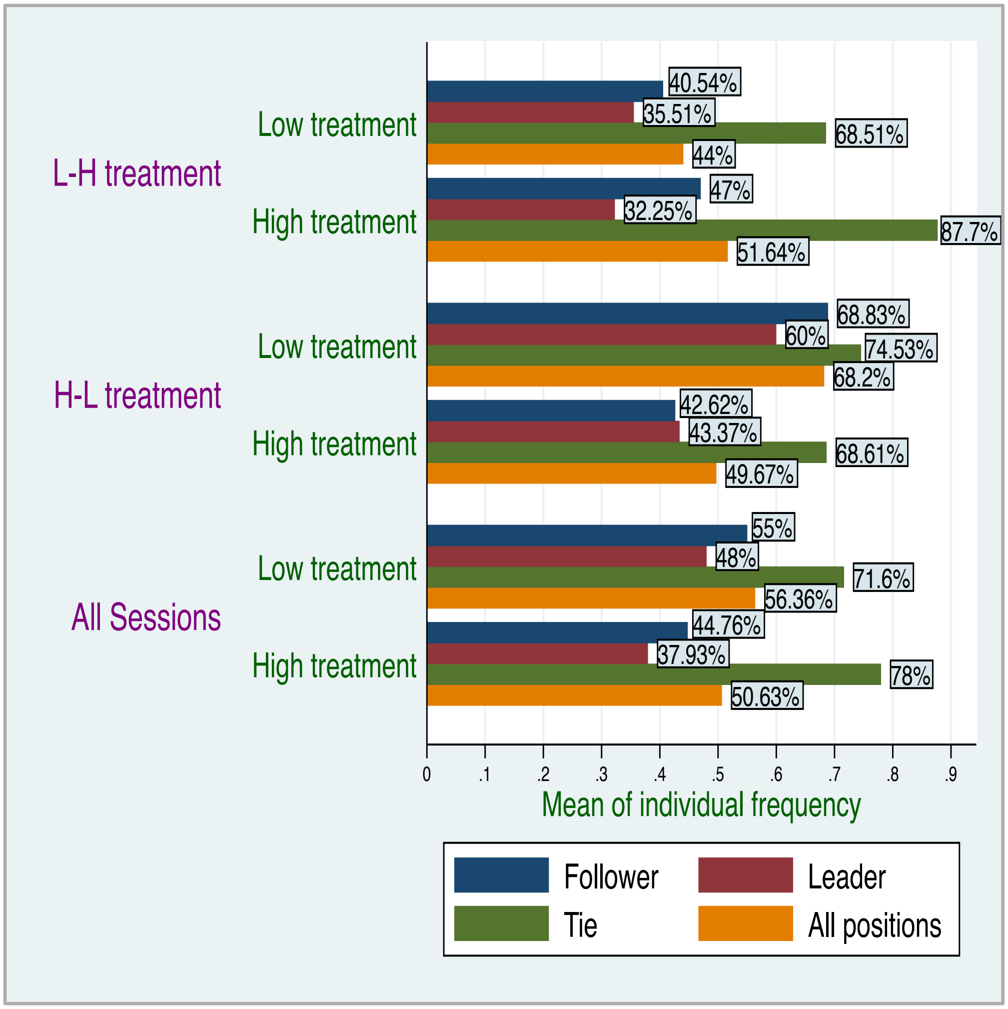


Table 2.11 reports the p-values of Wilcoxon signed rank tests on the individual frequency of consistency with the PSEP conditional or unconditional on players' position in each round between the low and high treatment. We found that, regardless of whether the order of the treatment is L-H or H-L, in general the players' behavior is more consistent with the PSEP in the second half of the sessions (p = 0.005 for L-H treatment and p = 0.000 for H-L treatment). In the L-H treatment, this learning effect is more significant for the followers (p = 0.098) and for the players in a tie position (p = 0.002). While in the H-L treatment, it is more significant for the followers and the leaders (p = 0.000 and 0.001 respectively).

Table 2.11: P-value of signed rank tests on individual frequency of consistency with the PSEP (low vs. high treatment)

	L-H treatment	H-L treatment	All sessions
Follower	0.098 Low<High	0.000 Low>High	0.009 Low>High
Leader	0.220 Low ≈ High	0.001 Low>High	0.001 Low>High
Tie	0.002 Low<High	0.729 Low ≈ High	0.078 Low<High
All positions	0.005 Low< High	0.000 Low>High	0.037 Low>High

Finding 6: Players' behavior is significantly more consistent with the pure strategy equilibrium predictions in the second half of the session than in the first half of the session, regardless of whether the treatment is L-H or H-L.

To examine whether the behaviors of the players are consistent with the PSEP in a multivariate framework, we use Probit regression models on the consistency level. The regression results from each model are presented in Table 2.12 below. The dependent variable "consistency" is a dummy variable that is equal to 1 if there is consistency with the equilibrium predictions and 0 otherwise. The independent variables are defined same as in Table 2.8.

The regression results show that, in the L-H treatment (Regression 1), only the behaviors of the leaders are less likely to be consistent with the PSEP in the high treatment compared to the low treatment. However, in the H-L (Regression 2), this result is true regardless of the players' position at the start of a round which means that there is a learning effect by moving from high to low initial stock of knowledge.

Conditional on all players' positions, Regression 4 indicates that there is an overall evidence of learning effect since regardless of whether the treatment is L-H or H-L, the second half of the session shows that the players learn over time about the equilibrium play.

Table 2.12: Probit regressions on the consistency level

	(1) L-H treatment		(2) H-L treatment		(3) All sessions		(4) All positions	
Dependent variable: **Consistency**	Coefficient	Marginal Effect	Coefficient	Marginal Effect	Coefficient	Marginal Effect	Coefficient	Marginal Effect
Leader	-1.304***	-0.456***	-0.871***	-0.294***	-1.058***	-0.389***		
	(0.249)	(0.075)	(0.233)	(0.082)	(0.167)	(0.059)		
Leader_ht	-0.923***	-0.323***	0.003	0.001	-0.399*	-0.147*		
	(0.276)	(0.099)	(0.264)	(0.089)	(0.199)	(0.073)		
Follower	-1.000***	-0.349***	-0.529**	-0.178*	-0.744***	-0.274***		
	(0.231)	(0.074)	(0.204)	(0.072)	(0.151)	(0.056)		
Follower_ht	-0.713*	-0.249*	-0.334	-0.112	-0.449*	-0.165*		
	(0.283)	(0.101)	(0.223)	(0.075)	(0.177)	(0.065)		
High_treatment	0.896***	0.313***	-0.560*	-0.189*	0.073	0.027	0.244**	0.090**
	(0.247)	(0.088)	(0.234)	(0.079)	(0.183)	(0.067)	(0.074)	(0.027)
High_low							0.763***	0.281***
							(0.153)	(0.050)
High_low* high_treatment							-0.854***	-0.315***
							(0.120)	(0.042)
Constant	0.834***		1.258***		1.053***		-0.157	
	(0.18)		(0.183)		(0.129)		(0.101)	
N	1918	1918	2470	2470	4388	4388	4388	4388

Clustered standard errors (by subject) in parentheses

Notes: * p<0.05, ** p<0.01, *** p<0.001

Subject-period as unit of observation

2.5.4 Efficiency Loss

In this part, we examine the efficiency level that the players achieve by comparing the actual total payoff at the group level in each race to the corresponding group efficient payoff which we present in Table 2.4. Based on the experimental results, we found that there is a deviation from the efficient outcome which may be caused by two distinct ways: the first one is through the learning effect since the players learn over time about the equilibrium play (Finding 4), and the second one is through the chosen inefficient effort that affects the players' payoffs (for example, based on the results from all the sessions, Table 2.5 shows that 5 races ended with no winner given that the zero effort was chosen by the two matched players for more than three consecutive rounds).

Based on the group-level percentage of efficiency loss of each race, we calculate the session-level average percentage of efficiency loss for every knowledge gap at the start of a race in the Low and High treatment of every session. In Figure 2.6, we present the mean of session-level average percentage of efficiency loss given the different initial positions (IP) at the start of the race in the low and high treatment.

Figure 2.6: Mean of session-level average percentage of efficiency loss

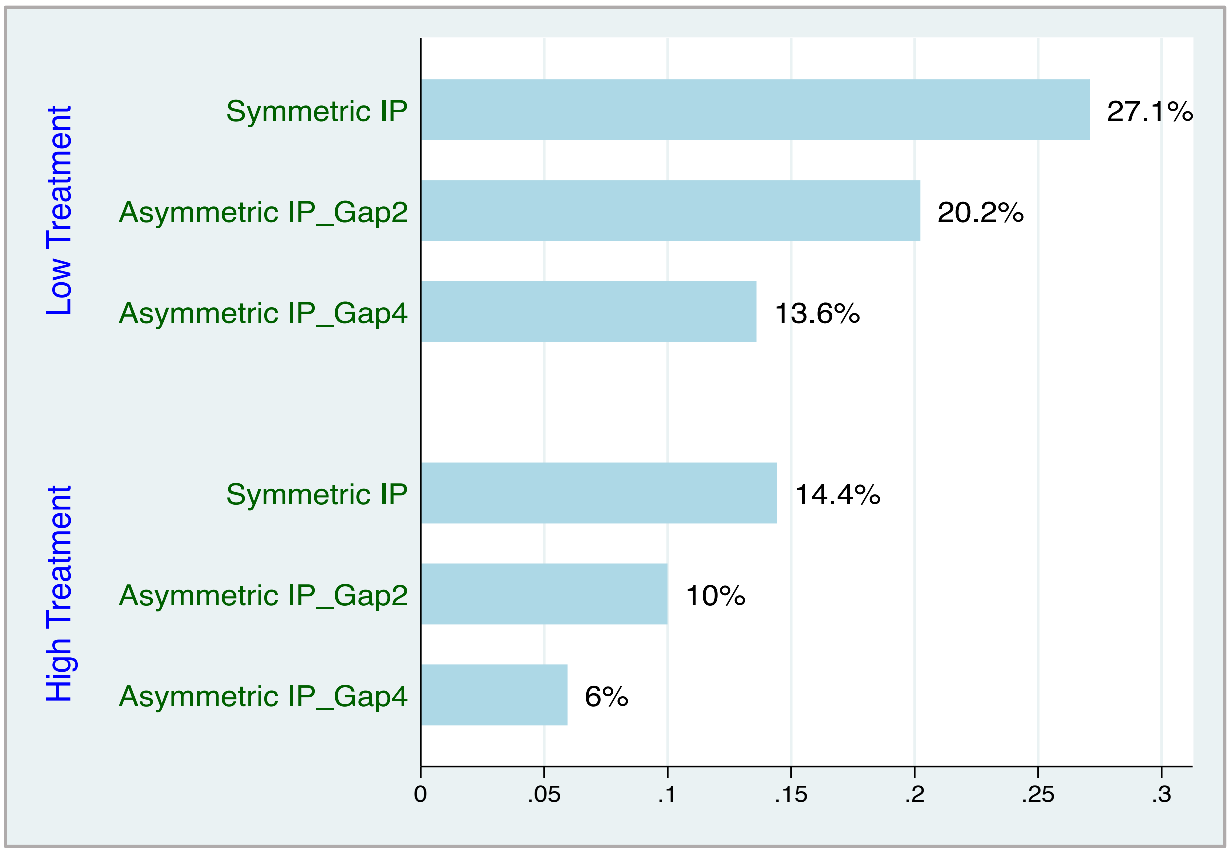


Figure 2.6 shows two patterns on the percentage of efficiency loss. First, fixing the initial knowledge gap, the percentage of efficiency loss is smaller in the high treatment than in the low treatment. Notice that this pattern does not support Hypothesis 7. Second, when the initial knowledge gap of the race becomes smaller from 4 to 2 and from 2 to the symmetric initial position, the percentage of efficiency loss becomes larger. In the low treatment, the highest percentage of efficiency loss occurs given the symmetric IP and reaches about 27%. This is probably due to the fact that there is too much competition between the players and indicates the investment behaviour is more consistent with the equilibrium strategy than the efficient strategy.

In order to examine whether these two patterns are significant, we conduct the pairwise Wilcoxon signed rank tests using the session-level average percentage of efficiency loss.[4] The test results confirm the significant difference between the low and high treatment conditional on each initial knowledge gap ($p < 0.05$ for all the two-tailed pair-wise tests, 8 obs.). Furthermore, given the low or high treatment, the average percentage of efficiency loss generally becomes significantly lower as the initial knowledge gap increases from the symmetric position to IP = 2 and from IP = 2 to IP = 4 ($p < 0.05$ for 5 two-tailed pair-wise tests, 8 obs.), except the comparison between IP = 2 and IP = 4 in the high treatment ($p = 12\%$, 8 obs.). We summarize these two patterns in Finding 7.

Finding 7: Conditional on each initial knowledge gap, the average percentage of efficiency loss is significantly higher in the low treatment compared to the high treatment. Conditional on the low or high treatment, the average percentage of efficiency loss decreases as the initial knowledge gap increases.

Intuitively, when the initial knowledge gap moves from 2 to 0, the frequency of choosing high effort increases which leads to an increase in terms of the efficiency loss and when the knowledge gap moves from 2 to 4, two possibilities occur. On the one hand, in some races, the players are behaving in a more consistent manner with the equilibrium outcome which reflects the evidence of learning effect about the equilibrium play. On the other hand, in other races, the players are behaving in more consistency with the efficient outcome where the followers are trying to catch up.

2.6 Conclusion

In this paper, we provided experimental evidence shedding light on how the introduction of different initial positions and different types to each player influences the competition behavior and outcomes in a setting of dynamic patent races with absorptive capacity. Our primary focus was on strategic decision-making within the competition. In every period, each subject made independent and simultaneous decisions on their R&D efforts to reach a predetermined level of

[4] In the z-Tree program, we randomly matched players and randomly assigned the group number to each pair of players in each race, so we cannot use the paired signed rank test using the group-level data on efficiency loss.

knowledge and win the prize. The overall R&D level of each firm is a summation of their own past efforts and a portion of their competitor's efforts, which spills over when both firms make R&D effort.

In our experiment, we had 8 experimental sessions, each involving subjects playing ten sequences (races). Since we do not know when the critical level of knowledge will be reached, each sequence had an unspecified number of periods for obtaining the prize. Within each sequence, participants were randomly and anonymously matched and their initial knowledge positions were either symmetric or asymmetric. We also included the variation in distances between the starting position and the target, categorizing them as "low treatment" for scenarios with a long distance to the target, and "high treatment" for scenarios with a short distance to the target. In order to investigate the impact of the order of the treatments (low and high) on the subjects' behavior, we divided the 8 experimental sessions into two groups: 4 sessions began with the low treatment for the first five sequences, followed by the high treatment for the next five, while another 4 sessions with the reversed order.

Upon analyzing the first and second halves of the sessions, we observed a general learning trend in the players' investment patterns. Towards the end of the experiment, the participants' actions closely align with pure strategy equilibrium predictions, no matter if the treatment is from low to high or high to low. As anticipated, the likelihood of winning the race is at highest for the leaders and lowest for the followers. By comparing the actual total payoff earned by the two matched players in each race to the corresponding group efficient outcome, we found that the efficiency loss exists throughout the session. Upon analyzing all the sequences, we showed that the positive spillover effect from the previous round notably boosts the motivation to make investment in the subsequent round.

In this paper, the experimental design does not consider asymmetric information and different monetary rewards. For future research, we are interested in investigating how different reward mechanisms affect players' competition behavior in dynamic patent races and how informational asymmetry between players influences the R&D outcome.

2.7 Appendix A: More details about the model

Dynamics of the knowledge stock

The experience for firm i at the beginning of period $(t+1)$, denoted by $\omega_i(t+1)$, is the summation of firm i's R&D investment and the spillovers in the past periods plus the initial experience (or initial knowledge stock), which can be represented by the following expression:

$$\omega_i(t+1) = \sum_{\tau=1}^{t}\{x_i(\tau) + \varphi(x_i(\tau), x_j(\tau))\} + \omega_i^1,$$

where $\omega_i^1 \geq 0$ is the firm i's initial experience.

Dynamics of the state variable

The state variable in period $(t+1)$ depends on the state variables in period t and the R&D investment and spillovers in period t by the following rule

$$(k_i(t+1), k_j(t+1)) = (\, k_i(t) - x_i(t) - \varphi[x_i(t), x_j(t)]\,, k_j(t) - x_j(t) - \varphi[x_i(t), x_j(t)]).$$

Stationary Markov strategy

The R&D choice of the firm is specified by a stationary Markov strategy χ_i that depends on the remaining stock of knowledge that the firm has to accumulate:

$$\chi_i\colon (k_i,\ k_j) \in \{1, \ldots, N\}^2 \to \chi_i(k_i,\ k_j) = x_i, \text{ with } x_i \in \{0, 1, 2\}.$$

Payoffs

The possible outcomes of firm i are determined by the following net profit function:

$$\pi_i(t) = \begin{cases} Y - AC_i(t) \quad if\ k_i(t+1) < k_j\,(t+1) \text{ and } k_i(t+1) \leq 0; & \textbf{(Case1)} \\ Y/2 - AC_i(t) \quad if\ \ k_i(t+1) = k_j(t+1) \text{ and } k_i(t+1) \leq 0; & \textbf{(Case 2)} \\ -AC_i(t) \quad if\ \{\, k_i(t+1) > \ k_j(t+1)\ \&\ k_i(t+1) \leq 0\} \text{ or } \{k_i(t+1) > 0\ \&\ k_j(t+1) \leq 0\} & \textbf{(Case 3)} \end{cases}$$

where $AC_i(t)$ is the accumulated cost of firm i up to time t with t denoting the time of discovery.

2.8 Appendix B: Payoffs and paths of choices

In this appendix, we present with details on how we calculate the different payoffs summarized in Table 2.4. Under each initial experience, we show the paths of choices for the two firms and we denote the payoff of each firm by V_i, i ={1,2}.

Case 1: $(\omega_1(1), \omega_2(1)) = (0, 0)$:

Table 2.13: Individual equilibrium payoffs for Case 1

t	1	2	3	4	5	6	7
$x_1(t)$	2	2	2	2	2	2	
$x_2(t)$	2	2	2	2	2	2	
$\omega_1(t)$	0	3	6	9	12	15	18
$k_1(t) = N - \omega_1(t)$	16	13	10	7	4	1	-2
$\omega_2(t)$	0	3	6	9	12	15	18
$k_2(t) = N - \omega_2(t)$	16	13	10	7	4	1	-2
$\varphi(x_1(t), x_2(t))$	1	1	1	1	1	1	
Cost for every player	c_2	c_2	c_2	c_2	c_2	c_2	

$V_1 = V_2 = \ 10 - (6 * 0.8) = 5.2$ ECU

Table 2.14: Individual Maximum payoff and Group efficient payoff for Case 1

t	1	2	3	4	5	6	7	8	9	10
$x_1(t)$	1	1	1	1	1	1	1	1	1	
$x_2(t)$	1	1	1	1	1	1	1	0	0	
$\omega_1(t)$	0	2	4	6	8	10	12	14	15	16
$k_1(t) = N - \omega_1(t)$	16	14	12	10	8	6	4	2	1	0
$\omega_2(t)$	0	2	4	6	8	10	12	14	14	14
$k_2(t) = N - \omega_2(t)$	16	14	12	10	8	6	4	2	2	2
$\varphi(x_1(t), x_2(t))$	1	1	1	1	1	1	1	0	0	
Cost for player 1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	

$V_1 = Y - (9 * c_1) = 20 - (9 * 0.2) = 18.2$ ECU , $\quad V_2 = -7 * 0.2 = -1.4$

$V_1 + V_2 = 16.8$: this is **the efficient outcome**.

Table 2.15: Individual Minimum payoff for Case 1

t	1	2	3	4	5	6	7
$x_1(t)$	2	2	2	2	2	2	
$x_2(t)$	2	2	2	2	2	1	
$\omega_1(t)$	0	3	6	9	12	15	18
$k_1(t) = N - \omega_1(t)$	16	13	10	7	4	1	-2
$\omega_2(t)$	0	3	6	9	12	15	17
$k_2(t) = N - \omega_2(t)$	16	13	10	7	4	1	-1
$\varphi(x_1(t), x_2(t))$	1	1	1	1	1	1	
Cost for player 2	c_2	c_2	c_2	c_2	c_2	c_1	

$V_2 = -(5 * c_2) - c_1 = -(5 * 0.8) - 0.2 = -4.2$ ECU

Case 2: $(\boldsymbol{\omega_1(1), \omega_2(1)}) = (\mathbf{8, 8})$:

Table 2.16: Individual equilibrium payoffs for Case 2

t	1	2	3	4
$x_1(t)$	2	2	2	
$x_2(t)$	2	2	2	
$\omega_1(t)$	8	11	14	17
$k_1(t) = N - \omega_1(t)$	8	5	2	-1
$\omega_2(t)$	8	11	14	17
$k_2(t) = N - \omega_2(t)$	8	5	2	-1
$\varphi(x_1(t), x_2(t))$	1	1	1	
Cost for every player	c_2	c_2	c_2	

$V_1 = V_2 = 10 - (3 * 0.8) = 7.6$ ECU

Table 2.17: Individual Maximum payoff and Group efficient payoff for Case 2

t	1	2	3	4	5	6
$x_1(t)$	1	1	1	1	1	
$x_2(t)$	1	1	1	0	0	
$\omega_1(t)$	8	10	12	14	15	16
$k_1(t) = N - \omega_1(t)$	8	6	4	2	1	0
$\omega_2(t)$	8	10	12	14	14	14
$k_2(t) = N - \omega_2(t)$	8	6	4	2	2	2
$\varphi(x_1(t), x_2(t))$	1	1	1	0	0	
Cost for player 1	c_1	c_1	c_1	c_1	c_1	

$V_1 = Y - (5 * c_1) = 20 - (5 * 0.2) = 19$ ECU , $V_2 = -3 * 0.2 = -0.6$

$V_1 + V_2 = 18.4$: this is **the efficient outcome**.

Table 2.18: Individual Minimum payoff for Case 2

t	1	2	3	4
$x_1(t)$	2	2	2	
$x_2(t)$	2	2	1	
$\omega_1(t)$	8	11	14	17
$k_1(t) = N - \omega_1(t)$	8	5	2	-1
$\omega_2(t)$	8	11	14	16
$k_2(t) = N - \omega_2(t)$	8	5	2	0
$\varphi(x_1(t), x_2(t))$	1	1	1	
Cost for player 2	c_2	c_2	c_1	

$V_2 = -(0.8 * 2) - 0.2 = -1.8$ ECU

Case 3: $(\omega_1(1), \omega_2(1)) = (2, 0)$:

Table 2.19: Individual equilibrium payoffs for Case 3

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x_1(t)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
$x_2(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
$\omega_1(t)$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$k_1(t) = N - \omega_1(t)$	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
$\omega_2(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_2(t) = N - \omega_2(t)$	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
$\varphi(x_1(t), x_2(t))$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Cost for player 1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	

$V_1 = Y - 14c_1 = 17.2$ ECU and $V_2 = 0$

Table 2.20: Individual Maximum payoff and Group efficient payoff for Case 3

t	1	2	3	4	5	6	7	8
$x_1(t)$	1	1	1	1	1	1	1	
$x_2(t)$	1	1	1	1	1	1	1	
$\omega_1(t)$	2	4	6	8	10	12	14	16
$k_1(t) = N - \omega_1(t)$	14	12	10	8	6	4	2	0
$\omega_2(t)$	0	2	4	6	8	10	12	14
$k_2(t) = N - \omega_2(t)$	16	14	12	10	8	6	4	2
$\varphi(x_1(t), x_2(t))$	1	1	1	1	1	1	1	
Cost for player 1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	

$V_1 = Y - (7 * c_1) = 20 - (7 * 0.2) = 18.6$ ECU, $V_2 = -7 * 0.2 = -1.4$

$V_1 + V_2 = 17.2$: this is **the efficient outcome**.

Table 2.21: Individual Minimum payoff for Case 3

t	1	2	3	4	5	6
$x_1(t)$	2	2	2	2	2	
$x_2(t)$	2	2	2	2	2	
$\omega_1(t)$	2	5	8	11	14	17
$k_1(t) = N - \omega_1(t)$	14	11	8	5	2	-1
$\omega_2(t)$	0	3	6	9	12	15
$k_2(t) = N - \omega_2(t)$	16	13	10	7	4	1
$\varphi(x_1(t), x_2(t))$	1	1	1	1	1	
Cost for player 2	c_2	c_2	c_2	c_2	c_2	

$V_2 = -(0.8 * 5) = -4$ ECU

<u>Case 4: $(\boldsymbol{\omega_1(1)}, \boldsymbol{\omega_2(1)}) = (\mathbf{4, 0})$:</u>

Table 2.22: Individual equilibrium payoffs for Case 4

t	1	2	3	4	5	6	7	8	9	10	11	12	13
x_1(t)	1	1	1	1	1	1	1	1	1	1	1	1	
$x_2(t)$	0	0	0	0	0	0	0	0	0	0	0	0	
$\omega_1(t)$	4	5	6	7	8	9	10	11	12	13	14	15	16
$k_1(t) = N - \omega_1(t)$	12	11	10	9	8	7	6	5	4	3	2	1	0
$\omega_2(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_2(t) = N - \omega_2(t)$	16	16	16	16	16	16	16	16	16	16	16	16	16
$\varphi(x_1(t), x_2(t))$	0	0	0	0	0	0	0	0	0	0	0	0	
Cost for player 1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	

$V_1 = Y - 12c_1 = 17.6$ ECU and $V_2 = 0$

Table 2.23: Individual Maximum payoff and Group efficient payoff for Case 4

t	1	2	3	4	5	6	7
$x_1(t)$	1	1	1	1	1	1	
$x_2(t)$	1	1	1	1	1	1	
$\omega_1(t)$	4	6	8	10	12	14	16
$k_1(t) = N - \omega_1(t)$	12	10	8	6	4	2	0
$\omega_2(t)$	0	2	4	6	8	10	12
$k_2(t) = N - \omega_2(t)$	16	14	12	10	8	6	4
$\varphi(x_1(t), x_2(t))$	1	1	1	1	1	1	
Cost for player 1	c_1	c_1	c_1	c_1	c_1	c_1	

$V_1 = Y - (6 * c_1) = 20 - (6 * 0.2) = 18.8$ ECU , $V_2 = -6 * 0.2 = -1.2$

$V_1 + V_2 = 17.6$: this is **the efficient outcome**.

Table 2.24: Individual Minimum payoff for Case 4

t	1	2	3	4	5
$x_1(t)$	2	2	2	2	
$x_2(t)$	2	2	2	2	
$\omega_1(t)$	4	7	10	13	16
$k_1(t) = N - \omega_1(t)$	12	9	6	3	0
$\omega_2(t)$	0	3	6	9	12
$k_2(t) = N - \omega_2(t)$	16	13	10	7	4
$\varphi(x_1(t), x_2(t))$	1	1	1	1	
Cost for player 2	c_2	c_2	c_2	c_2	

$V_2 = -(0.8 * 4) = -3.2$ ECU

Case 5: $(\omega_1(1), \omega_2(1)) = (10, 8)$:

Table 2.25: Individual equilibrium payoffs for Case 5

t	1	2	3	4	5	6	7
x_1(t)	1	1	1	1	1	1	
$x_2(t)$	0	0	0	0	0	0	
$\omega_1(t)$	10	11	12	13	14	15	16
$k_1(t) = N - \omega_1(t)$	6	5	4	3	2	1	0
$\omega_2(t)$	8	8	8	8	8	8	8
$k_2(t) = N - \omega_2(t)$	8	8	8	8	8	8	8
$\varphi(x_1(t), x_2(t))$	0	0	0	0	0	0	
Cost for player 1	c_1	c_1	c_1	c_1	c_1	c_1	

$V_1 = Y - 6c_1 = 18.8$ ECU and $V_2 = 0$

Table 2.26: Individual Maximum payoff and Group efficient payoff for Case 5

t	1	2	3	4
$x_1(t)$	1	1	1	
$x_2(t)$	1	1	1	
$\omega_1(t)$	10	12	14	16
$k_1(t) = N - \omega_1(t)$	6	4	2	0
$\omega_2(t)$	8	10	12	14
$k_2(t) = N - \omega_2(t)$	8	6	4	2
$\varphi(x_1(t), x_2(t))$	1	1	1	
Cost for player 1	c_1	c_1	c_1	

$V_1 = Y - (3 * c_1) = 20 - (3 * 0.2) = 19.4$ ECU , $V_2 = -3 * 0.2 = -0.6$

$V_1 + V_2 = 18.8$: this is **the efficient outcome**.

Table 2.27: Individual Minimum payoff for Case 5

t	1	2	3
$x_1(t)$	2	2	
$x_2(t)$	2	2	
$\omega_1(t)$	10	13	16
$k_1(t) = N - \omega_1(t)$	6	3	
$\omega_2(t)$	8	11	14
$k_2(t) = N - \omega_2(t)$	8	5	
$\varphi(x_1(t), x_2(t))$	1	1	
Cost for player 2	c_2	c_2	

$V_2 = -0.8 - 0.8 = -1.6$ ECU

Case 6: where $(\boldsymbol{\omega_1(1), \omega_2(1)}) = (\mathbf{12, 8})$:

Table 2.28: Individual equilibrium payoffs for Case 6

t	1	2	3	4	5
x_1(t)	1	1	1	1	
$x_2(t)$	0	0	0	0	
$\omega_1(t)$	12	13	14	15	16
$k_1(t) = N - \omega_1(t)$	4	3	2	1	0
$\omega_2(t)$	8	8	8	8	8
$k_2(t) = N - \omega_2(t)$	8	8	8	8	8
$\varphi(x_1(t), x_2(t))$	0	0	0	0	
Cost for player 1	c_1	c_1	c_1	c_1	

$V_1 = Y - 4c_1 = 19.2$ ECU and $V_2 = 0$

Table 2.29: Individual Maximum payoff and Group efficient payoff for Case 6

t	1	2	3
$x_1(t)$	1	1	
$x_2(t)$	1	1	
$\omega_1(t)$	12	14	16
$k_1(t) = N - \omega_1(t)$	4	2	0
$\omega_2(t)$	8	10	12
$k_2(t) = N - \omega_2(t)$	8	6	4
$\varphi(x_1(t), x_2(t))$	1	1	
Cost for player 1	c_1	c_1	

$V_1 = Y - (2 * c_1) = 20 - (2 * 0.2) = 19.6$ ECU , $V_2 = -2 * 0.2 = -0.4$, $V_1 + V_2 = 19.2$: this is **the efficient outcome**.

Table 2.30: Individual Minimum payoff for Case 6

t	1	2	3
$x_1(t)$	2	2	
$x_2(t)$	2	2	
$\omega_1(t)$	12	15	18
$k_1(t) = N - \omega_1(t)$	4	1	-2
$\omega_2(t)$	8	11	14
$k_2(t) = N - \omega_2(t)$	8	5	2
$\varphi(x_1(t), x_2(t))$	1	1	
Cost for player 2	c_2	c_2	

$V_2 = -0.8 - 0.8 = -1.6$

2.9 Appendix C: Instructions (L-H treatment)

This is an experiment in the economics of strategic decision making. You will receive $10 for showing up in the session. Your additional earnings will depend on your own decisions and other participants' decisions as explained below. The instructions are simple. If you follow them closely and make appropriate decisions, you can earn an appreciable amount of money. During the experiment, please remain silent and do not use your mobile device. If you have any questions, or need assistance of any kind, please raise your hand and an experimenter will come to assist you.

During the experiment, your additional earnings will be calculated using Experimental Currency Units (ECU). At the end of the experiment, the total amount of ECUs you have earned will be converted to Canadian Dollars at the rate of 1 ECU = $0.25 and will be paid to you in cash, in addition to the show-up fee.

Overview of the experiment

In this experiment, you will play 10 sequences in total. At the beginning of each sequence, you will be randomly and anonymously matched with another participant in this room with equal probability. You will not know whether or not you play with this participant in any previous sequence. Neither will you observe the choices made by your matched player in any previous sequence.

Specifics

Each sequence consists of multiple rounds, in which you and your matched player make investment decisions and accumulate points in order to compete for a prize that is worth 20 ECU. In each round, you and your matched player independently and simultaneously decide whether to invest 0, 1, or 2 points, which costs respectively c_0 = 0, c_1 = 0.2 and c_2= 0.8 ECU, as summarized in the following table:

Table 2.31: R&D effort levels with their associated costs

Points	Costs
0	c_0 = 0 ECU
1	c_1 = 0.2 ECU
2	c_2 = 0.8 ECU

At the end of each round, the points you invested in the round will be added to your accumulated points of the sequence. In each round, you and your matched player may earn one extra point from investment if both of you invest 1 or 2 points. If one of you or both of you choose to invest 0 point, no extra point will be earned. The following table shows all the possible cases for the extra point:

Table 2.32: The possible cases for the extra point

Your choice / Your partner's choice	0	1	2
0	(0,0) => no extra point	(0,1) => no extra point	(0,2) => no extra point
1	(1,0) => no extra point	(1,1) => one extra point	(1,2) => one extra point
2	(2,0) => no extra point	(2,1) => one extra point	(2,2) => one extra point

In each round, before you make your decision, you will be able to observe the choice of yourself and your matched player in the previous rounds, as well as your accumulated points and your matched player's accumulated points. After both you and your matched player submit the decision for the current round, you will observe the choice made by yourself and your matched player in this round, whether or not you receive the extra point, and the updated accumulated points.

When one player or both players in a pair reach or pass a level of accumulation of 16 points, a sequence ends. Otherwise, the sequence will continue into next round, in which the total points and the total costs accumulated in all previous rounds of the same sequence will carry over into the next round.

At the end of every sequence, you will receive the prize of 20 ECU if the following two conditions are satisfied:

(1) Your accumulation of points reaches or passes 16;

(2) Your accumulation of points is higher than the accumulation of points of your matched player.

In the case that you and your matched player reach or pass 16 points in the same round with the same accumulated points, you will share the prize and each of you receives 10 ECU.

<u>Examples:</u>

<u>Example 1:</u> At the end of the current round, you accumulated less than 16 points while your matched player accumulated at least 16 points, your matched player wins.

Example 2: At the end of the current round, you accumulated less than 16 points while your matched player accumulated 17 points, your matched player wins.

Example 3: At the end of the current round, you and your matched player both accumulated 16 points, both of you win and share the prize.

Example 4: At the end of the current round, you and your matched player both accumulated less than 16 points, the sequence continues to the next round.

Payoffs from a sequence:

- If you are the only winner, you will earn 20 ECU less the total costs you bared to accumulate 16 points.
- If both you and your matched player are winners, you will earn 10 ECU less the costs you bared to accumulate 16 points.
- If your matched player is the only winner, you get no prize from the game but still need to pay the costs that have occurred.
- Note that if neither you nor your matched player invest at least 1 point for three consecutive rounds, the sequence will be ended and neither of you will win the prize.

At the beginning of each sequence, you and your partner have an initial position, which is exogenously varied: In some sequences you will start in an asymmetric position, and in some other sequences you will start in a symmetric position. Your initial position will be shown on the screen when the sequence starts. The following table indicates the initial position for each pair in every sequence:

Table 2.33: Every player's initial position in each sequence

Sequences	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Initial position for every player in every sequence	(0,0)	(2,0)	(0,2)	(4,0)	(0,4)	(8,8)	(10,8)	(8,10)	(12,8)	(8,12)

Earnings:

At the end of the session, the program will calculate your entire earnings from participation in all the sequences. Your final payment will be equal to the earnings from the 10 sequences plus the $10 show-up fee. You will be paid in private and in cash at the end of the experiment.

Comments:

- You cannot change your choice once you have made it;
- Do not discuss your decisions or your results with anyone at any time during the experiment.
- Do not get up from your seat before the end of the experiment.
- Your ID# is private. Do not reveal it to anyone.

Now, please fill out the short enclosed quiz before we start the experiment. The aim of the quiz is to make sure that the key concepts covered in the instructions are clear for you. Your answers in the quiz will not affect your earnings directly.

Thank you for participating and good luck!

Quiz:

Please answer the following questions. If you have any questions, please contact one of the conductors of the study.

- You will be randomly and anonymously matched with another participant in the room in every round. True or False

- At the beginning of a round, you are randomly and anonymously paired with another participant. You choose one point to invest while your matched partner chooses 2 points. What is the value of the spillover in this case?
  What is your cost in this round?

- At the beginning of a round, you are randomly and anonymously paired with another participant. You choose zero point to invest while your matched partner chooses one point.

What is the value of the spillover in this case?
What is your cost in this round?

- At the end of a sequence, your accumulated points are greater than your partner's accumulated points and your accumulated cost is equal to 2.8 ECU.
  What is the value of the prize that you get?
  What is your payoff?

- At the end of a sequence, your accumulated points are equal to your partner's accumulated points and your accumulated cost is equal to 4.8 ECU.
  What is the value of the prize that you get?
  What is your payoff?

- At the end of a sequence, your accumulated points are lower than your partner's accumulated points and your accumulated cost is equal to 4.2 ECU.
  What is the value of the prize that you get?
  What is your payoff?

- Your final earnings will be determined by your payoffs in all the sequences. True or False

3 Government Subsidy and R&D Quality Investment

3.1 Introduction

Quality improvement and innovation are two key tools for businesses to outperform the competition, maximize future returns, and generate remarkable profits. Good quality allows firms to encourage the purchase of their products and therefore, to increase their profits (Stuebs and Sun, 2009).

In recent years, many consumers have developed their preferences for spending more money on higher-quality goods than on lower ones. Higher-quality products increase demand and boost the economic efficiency of the firms. For instance, when firm produces high quality, this will become widely known and the firm will acquire good reputation since the consumers' evaluation becomes known to other consumers.

Under the presence of product substitutability and demand spillover effect, I consider, in this paper, a duopoly market where the consumer's utility not only depends on the quantity of consumption and the quality of the good, but also the consumer' quality sensitivity.

Interestingly, it is possible that one firm's quality improving R&D investment increases the market share of the rival firm. For instance, in the context of environmental externalities, the invention of an eco-vehicle equipped with an electric motor might increase the willingness to pay of customers who are more concerned about the environmental issues than the brand of the car. Therefore, the market size might be expanded by the development of a new product or the improvement of the quality of an existing product. In that case, the rival firm's market share might be increased. Another relevant example occurs in the context of compatible network goods in the industries within technology sector such as computer hardware, telecommunications, software and so on. Specifically, customers who are using software and applications developed by some firms are willing to use software and applications developed by rival firms especially when these firms are aiming to improve the quality of their goods.

To stimulate innovation, the government use R&D subsidies as an industrial policy. Given the importance of determining the socially optimal subsidies, I consider, in this paper, a three-stage duopoly game. In the first stage, the government chooses the optimal R&D subsidy to subsidize the R&D investment in quality improvement. In the second stage, firms choose how much to spend on quality either cooperatively or non-cooperatively. In the final stage, firms decide how much to sell in the market either cooperatively or non-cooperatively.

The equilibrium solutions are obtained by backward induction, and the impact of consumers' quality sensitivity and demand spillovers on decision variables are investigated accordingly.

Three different cases are discussed. In the first case, the competition between firms occurs at the R&D stage as well as the production stage; in the second case, firms coordinate their R&D activities in order to maximize their joint profits while remain competitive in the final stage and in the last case, they fully share their R&D outcome. The last case deals with monopoly since the firms cooperate in the R&D stage as well as the production stage.

There are different real-life scenarios that illustrate how firms can interact with each other in different ways. For instance, in the 1990s, the Japanese semiconductor industry was characterized by intense competition between firms like Toshiba, Hitachi, and Sony. Each firm was engaged in a "winner-takes-all" game, where they invested heavily in R&D to develop the most advanced and cost-effective semiconductor technologies. This competition led to rapid innovation, with each firm trying to outdo the others in terms of quality, speed, and price. The competition also drove down costs and improved efficiency, making semiconductors more affordable and widely available. This case illustrates the first scenario, where firms compete at both the R&D and production stages.

In the early 2000s, the automotive industry began to shift towards electric vehicles (EVs) as a response to growing environmental concerns and government regulations. Several major automotive manufacturers, including Toyota, Honda, Nissan, Mazda and Subaru, recognized the importance of developing high-quality EV batteries to compete in the market. To address the significant R&D costs and challenges associated with developing advanced EV batteries, these firms decided to collaborate on their R&D activities to maximize their joint profits while remaining competitive in the final stage.[5] In 2020, BioNTech and Pfizer began collaborating to develop and produce the COVID-19 vaccine. This collaboration involved the R&D stage as well as the production stage. Specifically, the firms fully shared their R&D outcome, illustrating the third scenario where companies collaborate on innovation. By doing so, they were able to provide a

[5] https://www.topgear.com.ph/news/industry-news/honda-nissan-mazda-toyota-alliance-a962-20210928

higher-quality vaccine that was more effective in preventing the disease, ultimately benefiting public health.[6]

In this paper, I investigate the implications of consumers' quality sensitivity on the optimal non-cooperative and cooperative quality investment, and I examine whether the optimal subsidy decreases or increases with the increase of consumers' quality sensitivity. In addition to that, I investigate whether the impact of this sensitivity on the optimal quality investment differs when the scenario changes from full competition to partial cooperation and from partial to full cooperation. Under this framework, I consider a Cournot duopoly where two firms choose their quality investment levels given the government's R&D policy. Taking the non-cooperative and the cooperative scenarios into consideration, the firms' optimal decisions are compared and analyzed and the R&D subsidies are solved in a way to maximize the social welfare. Then I examine the changes of the firms' profit, quality investment and social welfare before and after the implementation of the socially optimal subsidy policy. In addition to that, I conduct a comparative static analysis on how the optimal quality investment and optimal subsidy change with demand spillovers and consumers' quality sensitivity.

The results show that, when there is no subsidy, cooperation in both quality and quantity leads mostly to the highest quality investment level in comparison to the non-cooperative and partial cooperative scenario. In most cases and compared to the full cooperative and non-cooperative regime, the partial cooperative regime shows that an increase in the R&D optimal subsidy encourages firms to cooperate in quality investment.

By considering the ranking of the profits under each regime, the results show that when there is no support from the government and when the demand spillover is different from the critical threshold, firms have more incentive to cooperate in the R&D stage as well as the production stage since the profit is the highest under this case.

Regarding the consumers' quality sensitivity, I find that it has an influence on the optimal subsidy under non-cooperation only and that its impact on the quality investment level is the same when comparing the non-cooperative and the partial cooperative scenario. However, this effect is not the same when comparing the partial and the full cooperative scenario.

[6] Information collected from the following website: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC9433349/#:~:text=To%20that%20end%2C%20on%20March,and%20resources%20of%20both%20companies.

When demand spillovers are small, the optimal R&D subsidy under partial cooperation is higher than the optimal subsidy under full competition. In addition to that, the results show that subsidizing partial cooperative quality has a positive effect on the profit of the firm. However, when demand spillovers are large, the government needs to provide more support for non-cooperative quality investment so that the firm's profit under non-cooperation is improved.

From a social welfare perspective, the results indicate that subsidizing optimally the non-cooperative or the partial cooperative quality investment is more effective than allowing for full cooperation with subsidization. For that reason, the last regime should not be encouraged.

The remainder of this paper is organized as follows. Section 3.2 discusses the related literature. Section 3.3 presents the theoretical model. Section 3.4 considers three kinds of R&D strategies such that the non-cooperative R&D (where there is a Cournot-type competition at the R&D stage as well as at the production stage), the partial cooperative R&D (where the cooperation occurs at the R&D stage only) and the full cooperative R&D (where the R&D results are fully shared and the R&D activities are coordinated). Section 3.5 compares and discusses the outcomes of these scenarios and Section 3.6 concludes. In the appendix, I provide the mathematical proofs.

3.2 Literature review

A number of theoretical studies have examined the R&D policies and the welfare implications of R&D activities in light of governmental intervention. As a seminal work, Leahy and Neary (1997) developed a duopoly model with R&D subsidies. They found that, under Cournot competition without or even with low enough R&D spillovers, the R&D should be taxed given that firms over-invest in R&D; however, under Bertrand competition, R&D should always be subsidized. Taking into consideration a two stage asymmetric Cournot duopoly model that allows for asymmetry in the marginal cost level, Lahiri and Ono (1999) found that the optimal structure of R&D policy is to tax the R&D activities of the firm with lower market share and to subsidize the firm with higher market share so as to maximize the global welfare; however, in the symmetric duopoly case, the optimal structure of R&D policy depends on the shape of the demand function. Specifically, when the latter is concave (convex), the optimal R&D policy is to tax (subsidize) the R&D activities; while, in the case of linear demand, the best policy is that the government does not intervene. In a mixed and a private duopoly market, Gil-Molto, Poyago-Theotoky and Zikos (2011) showed that the optimal R&D subsidy increases with spillovers but it is greater in the mixed market (where one of the two firms is public) relative to the private market. In a mixed duopoly, Lee et al. (2017) studied output and R&D subsidies without spillovers. They found that the output subsidy is more socially beneficial than the R&D subsidy, regardless of the degree of privatization. Moreover, compared to the output subsidy, the government has a lower incentive to privatize the public firm under the R&D subsidy. The results of Lee et al. (2017) show that the optimal rate of output subsidy is constant; however, the optimal rate of R&D subsidy is negative. By considering a three-stage duopoly market without spillovers, Yang (2014) examined the impact of government preference on R&D subsidy policies. He found that greater preference to the welfare of customers results in a higher subsidy level that not only brings benefit to consumers, but also creates higher net profits to firms as their production sales increase and their costs decrease. By comparing the output and the R&D subsidy in a mixed duopoly, Kesavayuth and Zikos (2013) showed that the R&D subsidy outperforms the output subsidy only if the technological spillovers are sufficiently high. Cabon-Dhersin and Gibert (2020) considered a model that follows a three-stage duopoly game composed of a regulator and n firms. They showed that R&D subsidies that stimulate the cooperation between firms do not necessarily increase social welfare or research output in comparison with a non-cooperative R&D scenario. In comparison with a policy that optimally subsidizes just private or just public research, they demonstrated that a research funding policy that optimally encourages both public and private research investments results in a better performance in terms of consumer surplus and social welfare. In addition to that, they showed that as the level of inter-firm spillovers increases, the proportion of public R&D funding allocated to private sector increases as well.

Without considering the spillover effects, Chen and Lee (2023) did an examination and a comparison in terms of the output and the R&D subsidy policies between Cournot and Bertrand competitions. They demonstrated that under Cournot competition with output subsidy policies, firms provide more R&D investment and receive greater subsidies than under Bertrand competition. With R&D subsidy policies, the results are, however, the opposite. They further demonstrated that with production subsidy policies, both Cournot and Bertrand competitions produce the same social welfare; however, with R&D subsidy policies, Bertrand competition produces better social benefit than Cournot competition. By considering a quality differentiated Cournot/Bertrand duopoly model without spillover effects, Toshimitsu (2003) investigated the optimal R&D tax/subsidy policies applied to R&D investment in quality improvement. He found that, under Cournot (Bertrand) competition, taxing (subsidizing) the lower quality firm leads to an improvement in terms of the social welfare; however, subsidizing the higher quality firm results in an increased social welfare, regardless of whether the competition is Bertrand or Cournot.

To the best of my knowledge, none of the previous studies has explored the government R&D subsidy based on the quality level by considering the consumers' quality sensitivity and the demand spillover effects under the non-cooperative, the partial and the full cooperative scenarios. Examining these effects is important and helpful for the policy implications on how to protect the R&D activities and improve social welfare.

In this paper, I investigate the impact of R&D subsidies on firms and social welfare in a Cournot duopoly market by considering the non-cooperative and the cooperative scenarios. I adopt a consumer-driven model in which three key components are explicitly characterized: the intensity of competition (or the degree of product substitutability) between the two firms/products, the consumers' quality sensitivity and the demand spillovers. Through these elements, the duopoly model can characterize the impact of R&D subsidies on the firms' investment quality decisions and profits. Taking into consideration the non-cooperative, the partial and the full cooperative scenarios, I solve the optimal level of R&D subsidies and compare the changes of firms' profit, quality investment and social welfare before and after the implementation of the socially optimal subsidy policy. Moreover, I investigate whether there is a relationship between the consumer's quality sensitivity and the optimal subsidy. In addition to that, I examine the effect of this sensitivity on the firm's optimal quality investment and whether this effect differs when the regime is changed from full competition to partial cooperation and from partial to full cooperation and whether the optimal quality investment and optimal subsidy change with demand spillovers and consumers' quality sensitivity.

3.3 Model

The model follows a three-stage duopoly game composed of a regulator and two firms. In the first stage, the government decides the socially optimal R&D subsidy applied to investment in quality improvement. The objective is to maximize the well-being of the nation (consumers and firms). In the second stage, two firms, denoted by i and j respectively, who produce differentiated products, simultaneously choose the quality investment level (either cooperatively or non-cooperatively), given the government's R&D policy. In the third stage, the two firms compete or cooperate in quantities. The game is solved by backward induction in order to find the subgame perfect Nash equilibrium.

The basic model shares some features with the model of Singh and Vives (1984) and the model of Toshimitsu (2014). These two papers consider a number of consumers who consume two goods, good i and good j, and who have a set of preferences over the consumption of goods produced by the duopoly. Different from the research that these authors performed, I include the quality sensitivity parameter in the consumers' utility function that interacts with the variable of the product quality and has an impact on the market demand for the good.

Given that both quantities $q = (q_i, q_j)$ and qualities $d = (d_i, d_j)$ impact consumers' utility, I consider the following utility function for the representative consumer, in which quality is complementary to consumption:

$$u(q_0, q_i, q_j) = (a + \lambda d_i + \theta d_j)q_i + (a + \lambda d_j + \theta d_i)q_j - \left(\frac{q_i^2}{2} + \frac{q_j^2}{2} + \gamma q_i q_j\right) + q_0\,,$$
$$q_0 = R - p_i q_i - p_j q_j.$$

Specifically, the representative consumer is assumed to have a utility function that is quadratic, strictly concave, symmetric in q_i and q_j, linear and separable in some numeraire good, similar to Singh and Vives (1984). The parameter $\gamma \in (0,1]$ provides a convenient parametrization of the substitutability between the two quantities q_i and q_j. The interaction term $\gamma q_i q_j$ accounts for the relationship between the two goods in terms of how consumption of one affects the marginal utility of the other. Importantly, the demands for these goods are independent whenever $\gamma = 0$, while the goods are substitutes whenever $\gamma > 0$ which means that increasing consumption of one reduces the marginal utility of consuming the other; however, the goods are homogeneous (perfect substitutes) whenever $\gamma = 1$. λ represents consumers' sensitivity to quality and is assumed to be the same for the two firms. This parameter is assumed to be positive. In other words, an increase in the quality level of product i increases the marginal utility for the same product $\partial(\partial U/\partial q_i)/\partial d_i > 0$.

In this paper, I assume that all quality improvements can be observed and verified by consumers. R is the income, q_0 is the numeraire and α is constant. The quadratic preferences (q_i^2 or q_j^2) in the utility function imply linear demand so that first order condition ensures utility maximization. The following inverse demand function of good i is determined by considering a zero marginal utility:

$$\frac{\partial u}{\partial q_i} = a + \lambda d_i + \theta d_j - q_i - \gamma q_j - p_i = 0$$
$$\Leftrightarrow p_i = a + \lambda d_i + \theta d_j - q_i - \gamma q_j.$$

I assume there is demand spillover between the two products. Particularly, an increase in firm j's quality investment increases the willingness to pay for the product of firm i. Such effect is captured by $(\partial p_i/\partial d_j) = \theta \in (0,1)$.

For the marginal cost of firm i, it is defined as $c_i = c + \Omega d_i > 0$ with $|\Omega| < 1$, where $c_i = c$ if no innovative investments are undertaken, while if there are R&D investments in quality improvement, the firm's marginal cost may be increased or decreased by the term Ω (Kesteloot and Voet, 1998). This reduction may be due, for instance, to better use of available technological capacity. It might be the case where the quality improvement generates higher production costs. For example, the invention of CT (Computed tomography) scanner complementary to RX for imaging may improve the quality but it may increase the cost.

From the inverse demand function and the firm' marginal cost, I get the following expression:

$$p_i - c_i = a + \lambda d_i + \theta d_j - q_i - \gamma q_j - c - \Omega d_i = a - c + (\lambda - \Omega) d_i + \theta d_j - q_i - \gamma q_j$$
$$= \alpha + (\lambda - \Omega) d_i + \theta d_j - (q_i + \gamma q_j) \text{ with } \alpha = a - c > 0.$$

A rational firm can implement cost increasing quality improvements only when their net effect on the objective function is positive. Consequently, $(\lambda - \Omega)$ is assumed to be positive for the implemented quality improvements.

The quality investments exhibit diminishing returns such that the total investment cost in quality improvement is $d_i^2/2$. Regarding the R&D input subsidy strategy, the government provides $sd_i^2/2$ as a proportion of the investment cost in quality improvement. Therefore, the profit of firm i is defined as follows:

$$\pi_i = (p_i - c_i) q_i - \frac{d_i^2}{2} + \frac{sd_i^2}{2} = \left[\alpha + (\lambda - \Omega) d_i + \theta d_j - (q_i + \gamma q_j)\right] q_i - \frac{d_i^2}{2} + \frac{sd_i^2}{2}. \quad \mathbf{(1)}$$

Based on the utility function and the inverse demand function, the consumer's surplus is defined as:

$$CS = u(q_0, q_i, q_j) - (p_i q_i + p_j q_j + q_0)$$

$$= (a + \lambda d_i + \theta d_j)q_i + (a+\lambda d_j + \theta d_i)q_j - \left(\frac{q_i^2}{2} + \frac{q_j^2}{2} + \gamma q_i q_j\right) - p_i q_i - p_j q_j$$

$$= \frac{q_i^2}{2} + \frac{q_j^2}{2} + \gamma q_i q_j.$$

As it is mentioned in the above expression, the consumer's surplus, that measures the net benefit from consumption, depends only on quantities q_i and q_j; however, the quality parameters d_i and d_j are cancelled out. This is because the prices p_i and p_j incorporate these quality terms. Specifically, when calculating the net benefit from consumption, the additional utility from quality and the additional cost cancel each other out, leaving the consumer surplus dependent only on the quantities and their interaction terms.

In the following analysis, I treat the government as an active player who chooses the policy that maximizes the social welfare function SW which is defined as the sum of consumers and producers surpluses minus the cost of R&D funding policy for quality (denoted by SC).

$$SW = CS + \pi_i + \pi_j - SC,$$

where

Producers' surplus = $\pi_i + \pi_j$,

and

$$SC = \frac{sd_i^2}{2} + \frac{sd_j^2}{2}.$$

3.4 Cooperative and non-cooperative quality investment with subsidy

Under the provision of R&D subsidies, I solve the non-cooperative (NC), the partial cooperative (C), and the full cooperative (M) regimes.

3.4.1 Non-cooperative scenario

Using backward induction, I first solve the equilibrium quantity of the output stage then I solve the equilibrium quality investment of the R&D stage.

In the third stage, given the observed d_i and d_j, each firm selects the quantity that maximizes its objective function π_i (equation **(1)**) by taking the quantity of the other as given. The first order conditions are given by:

$$\frac{\partial \pi_i}{\partial q_i} = 0, \qquad \frac{\partial \pi_j}{\partial q_j} = 0.$$

I obtain the following two best response functions:

$$q_i = \frac{\alpha + (\lambda - \Omega)d_i + \theta d_j - \gamma q_j}{2}, \quad q_j = \frac{\alpha + (\lambda - \Omega)d_j + \theta d_i - \gamma q_i}{2}.$$

Conditional on d_i and d_j, the optimal production functions at equilibrium are as follows:

$$q_i^*(d_i, d_j) = \frac{\alpha(2-\gamma) + [2(\lambda - \Omega) - \theta\gamma]d_i + [2\theta - \gamma(\lambda - \Omega)]d_j}{(4-\gamma^2)},$$

$$q_j^*(d_i, d_j) = \frac{\alpha(2-\gamma) + [2(\lambda - \Omega) - \theta\gamma]d_j + [2\theta - \gamma(\lambda - \Omega)]d_i}{(4-\gamma^2)}.$$

Recall that $\gamma \in (0,1], \theta \in (0,1)$ and $(\lambda - \Omega) > 0$. Given the range of the degree of product substitutability, it is clear that $(2-\gamma) > 0$ and $(4-\gamma^2) > 0$ as well. I will discuss the sign of $[2(\lambda - \Omega) - \theta\gamma]$ and $[2\theta - \gamma(\lambda - \Omega)]$ when I solve the optimal quality investments.

Equation **(1)** can be written as follows:

$$\pi_i^* = (p_i^* - c_i)q_i^* - \frac{d_i^2}{2} + \frac{sd_i^2}{2} = \left[\alpha + (\lambda - \Omega)d_i + \theta d_j - \left(q_i^* + \gamma q_j^*\right)\right]q_i^* - \frac{d_i^2}{2} + \frac{sd_i^2}{2}.$$

The best response functions of the optimal quantities imply that

$$\alpha + (\lambda - \Omega)d_i + \theta d_j - q_i^* - \gamma q_j^* = q_i^* = p_i^* - c_i,$$

therefore,

$$\pi_i^* = (p_i^* - c_i^*)q_i^* - \frac{d_i^2}{2} + \frac{sd_i^2}{2} = (q_i^*)^2 - \frac{d_i^2}{2} + \frac{sd_i^2}{2}.$$

In the second stage, given the observed proportion of the R&D subsidy "s", each firm determines its quality investment to maximize its profit:

$$\text{Max}\,\pi_i^* = (q_i^*)^2 - \frac{d_i^2}{2} + \frac{sd_i^2}{2}$$

$$\text{subject to } d_i, d_j, \pi_i^* \geq 0.$$

$$\frac{\partial \pi_i^*}{\partial d_i} = 2q_i^* \frac{\partial q_i^*}{\partial d_i} - (1-s)d_i.$$

Since

$$q_i^* = \frac{\alpha(2-\gamma) + [2(\lambda-\Omega) - \theta\gamma]d_i + [2\theta - \gamma(\lambda-\Omega)]d_j}{(4-\gamma^2)},$$

$$\frac{\partial \pi_i^*}{\partial d_i} = 2\left[\frac{\alpha(2-\gamma) + [2(\lambda-\Omega) - \theta\gamma]d_i + [2\theta - \gamma(\lambda-\Omega)]d_j}{(4-\gamma^2)}\right]\left[\frac{[2(\lambda-\Omega) - \theta\gamma]}{(4-\gamma^2)}\right] - (1-s)d_i,$$

and

$$\frac{\partial^2 \pi_i^*}{\partial d_i^2} = \frac{2[2(\lambda-\Omega) - \theta\gamma]^2}{(4-\gamma^2)^2} - (1-s).$$

To ensure the concavity of π_i^* with respect to d_i, I place a restriction on "s", such that:

$$2[2(\lambda-\Omega) - \theta\gamma]^2 < (1-s)(4-\gamma^2)^2, \qquad (\boldsymbol{I})$$

or equivalently,

$$0 < s < 1 - \frac{2[2(\lambda-\Omega) - \theta\gamma]^2}{(4-\gamma^2)^2}.$$

The above condition implies the inequality:

$$[2(\lambda-\Omega) - \theta\gamma]^2 < \frac{(4-\gamma^2)^2}{2}.$$

For the interior solution, given $\partial \pi_i^*/\partial d_i = 0$, the best responses for quality investment decisions are as follows:

$$d_i(d_j) = \frac{2[2(\lambda-\Omega) - \theta\gamma](2-\gamma)\alpha + 2[2(\lambda-\Omega) - \theta\gamma][2\theta - \gamma(\lambda-\Omega)]d_j}{(1-s)(4-\gamma^2)^2 - 2[2(\lambda-\Omega) - \theta\gamma]^2}, \qquad (\mathbf{2})$$

$$d_j(d_i) = \frac{2[2(\lambda-\Omega) - \theta\gamma](2-\gamma)\alpha + 2[2(\lambda-\Omega) - \theta\gamma][2\theta - \gamma(\lambda-\Omega)]d_i}{(1-s)(4-\gamma^2)^2 - 2[2(\lambda-\Omega) - \theta\gamma]^2}.$$

Given the restriction on "s", the denominator of the above expression is positive. The symmetry and the linearity of the response functions which are $d_i(d_j)$ and $d_j(d_i)$ imply that both firms produce the same level of quality investment at equilibrium. Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I have the following condition:

$$(1-s)(4-\gamma^2)^2 > 2[2(\lambda-\Omega)-\theta\gamma]^2 + 2[2(\lambda-\Omega)-\theta\gamma][2\theta-\gamma(\lambda-\Omega)]$$
$$\Leftrightarrow (1-s)(4-\gamma^2)^2 > 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega]. \qquad (\boldsymbol{II})$$

In all the results in this subsection and the corresponding proofs, I assume both concavity conditions (I) and (II) of the profit function.

<u>Proposition 1:</u> The optimal non-cooperative quality investment $d_i^* = d_j^* = d^{NC}$ is expressed as follows:

$$d^{NC} = \begin{cases} 0 \; if \; \lambda \leq \Omega + \dfrac{\gamma\theta}{2}, \\ \dfrac{2\alpha[2(\lambda-\Omega)-\theta\gamma](2-\gamma)}{(2-\gamma)^2(2+\gamma)^2(1-s)-2(2-\gamma)(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]} \; if \; \lambda > \Omega + \dfrac{\gamma\theta}{2}. \end{cases}$$

The proof of Proposition 1 is provided in the appendix.

Proposition 1 shows that the amount of quality investment in equilibrium depends on whether the quality sensitivity is large or small. When the consumer's quality sensitivity λ is small such that $\lambda \leq \Omega + \frac{\gamma\theta}{2}, d^{NC} = 0$; however, when the consumer's quality sensitivity λ is large, $d^{NC} > 0$. Notice that under these two cases, the profit of firm i is positive.

Given Proposition 1, I will focus on the case where the non-cooperative quality investment is positive, where $2(\lambda-\Omega)-\theta\gamma > 0$, and given the concavity condition of π_i^* with respect to d_i such that: $0 < s < 1 - \frac{2[2(\lambda-\Omega)-\theta\gamma]^2}{(4-\gamma^2)^2}$ so that $\frac{2[2(\lambda-\Omega)-\theta\gamma]^2}{(4-\gamma^2)^2} < 1$, I derive other results by imposing the following assumption.

Assumption 1: $\frac{\gamma\theta}{2} < \lambda - \Omega < \frac{(4-\gamma^2)}{2\sqrt{2}} + \frac{\gamma\theta}{2}$.

Under the above assumption, the following expression, which is derived from equation **(2),** depends only on the sign of $[2\theta - \gamma(\lambda - \Omega)]$:

$$\frac{\partial d_i(d_j)}{\partial d_j} = \frac{2[2(\lambda - \Omega) - \theta\gamma][2\theta - \gamma(\lambda - \Omega)]}{(1 - s)(4 - \gamma^2)^2 - 2[2(\lambda - \Omega) - \theta\gamma]^2}.$$

Taking into consideration the above relationship between d_i and d_j, it is interesting to determine whether they are strategic substitutes or strategic complements. According to Bulow et al. (1985), when firms compete in prices (quantities), the strategic variables are called strategic substitutes (complements) when they correspond to the assumption of downward (upward) sloping reaction curves in the price (quantity). Consequently, d_i and d_j are called strategic complement (substitute) when the interaction between them is positive (negative).

<u>Proposition 2:</u> Under Assumption 1, when the degree of product substitutability γ is small enough and/or the demand spillovers θ are large enough such that $\theta > \frac{\gamma(\lambda-\Omega)}{2}$, d_i and d_j are strategic complements and $\left[\partial^2\pi_i^*/\partial d_i \partial d_j\right] > 0$; otherwise, they are strategic substitutes.

Proposition 2 states that when θ is large enough and product substitutability is small enough (indicating low competition in the market), an increase in quality investment by a firm always increases the marginal profit of quality investment to its rival since the sign of $\left[\partial d_i(d_j)/d_j\right]$ is the same as the sign of $\left[\partial^2\pi_i^*/\partial d_i \partial d_j\right] = \left[\partial(\partial\pi_i^*/\partial d_i)/\partial d_j\right]$. However, when θ is small enough such that $\theta < \frac{\gamma(\lambda-\Omega)}{2}$ and the competition is intense such that γ is large enough, an increase in quality investment by a firm reduces the marginal profit of quality investment to the rival, inducing the rival to invest less in quality. In this case, d_i and d_j are strategic substitutes.

Next, I conduct a comparative static analysis on how the optimal quality investment and optimal subsidy change with demand spillovers, consumers' quality sensitivity and degree of product substitutability.

<u>Lemma 1:</u> Under Assumption 1 and condition (II),

$$\frac{\partial d^{NC}}{\partial s} > 0, \frac{\partial d^{NC}}{\partial \lambda} > 0,$$

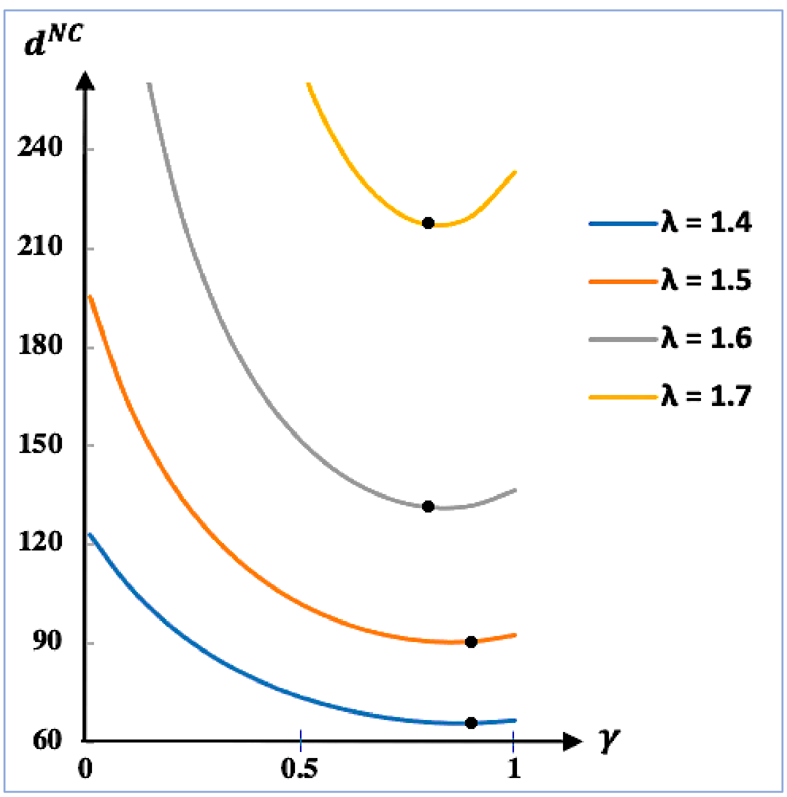
$$\frac{\partial d^{NC}}{\partial \gamma}\begin{cases}\geq 0 \text{ for } \gamma \in \left(\frac{2}{3},1\right], \quad 0<\theta<\frac{(3\gamma-2)}{\sqrt{2}}, \quad \text{and } \lambda-\Omega \geq \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}, \\ < 0 \text{ otherwise.}\end{cases}$$

The proof of Lemma 1 is provided in the appendix.

Lemma 1 shows that the R&D subsidies and consumer's quality sensitivity encourage firms to invest in quality. However, more intense competition does not necessarily prompt firm to provide more investment in non-cooperative quality. This result is illustrated graphically and evaluated at discrete values of the parameters. Specifically, in Figures 3.1 and 3.2, I provide numerical examples given $\theta = 0.3, \Omega = 0.5, s = 0.1$ and $\alpha = 100$. In addition to that, I allow λ to vary within the interval of [1.4, 1.7] so Assumption 1 and condition (II) are always satisfied.

Given different quality sensitivities λ, the following figure shows that the equilibrium quality effort under non-cooperation may increase or decrease with the intensification of market competition.

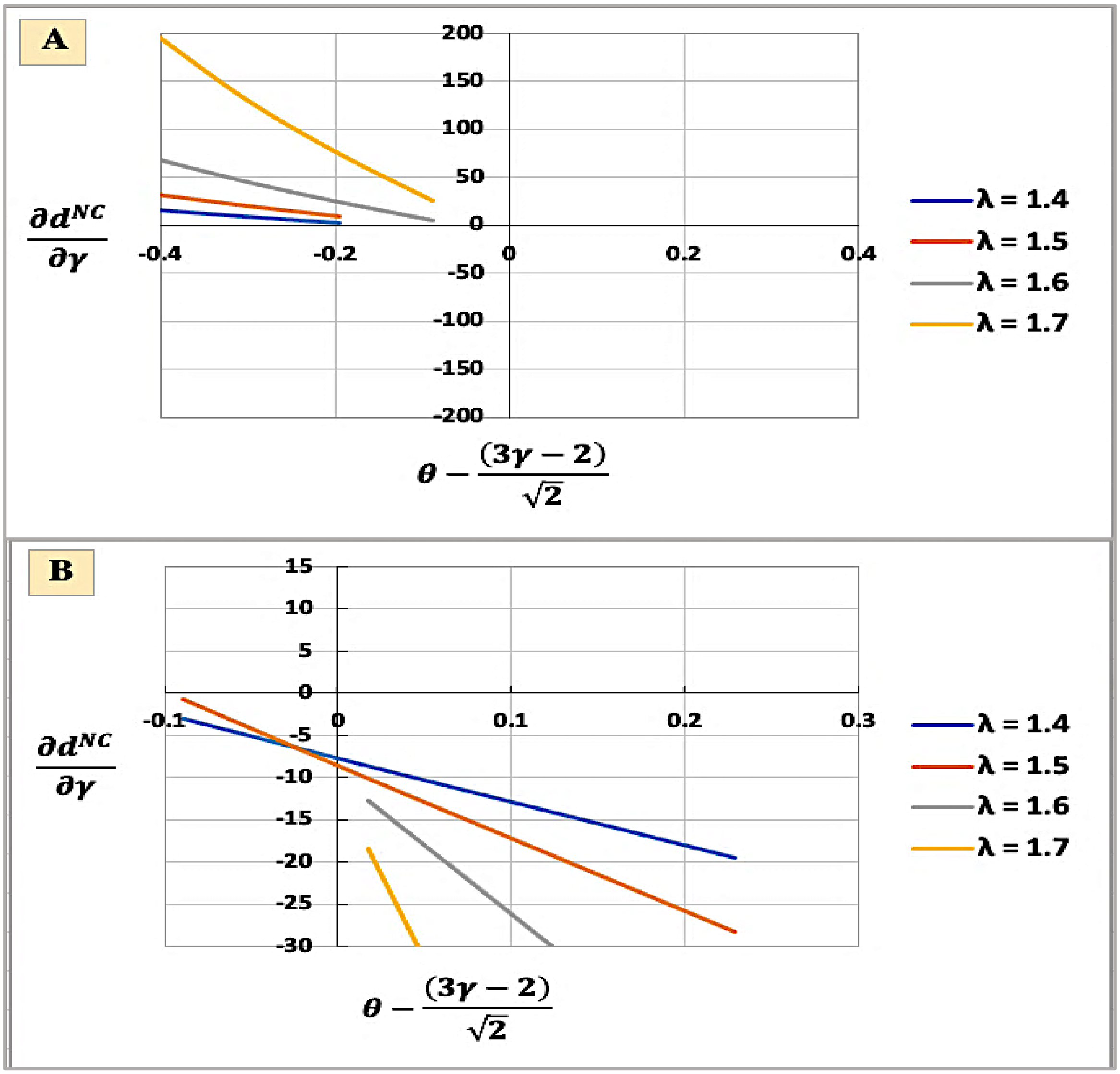
Figure 3.1: Optimal quality investment with respect to γ under different quality sensitivities (for $\boldsymbol{\theta = 0.3, \Omega = 0.5, s = 0.1}$ and $\boldsymbol{\alpha = 100}$)



In Figure 3.2, I show that, for all $\gamma \in \left(\frac{2}{3}, 1\right]$, the consumer's quality sensitivity λ and the gap between the demand spillover θ and the critical threshold $(\theta - \frac{(3\gamma-2)}{\sqrt{2}})$ might increase or decrease the effect of substitutability on the optimal quality investment under non-cooperation $(\frac{\partial d^{NC}}{\partial \gamma})$.

Figure 3.2: Derivative of the optimal quality investment with respect to γ

(for $\theta = 0.3, \Omega = 0.5, s = 0.1, \alpha = 100$ and $\gamma \in \left(\frac{2}{3}, 1\right]$)



Notes: In Figure 3.2(**A**), $\lambda - \Omega \geq \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}$ while in Figure 3.2(**B**), $\lambda - \Omega < \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}$. Conditional on Assumption 1, condition (II) and mathematical proofs, it is impossible to have $\lambda - \Omega \geq \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}$ and $\theta > \frac{(3\gamma-2)}{\sqrt{2}}$ at the same time.

Figure 3.2(**A**) indicates that the positive effect of substitutability on the optimal quality investment under non-cooperation ($\frac{\partial d^{NC}}{\partial \gamma} > 0$) is due to three factors: the intense competition ($\gamma \in \left(\frac{2}{3}, 1\right]$), the small demand spillover such that θ is lower than the critical threshold $\frac{(3\gamma-2)}{\sqrt{2}}$ and the large consumer's quality sensitivity ($\lambda \geq \Omega + \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}$). However, as it is shown in Figure 3.2(**B**), when the latter becomes small ($\lambda < \Omega + \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}$), the effect of substitutability on the optimal quality investment under non-cooperation becomes negative regardless of whether the demand spillover is smaller or larger than the critical threshold.

Regarding the social welfare, I already have

$$q_i^* = \frac{\alpha(2-\gamma) + [2(\lambda-\Omega) - \theta\gamma]d_i + [2\theta - \gamma(\lambda-\Omega)]d_j}{(4-\gamma^2)},$$

$$d_i = d_j = d^{NC} \Longrightarrow q^{NC} = \frac{\alpha + (\theta+\lambda-\Omega)d^{NC}}{(2+\gamma)}.$$

The Subgame Perfect Nash Equilibrium is as follows:

$$D^{NC} = 2d^{NC} = \frac{4\alpha[2(\lambda-\Omega)-\theta\gamma]}{(2-\gamma)(2+\gamma)^2(1-s) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]},$$

$$Q^{NC} = 2q^{NC} = \frac{2\alpha + 2(\theta+\lambda-\Omega)d^{NC}}{(2+\gamma)}.$$

The firm's individual profit at equilibrium is as follows:

$$\pi^{NC} = (q^{NC})^2 - \frac{(1-s)(d^{NC})^2}{2}.$$

At the non-cooperative equilibrium, the consumers and the producers' surpluses (CS and PS) as well the social welfare (SW) are expressed as follows:

$$CS^{NC} = \frac{(q^{NC})^2}{2} + \frac{(q^{NC})^2}{2} + \gamma(q^{NC})^2 = (1+\gamma)(q^{NC})^2,$$

and

$$PS^{NC} = 2\pi^{NC} = 2(q^{NC})^2 - (1-s^{NC})(d^{NC})^2.$$

$$SW^{NC} = (3+\gamma)(q^{NC})^2 - (1-s^{NC})(d^{NC})^2 - \frac{s^{NC}}{2}(d^{NC})^2 - \frac{s^{NC}}{2}(d^{NC})^2$$

$$= (3+\gamma)(q^{NC})^2 - (d^{NC})^2.$$

Given that d^{NC} as well q^{NC}depend on "s", maximizing the social welfare with respect to the proportion of the R&D subsidy results in an optimal subsidy under non-cooperation, which is denoted by s^{NC}.

<u>Proposition 3:</u> Under Assumption 1 and condition (II),

$$s^{NC} = 1 - \frac{2[2(\lambda - \Omega) - \theta\gamma]}{(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)},$$

$$\frac{\partial s^{NC}}{\partial \gamma} \begin{cases} < 0 \; for \;\; \theta < \dfrac{2(\lambda - \Omega)(1+2\gamma)}{(3+\gamma)(2-\gamma) + \gamma(1+2\gamma)}, \\ \geq 0 \; otherwise. \end{cases}$$

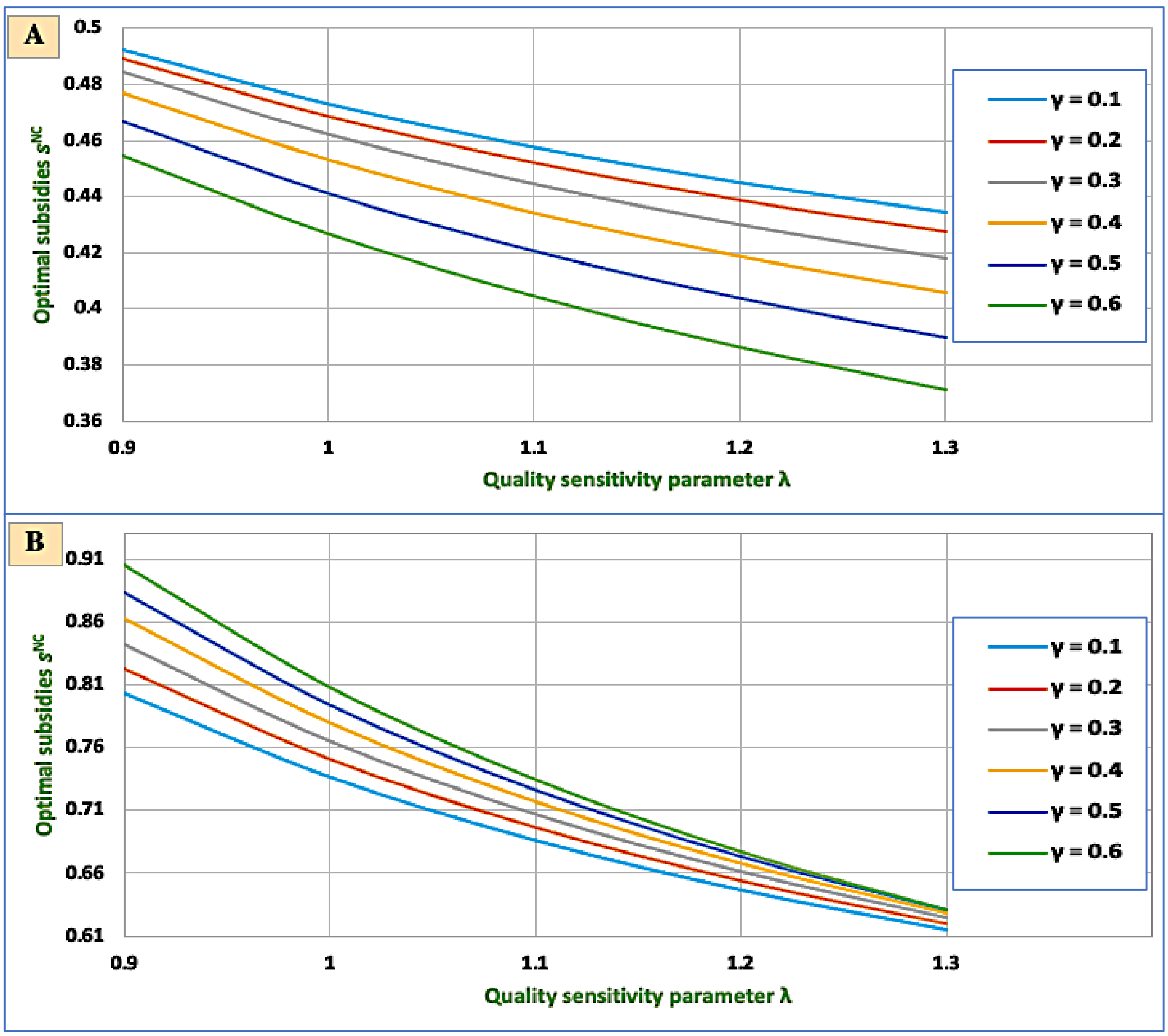
$$\frac{\partial s^{NC}}{\partial \theta} > 0, \frac{\partial s^{NC}}{\partial \lambda} < 0.$$

The proof of Proposition 3 is provided in the appendix.

When products are highly substitutable and when the spillovers are small enough (i.e., γ is large and θ is small), the government does not need to subsidize firms to stimulate the quality, because firms will be urged to proactively make efforts due to the intense competition and due to the fact that the size of the market is not expanded. As it is shown in Proposition 3, the optimal subsidy under non-cooperation s^{NC} and the degree of product substitutability γ can be positively or negatively correlated depending on whether the demand spillover θ is larger or smaller than $\frac{2(\lambda-\Omega)(1+2\gamma)}{(3+\gamma)(2-\gamma)+\gamma(1+2\gamma)}$. For all γ, Proposition 3 shows that, as demand spillovers θ increase, the R&D subsidy under non-cooperation increases as well which means that the government needs to provide more support for non-cooperative quality investments, through higher subsidies s, when spillovers are large. As quality sensitivity λ increases, the market forces on the demand side will stimulate firms to make more quality investment, therefore the government can lower subsidy spending regardless of the demand spillovers and regardless of the degree of product substitutability.

Figure 3.3 shows the negative relationship between the quality sensitivity parameter and the optimal subsidy under non-cooperation. In Figure 3.3(**A**), I fix Ω at 0.3 and θ at 0.19 so that $\theta < min\left\{\frac{2(\lambda-\Omega)(1+2\gamma)}{(3+\gamma)(2-\gamma)+\gamma(1+2\gamma)}\right\}$ and therefore, s^{NC} and γ are negatively correlated while, in Figure 3.3(**B**), I fix Ω at 0.7 and θ at 0.42 so that $\theta \geq max\left\{\frac{2(\lambda-\Omega)(1+2\gamma)}{(3+\gamma)(2-\gamma)+\gamma(1+2\gamma)}\right\}$ and therefore, s^{NC} and γ are positively correlated. In addition to that, I allow λ to vary within the interval of [0.9, 1.3] so Assumption 1 and Condition (II) are always satisfied given the numerical values of θ.

Figure 3.3: Optimal subsidy under non-cooperation as a function of quality sensitivity parameter under different degrees of substitutability



Notes: In Figure 3.3(**A**), $\Omega = 0.3$ and $\theta = 0.19$ such that $\theta < min\left\{\frac{2(\lambda-\Omega)(1+2\gamma)}{(3+\gamma)(2-\gamma)+\gamma(1+2\gamma)}\right\}$ while in Figure 3.3(**B**), $\Omega = 0.7$ and $\theta = 0.42 \geq max\left\{\frac{2(\lambda-\Omega)(1+2\gamma)}{(3+\gamma)(2-\gamma)+\gamma(1+2\gamma)}\right\}$. I focus on λ in [0.9, 1.3] so that Assumption 1 and Condition (II) are always satisfied given the value of θ.

Since I consider the quality sensitivity parameter, the optimal subsidy under non-cooperation and the degree of substitution in the above figure, it is important to mention that, for all θ positive, substitutability reduces the effect of quality sensitivity on optimal subsidy under non-cooperation since $\partial^2 s^{NC}/\partial\lambda\partial\gamma = \partial(\partial s^{NC}/\partial\lambda)/\partial\gamma < 0$.

3.4.2 Partial cooperative scenario

In this subsection, I consider an R&D cartel without cooperation in the product market, so the firms cooperate in the second stage but compete in the third stage. In this scenario, the production stage remains the same compared to the noncooperative case developed previously. The only change concerns the R&D stage. In this stage, firms now jointly choose a quality investment level that maximizes the sum of profits:

$$\text{Max } \pi_i^* + \pi_j^* = (q_i^*)^2 + (q_j^*)^2 - \left(\frac{d_i^2}{2} - \frac{sd_i^2}{2}\right) - \left(\frac{d_j^2}{2} - \frac{sd_j^2}{2}\right)$$

subject to $d_i, d_j, \pi_i^*, \pi_j^* \geq 0$.

$$\frac{\partial(\pi_i^* + \pi_j^*)}{\partial d_i} = 2q_i \frac{\partial q_i^*}{\partial d_i} + 2q_j \frac{\partial q_j^*}{\partial d_i} - (1-s)d_i = 0.$$

Conditional on d_i and d_j, the optimal production functions at equilibrium are:

$$q_i^*(d_i, d_j) = \frac{\alpha(2-\gamma) + [2(\lambda - \Omega) - \gamma\theta]d_i + [2\theta - \gamma(\lambda - \Omega)]d_j}{(4-\gamma^2)},$$

$$q_j^*(d_i, d_j) = \frac{\alpha(2-\gamma) + [2(\lambda - \Omega) - \gamma\theta]d_j + [2\theta - \gamma(\lambda - \Omega)]d_i}{(4-\gamma^2)}.$$

Therefore,

$$\frac{\partial q_i^*}{\partial d_i} = \frac{[2(\lambda - \Omega) - \gamma\theta]}{(4-\gamma^2)}, \qquad \frac{\partial q_j^*}{\partial d_i} = \frac{[2\theta - \gamma(\lambda - \Omega)]}{(4-\gamma^2)},$$

$$\frac{\partial(\pi_i^* + \pi_j^*)}{\partial d_i} = 2q_i \frac{[2(\lambda - \Omega) - \gamma\theta]}{(4-\gamma^2)} + 2q_j \frac{[2\theta - \gamma(\lambda - \Omega)]}{(4-\gamma^2)} - (1-s)d_i,$$

$$\frac{\partial^2(\pi_i^* + \pi_j^*)}{\partial d_i^2} = 2\frac{[2(\lambda - \Omega) - \gamma\theta]^2}{(4-\gamma^2)^2} + 2\frac{[2\theta - \gamma(\lambda - \Omega)]^2}{(4-\gamma^2)^2} - (1-s).$$

To ensure the concavity of $(\pi_i^* + \pi_j^*)$ with respect to d_i, I place a restriction on "s" such that:

$$2\frac{[2(\lambda - \Omega) - \gamma\theta]^2}{(4-\gamma^2)^2} + 2\frac{[2\theta - \gamma(\lambda - \Omega)]^2}{(4-\gamma^2)^2} < (1-s). \qquad (\boldsymbol{III})$$

By solving the first order conditions, the best responses for quality investment decisions are expressed as follows:

$$d_i(d_j) = \frac{2\alpha(2-\gamma)^2(\theta+\lambda-\Omega) + 4[2(\lambda-\Omega)-\gamma\theta][2\theta-\gamma(\lambda-\Omega)]d_j}{(4-\gamma^2)^2(1-s)-2[[2(\lambda-\Omega)-\gamma\theta]^2+[2\theta-\gamma(\lambda-\Omega)]^2]},$$

$$d_j(d_i) = \frac{2\alpha(2-\gamma)^2(\theta+\lambda-\Omega) + 4[2(\lambda-\Omega)-\gamma\theta][2\theta-\gamma(\lambda-\Omega)]d_i}{(4-\gamma^2)^2(1-s)-2[[2(\lambda-\Omega)-\gamma\theta]^2+[2\theta-\gamma(\lambda-\Omega)]^2]}.$$

The symmetry and the linearity of the above response functions imply that both firms produce the same level of quality investment at equilibrium. By solving the system of these reaction functions, I obtain the unique symmetric equilibrium under partial cooperation, which is positive given the concavity of the profit with respect to d_i, or equivalently, by assuming

$$2[\theta+\lambda-\Omega]^2 < (1-s)(2+\gamma)^2. \qquad (\boldsymbol{IV})$$

In all the results in this subsection and the corresponding proofs, I assume both concavity conditions *(III)* and *(IV)* of the profit function.

<u>Proposition 4:</u> The optimal quality investment $d_i^* = d_j^* = d^c$ in response to the partial cooperative scenario is expressed as follows:

$$d^C = \frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2(1-s)-2(\theta+\lambda-\Omega)^2}.$$

The proof of Proposition 4 is provided in the appendix.

<u>Lemma 2:</u>

$$\frac{\partial d^C}{\partial s} > 0, \frac{\partial d^C}{\partial \lambda} > 0, \frac{\partial d^C}{\partial \gamma} < 0.$$

The proof of Lemma 2 is provided in the appendix.

Lemma 2 shows that R&D subsidies and higher quality sensitivity encourage firms to invest in cooperative quality. However, regardless of the demand spillovers and the quality sensitivity level, more intense competition (when γ increases) does not prompt firms to provide more effort in cooperative quality investment.

At equilibrium, the consumer surplus (CS), the joint profit and the social welfare (SW) under partial cooperation are expressed as follows:

$$CS^C = \frac{(q^C)^2}{2} + \frac{(q^C)^2}{2} + \gamma(q^C)^2 = (1+\gamma)(q^C)^2,$$

where

$$q^C = \frac{\alpha(2-\gamma) + [2(\lambda-\Omega) - \gamma\theta]d^C + [2\theta - \gamma(\lambda-\Omega)]d^C}{(4-\gamma^2)} = \frac{\alpha + (\theta+\lambda-\Omega)d^C}{(2+\gamma)},$$

and

$$2\pi^C = 2(q^C)^2 - (1-s^C)(d^C)^2.$$

$$SW^C = (3+\gamma)(q^C)^2 - (1-s^C)(d^C)^2 - \frac{s^C}{2}(d^C)^2 - \frac{s^C}{2}(d^C)^2$$
$$= (3+\gamma)(q^C)^2 - (d^C)^2.$$

Given that d^C as well q^Cdepend on "s", maximizing the social welfare with respect to the proportion of the R&D subsidy results in an optimal subsidy that is denoted by s^C.

Proposition 5: Under partial cooperation, $s^C = \frac{(1+\gamma)}{(3+\gamma)}$. Furthermore, $[\partial s^C/\partial\gamma] > 0$.

The proof of Proposition 5 is provided in the appendix.

Proposition 5 implies that it is optimal to subsidize 50% of the costs of partial cooperative quality investment when the final goods are homogeneous ($\gamma = 1$). As the degree of product substitutability increases, firms reduce their cooperative quality investment and to correct for this underinvestment, the government finds that it is optimal to increase the subsidy. In contrast with the non-cooperative scenario, the optimal subsidy s^Cdoes not depend on the the demand spillovers and the quality sensitivity level and increases only with γ. This kind of simplification arises in the literature. By considering two scenarios, Cabon-Dhersin and Gibert (2020) found that the optimal subsidy under non-cooperation depends on the technological spillover as well as the number of firms in the industry while the optimal subsidy under partial cooperation depends only on the number of cooperating firms. When there are two firms, their result shows that it is optimal to subsidize 50% of the costs of partial cooperative investment, which is consistent with my result when the degree of product substitutability is equal to 1.

3.4.3 Full cooperative scenario

Firms collectively act as monopolist if they cooperate in their production stage as well as their R&D stage. Under the monopolistic situation, the joint profit maximization is defined as follows:

$$\text{Max}\, \pi_i + \pi_j = \left(\alpha + \lambda d_i - \Omega d_i + \theta d_j - (q_i + \gamma q_j)\right)q_i - \left(\frac{d_i^2}{2} - \frac{s d_i^2}{2}\right)$$

$$+\left(\alpha + \lambda d_j - \Omega d_j + \theta d_i - (q_j + \gamma q_i)\right)q_j - \left(\frac{d_j^2}{2} - \frac{s d_j^2}{2}\right)$$

subject to $q_i, q_j, d_i, d_j, \pi_i, \pi_j \geq 0$.

By using backward induction, I first solve the equilibrium quantity (which is different from the one that I found in the non-cooperative and partial cooperative scenarios) of the third stage then I solve the quality investment of the second stage. The first order conditions imply

$$\frac{\partial(\pi_i + \pi_j)}{\partial q_i} = \frac{\partial(\pi_i + \pi_j)}{\partial q_j} = 0.$$

The equilibrium quantities of firm i and j are expressed as follows:

$$\tilde{q}_i = \frac{\alpha(1-\gamma) + [(\lambda - \Omega) - \gamma\theta]d_i + [\theta - \gamma(\lambda - \Omega)]d_j}{2(1-\gamma^2)},$$

$$\tilde{q}_j = \frac{\alpha(1-\gamma) + [(\lambda - \Omega) - \gamma\theta]d_j + [\theta - \gamma(\lambda - \Omega)]d_i}{2(1-\gamma^2)}.$$

Therefore,

$$\frac{\partial(\tilde{\pi}_i + \tilde{\pi}_j)}{\partial d_i} = \frac{\begin{bmatrix}\alpha(\lambda - \Omega + \theta)(1-\gamma) + [(\lambda - \Omega)^2 - 2\gamma\theta(\lambda - \Omega) + \theta^2]d_i \\ +[2(\lambda - \Omega)\theta - \gamma\theta^2 - \gamma(\lambda - \Omega)^2]d_j\end{bmatrix}}{2(1-\gamma^2)} - (1-s)d_i,$$

$$\frac{\partial^2(\tilde{\pi}_i + \tilde{\pi}_j)}{\partial d_i^2} = \frac{[(\lambda - \Omega)^2 - 2\gamma\theta(\lambda - \Omega) + \theta^2]}{2(1-\gamma^2)} - (1-s).$$

To ensure the concavity of $(\tilde{\pi}_i + \tilde{\pi}_j)$ with respect to d_i, I place a restriction on "s" such that:

$$\frac{[(\lambda - \Omega)^2 - 2\gamma\theta(\lambda - \Omega) + \theta^2]}{2(1-\gamma^2)} < (1-s). \qquad (\boldsymbol{V})$$

I obtain the unique optimal solution d^M under the full cooperative scenario by solving the first order conditions of the joint profit function with respective to d_i and d_j after taking into account the optimal quantities in the third stage,

$$\frac{\partial\left(\tilde{\pi}_i + \tilde{\pi}_j\right)}{\partial d_i} = \frac{\partial\left(\tilde{\pi}_i + \tilde{\pi}_j\right)}{\partial d_j} = 0,$$

which implies the following system of reaction functions:

$$\begin{cases} d_i(d_j) = \dfrac{\alpha(\lambda-\Omega+\theta)(1-\gamma)+[2(\lambda-\Omega)\theta-\gamma\theta^2-\gamma(\lambda-\Omega)^2]d_j}{2(1-s)(1-\gamma^2)-[(\lambda-\Omega)^2-2\gamma\theta(\lambda-\Omega)+\theta^2]}, \\ d_j(d_i) = \dfrac{\alpha(\lambda-\Omega+\theta)(1-\gamma)+[2(\lambda-\Omega)\theta-\gamma\theta^2-\gamma(\lambda-\Omega)^2]d_i}{2(1-s)(1-\gamma^2)-[(\lambda-\Omega)^2-2\gamma\theta(\lambda-\Omega)+\theta^2]}. \end{cases}$$

The symmetry and the linearity of the above response functions imply that both firms produce the same level of quality investment at equilibrium. Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I have the following condition:

$$[\theta+\lambda-\Omega]^2 < 2(1-s)(1+\gamma). \qquad (\boldsymbol{VI})$$

In all the results in this subsection and the corresponding proofs, I assume both concavity conditions *(V)* and *(VI)* of the profit function.

<u>Proposition 6:</u> The optimal quality investment $d_i^* = d_j^* = d^M$ in response to the full cooperative scenario is expressed as follows:

$$d^M = \frac{\alpha(\theta+\lambda-\Omega)}{2(1-s)(1+\gamma)-(\theta+\lambda-\Omega)^2}.$$

The proof of Proposition 6 is provided in the appendix.

<u>Lemma 3:</u>

$$\frac{\partial d^M}{\partial s} > 0, \frac{\partial d^M}{\partial \lambda} > 0, \frac{\partial d^M}{\partial \gamma} < 0.$$

The proofs are provided in the appendix.

Notice that the results in Lemma 3 are the same as the results in Lemma 2. In the proof of Proposition 6, I show that, under the monopolistic situation,

$$\tilde{\pi}_i + \tilde{\pi}_j = \tilde{q}_i^2 + \tilde{q}_j^2 + 2\gamma\tilde{q}_i\tilde{q}_j - \frac{(1-s)}{2}\left(d_i^2 + d_j^2\right).$$

At equilibrium, $\tilde{q}_i = \tilde{q}_j = q^M$ and $d_i = d_j = d^M$; therefore, the joint profit (π), the consumer surplus (CS) and the social welfare (SW) under full cooperation are expressed as follows:

$$\pi = 2(1+\gamma)(q^M)^2 - (1-s^M)(d^M)^2,$$

$$CS^M = \frac{(q^M)^2}{2} + \frac{(q^M)^2}{2} + \gamma(q^M)^2 = (1+\gamma)(q^M)^2,$$

$$SW^M = (1+\gamma)(q^M)^2 + 2(1+\gamma)(q^M)^2 - (1-s^M)(d^M)^2 - s^M(d^M)^2$$
$$= 3(1+\gamma)(q^M)^2 - (d^M)^2.$$

Given that d^M and q^M depend on "s", maximizing the social welfare with respect to the proportion of the R&D subsidy results in an optimal subsidy under full cooperation that is denoted by s^M. In contrast with the non-cooperative scenario, Proposition 7 shows that the optimal strategy for the government is to subsidize one third of the costs of full cooperative quality investment, independently of the degree of product substitutability, the demand spillovers and the quality sensitivity level.

Proposition 7: Under the monopolistic situation, $s^M = \frac{1}{3}$.

The proof of Proposition 7 is provided in the appendix.

3.5 Comparisons and results

In this section, I compare and discuss the outcomes of the three scenarios stated above under which Assumption 1 and all the concavity conditions of the profit functions are satisfied. Specifically, I investigate whether the ranking of the subsidized (S) and the non-subsidized (NS) optimal quality expenditures and profits as well as the ranking of the optimal subsidies depend on the degree of product substitutability, the demand spillovers, the quality sensitivity level.

3.5.1 Non-subsidized equilibrium outcomes

The non-subsidized (NS) optimal quality investments and profits under the three different scenarios are identified as follows:

$$\begin{cases} d_{NS}^{NC} = \dfrac{2\alpha[2(\lambda-\Omega)-\theta\gamma]}{(2-\gamma)(2+\gamma)^2-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]}, \\ d_{NS}^{C} = \dfrac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2-2(\theta+\lambda-\Omega)^2}, \\ d_{NS}^{M} = \dfrac{\alpha(\lambda-\Omega+\theta)}{2(1+\gamma)-(\theta+\lambda-\Omega)^2}. \end{cases}$$

$$\begin{cases} \pi_{NS}^{NC} = \dfrac{\alpha^2[(2-\gamma)^2(2+\gamma)^2-2[2(\lambda-\Omega)-\theta\gamma]^2]}{\left[(2-\gamma)(2+\gamma)^2-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]^2}, \\ \pi_{NS}^{C} = \dfrac{\alpha^2}{[(2+\gamma)^2-2(\theta+\lambda-\Omega)^2]}, \\ \pi_{NS}^{M} = \dfrac{\alpha^2}{2[2(1+\gamma)-(\theta+\lambda-\Omega)^2]}. \end{cases}$$

Proposition 8: For all $\theta, (\lambda-\Omega)$ and $\gamma > 0,$ the rankings of the non-subsidized (NS) optimal quality investments and profits are based on the following conditions:

$d_{NS}^{NC} > d_{NS}^{M} > d_{NS}^{C}$ for all $0 < \theta < \dfrac{(\lambda-\Omega)[8(1+\gamma)-(2-\gamma)(2+\gamma)^2]}{4\gamma(1+\gamma)+(2-\gamma)(2+\gamma)^2}$,

$d_{NS}^{M} > d_{NS}^{NC} > d_{NS}^{C}$ for all $\dfrac{(\lambda-\Omega)[8(1+\gamma)-(2-\gamma)(2+\gamma)^2]}{4\gamma(1+\gamma)+(2-\gamma)(2+\gamma)^2} < \theta < \dfrac{\gamma(\lambda-\Omega)}{2}$,

$d_{NS}^{M} > d_{NS}^{C} > d_{NS}^{NC}$ for all $\dfrac{\gamma(\lambda-\Omega)}{2} < \theta < 1$,

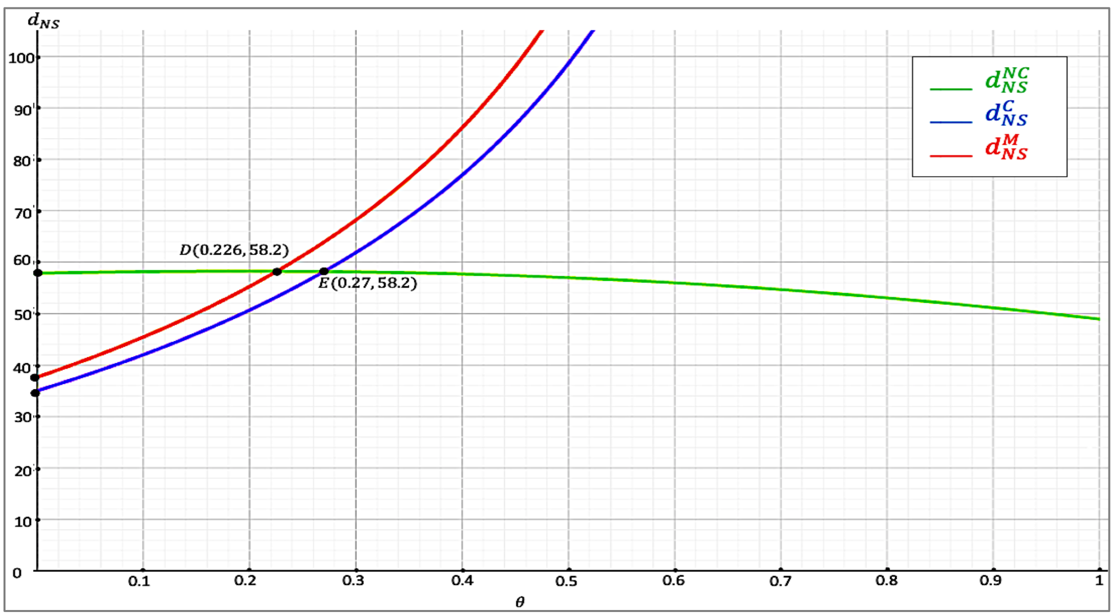
$\pi_{NS}^{M} > \pi_{NS}^{C} > \pi_{NS}^{NC}$ for all $\theta \neq \dfrac{\gamma(\lambda-\Omega)}{2}$,

$\pi_{NS}^{M} > \pi_{NS}^{C} = \pi_{NS}^{NC}$ for $\theta = \dfrac{\gamma(\lambda-\Omega)}{2}$.

The proof is provided in the appendix.

As it is shown in Proposition 8, the ranking of the non-subsidized optimal quality investments depends on the magnitude of the demand spillovers, the degree of product differentiation (in other words, the degree of competition) and the net effect of quality sensitivity $(\lambda-\Omega)$. Interestingly, regardless of whether the demand spillover is small or large, the firm's quality investment under the full cooperative scenario d_{NS}^{M} is always larger than d_{NS}^{C}. This is due to the fact that the collaborative nature of full cooperation always allows for greater quality investment compared to partial cooperation. Regarding the firm's quality investment d_{NS}^{NC}, it is larger than d_{NS}^{C} and d_{NS}^{M} for small demand spillover ($0 < \theta < \frac{(\lambda-\Omega)[8(1+\gamma)-(2-\gamma)(2+\gamma)^2]}{4\gamma(1+\gamma)+(2-\gamma)(2+\gamma)^2}$) and this stems from the fact that the individual returns to quality investment are very large when θ is small. However, when there is a large demand spillover ($\frac{\gamma(\lambda-\Omega)}{2} < \theta < 1$), d_{NS}^{NC} is smaller than d_{NS}^{C} and d_{NS}^{M}. This result is explained by the "free-riding" effect where at some point, the presence of externalities discourages the non-cooperating firm from investing in quality. However, when firms engage in partial or full cooperation, the free riding effect does not occur since the presence of demand spillovers encourages them to exert additional efforts and invest more in quality. This result is consistent with Figure 3.4 since as θ increases, d_{NS}^{C} and d_{NS}^{M} increase as well.

Figure 3.4: Non-subsidized optimal quality investment as a function of demand spillover



Notes: $\alpha = 100$: the size of the market; $\lambda = 1.5$; $\Omega = 0.6$; $\gamma = 0.6$.

Regarding the profits under the partial and full cooperative scenarios (π_{NS}^{C} and π_{NS}^{M}), Figure 3.5 shows that they are increasing with demand spillover.[7] In other words, the firms that are engaging in partial or full cooperative agreement are more willing to increase their effort for large spillover and to share the results of their research.[8]

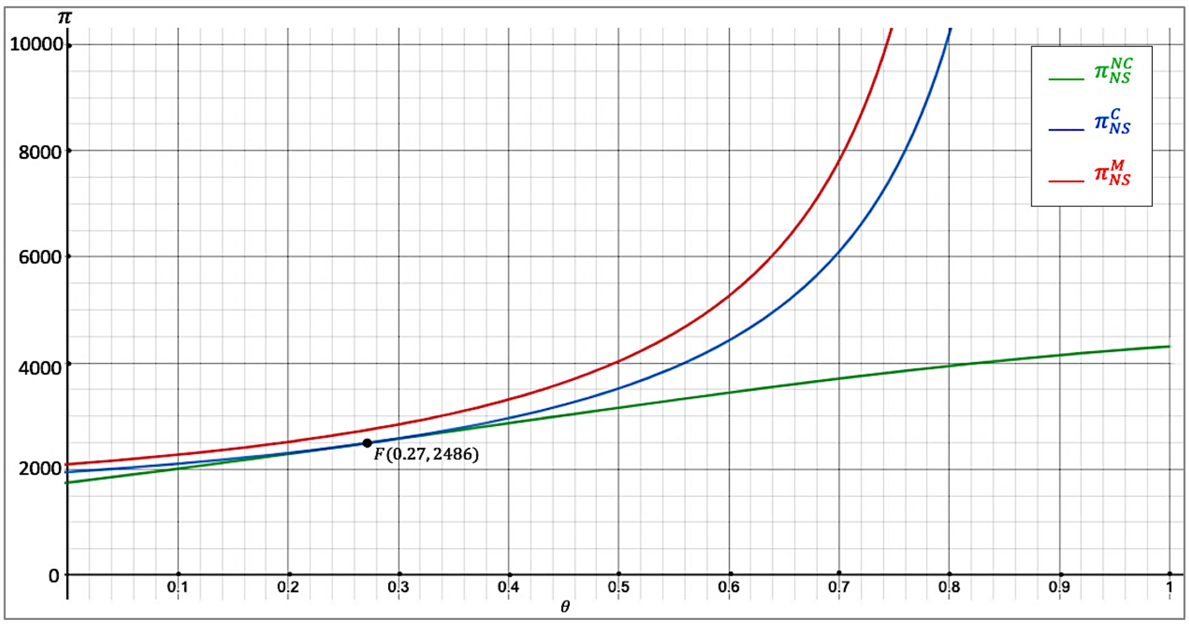
---

[7] $(\partial\pi_{NS}^{C}/\partial\theta) = 4\alpha^2(\theta+\lambda-\Omega)/[(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2]^2 > 0$,
$(\partial\pi_{NS}^{M}/\partial\theta) = \alpha^2(\theta+\lambda-\Omega)/(2[2(1+\gamma) - (\theta+\lambda-\Omega)^2]^2) > 0$.

[8] $(\partial d_{NS}^{C}/\partial\theta) = [2\alpha(2+\gamma)^2 + 4\alpha(\theta+\lambda-\Omega)^2]/[(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2]^2 > 0$,
$(\partial d_{NS}^{M}/\partial\theta) = [2\alpha(1+\gamma) + \alpha(\theta+\lambda-\Omega)^2]/[2(1+\gamma) - (\theta+\lambda-\Omega)^2]^2 > 0$.

By considering the ranking of the profits under each regime, Proposition 8 shows that regardless of whether the demand spillover is small or large, firms are motivated to cooperate in as many stages as allowed to. This inclination towards more cooperation can stem from various reasons, such as the desire to leverage complementary resources and expertise, reduce costs, increase efficiency, and access new markets or technologies. Consequently, the full cooperative regime is the best market outcome ($\pi_{NS}^{M} > \pi_{NS}^{C} > \pi_{NS}^{NC}$ for all $\theta \neq \frac{\gamma(\lambda-\Omega)}{2}$). This result is consistent with Figure 3.5.

Figure 3.5: Non-subsidized optimal profit as a function of demand spillover



Notes: $\alpha = 100$: the size of the market; $\lambda = 1.5$; $\Omega = 0.6$; $\gamma = 0.6$.

For the social welfares under the case where there is no subsidy, they are identified as follows:

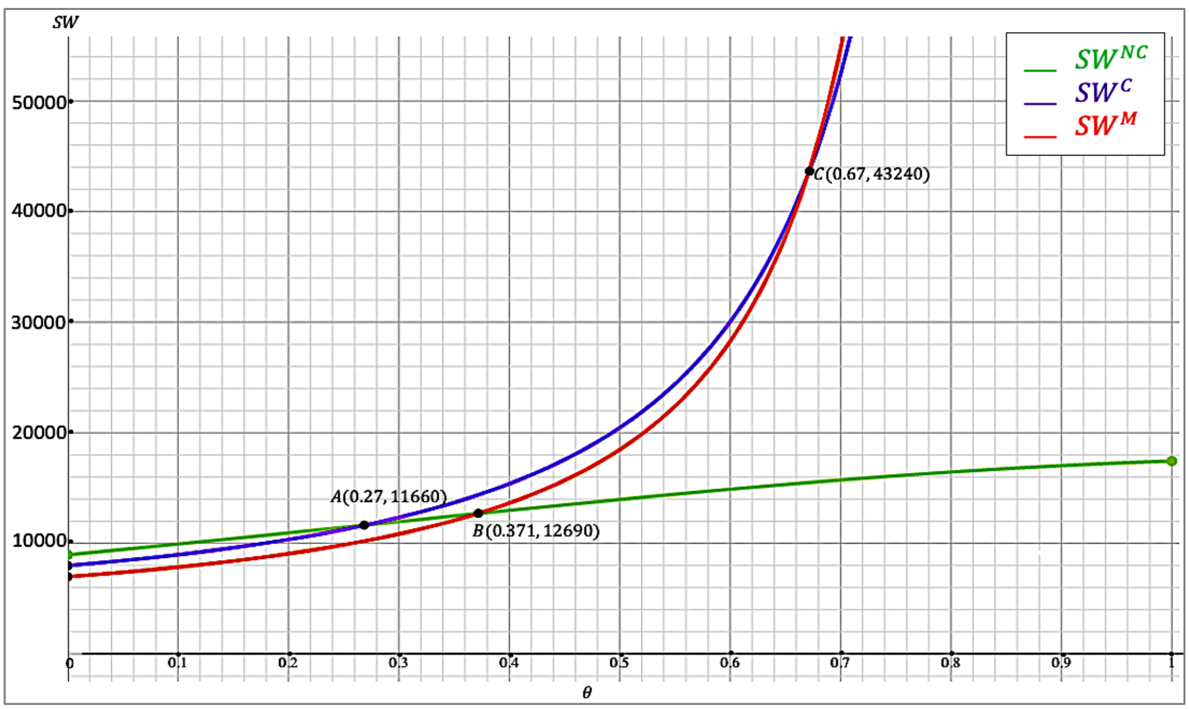
$$\begin{cases} SW_{NS}^{NC} = (3+\gamma)(q_{NS}^{NC})^2 - (d_{NS}^{NC})^2 = \dfrac{\alpha^2(2-\gamma)^2(2+\gamma)^2(3+\gamma) - 4\alpha^2[2(\lambda-\Omega)-\theta\gamma]^2}{\left[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]^2}, \\ SW_{NS}^{C} = (3+\gamma)(q_{NS}^{C})^2 - (d_{NS}^{C})^2 = \dfrac{(3+\gamma)\alpha^2(2+\gamma)^2 - 4\alpha^2[\theta+\lambda-\Omega]^2}{[(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2]^2}, \\ SW_{NS}^{M} = (1+\gamma)(q_{NS}^{M})^2 - (d_{NS}^{M})^2 = \dfrac{3(1+\gamma)\alpha^2 - \alpha^2(\theta+\lambda-\Omega)^2}{[2(1+\gamma) - (\theta+\lambda-\Omega)^2]^2}. \end{cases}$$

such that $q_{NS}^{NC} = \frac{\alpha+(\theta+\lambda-\Omega)d_{NS}^{NC}}{(2+\gamma)}$, $q_{NS}^{C} = \frac{\alpha+(\theta+\lambda-\Omega)d_{NS}^{C}}{(2+\gamma)}$ and $q_{NS}^{M} = \frac{\alpha+(\theta+\lambda-\Omega)d_{NS}^{M}}{2(1+\gamma)}$.

Regarding the ranking of these social welfares, the mathematical proofs are complicated; therefore, I rely on the numerical simulations and present the following Figure 3.6 that plots the social welfare under each scenario as a function of the demand spillovers. As Figure 3.6 shows, it is not always socially efficient when firms extend their research cooperative agreement from the R&D stage to the production stage since the ranking of SW_{NS}^{M} depends on the size of the demand spillovers.

For a small value of θ such that $\theta < 0.371$, the results indicate that it is always harmful when firms choose a full cooperative agreement (SW_{NS}^{M} is the lowest). This is due to the fact these firms do not share enough external information. While for $\theta > 0.371$, the results indicate that it is always harmful when firms choose to fully compete with each other (SW_{NS}^{NC} is the lowest) since the presence of large spillovers discourages them from investing in quality.

Figure 3.6: Non-subsidized Social welfare as a function of demand spillover



Notes: $\alpha = 100$: the size of the market; $\lambda = 1.5$; $\Omega = 0.6$; $\gamma = 0.6$. In the numerical example presented in Figure 3.6, the social welfare under the case where there is no subsidy is based on the following rankings:

$SW_{NS}^{NC} > SW_{NS}^{C} > SW_{NS}^{M}$ for all $0 < \theta < 0.27$,

$SW_{NS}^{C} > SW_{NS}^{NC} > SW_{NS}^{M}$ for all $0.27 < \theta < 0.371$,

$SW_{NS}^{C} > SW_{NS}^{M} > SW_{NS}^{NC}$ for all $0.371 < \theta < 0.67$,

$SW_{NS}^{M} > SW_{NS}^{C} > SW_{NS}^{NC}$ for all $0.67 < \theta < 1$,

$SW_{NS}^{C} = SW_{NS}^{NC} > SW_{NS}^{M}$ for $\theta = 0.27$,

$SW_{NS}^{C} > SW_{NS}^{M} = SW_{NS}^{NC}$ for $\theta = 0.371$,

$SW_{NS}^{M} = SW_{NS}^{C} > SW_{NS}^{NC}$ for $\theta = 0.67$.

3.5.2 Optimal subsidies

For the optimal subsidies under each scenario, I already found the following expressions:

$$\begin{cases} s^{NC} = 1 - \dfrac{2[2(\lambda - \Omega) - \theta\gamma]}{(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)}, \\ s^{C} = \dfrac{(1+\gamma)}{(3+\gamma)}, \\ s^{M} = \dfrac{1}{3}. \end{cases}$$

Proposition 9: For all $\theta, (\lambda - \Omega)$ and $\gamma > 0,$ the rankings of the different optimal subsidies are based on the following conditions:

$$s^C > s^M > s^{NC} \text{ for all } 0 < \theta < \frac{\gamma(1+\gamma)(\lambda - \Omega)}{(6 + 2\gamma - \gamma^2)},$$

$$s^C > s^{NC} > s^M \text{ for all } \frac{\gamma(1+\gamma)(\lambda - \Omega)}{(6 + 2\gamma - \gamma^2)} < \theta < \frac{\gamma(\lambda - \Omega)}{2},$$

$$s^{NC} > s^C > s^M \text{ for all } \frac{\gamma(\lambda - \Omega)}{2} < \theta < 1.$$

The proof is provided in the appendix.

When demand spillovers are small such that $\theta < \frac{\gamma(\lambda-\Omega)}{2}$, the optimal subsidy under partial cooperation is higher than the optimal subsidy under full competition ($s^C > s^{NC}$) . However, when demand spillovers are large such that $\theta > \frac{\gamma(\lambda-\Omega)}{2}$, the government needs to provide more support for non-cooperative quality investment. For a relatively large demand spillovers such that $\left\{\theta \epsilon (\frac{\gamma(1+\gamma)(\lambda-\Omega)}{(6+2\gamma-\gamma^2)}, 1)\right\} \setminus \left\{\frac{\gamma(\lambda-\Omega)}{2}\right\}$, the optimal R&D subsidy for the quality improvement which is destinated to the monopolist is the lowest when comparing to the partial or the full competitive scenario. Proposition 8 states that, for small and relatively small (large) demand spillover, the non-subsidized optimal quality investment under partial cooperation (non-cooperation) is the lowest compared to the other scenarios; therefore, the government needs to increase its subsidies $s^C (s^{NC})$.

3.5.3 Subsidized equilibrium outcomes

By substituting s^{NC}, s^{C} and s^{M} in the expressions of d^{NC}, d^{C} and d^{M} respectively, I get the following results:

$$\begin{cases} d_S^{NC} = \dfrac{2\alpha[2(\lambda-\Omega)-\theta\gamma]}{(1-s^{NC})(2-\gamma)(2+\gamma)^2-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]} \\ \quad\quad = \dfrac{\alpha(3+\gamma)(\theta+\lambda-\Omega)}{(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2}, \\ d_S^{C} = \dfrac{2\alpha(\theta+\lambda-\Omega)}{(1-s^{C})(2+\gamma)^2-2(\theta+\lambda-\Omega)^2} = \dfrac{\alpha(\theta+\lambda-\Omega)(3+\gamma)}{(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2}, \\ d_S^{M} = \dfrac{\alpha(\lambda-\Omega+\theta)}{2(1-s^{M})(1+\gamma)-(\theta+\lambda-\Omega)^2} = \dfrac{3\alpha(\theta+\lambda-\Omega)}{4(1+\gamma)-3(\theta+\lambda-\Omega)^2}. \end{cases}$$

<u>Proposition 10:</u>

The subsidized equilibrium outcomes satisfy the following conditions:[9]

$d_S^{NC} = d_S^{C} > d_S^{M}$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$,

$\dfrac{\partial d_S^{NC}}{\partial\lambda} = \dfrac{\partial d_S^{C}}{\partial\lambda} \neq \dfrac{\partial d_S^{M}}{\partial\lambda}$,

$\pi_S^{C} > (<)\ \pi_S^{M}$ if $\theta > (<) \sqrt{\gamma} - (\lambda-\Omega)$,

$\pi_S^{C} > (<)\ \pi_S^{NC}$ if $\theta < (>) \dfrac{\gamma(\lambda-\Omega)}{2}$,

$SW_S^{NC} = SW_S^{C} > SW_S^{M}$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

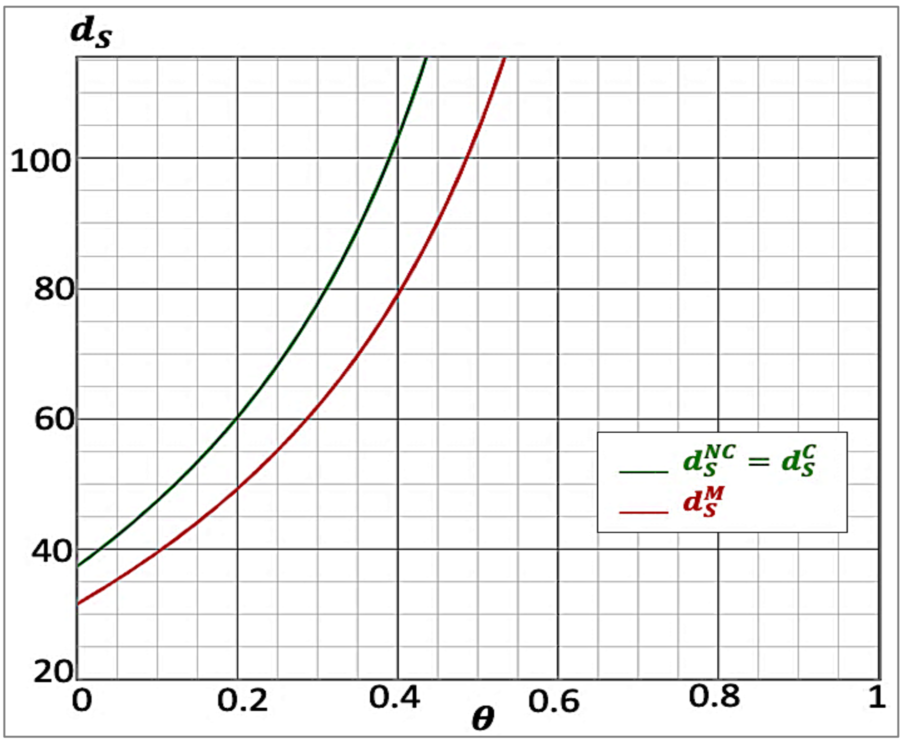
The proof is provided in the appendix.

Proposition 10 states that the noncooperative and the partial cooperative scenarios lead to the same subsidized equilibrium outcomes in terms of quality investment levels and social welfares ($d_S^{NC} = d_S^{C}$ and $SW_S^{NC} = SW_S^{C}$). This is due to the determination of the level of the optimal subsidy under non-cooperation that fully offsets the demand spillover effect. In other words, the public decision-maker must support the non-cooperative quality investment more intensively since without the public intervention, the non-cooperative quality investment is smaller than the partial cooperative quality investment for high degree of demand spillover ($d_{NS}^{NC} < d_{NS}^{C}$ for all $\theta > \frac{\gamma(\lambda-\Omega)}{2}$ as it is shown in Proposition 8) which is due to the internalization of externalities through cooperation.

[9] To determine the critical threshold for the difference between π_S^{M} and π_S^{NC}, the mathematical proofs are complicated, so I relied on the numerical simulations.

The greater support for non-cooperative quality investment offsets this spillover effect and d_S^{NC} becomes as efficient as d_S^C. In addition to that, Proposition 8 states that the non-subsidized optimal quality investment under full cooperation (d_{NS}^M) is always larger than d_{NS}^C; therefore, the government needs to reduce its subsidies s^M compared to s^C. Since the optimal subsidy under full cooperation is smaller than s^C (as it is shown in Proposition 9), the subsidized d_S^M becomes the least efficient one compared to the other quality investments (as it is shown in Proposition 10). This is consistent with Figure 3.7.

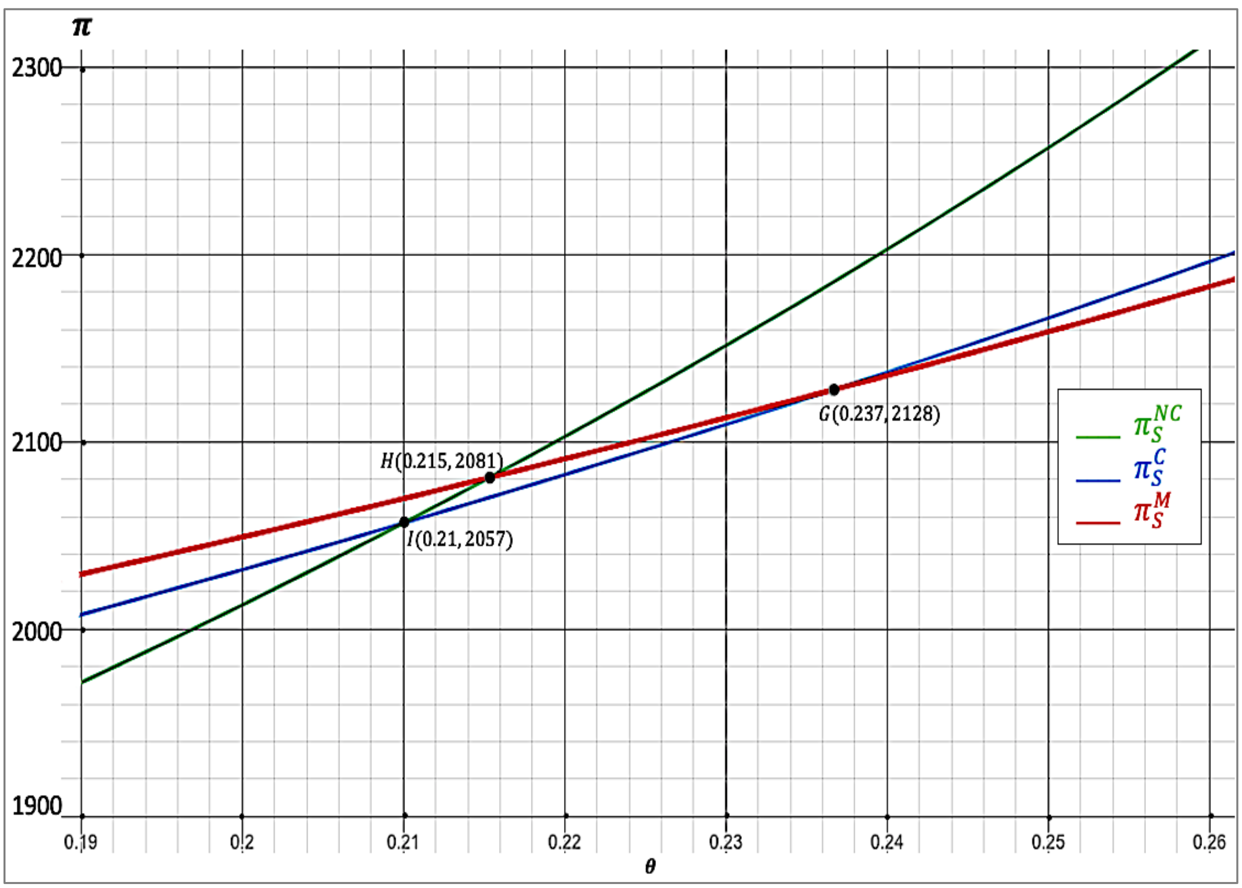
Figure 3.7: Subsidized optimal quality investment as a function of demand spillover



Notes: $\alpha = 100$: the size of the market; $\lambda = 1.2$; $\Omega = 0.6$; $\gamma = 0.7$.

Regardless of whether the demand spillover is small or large and for all $s \neq 0$, the comparison between the non-cooperative and the partial cooperative scenario shows that the impact of quality sensitivity on the quality investment is the same $(\frac{\partial d_S^{NC}}{\partial \lambda} = \frac{\partial d_S^C}{\partial \lambda})$. However, for all $\gamma \neq 0$, this effect is not the same when comparing the partial and the full cooperative scenario $((\partial d_S^C/\partial \lambda) \neq (\partial d_S^M/\partial \lambda))$ since the difference between $(\partial d_S^C/\partial \lambda)$ and $(\partial d_S^M/\partial \lambda)$ depends on the demand spillovers, the degree of product substitutability and the net effect of quality improvement (as it is mentioned in the proof of Proposition 10 in the appendix). For all $\{\theta > \sqrt{\gamma} - (\lambda - \Omega)\}$ and for $s \neq 0$, the firm's profit is larger when it decides to cooperate in the R&D stage only compared to the case where it decides to cooperate in the R&D stage as well as the production stage ($\pi_S^C > \pi_S^M$).[10] This is consistent, at some point, with Proposition 9 since $s^C > s^M$ for large demand spillover. In addition to that, Proposition 9 shows that, when demand spillovers are small such that $\theta < \frac{\gamma(\lambda-\Omega)}{2}$, the optimal subsidy under partial cooperation is larger than the optimal subsidy under full competition ($s^C > s^{NC}$) . In that case and as it is mentioned in Proposition 10, subsidizing partial cooperative quality affects positively the profit of the firms since ($\pi_S^C > \pi_S^{NC}$ for all $\theta < \frac{\gamma(\lambda-\Omega)}{2}$). The rankings of these profits under the different scenarios are consistent with the following figure.

Figure 3.8: Subsidized optimal profit as a function of demand spillover

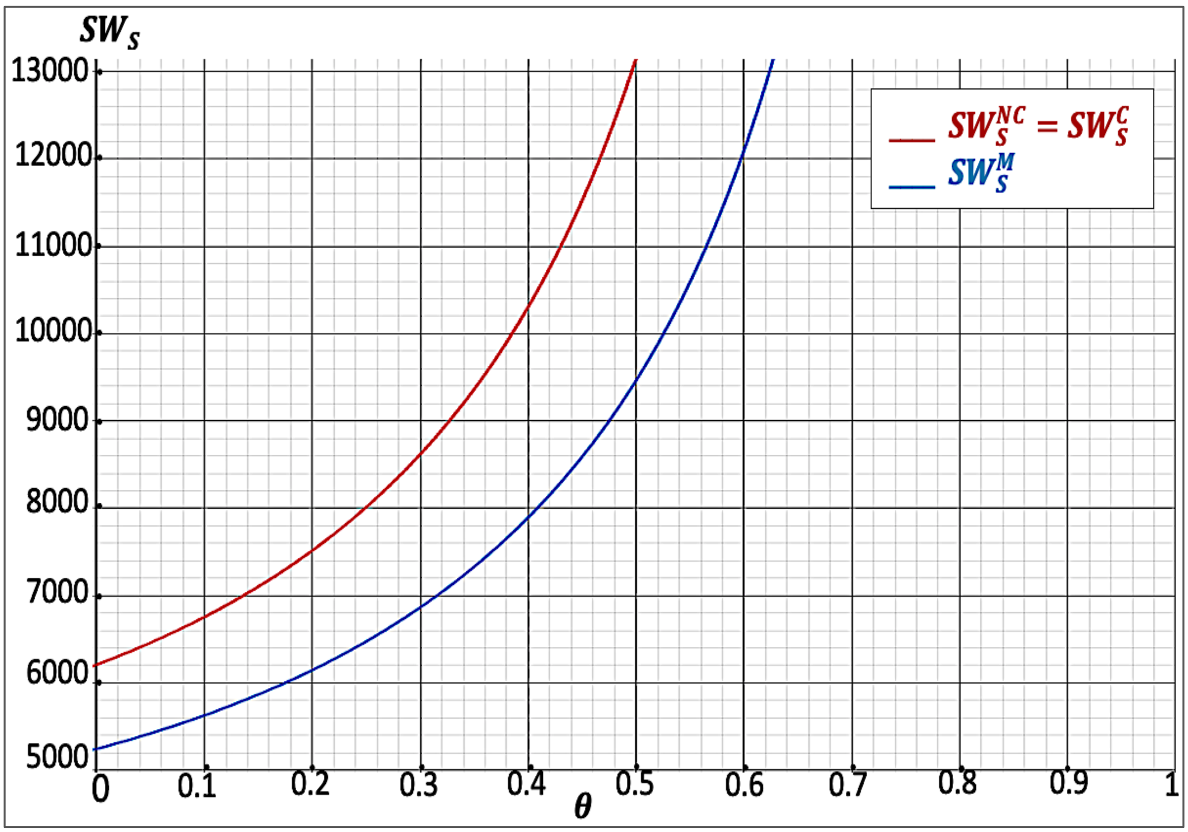


Notes: $\alpha = 100$: the size of the market; $\lambda = 1.2$; $\Omega = 0.6$; $\gamma = 0.7$, $\frac{\gamma(\lambda-\Omega)}{2} = 0.21$, $(\sqrt{\gamma} - (\lambda - \Omega)) = 0.237$.

[10] I assume that $[\sqrt{\gamma} - (\lambda - \Omega)] \in (0,1)$ so that I can avoid the case where $\theta < 0$.

The results of Proposition 10 show that subsidizing the non-cooperative or the partial cooperative quality investment leads to the same social welfare for all $\theta, (\lambda - \Omega)$ and $\gamma > 0$. Furthermore, compared to the full cooperative scenario, it is socially more efficient to increase the subsidy when firms coordinate their R&D activities or when they fully compete each other ($W_S^{NC} = SW_S^C > SW_S^M$). Therefore, only the non-cooperative and the partial cooperative scenarios should be encouraged since the monopoly welfare is the lowest one. This result is consistent with the following figure.

Figure 3.9: Subsidized social welfare as a function of demand spillover



Notes: $\alpha = 100$: the size of the market; $\lambda = 1.2$; $\Omega = 0.6$; $\gamma = 0.7$.

3.5.4 "Subsidy" versus "No subsidy"

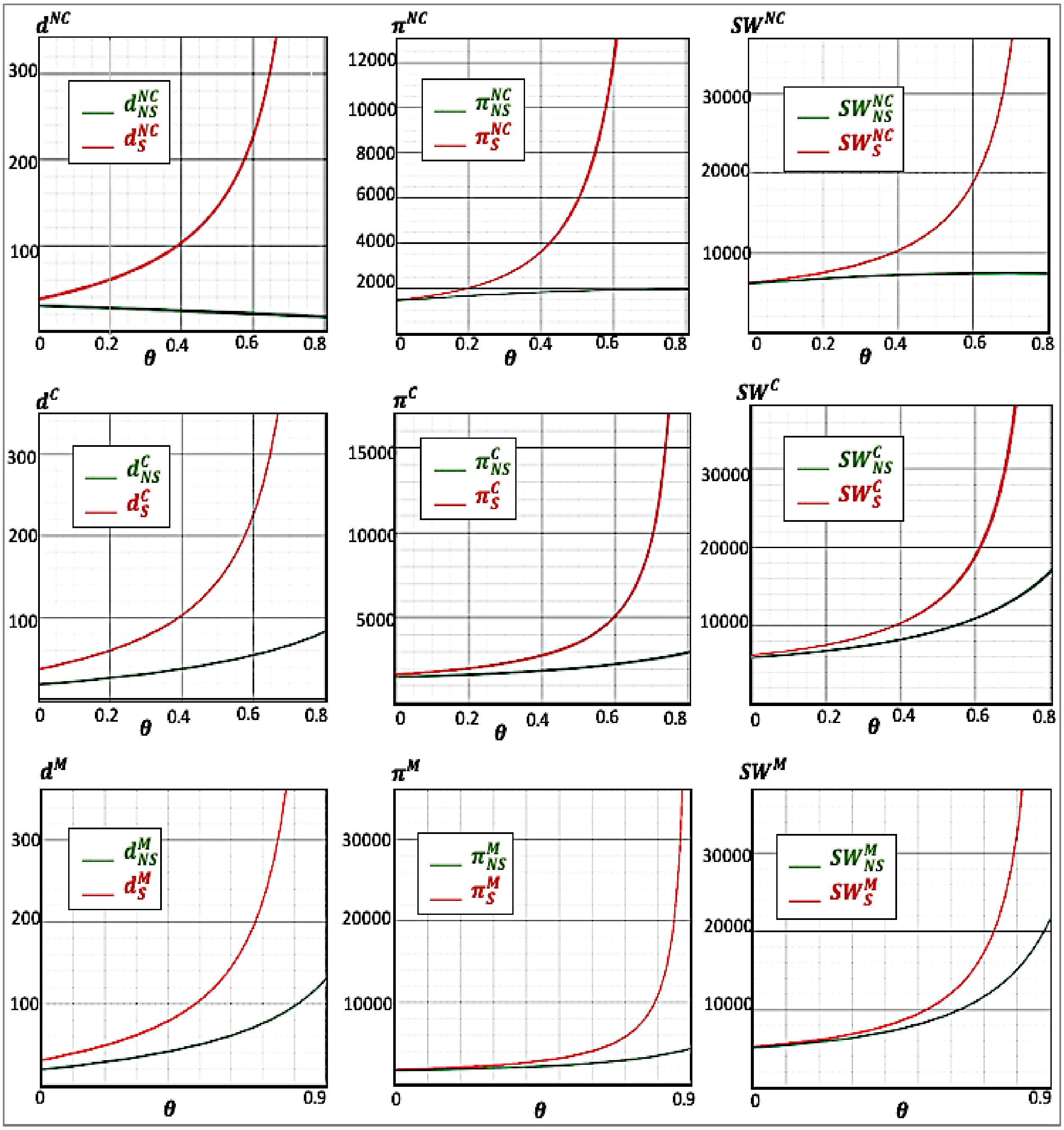
The comparison between the subsidized (S) and the non-subsidized (NS) equilibrium outcomes leads to the following proposition.[11]

Proposition 11: For all $\theta, (\lambda - \Omega)$ and $\gamma > 0$,
$d_S^{NC} > d_{NS}^{NC}, d_S^C > d_{NS}^C, d_S^M > d_{NS}^M$,
$\pi_S^C > \pi_{NS}^C, \pi_S^M > \pi_{NS}^M, \pi_S^{NC} > \pi_{NS}^{NC}$ for $\theta > \frac{\gamma(\lambda-\Omega)}{2}$,
$SW_S^{NC} > SW_{NS}^{NC}, SW_S^C > SW_{NS}^C, SW_S^M > SW_{NS}^M$.
The proof is provided in the appendix.

In each scenario and compared to the case where there is no subsidy, Proposition 11 shows that the optimal R&D subsidy allows firms to generate more quality investment $(d_S > d_{NS})$ and that subsidizing optimally the R&D investment in quality improvement is socially more effective $(SW_S > SW_{NS})$ which is obvious since the optimal R&D subsidy level is solved in a way to maximize the social welfare. Moreover, the mathematical proofs and the numerical simulations show that, under each scenario, the optimal subsidies can help firms increase their profits $(\pi_S > \pi_{NS})$. The results stated above are consistent with Figure 3.10, which shows the relationship between θ and $d_S, d_{NS}, \pi_S, \pi_{NS}, SW_S, SW_{NS}$ under the non-cooperative, the partial and the full cooperative case.
Compared to d_{NS}^M or d_{NS}^C, the results in Proposition 10 and Proposition 11 indicate that subsidizing optimally the non-cooperative R&D efforts encourages firms to generate more quality investment $(d_S^{NC} > d_S^M > d_{NS}^M$ and $d_S^{NC} = d_S^C > d_{NS}^C)$, resulting in greater benefits for the society as a whole since $SW_S^{NC} > SW_S^M > SW_{NS}^M$ and $SW_S^{NC} = SW_S^C > SW_{NS}^C$. R&D subsidies is alwaysneficial.
Regarding the profit under the non-cooperative case, Proposition 9 shows that when demand spillovers are large such that $\theta > \frac{\gamma(\lambda-\Omega)}{2}$, the government needs to provide more support for non-cooperative quality investment $(s^{NC} > s^C > s^M)$ so that the firm's profit is improved since $\pi_S^{NC} > \pi_S^C > \pi_{NS}^C > \pi_{NS}^{NC}$ (as it is mentioned in Propositions 8, 10 and 11). Interestingly, this ranking shows that the difference between π_S and π_{NS} is larger under the non-cooperative scenario compared to the partial cooperative case.

[11] I couldn't determine the sign of $(\pi_S^{NC} - \pi_{NS}^{NC})$ for small demand spillover; however, the numerical simulations show that π_S^{NC} is always greater than π_{NS}^{NC}.

Figure 3.10: “Subsidy” versus “No subsidy”



Notes: $\alpha = 100$: the size of the market; $\lambda = 1.2$; $\Omega = 0.6$; $\gamma = 0.7$.
Given Assumption 1 and the numerical values, $2(\lambda - \Omega) - \theta\gamma = 2(0.6) - 0.7\theta > 0 \Leftrightarrow \theta < 1.714$, which is always true. In the denominator of $d_S^{NC}, d_S^C, \pi_S^{NC}, \pi_S^C, SW_S^{NC}$ and SW_S^C, $[(2+\gamma)^2 - (3+\gamma)(\theta + \lambda - \Omega)^2]$ should be positive so that the concavity condition is satisfied. By using the numerical values, this condition is equivalent to:
$[(2.7)^2 - (3.7)(\theta + 0.6)^2] > 0 \Leftrightarrow \theta < 0.8$. $\lim_{\theta \to 0.8^-} d_S^{NC} = +\infty$, $\lim_{\theta \to 0.8^-} d_S^C = +\infty$, $\lim_{\theta \to 0.8^-} \pi_S^{NC} = +\infty$, $\lim_{\theta \to 0.8^-} \pi_S^C = +\infty$, $\lim_{\theta \to 0.8^-} SW_S^{NC} = +\infty$, $\lim_{\theta \to 0.8^-} SW_S^C = +\infty$.
In the denominator of d_S^M, π_S^M and SW_S^M, $[4(1+\gamma) - 3(\theta + \lambda - \Omega)^2]$ should be positive so that the concavity condition is satisfied. By using the numerical values, this condition is equivalent to:
$[4(1.7) - 3(\theta + 0.6)^2] > 0 \Leftrightarrow \theta < 0.9$. $\lim_{\theta \to 0.9^-} d_S^M = +\infty$, $\lim_{\theta \to 0.9^-} \pi_S^M = +\infty$, $\lim_{\theta \to 0.9^-} SW_S^M = +\infty$.

3.6 Conclusion

In this paper, I considered the full and the partial cooperative R&D agreements in quality investment in addition to the non-cooperative quality decisions and I examined the importance of quality sensitivity level, demand spillovers and degree of product differentiation in ranking the investment level, the profit, the optimal subsidy and the social welfare.

The strategic non-cooperative, the partial cooperative and the full cooperative quality decisions have been examined in the presence of different parameters in a Cournot model in terms of a three-stage game. In the first stage, the government chooses optimally the R&D subsidy based on the quality level with the goal of maximizing the social welfare. In the second stage, the firms choose their quality investment levels in a non-cooperative or cooperative setting and in the third stage, they choose their output levels whether cooperatively or non-cooperatively.

Under each scenario, the results show that the consumers' quality sensitivity increases the investment in quality regardless of whether the demand spillover is small or large. In addition to that, the results indicate that this quality sensitivity solely affects the optimal subsidy under non-cooperation, and that its influence on the subsidized quality investment is the same when comparing the non-cooperative and the partial cooperative scenario; however, this effect is not the same when comparing the partial and the full cooperative scenario.

Conditional on the absence of subsidy and for large and relatively large demand spillover, the optimal quality investment level under the full cooperative scenario is greater than under the non-cooperative and the partial cooperative scenarios. With the government' intervention, the profit under each scenario is improved regardless of the demand spillover. At the optimal level, firms should be encouraged to fully compete or to partially cooperate between each other given that these two options are socially more effective compared to the full cooperative scenario.

For future reseach, I am interested in considering a more general utility function that allows for non-linear interactions between quality and quantity and that includes additional factors such as income effects, network externalities and hetererogeneous consumers' quality sensitivity to better capture the complexities of real-world consumer behavior and market dynamics. Additionally, I aim to analyse how the intervention of the government impacts overall social welfare and market equilibrium in a more comprehensive framework.

3.7 Appendix

<u>Proof of Proposition 1:</u>

There are two cases regarding the sign of d^{NC}:

<u>Case 1</u>: I first consider the case where there is an interior solution $d^{NC} > 0$, so $\left(\frac{\partial \pi_i^*}{\partial d_i}\right) = 0$.

The best responses for quality investment decisions are as follows:

$$d_i(d_j) = \frac{2[2(\lambda - \Omega) - \theta\gamma](2-\gamma)\alpha + 2[2(\lambda - \Omega) - \theta\gamma][2\theta - \gamma(\lambda - \Omega)]d_j}{(1-s)(4-\gamma^2)^2 - 2[2(\lambda - \Omega) - \theta\gamma]^2}.$$

Since the best response functions are linear and symmetric, the equilibrium is unique and symmetric. Therefore, at equilibrium, $d_i = d_j$:

$$d_i^* = \frac{2[2(\lambda - \Omega) - \theta\gamma](2-\gamma)\alpha}{(1-s)(4-\gamma^2)^2 - 2[2(\lambda - \Omega) - \theta\gamma]^2 - 2[2(\lambda - \Omega) - \theta\gamma][2\theta - \gamma(\lambda - \Omega)]}, \quad (\mathbf{3})$$

$$d_i^* = \frac{2[2(\lambda - \Omega) - \theta\gamma](2-\gamma)\alpha}{(1-s)(4-\gamma^2)^2 - 2[2(\lambda - \Omega) - \theta\gamma]\big[[2(\lambda - \Omega) - \theta\gamma] + [2\theta - \gamma(\lambda - \Omega)]\big]},$$

$$d_i^* = \frac{2[2(\lambda - \Omega) - \theta\gamma](2-\gamma)\alpha}{(1-s)(2-\gamma)^2(2+\gamma)^2 - 2[2(\lambda - \Omega) - \theta\gamma][2-\gamma][\theta + \lambda - \Omega]} \quad (\mathbf{4})$$

$$= \frac{2[2(\lambda - \Omega) - \theta\gamma]\alpha}{(1-s)(2-\gamma)(2+\gamma)^2 - 2[2(\lambda - \Omega) - \theta\gamma][\theta + \lambda - \Omega]} = d^{NC}.$$

Given that $(1-s)(4-\gamma^2)^2 - 2[2(\lambda - \Omega) - \theta\gamma]^2 > 0$ and $[2(\lambda - \Omega) - \theta\gamma] > 0$, the sign of the denominator in expression $(\mathbf{3})$ depends on the sign of $[2\theta - \gamma(\lambda - \Omega)]$:

1-1- When $[2(\lambda - \Omega) - \theta\gamma] > 0$ and $[2\theta - \gamma(\lambda - \Omega)] < 0$, $d^{NC} > 0$.

1-2- When $[2(\lambda - \Omega) - \theta\gamma] > 0$ and $[2\theta - \gamma(\lambda - \Omega)] = 0$, $d^{NC} > 0$.

1-3- Now, I need to prove that $d^{NC} > 0$ when $[2(\lambda - \Omega) - \theta\gamma] > 0$ and $[2\theta - \gamma(\lambda - \Omega)] > 0$:

By considering the symmetric case such that $d_i = d_j$,

$$q_i^* = \frac{\alpha(2-\gamma) + (2-\gamma)[\theta + \lambda - \Omega]d_i}{(4-\gamma^2)}.$$

$$\frac{\partial \pi_i^*}{\partial d_i} = 2\left[\frac{\alpha(2-\gamma) + (2-\gamma)[\theta + \lambda - \Omega]d_i}{(4-\gamma^2)}\right]\left[\frac{(2-\gamma)[\theta + \lambda - \Omega]}{(4-\gamma^2)}\right] - (1-s)d_i,$$

and

$$\frac{\partial^2 \pi_i^*}{\partial d_i^2} = \frac{2[2-\gamma]^2[\theta + \lambda - \Omega]^2}{(4-\gamma^2)^2} - (1-s).$$

To ensure the concavity of π_i with respect to d_i in the symmetric case, I place a restriction on "s", such that:

$$2[2-\gamma]^2[\theta+\lambda-\Omega]^2 < (1-s)(4-\gamma^2)^2$$

$$\Leftrightarrow (1-s)(2-\gamma)^2(2+\gamma)^2 - 2[2-\gamma]^2[\theta+\lambda-\Omega]^2 > 0.$$

Expression (**4**) can be written as follows:

$$d^{NC} = \frac{2[2(\lambda-\Omega)-\theta\gamma](2-\gamma)\alpha}{\begin{bmatrix} (1-s)(2-\gamma)^2(2+\gamma)^2 - 2[2-\gamma]^2[\theta+\lambda-\Omega]^2 \\ +2[2-\gamma]^2[\theta+\lambda-\Omega]^2 - 2[2(\lambda-\Omega)-\theta\gamma](2-\gamma)[\theta+\lambda-\Omega] \end{bmatrix}}.$$

Now, I need to determine the sign of:

$$2[2-\gamma]^2[\theta+\lambda-\Omega]^2 - 2[2(\lambda-\Omega)-\theta\gamma](2-\gamma)[\theta+\lambda-\Omega],$$

$$[2\theta-\gamma(\lambda-\Omega)] > 0 \ \Leftrightarrow [2\theta-\theta\gamma+\theta\gamma-\gamma(\lambda-\Omega)+2(\lambda-\Omega)-2(\lambda-\Omega)] > 0$$

$$\Leftrightarrow [2\theta-\theta\gamma+\theta\gamma-\gamma(\lambda-\Omega)+2(\lambda-\Omega)-2(\lambda-\Omega)] > 0$$

$$\Leftrightarrow [2\theta-\theta\gamma-\gamma(\lambda-\Omega)+2(\lambda-\Omega)] > [2(\lambda-\Omega)-\theta\gamma]$$

$$\Leftrightarrow [\theta(2-\gamma)+(2-\gamma)(\lambda-\Omega)] > [2(\lambda-\Omega)-\theta\gamma]$$

$$\Leftrightarrow [\theta(2-\gamma)+(2-\gamma)(\lambda-\Omega)] > [2(\lambda-\Omega)-\theta\gamma]$$

$$\Leftrightarrow (2-\gamma)[\theta+\lambda-\Omega] > [2(\lambda-\Omega)-\theta\gamma] > 0$$

$$\Leftrightarrow 2[2-\gamma]^2[\theta+\lambda-\Omega]^2 > 2[2(\lambda-\Omega)-\theta\gamma](2-\gamma)[\theta+\lambda-\Omega]$$

$$\Leftrightarrow 2[2-\gamma]^2[\theta+\lambda-\Omega]^2 - 2[2(\lambda-\Omega)-\theta\gamma](2-\gamma)[\theta+\lambda-\Omega] > 0.$$

Therefore, the denominator of $d^{NC} > 0$ is positive.

Consequently, when $[2(\lambda-\Omega)-\theta\gamma] > 0$ and $[2\theta-\gamma(\lambda-\Omega)] > 0$, $d^{NC} > 0$.

Since cases (**1.1**), (**1.2**) and (**1.3**) are verified, I can conclude that $d^{NC} > 0$ when $[2(\lambda-\Omega)-\theta\gamma] > 0$ regardless of whether $[2\theta-\gamma(\lambda-\Omega)] > 0$ or ≤ 0.

<u>Case 2:</u> I consider a possible corner solution with $d_j = 0$ and show below if $[2(\lambda-\Omega)-\theta\gamma] \leq 0$, then $(d_i^*, d_j^*) = (0, 0)$.

If $d_j = 0, \pi_i^* = \left[\frac{\alpha(2-\gamma)+[2(\lambda-\Omega)-\theta\gamma]d_i}{(4-\gamma^2)}\right]^2 - \frac{d_i^2}{2} + \frac{s d_i^2}{2}$,

$$\frac{\partial \pi_i^*}{\partial d_i} = 2\left[\frac{\alpha(2-\gamma)+[2(\lambda-\Omega)-\theta\gamma]d_i}{(4-\gamma^2)}\right]\left[\frac{[2(\lambda-\Omega)-\theta\gamma]}{(4-\gamma^2)}\right] - (1-s)d_i,$$

$$\frac{\partial \pi_i^*}{\partial d_i} = \left[\frac{2\alpha(2-\gamma)[2(\lambda-\Omega)-\theta\gamma] + [2[2(\lambda-\Omega)-\theta\gamma]^2 - (1-s)(4-\gamma^2)^2]\, d_i}{(4-\gamma^2)^2}\right].$$

Given $[2(\lambda-\Omega)-\theta\gamma] \leq 0$ and $2[2(\lambda-\Omega)-\theta\gamma]^2 - (1-s)(4-\gamma^2)^2 < 0$,

$(\partial \pi_i^* / \partial d_i) < 0$ for $d_i \geq 0$. In other words, as the quality level of product i increases, the profit of firm i decreases. Therefore, if $d_j = 0$ and $[2(\lambda - \Omega) - \theta\gamma] \leq 0, d_i^* = 0$.

Proof of Lemma 1:

$$d^{NC} = \frac{2[2(\lambda - \Omega) - \theta\gamma]\alpha}{(1-s)(2-\gamma)(2+\gamma)^2 - 2[2(\lambda - \Omega) - \theta\gamma][\theta + \lambda - \Omega]},$$

$$\frac{\partial d^{NC}}{\partial s} = \frac{2[2(\lambda - \Omega) - \theta\gamma]\alpha(2-\gamma)(2+\gamma)^2}{\big[(1-s)(2-\gamma)(2+\gamma)^2 - 2[2(\lambda - \Omega) - \theta\gamma][\theta + \lambda - \Omega]\big]^2} > 0.$$

Assume $F = 2\alpha[2(\lambda - \Omega) - \theta\gamma] \rightarrow \frac{\partial F}{\partial \lambda} = 4\alpha$.

Assume $G = (2-\gamma)(2+\gamma)^2(1-s) - 2(\theta + \lambda - \Omega)[2(\lambda - \Omega) - \theta\gamma]$,

$\frac{\partial G}{\partial \lambda} = -4(\theta + \lambda - \Omega) - 2[2(\lambda - \Omega) - \theta\gamma]$ therefore:

$$\frac{\partial d^{NC}}{\partial \lambda} = \frac{G\frac{\partial F}{\partial \lambda} - F\frac{\partial G}{\partial \lambda}}{G^2} = \frac{4\alpha[(1-s)(2-\gamma)(2+\gamma)^2 + [2(\lambda - \Omega) - \theta\gamma]^2]}{G^2} > 0.$$

$$\frac{\partial F}{\partial \gamma} = -2\alpha\theta, \quad \frac{\partial G}{\partial \gamma} = -(2+\gamma)^2(1-s) + 2(2+\gamma)(2-\gamma)(1-s) + 2\theta(\theta + \lambda - \Omega),$$

$$\frac{\partial G}{\partial \gamma} = (2+\gamma)(1-s)[4 - 2\gamma - 2 - \gamma] + 2\theta(\theta + \lambda - \Omega)$$

$$= (2+\gamma)(1-s)[2 - 3\gamma] + 2\theta(\theta + \lambda - \Omega).$$

$$\frac{\partial d^{NC}}{\partial \gamma} = \frac{G\frac{\partial F}{\partial \gamma} - F\frac{\partial G}{\partial \gamma}}{G^2} = \frac{\begin{bmatrix} -2\alpha\theta(2-\gamma)(2+\gamma)^2(1-s) + 4\alpha\theta(\theta + \lambda - \Omega)[2(\lambda - \Omega) - \theta\gamma] \\ -2\alpha[2(\lambda - \Omega) - \theta\gamma](2+\gamma)(1-s)[2-3\gamma] \\ -4\alpha\theta(\theta + \lambda - \Omega)[2(\lambda - \Omega) - \theta\gamma] \end{bmatrix}}{G^2},$$

$$\frac{\partial d^{NC}}{\partial \gamma} = \frac{G\frac{\partial F}{\partial \gamma} - F\frac{\partial G}{\partial \gamma}}{G^2} = \frac{-2\alpha(2+\gamma)(1-s)\big[\theta(2-\gamma)(2+\gamma) + [2(\lambda - \Omega) - \theta\gamma][2-3\gamma]\big]}{G^2}.$$

The sign of $(\partial d^{NC}/\partial \gamma)$ depends on the sign of $\big[\theta(2-\gamma)(2+\gamma) + [2(\lambda - \Omega) - \theta\gamma][2-3\gamma]\big]$.

For $[2 - 3\gamma] \geq 0$ which is equivalent to $\gamma \in \left(0, \frac{2}{3}\right]$, $(\partial d^{NC}/\partial \gamma) < 0$ since

$[2(\lambda - \Omega) - \theta\gamma] > 0$.

For $[2 - 3\gamma] < 0$ which is equivalent to $\gamma \in \left(\frac{2}{3}, 1\right]$, $(\partial d^{NC}/\partial \gamma) \geq 0$ if

$\big[\theta(2-\gamma)(2+\gamma) + [2(\lambda - \Omega) - \theta\gamma][2-3\gamma]\big] \leq 0$.

The above condition is equivalent to $[2(\lambda - \Omega) - \theta\gamma][3\gamma - 2] \geq \theta(2-\gamma)(2+\gamma)$

$\Leftrightarrow 2(\lambda - \Omega)[3\gamma - 2] \geq \theta(2-\gamma)(2+\gamma) + \theta\gamma[3\gamma - 2]$

$\Leftrightarrow \lambda - \Omega \geq \dfrac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}.$

Given that $\frac{\gamma\theta}{2} < \lambda - \Omega < \frac{(4-\gamma^2)}{2\sqrt{2}} + \frac{\gamma\theta}{2}$, I have $\frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)} \leq \lambda - \Omega < \frac{(4-\gamma^2)}{2\sqrt{2}} + \frac{\gamma\theta}{2}$ for all $\gamma \in \left(\frac{2}{3}, 1\right]$.

$$\frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)} - \frac{\gamma\theta}{2} = \frac{2\theta(2-\gamma+\gamma^2)}{2(3\gamma-2)} - \frac{\gamma\theta(3\gamma-2)}{2(3\gamma-2)}$$
$$= \frac{4\theta - 2\theta\gamma + 2\theta\gamma^2 - 3\theta\gamma^2 + 2\gamma\theta}{2(3\gamma-2)} = \frac{(4-\gamma^2)\theta}{2(3\gamma-2)} > 0 \text{ for all } \gamma \in \left(\frac{2}{3}, 1\right].$$

$$\frac{(4-\gamma^2)}{2\sqrt{2}} + \frac{\gamma\theta}{2} > \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}$$
$$\Leftrightarrow \frac{(4-\gamma^2)}{2\sqrt{2}} > \frac{2\theta(2-\gamma+\gamma^2)}{2(3\gamma-2)} - \frac{\gamma\theta(3\gamma-2)}{2(3\gamma-2)}$$
$$\Leftrightarrow \frac{(4-\gamma^2)}{2\sqrt{2}} > \frac{2\theta(2-\gamma+\gamma^2) - \gamma\theta(3\gamma-2)}{2(3\gamma-2)}$$
$$\Leftrightarrow \frac{(4-\gamma^2)}{2\sqrt{2}} > \frac{4\theta - 2\theta\gamma + 2\theta\gamma^2 - 3\theta\gamma^2 + 2\gamma\theta}{2(3\gamma-2)}$$
$$\Leftrightarrow \frac{(4-\gamma^2)}{2\sqrt{2}} > \frac{\theta(4-\gamma^2)}{2(3\gamma-2)}$$
$$\Leftrightarrow \frac{(3\gamma-2)}{\sqrt{2}} > \theta$$
$$\Leftrightarrow \theta < \frac{(3\gamma-2)}{\sqrt{2}}.$$

If $\theta \geq \frac{(3\gamma-2)}{\sqrt{2}}$, then $\frac{(4-\gamma^2)}{2\sqrt{2}} + \frac{\gamma\theta}{2} \leq \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}$, so $\frac{\partial d^{NC}}{\partial\gamma} < 0$ under Assumption 1.
I conclude that:

$$\frac{\partial d^{NC}}{\partial \gamma}\begin{cases} < 0 \; for \; \gamma \in \left(0, \frac{2}{3}\right], \\ < 0 \; for \;\; \gamma \in \left(\frac{2}{3}, 1\right], 0 < \theta < \frac{(3\gamma-2)}{\sqrt{2}} \, and \; \frac{\gamma\theta}{2} < \lambda - \Omega < \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)} < \frac{(4-\gamma^2)}{2\sqrt{2}} + \frac{\gamma\theta}{2}, \\ < 0 \; for \;\; \gamma \in \left(\frac{2}{3}, 1\right], \frac{(3\gamma-2)}{\sqrt{2}} \leq \theta < 1 \; and \; \frac{\gamma\theta}{2} < \lambda - \Omega < \frac{(4-\gamma^2)}{2\sqrt{2}} + \frac{\gamma\theta}{2} \leq \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)}, \\ \geq 0 \; for \; \gamma \in \left(\frac{2}{3}, 1\right], 0 < \theta < \frac{(3\gamma-2)}{\sqrt{2}} \, and \; \frac{\gamma\theta}{2} < \frac{\theta(2-\gamma+\gamma^2)}{(3\gamma-2)} \leq \lambda - \Omega < \frac{(4-\gamma^2)}{2\sqrt{2}} + \frac{\gamma\theta}{2}. \end{cases}$$

Proof of Proposition 3:

$SW^{NC} = (3+\gamma)(q^{NC})^2 - (d^{NC})^{\mathbf{2}},$

$$\frac{\partial SW^{NC}}{\partial s} = 2(3+\gamma)\frac{\partial q^{NC}}{\partial s}q^{NC} - 2d^{NC}\frac{\partial d^{NC}}{\partial s} = 0$$

$$\Leftrightarrow 2(3+\gamma)\frac{\partial d^{NC}}{\partial s}\left[\frac{(\theta+\lambda-\Omega)}{(2+\gamma)}\right]\left[\frac{\alpha+(\theta+\lambda-\Omega)d^{NC}}{(2+\gamma)}\right] = 2d^{NC}\frac{\partial d^{NC}}{\partial s}$$

$$\Leftrightarrow \left[\frac{\alpha(3+\gamma)(\theta+\lambda-\Omega)}{(2+\gamma)^2}\right] + \left[\frac{(3+\gamma)(\theta+\lambda-\Omega)^2 d^{NC}}{(2+\gamma)^2}\right] = d^{NC}$$

$$\Leftrightarrow \left[\frac{\alpha(3+\gamma)(\theta+\lambda-\Omega)}{(2+\gamma)^2}\right] = d^{NC} - \left[\frac{(3+\gamma)(\theta+\lambda-\Omega)^2 d^{NC}}{(2+\gamma)^2}\right]$$

$$\Leftrightarrow \left[\frac{\alpha(3+\gamma)(\theta+\lambda-\Omega)}{(2+\gamma)^2}\right] = \left[\frac{(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2}{(2+\gamma)^2}\right]d^{NC}$$

$$\Leftrightarrow [\alpha(3+\gamma)(\theta+\lambda-\Omega)] = [(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]d^{NC}. \qquad (\mathbf{5})$$

Given

$$d^{NC} = \frac{2\alpha[2(\lambda-\Omega)-\theta\gamma]}{(2-\gamma)(2+\gamma)^2(1-s) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]},$$

expression $(\mathbf{5})$ can be written as follows:

$$[(3+\gamma)(\theta+\lambda-\Omega)](2-\gamma)(2+\gamma)^2(1-s) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma][(3+\gamma)(\theta+\lambda-\Omega)]$$

$$= 2(2+\gamma)^2[2(\lambda-\Omega)-\theta\gamma] - 2(3+\gamma)[2(\lambda-\Omega)-\theta\gamma](\theta+\lambda-\Omega)^2$$

$$\Leftrightarrow [(3+\gamma)(\theta+\lambda-\Omega)](2-\gamma)(1-s) = 2[2(\lambda-\Omega)-\theta\gamma]$$

$$\Leftrightarrow s^{NC} = 1 - \frac{2[2(\lambda-\Omega)-\theta\gamma]}{(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)}.$$

$$\frac{\partial s^{NC}}{\partial \gamma} = \frac{2\theta(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma) - 2(\theta+\lambda-\Omega)(2\gamma+1)[2(\lambda-\Omega)-\theta\gamma]}{[(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)]^2}$$

$$= \frac{2(\theta+\lambda-\Omega)\big[\theta(3+\gamma)(2-\gamma) - (2\gamma+1)[2(\lambda-\Omega)-\theta\gamma]\big]}{[(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)]^2},$$

$$\frac{\partial s^{NC}}{\partial \gamma} < 0 \text{ for all } 0 < \theta < \frac{2(\lambda-\Omega)(1+2\gamma)}{(3+\gamma)(2-\gamma)+\gamma(1+2\gamma)}.$$

$$\frac{\partial s^{NC}}{\partial \gamma} \geq 0 \text{ for all } \frac{2(\lambda-\Omega)(1+2\gamma)}{(3+\gamma)(2-\gamma)+\gamma(1+2\gamma)} \leq \theta < 1$$

$$\Rightarrow 2(\lambda-\Omega)(1+2\gamma) < (3+\gamma)(2-\gamma) + \gamma(1+2\gamma)$$

$$\Leftrightarrow (\lambda-\Omega) < \frac{6+\gamma^2}{2(1+2\gamma)}.$$

$$\frac{\partial s^{NC}}{\partial \lambda} = \frac{-4(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma) + 2(3+\gamma)(2-\gamma)[2(\lambda-\Omega)-\theta\gamma]}{[(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)]^2}$$

$$= \frac{2(3+\gamma)(2-\gamma)[-2\theta - 2\lambda + 2\Omega + 2\lambda - 2\Omega - \theta\gamma]}{[(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)]^2} = \frac{-2\theta[2+\gamma]}{(3+\gamma)(2-\gamma)[\theta+\lambda-\Omega]^2} < 0.$$

$$\frac{\partial s^{NC}}{\partial \theta} = \frac{2\gamma(3+\gamma)(2-\gamma)(\theta+\lambda-\Omega) + 2(3+\gamma)(2-\gamma)[2(\lambda-\Omega)-\theta\gamma]}{[(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)]^2},$$

$$\frac{\partial s^{NC}}{\partial \theta} = \frac{2(3+\gamma)(2-\gamma)(2+\gamma)(\lambda-\Omega)}{[(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)]^2} > 0.$$

$$\frac{\partial^2 s^{NC}}{\partial \lambda \partial \gamma} = \frac{\partial\left(\frac{\partial s^{NC}}{\partial \lambda}\right)}{\partial \gamma} = \frac{-2\theta[(3+\gamma)(2-\gamma) + (1+2\gamma)(2+\gamma)]}{[(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)]^2} < 0.$$

Proof of Proposition 4:

$$\frac{\partial(\pi_i^* + \pi_j^*)}{\partial d_i} = 2q_i \frac{\partial q_i^*}{\partial d_i} + 2q_j \frac{\partial q_j^*}{\partial d_i} - (1-s)d_i$$

$$= 2\left[\begin{array}{l}\left[\frac{\alpha(2-\gamma) + [2(\lambda-\Omega)-\gamma\theta]d_i + [2\theta-\gamma(\lambda-\Omega)]d_j}{(4-\gamma^2)}\right]\left[\frac{[2(\lambda-\Omega)-\gamma\theta]}{(4-\gamma^2)}\right] \\ + \left[\frac{\alpha(2-\gamma) + [2(\lambda-\Omega)-\gamma\theta]d_j + [2\theta-\gamma(\lambda-\Omega)]d_i}{(4-\gamma^2)}\right]\left[\frac{[2\theta-\gamma(\lambda-\Omega)]}{(4-\gamma^2)}\right]\end{array}\right]$$

$$-(1-s)d_i = 0,$$

$$\frac{\partial(\pi_i^* + \pi_j^*)}{\partial d_i} = 2\frac{\begin{bmatrix}\alpha(2-\gamma)^2(\theta+\lambda-\Omega) + 2[2(\lambda-\Omega)-\gamma\theta][2\theta-\gamma(\lambda-\Omega)]d_j \\ +[[2(\lambda-\Omega)-\gamma\theta]^2 + [2\theta-\gamma(\lambda-\Omega)]^2]d_i\end{bmatrix}}{(4-\gamma^2)^2}$$

$$-(1-s)d_i = 0,$$

$$\frac{\partial(\pi_i^* + \pi_j^*)}{\partial d_j} = 2\frac{\begin{bmatrix}\alpha(2-\gamma)^2(\theta+\lambda-\Omega) + 2[2(\lambda-\Omega)-\gamma\theta][2\theta-\gamma(\lambda-\Omega)]d_i \\ +[[2(\lambda-\Omega)-\gamma\theta]^2 + [2\theta-\gamma(\lambda-\Omega)]^2]d_j\end{bmatrix}}{(4-\gamma^2)^2}$$

$$-(1-s)d_j = 0.$$

$$d_i(d_j) = \frac{2\alpha(2-\gamma)^2(\theta+\lambda-\Omega) + 4[2(\lambda-\Omega)-\gamma\theta][2\theta-\gamma(\lambda-\Omega)]d_j}{(4-\gamma^2)^2(1-s) - 2[[2(\lambda-\Omega)-\gamma\theta]^2 + [2\theta-\gamma(\lambda-\Omega)]^2]},$$

$$d_j(d_i) = \frac{2\alpha(2-\gamma)^2(\theta+\lambda-\Omega) + 4[2(\lambda-\Omega)-\gamma\theta][2\theta-\gamma(\lambda-\Omega)]d_i}{(4-\gamma^2)^2(1-s) - 2[[2(\lambda-\Omega)-\gamma\theta]^2 + [2\theta-\gamma(\lambda-\Omega)]^2]}.$$

Assume $L = 2\alpha(2-\gamma)^2(\theta+\lambda-\Omega)$, $M = 4[2(\lambda-\Omega)-\gamma\theta][2\theta-\gamma(\lambda-\Omega)]$ and $N = (4-\gamma^2)^2(1-s) - 2[[2(\lambda-\Omega)-\gamma\theta]^2 + [2\theta-\gamma(\lambda-\Omega)]^2]$,

$$d_j = \frac{L + M\left[\frac{L + Md_j}{N}\right]}{N} \Leftrightarrow Nd_j = \frac{LN + LM + M^2 d_j}{N} \Leftrightarrow (N^2 - M^2)d_j = L(N + M)$$

$$\Leftrightarrow d_j = \frac{L}{N - M},$$

$$N - M = (4 - \gamma^2)^2(1 - s) - 2[[2(\lambda - \Omega) - \gamma\theta]^2 + 2[2(\lambda - \Omega) - \gamma\theta][2\theta - \gamma(\lambda - \Omega)]$$
$$+[2\theta - \gamma(\lambda - \Omega)]^2] = (2 - \gamma)^2[(2 + \gamma)^2(1 - s) - 2(\theta + \lambda - \Omega)^2].$$

Therefore,

$$d_j^* = d_i^* = d^C = \frac{2\alpha(\theta + \lambda - \Omega)}{(2 + \gamma)^2(1 - s) - 2(\theta + \lambda - \Omega)^2}.$$

Proof of Lemma 2:

$$d^C = \frac{2\alpha(\theta + \lambda - \Omega)}{(2 + \gamma)^2(1 - s) - 2(\theta + \lambda - \Omega)^2},$$

$$\frac{\partial d^C}{\partial s} = \frac{2\alpha(\theta + \lambda - \Omega)}{[(2 + \gamma)^2(1 - s) - 2(\theta + \lambda - \Omega)^2]^2} > 0.$$

$$\frac{\partial d^C}{\partial \gamma} = \frac{-4\alpha(2 + \gamma)(1 - s)(\theta + \lambda - \Omega)}{[(2 + \gamma)^2(1 - s) - 2(\theta + \lambda - \Omega)^2]^2} < 0.$$

$$\frac{\partial d^C}{\partial \lambda} = \frac{2\alpha(2 + \gamma)^2(1 - s) + 4\alpha(\theta + \lambda - \Omega)^2}{[(2 + \gamma)^2(1 - s) - 2(\theta + \lambda - \Omega)^2]^2} > 0.$$

Proof of Proposition 5:

$$SW^C = (3 + \gamma)(q^C)^2 - (d^C)^2,$$

$$\frac{\partial SW^C}{\partial s} = 2(3 + \gamma)\frac{\partial q^C}{\partial s}q^C - 2d^C\frac{\partial d^C}{\partial s} = 0$$

$$\Leftrightarrow 2(3 + \gamma)\frac{\partial d^C}{\partial s}\left[\frac{(\theta + \lambda - \Omega)}{(2 + \gamma)}\right]\left[\frac{\alpha + (\theta + \lambda - \Omega)d^C}{(2 + \gamma)}\right] = 2d^C\frac{\partial d^C}{\partial s}$$

$$\Leftrightarrow \left[\frac{2\alpha(3 + \gamma)(\theta + \lambda - \Omega)}{(2 + \gamma)^2}\right] + \left[\frac{2(3 + \gamma)(\theta + \lambda - \Omega)^2 d^C}{(2 + \gamma)^2}\right] = 2d^C$$

$$\Leftrightarrow \left[\frac{2\alpha(3 + \gamma)(\theta + \lambda - \Omega)}{(2 + \gamma)^2}\right] = 2d^C - \left[\frac{2(3 + \gamma)(\theta + \lambda - \Omega)^2 d^C}{(2 + \gamma)^2}\right]$$

$$\Leftrightarrow [\alpha(3 + \gamma)(\theta + \lambda - \Omega)] = [(2 + \gamma)^2 - (3 + \gamma)(\theta + \lambda - \Omega)^2]d^C. \qquad (6)$$

Given

$$d^C = \frac{2\alpha(\theta + \lambda - \Omega)}{(2+\gamma)^2(1-s) - 2(\theta + \lambda - \Omega)^2},$$

expression $(\mathbf{7})$ can be written as follows:

$$[\alpha(3+\gamma)(\theta+\lambda-\Omega)](2+\gamma)^2(1-s) - [2\alpha(3+\gamma)(\theta+\lambda-\Omega)^3]$$

$$= 2\alpha(2+\gamma)^2(\theta+\lambda-\Omega) - 2\alpha(3+\gamma)(\theta+\lambda-\Omega)^3$$

$$\Leftrightarrow (3+\gamma)(1-s) = 2 \Leftrightarrow s^C = \frac{1+\gamma}{3+\gamma}.$$

$$\frac{\partial s^C}{\partial \gamma} = \frac{2}{(3+\gamma)^2} > 0.$$

Proof of Proposition 6:

$$\pi_i = (p_i - c_i)q_i - \frac{d_i^2}{2} + \frac{sd_i^2}{2} = \left[\alpha + (\lambda - \Omega)d_i + \theta d_j - (q_i + \gamma q_j)\right]q_i - \frac{d_i^2}{2} + \frac{sd_i^2}{2},$$

$$\pi_i + \pi_j = \left(\alpha + \lambda d_i - \Omega d_i + \theta d_j - (q_i + \gamma q_j)\right)q_i - \left(\frac{d_i^2}{2} - \frac{sd_i^2}{2}\right)$$
$$+\left(\alpha + \lambda d_j - \Omega d_j + \theta d_i - (q_j + \gamma q_i)\right)q_j - \left(\frac{d_j^2}{2} - \frac{sd_j^2}{2}\right). \qquad (\mathbf{7})$$

$$\frac{\partial(\pi_i + \pi_j)}{\partial q_i} = \alpha + \lambda d_i - \Omega d_i + \theta d_j - 2q_i - \gamma q_j - \gamma q_j = 0$$

$$\Leftrightarrow \alpha + (\lambda - \Omega)d_i + \theta d_j = 2(\tilde{q}_i + \gamma \tilde{q}_j)$$

$$\Leftrightarrow (\tilde{q}_i + \gamma \tilde{q}_j) = \frac{\alpha + (\lambda - \Omega)d_i + \theta d_j}{2}. \qquad (\mathbf{8})$$

$$\frac{\partial(\pi_i + \pi_j)}{\partial q_j} = 0 \Leftrightarrow \alpha + (\lambda - \Omega)d_j + \theta d_i = 2(\tilde{q}_j + \gamma \tilde{q}_i)$$

$$\Leftrightarrow (\tilde{q}_j + \gamma \tilde{q}_i) = \frac{\alpha + (\lambda - \Omega)d_j + \theta d_i}{2}. \qquad (\mathbf{9})$$

$$\left\{\begin{array}{ll} (\gamma\tilde{q}_i + \gamma^2\tilde{q}_j) = \dfrac{\gamma\alpha + \gamma(\lambda - \Omega)d_i + \gamma\theta d_j}{2}, & (\mathbf{10}) \\ (\tilde{q}_j + \gamma\tilde{q}_i) = \dfrac{\alpha + (\lambda - \Omega)d_j + \theta d_i}{2}. & (\mathbf{9}) \end{array}\right\}$$

$(\mathbf{9}) - (\mathbf{10}) \Longrightarrow$

$$\tilde{q}_j = \frac{\alpha(1-\gamma) + [(\lambda - \Omega) - \gamma\theta]d_j + [\theta - \gamma(\lambda - \Omega)]d_i}{2(1-\gamma^2)}. \qquad (\mathbf{11})$$

$$\begin{cases}\left(\gamma\tilde{q}_j+\gamma^2\tilde{q}_i\right)=\dfrac{\gamma\alpha+\gamma(\lambda-\Omega)d_j+\gamma\theta d_i}{2}, & (\mathbf{12})\\ \left(\tilde{q}_i+\gamma\tilde{q}_j\right)=\dfrac{\alpha+(\lambda-\Omega)d_i+\theta d_j}{2}. & (\mathbf{8})\end{cases}$$

$(\mathbf{8})-(\mathbf{12})\Longrightarrow$

$$\tilde{q}_i=\frac{\alpha(1-\gamma)+[(\lambda-\Omega)-\gamma\theta]d_i+[\theta-\gamma(\lambda-\Omega)]d_j}{2(1-\gamma^2)}. \qquad (\mathbf{13})$$

$(\mathbf{8})$ and $(\mathbf{9})$ in $(\mathbf{7})\Longrightarrow$

$$\begin{aligned}\tilde{\pi}_i+\tilde{\pi}_j&=\left(\alpha+\lambda d_i-\Omega d_i+\theta d_j-\left(\frac{\alpha+(\lambda-\Omega)d_i+\theta d_j}{2}\right)\right)q_i-\left(\frac{d_i^2}{2}-\frac{sd_i^2}{2}\right)\\&\quad+\left(\alpha+\lambda d_j-\Omega d_j+\theta d_i-\left(\frac{\alpha+(\lambda-\Omega)d_j+\theta d_i}{2}\right)\right)q_j-\left(\frac{d_j^2}{2}-\frac{sd_j^2}{2}\right)\\&=\left(\frac{\alpha+(\lambda-\Omega)d_i+\theta d_j}{2}\right)q_i-\left(\frac{d_i^2}{2}-\frac{sd_i^2}{2}\right)\\&\quad+\left(\frac{\alpha+(\lambda-\Omega)d_j+\theta d_i}{2}\right)q_j-\left(\frac{d_j^2}{2}-\frac{sd_j^2}{2}\right).\end{aligned}$$

$$\begin{aligned}\tilde{\pi}_i+\tilde{\pi}_j&=\left(\tilde{q}_i+\gamma\tilde{q}_j\right)\tilde{q}_i+\left(\tilde{q}_j+\gamma\tilde{q}_i\right)\tilde{q}_j-\frac{(1-s)}{2}(d_i^2+d_j^2)\\&=\tilde{q}_i^2+\tilde{q}_j^2+2\gamma\tilde{q}_i\tilde{q}_j-\frac{(1-s)}{2}\left(d_i^2+d_j^2\right).\end{aligned}$$

$$\frac{\partial(\tilde{\pi}_i+\tilde{\pi}_j)}{\partial d_i}=2\tilde{q}_i\frac{\partial\tilde{q}_i}{\partial d_i}+2\tilde{q}_j\frac{\partial\tilde{q}_j}{\partial d_i}+2\gamma\left[\tilde{q}_i\frac{\partial\tilde{q}_j}{\partial d_i}+\tilde{q}_j\frac{\partial\tilde{q}_i}{\partial d_i}\right]-(1-s)d_i.$$

Given $(\mathbf{11})$ and $(\mathbf{13})$,

$$\begin{aligned}\frac{\partial\left(\tilde{\pi}_i+\tilde{\pi}_j\right)}{\partial d_i}&=\left[2\left(\tilde{q}_i+\gamma\tilde{q}_j\right)\left[\frac{[(\lambda-\Omega)-\gamma\theta]}{2(1-\gamma^2)}\right]\right]+2\left(\tilde{q}_j+\gamma\tilde{q}_i\right)\left[\frac{[\theta-\gamma(\lambda-\Omega)]}{2(1-\gamma^2)}\right]\\&\quad-(1-s)d_i\\&=\tilde{q}_i\left[\frac{[(\lambda-\Omega)-\gamma\theta+\gamma\theta-\gamma^2(\lambda-\Omega)]}{(1-\gamma^2)}\right]\\&\quad+\tilde{q}_j\left[\frac{[\theta-\gamma(\lambda-\Omega)+\gamma(\lambda-\Omega)-\theta\gamma^2]}{(1-\gamma^2)}\right]-(1-s)d_i\\&=(\lambda-\Omega)\tilde{q}_i+\theta\tilde{q}_j-(1-s)d_i.\end{aligned}$$

Given $(\mathbf{11})$ and $(\mathbf{13})$,

$(\lambda-\Omega)\tilde{q}_i+\theta\tilde{q}_j$

$$= \frac{\begin{bmatrix} \alpha(\lambda-\Omega)(1-\gamma) + [(\lambda-\Omega)^2 - \gamma\theta(\lambda-\Omega)]d_i + (\lambda-\Omega)[\theta - \gamma(\lambda-\Omega)]d_j \\ +\alpha\theta(1-\gamma) + [(\lambda-\Omega)\theta - \gamma\theta^2]d_j + \theta[\theta-\gamma(\lambda-\Omega)]d_i \end{bmatrix}}{2(1-\gamma^2)},$$

$$\frac{\partial(\tilde{\pi}_i + \tilde{\pi}_j)}{\partial d_i}$$

$$= \frac{\begin{bmatrix} \alpha(\lambda-\Omega+\theta)(1-\gamma) + [(\lambda-\Omega)^2 - 2\gamma\theta(\lambda-\Omega) + \theta^2]d_i \\ +[2(\lambda-\Omega)\theta - \gamma\theta^2 - \gamma(\lambda-\Omega)^2]d_j \end{bmatrix}}{2(1-\gamma^2)} - (1-s)d_i = 0,$$

$$\frac{\partial(\tilde{\pi}_i + \tilde{\pi}_j)}{\partial d_j} = (\lambda-\Omega)\tilde{q}_j + \theta\tilde{q}_i - (1-s)d_j$$

$$= \frac{\begin{bmatrix} \alpha(\lambda-\Omega+\theta)(1-\gamma) + [(\lambda-\Omega)^2 - 2\gamma\theta(\lambda-\Omega) + \theta^2]d_j \\ +[2(\lambda-\Omega)\theta - \gamma\theta^2 - \gamma(\lambda-\Omega)^2]d_i \end{bmatrix}}{2(1-\gamma^2)} - (1-s)d_j = 0.$$

Therefore, the best response functions under full cooperation are expressed as follows:

$$\begin{cases} d_i(d_j) = \dfrac{\alpha(\lambda-\Omega+\theta)(1-\gamma) + [2(\lambda-\Omega)\theta - \gamma\theta^2 - \gamma(\lambda-\Omega)^2]d_j}{2(1-s)(1-\gamma^2) - [(\lambda-\Omega)^2 - 2\gamma\theta(\lambda-\Omega) + \theta^2]}, \\ \\ d_j(d_i) = \dfrac{\alpha(\lambda-\Omega+\theta)(1-\gamma) + [2(\lambda-\Omega)\theta - \gamma\theta^2 - \gamma(\lambda-\Omega)^2]d_i}{2(1-s)(1-\gamma^2) - [(\lambda-\Omega)^2 - 2\gamma\theta(\lambda-\Omega) + \theta^2]}. \end{cases}$$

Assume $P = \alpha(\lambda-\Omega+\theta)(1-\gamma), Q = [2(\lambda-\Omega)\theta - \gamma\theta^2 - \gamma(\lambda-\Omega)^2]$ and $R = 2(1-s)(1-\gamma^2) - [(\lambda-\Omega)^2 - 2\gamma\theta(\lambda-\Omega) + \theta^2]$,

$$d_j = \frac{P + Q\left[\frac{P+Qd_j}{R}\right]}{R} \Leftrightarrow Rd_j = \frac{PR + QP + Q^2 d_j}{R} \Leftrightarrow (R^2 - Q^2)d_j = P(R+Q)$$

$$\Leftrightarrow d_j = \frac{P}{R-Q}.$$

$$R - Q = 2(1-s)(1-\gamma^2) - [(\lambda-\Omega)^2 - 2\gamma\theta(\lambda-\Omega) + \theta^2 + 2(\lambda-\Omega)\theta - \gamma\theta^2 - \gamma(\lambda-\Omega)^2]$$

$$= 2(1-s)(1-\gamma^2) - (1-\gamma)[(\lambda-\Omega)^2 + \theta^2 + 2(\lambda-\Omega)\theta]$$

$$= 2(1-s)(1-\gamma)(1+\gamma) - (1-\gamma)(\theta+\lambda-\Omega)^2.$$

Therefore,

$$d_j^* = d_i^* = d^M = \frac{\alpha(\lambda-\Omega+\theta)(1-\gamma)}{2(1-s)(1-\gamma)(1+\gamma) - (1-\gamma)[(\theta+\lambda-\Omega)^2]} = \frac{\alpha(\lambda-\Omega+\theta)}{2(1-s)(1+\gamma) - (\theta+\lambda-\Omega)^2}.$$

By considering the symmetric case such that $d_i = d_j$,

$$\frac{\partial(\tilde{\pi}_i+\tilde{\pi}_j)}{\partial d_i}=\frac{\begin{bmatrix}\alpha(\lambda-\Omega+\theta)(1-\gamma)\\ +\begin{bmatrix}(\lambda-\Omega)^2-2\gamma\theta(\lambda-\Omega)\\ +\theta^2+2(\lambda-\Omega)\theta-\gamma\theta^2-\gamma(\lambda-\Omega)^2\end{bmatrix}d_i\end{bmatrix}}{2(1-\gamma^2)}-(1-s)d_i$$

$$=\frac{[\alpha(\lambda-\Omega+\theta)(1-\gamma)+(1-\gamma)[(\lambda-\Omega)^2+\theta^2+2(\lambda-\Omega)\theta]d_i]}{2(1-\gamma^2)}-(1-s)d_i$$

$$=\frac{[\alpha(\lambda-\Omega+\theta)(1-\gamma)+(1-\gamma)[(\lambda-\Omega)^2+\theta^2+2(\lambda-\Omega)\theta]d_i]}{2(1-\gamma^2)}-(1-s)d_i$$

$$=\frac{[\alpha(\lambda-\Omega+\theta)(1-\gamma)+(1-\gamma)[\theta+\lambda-\Omega]^2d_i]}{2(1-\gamma^2)}-(1-s)d_i$$

$$=\frac{\alpha(\lambda-\Omega+\theta)+[\theta+\lambda-\Omega]^2d_i}{2(1+\gamma)}-(1-s)d_i,$$

$$\frac{\partial^2(\tilde{\pi}_i+\tilde{\pi}_j)}{\partial d_i^2}=\frac{[\theta+\lambda-\Omega]^2}{2(1+\gamma)}-(1-s).$$

To ensure the concavity of the profit with respect to d_i in the symmetric case, I place a restriction on "s", such that:

$$[\theta+\lambda-\Omega]^2<2(1-s)(1+\gamma).$$

<u>Proof of Lemma 3:</u>

$$d^M=\frac{\alpha(\theta+\lambda-\Omega)}{2(1-s)(1+\gamma)-(\theta+\lambda-\Omega)^2}.$$

$$\frac{\partial d^M}{\partial s}=\frac{2\alpha(\theta+\lambda-\Omega)}{[2(1-s)(1+\gamma)-(\theta+\lambda-\Omega)^2]^2}>0.$$

$$\frac{\partial d^M}{\partial \lambda}=\frac{2\alpha(1+\gamma)(1-s)+\alpha(\theta+\lambda-\Omega)^2}{[2(1-s)(1+\gamma)-(\theta+\lambda-\Omega)^2]^2}>0.$$

$$\frac{\partial d^M}{\partial \gamma}=\frac{-2\alpha(1-s)(\theta+\lambda-\Omega)}{[2(1-s)(1+\gamma)-(\theta+\lambda-\Omega)^2]^2}<0.$$

<u>Proof of Proposition 7:</u>

$$SW^M=3(1+\gamma)(q^M)^2-(d^M)^2,$$

$$\frac{\partial SW^M}{\partial s}=(3)(2)(1+\gamma)\frac{\partial q^M}{\partial s}q^M-2d^M\frac{\partial d^M}{\partial s}=0$$

$$\Leftrightarrow (3)(2)(1+\gamma)\frac{\partial d^M}{\partial s}\left[\frac{(\theta+\lambda-\Omega)}{2(1+\gamma)}\right]\left[\frac{\alpha+(\theta+\lambda-\Omega)d^M}{2(1+\gamma)}\right]=2d^M\frac{\partial d^M}{\partial s}$$

$$\Leftrightarrow 3[(\theta+\lambda-\Omega)][\alpha+(\theta+\lambda-\Omega)d^M]=4(1+\gamma)d^M$$

$$\Leftrightarrow 3\alpha(\theta+\lambda-\Omega)=[4(1+\gamma)-3(\theta+\lambda-\Omega)^2]\left[\frac{\alpha(\theta+\lambda-\Omega)}{2(1-s)(1+\gamma)-(\theta+\lambda-\Omega)^2}\right]$$

$$\Leftrightarrow 6\alpha(\theta+\lambda-\Omega)(1-s)(1+\gamma)-3\alpha(\theta+\lambda-\Omega)^2=4(1+\gamma)\alpha(\theta+\lambda-\Omega)-3\alpha(\theta+\lambda-\Omega)^2$$

$$\Leftrightarrow 3(1-s)=2\Leftrightarrow s^M=\frac{1}{3}.$$

Proof of Proposition 8:

For $s=0$, the expressions that I found for d^{NC}, d^{C} and d^{M} can be written as follows:

$$\begin{cases} d_{NS}^{NC}=\dfrac{2\alpha[2(\lambda-\Omega)-\theta\gamma]}{(2-\gamma)(2+\gamma)^2-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]}, \\ d_{NS}^{C}=\dfrac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2-2(\theta+\lambda-\Omega)^2}, \\ d_{NS}^{M}=\dfrac{\alpha(\lambda-\Omega+\theta)}{2(1+\gamma)-(\theta+\lambda-\Omega)^2}. \end{cases}$$

- **I will determine the rankings of the non-subsidized optimal quality investments:**

$$d_{NS}^{C}-d_{NS}^{NC}=\begin{bmatrix}\dfrac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2-2(\theta+\lambda-\Omega)^2}\\ -\dfrac{2\alpha[2(\lambda-\Omega)-\theta\gamma]}{(2-\gamma)(2+\gamma)^2-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]}\end{bmatrix},$$

$$=\frac{\begin{bmatrix}2\alpha(\theta+\lambda-\Omega)\big[(2-\gamma)(2+\gamma)^2-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big]\\ -2\alpha[2(\lambda-\Omega)-\theta\gamma][(2+\gamma)^2-2(\theta+\lambda-\Omega)^2]\end{bmatrix}}{[(2+\gamma)^2-2(\theta+\lambda-\Omega)^2]\big[(2-\gamma)(2+\gamma)^2-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big]}.$$

The numerator of $(d_{NS}^{C}-d_{NS}^{NC})$ is equal to:

$$2\alpha(\theta+\lambda-\Omega)(2-\gamma)(2+\gamma)^2-4\alpha(\theta+\lambda-\Omega)^2[2(\lambda-\Omega)-\theta\gamma]$$
$$-2\alpha[2(\lambda-\Omega)-\theta\gamma](2+\gamma)^2+4\alpha(\theta+\lambda-\Omega)^2[2(\lambda-\Omega)-\theta\gamma]$$
$$=2\alpha(2+\gamma)^2\big[(\theta+\lambda-\Omega)(2-\gamma)-[2(\lambda-\Omega)-\theta\gamma]\big]$$
$$=2\alpha(2+\gamma)^2[2\theta-\gamma(\lambda-\Omega)]>0 \text{ for all } \theta>\frac{\gamma(\lambda-\Omega)}{2}.$$

Based on the concavity of the profit with respect to d_i when $d_i=d_j$, I already found the following conditions (for $s=0$):

$(2+\gamma)^2-2(\theta+\lambda-\Omega)^2>0$ ($\boldsymbol{IV}$) and

$(4-\gamma^2)^2-2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega]>0$ ($\boldsymbol{II}$)

$$\Leftrightarrow (2-\gamma)\big[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big] > 0$$

$$\Leftrightarrow \big[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big] > 0.$$

Therefore $(d_{NS}^{C} - d_{NS}^{NC}) > 0$ for all $\theta > \frac{\gamma(\lambda-\Omega)}{2}$. $\quad(\mathbf{14})$

$$d_{NS}^{C} - d_{NS}^{M} = \left[\frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2} - \frac{\alpha(\lambda-\Omega+\theta)}{2(1+\gamma)-(\theta+\lambda-\Omega)^2}\right]$$

$$= \frac{\begin{bmatrix} 2\alpha(\theta+\lambda-\Omega)[2(1+\gamma)-(\theta+\lambda-\Omega)^2] \\ -\alpha(\lambda-\Omega+\theta)[(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2]\end{bmatrix}}{[(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2][2(1+\gamma)-(\theta+\lambda-\Omega)^2]}.$$

The numerator of$(d_{NS}^{C} - d_{NS}^{M})$ is equal to:

$4\alpha(\theta+\lambda-\Omega)(1+\gamma) - 2\alpha(\theta+\lambda-\Omega)^3 - \alpha(2+\gamma)^2(\theta+\lambda-\Omega) + 2\alpha(\theta+\lambda-\Omega)^3$

$= (\theta+\lambda-\Omega)\alpha[4(1+\gamma)-(2+\gamma)^2] = -\gamma^2(\theta+\lambda-\Omega)\alpha < 0$ for all $\theta, (\lambda-\Omega) > 0$ and $\gamma \in (0,1]$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions (for $s = 0$):

$(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2 > 0$ $(\boldsymbol{IV})$ and $2(1+\gamma)-(\theta+\lambda-\Omega)^2 > 0.(\boldsymbol{VI})$

Therefore $d_{NS}^{C} < d_{NS}^{M}$ for all $\theta, (\lambda-\Omega) > 0$ and $\gamma \in (0,1]$. $\quad(\mathbf{15})$

From $(\mathbf{14})$ and $(\mathbf{15})$, I conclude that $d_{NS}^{M} > d_{NS}^{C} > d_{NS}^{NC}$ for all $\frac{\gamma(\lambda-\Omega)}{2} < \theta < 1$.

$$d_{NS}^{NC} - d_{NS}^{M} = \begin{bmatrix} \dfrac{2\alpha[2(\lambda-\Omega)-\theta\gamma]}{(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]} \\ -\dfrac{\alpha(\lambda-\Omega+\theta)}{2(1+\gamma)-(\theta+\lambda-\Omega)^2}\end{bmatrix}$$

$$= \frac{\begin{bmatrix} 2\alpha[2(\lambda-\Omega)-\theta\gamma][2(1+\gamma)-(\theta+\lambda-\Omega)^2] \\ -\alpha(\lambda-\Omega+\theta)\big[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big]\end{bmatrix}}{\begin{bmatrix}(2-\gamma)(2+\gamma)^2 \\ -2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\end{bmatrix}[2(1+\gamma)-(\theta+\lambda-\Omega)^2]}.$$

The numerator of $(d_{NS}^{NC} - d_{NS}^{M})$ is equal to:

$2\alpha[2(\lambda-\Omega)-\theta\gamma]\,[2(1+\gamma)-(\theta+\lambda-\Omega)^2]$

$-\,\alpha(\theta+\lambda-\Omega)(2-\gamma)(2+\gamma)^2 + 2\alpha(\theta+\lambda-\Omega)^2[2(\lambda-\Omega)-\theta\gamma]$

$= 4\alpha[2(\lambda-\Omega)-\theta\gamma](1+\gamma) - 2\alpha[2(\lambda-\Omega)-\theta\gamma]\,(\theta+\lambda-\Omega)^2$

$-\,\alpha(\theta+\lambda-\Omega)(2-\gamma)(2+\gamma)^2 + 2\alpha(\theta+\lambda-\Omega)^2[2(\lambda-\Omega)-\theta\gamma]$

$= 4\alpha[2(\lambda-\Omega)-\theta\gamma](1+\gamma) - \alpha(\theta+\lambda-\Omega)(2-\gamma)(2+\gamma)^2$

$= \alpha[4[2(\lambda-\Omega)-\theta\gamma](1+\gamma) - (\theta+\lambda-\Omega)(2-\gamma)(2+\gamma)^2]$

$= \alpha\big[(\lambda - \Omega)[8(1+\gamma) - (2-\gamma)(2+\gamma)^2] - \theta[4\gamma(1+\gamma) + (2-\gamma)(2+\gamma)^2]\big] > 0$ for

$\theta < \dfrac{(\lambda - \Omega)[8(1+\gamma) - (2-\gamma)(2+\gamma)^2]}{4\gamma(1+\gamma) + (2-\gamma)(2+\gamma)^2}$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions (for $s = 0$):

$(4-\gamma^2)^2 - 2(2-\gamma)[2(\lambda-\Omega) - \theta\gamma][\theta + \lambda - \Omega] > 0 \quad (\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\big[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega) - \theta\gamma]\big] > 0$

$\Leftrightarrow \big[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega) - \theta\gamma]\big] > 0.$

$$2(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0. \quad (\boldsymbol{VI})$$

Therefore $d_{NS}^{NC} > d_{NS}^{M}$ for all $0 < \theta < \frac{(\lambda-\Omega)[8(1+\gamma)-(2-\gamma)(2+\gamma)^2]}{4\gamma(1+\gamma)+(2-\gamma)(2+\gamma)^2}$.

Since $d_{NS}^{M} > d_{NS}^{C}$ for all $\theta, (\lambda - \Omega) > 0$ and $\gamma \in (0,1]$, $d_{NS}^{NC} > d_{NS}^{M} > d_{NS}^{C}$ for all

$0 < \theta < \dfrac{(\lambda - \Omega)[8(1+\gamma) - (2-\gamma)(2+\gamma)^2]}{4\gamma(1+\gamma) + (2-\gamma)(2+\gamma)^2}$.

Note that $8(1+\gamma) - (2-\gamma)(2+\gamma)^2 > 0$.

- **For $s = 0$, I will show that $\pi_{NS}^{M} > \pi_{NS}^{C} > \pi_{NS}^{NC}$ for all $\theta \neq \frac{\gamma(\lambda-\Omega)}{2}$:**

$$q_{NS}^{NC} = \frac{\alpha}{(2+\gamma)} + \frac{(\theta+\lambda-\Omega)}{(2+\gamma)} d_{NS}^{NC}$$

$$= \frac{\alpha}{(2+\gamma)} + \frac{(\theta+\lambda-\Omega)}{(2+\gamma)}\left[\frac{2\alpha[2(\lambda-\Omega) - \theta\gamma]}{(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega) - \theta\gamma]}\right]$$

$$= \frac{\begin{bmatrix}\alpha(2-\gamma)(2+\gamma)^2 - 2\alpha[2(\lambda-\Omega) - \theta\gamma](\theta+\lambda-\Omega) \\ +2\alpha[2(\lambda-\Omega) - \theta\gamma](\theta+\lambda-\Omega)\end{bmatrix}}{(2+\gamma)\big[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega) - \theta\gamma]\big]}$$

$$= \frac{\alpha(2-\gamma)(2+\gamma)}{\big[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega) - \theta\gamma]\big]},$$

$$\pi_{NS}^{NC} = (q_{NS}^{NC})^2 - \frac{1}{2}(d_{NS}^{NC})^2 = \frac{\alpha^2[(2-\gamma)^2(2+\gamma)^2 - 2[2(\lambda-\Omega) - \theta\gamma]^2]}{\big[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega) - \theta\gamma]\big]^2}.$$

$$\pi_{NS}^{C} - \pi_{NS}^{NC} = (q_{NS}^{C} - q_{NS}^{NC})(q_{NS}^{C} + q_{NS}^{NC}) - \frac{1}{2}[(d_{NS}^{C})^2 - (d_{NS}^{NC})^2],$$

$$(q_{NS}^{C} - q_{NS}^{NC}) = \frac{(\theta+\lambda-\Omega)}{(2+\gamma)}[d_{NS}^{C} - d_{NS}^{NC}],$$

$$(q_{NS}^{C} + q_{NS}^{NC}) = \frac{2\alpha}{(2+\gamma)} + \frac{(\theta+\lambda-\Omega)}{(2+\gamma)}[d_{NS}^{C} + d_{NS}^{NC}],$$

$$\pi_{NS}^{C} - \pi_{NS}^{NC} = \frac{(\theta+\lambda-\Omega)}{(2+\gamma)}[d_{NS}^{C} - d_{NS}^{NC}]\left[\frac{2\alpha}{(2+\gamma)} + \frac{(\theta+\lambda-\Omega)}{(2+\gamma)}[d_{NS}^{C} + d_{NS}^{NC}]\right]$$

$$-\frac{1}{2}[d_{NS}^{C} + d_{NS}^{NC}][d_{NS}^{C} - d_{NS}^{NC}]$$

$$= [d_{NS}^{C} - d_{NS}^{NC}]\left[\frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2} + \left[\frac{(\theta+\lambda-\Omega)^2}{(2+\gamma)^2} - \frac{1}{2}\right][d_{NS}^{C} + d_{NS}^{NC}]\right]$$

$$= [d_{NS}^{C} - d_{NS}^{NC}]\left[\begin{array}{c}\frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2} \\ -\left[\frac{(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2}{2(2+\gamma)^2}\right][d_{NS}^{C} + d_{NS}^{NC}]\end{array}\right].$$

Given

$$d_{NS}^{C} = \frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2} \text{ for } s = 0,$$

$$(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2 = \frac{2\alpha(\theta+\lambda-\Omega)}{d_{NS}^{C}} \Rightarrow$$

$$\pi_{NS}^{C} - \pi_{NS}^{NC} = [d_{NS}^{C} - d_{NS}^{NC}]\left[\begin{array}{c}\frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2} \\ -\left[\frac{2\alpha(\theta+\lambda-\Omega)}{2(2+\gamma)^2 d^{C}}\right][d_{NS}^{C} + d_{NS}^{NC}]\end{array}\right]$$

$$= [d_{NS}^{C} - d_{NS}^{NC}]\left[\begin{array}{c}\frac{\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2} \\ -\left[\frac{\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2 d^{C}}\right]d_{NS}^{NC}\end{array}\right]$$

$$= [d_{NS}^{C} - d_{NS}^{NC}]\frac{\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2}\left[\frac{d_{NS}^{C} - d_{NS}^{NC}}{d_{NS}^{C}}\right] = \frac{\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2 d^{C}}(d_{NS}^{C} - d_{NS}^{NC})^2,$$

$\pi_{NS}^{C} - \pi_{NS}^{NC} > 0$ for all $\theta \neq \frac{\gamma(\lambda-\Omega)}{2}$ **(16)**

since I already show that $d_{NS}^{C} \neq d_{NS}^{NC}$ for $\theta \neq \frac{\gamma(\lambda-\Omega)}{2}$.

Now, I will show that $\pi_{NS}^{M} - \pi_{NS}^{C} > 0$ for $s = 0$:

$$\pi_{NS}^{C} = (q_{NS}^{C})^2 - \frac{1}{2}(d_{NS}^{C})^2 = \left(\frac{\alpha + (\theta+\lambda-\Omega)d_{NS}^{C}}{(2+\gamma)}\right)^2 - \frac{1}{2}(d_{NS}^{C})^2$$

$$= \frac{\alpha^2 + (\theta + \lambda - \Omega)^2 (d_{NS}^C)^2 + 2\alpha(\theta + \lambda - \Omega) d_{NS}^C}{(2+\gamma)^2} - \frac{1}{2}(d_{NS}^C)^2$$

$$= \frac{\alpha^2}{(2+\gamma)^2} + \frac{2\alpha(\theta + \lambda - \Omega) d_{NS}^C}{(2+\gamma)^2} - \left[\frac{(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2}{2(2+\gamma)^2}\right](d_{NS}^C)^2$$

$$= \frac{\alpha^2}{(2+\gamma)^2} + \frac{2\alpha(\theta + \lambda - \Omega) d_{NS}^C}{(2+\gamma)^2} - \left[\frac{2\alpha(\theta + \lambda - \Omega)}{2(2+\gamma)^2\, d^C}\right](d_{NS}^C)^2$$

$$= \frac{\alpha^2}{(2+\gamma)^2} + \frac{\alpha(\theta + \lambda - \Omega) d_{NS}^C}{(2+\gamma)^2}$$

$$= \frac{\alpha^2}{(2+\gamma)^2} + \frac{2\alpha^2(\theta + \lambda - \Omega)^2}{(2+\gamma)^2[(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2]}$$

$$= \frac{\alpha^2[(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2] + 2\alpha^2(\theta + \lambda - \Omega)^2}{(2+\gamma)^2[(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2]},$$

$$\pi_{NS}^C = \frac{\alpha^2}{[(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2]}.$$

$$\frac{\partial \pi_{NS}^C}{\partial \theta} = \frac{4\alpha^2(\theta + \lambda - \Omega)}{[(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2]^2} > 0.$$

For $s = 0$, the profit of the monopolist is expressed as follows:

$$\pi = 2(1+\gamma)(q_{NS}^M)^2 - (d_{NS}^M)^2$$

$$= 2(1+\gamma)\left[\frac{\alpha + (\theta + \lambda - \Omega) d_{NS}^M}{2(1+\gamma)}\right]^2 - (d_{NS}^M)^2$$

$$= 2(1+\gamma)\left[\frac{\alpha^2 + (\theta + \lambda - \Omega)^2 (d_{NS}^M)^2 + 2\alpha(\theta + \lambda - \Omega) d_{NS}^M}{4(1+\gamma)^2}\right] - (d_{NS}^M)^2$$

$$= 2(1+\gamma)\left[\frac{\alpha^2 + (\theta + \lambda - \Omega)^2 (d_{NS}^M)^2 + 2\alpha(\theta + \lambda - \Omega) d_{NS}^M}{4(1+\gamma)^2}\right] - (d_{NS}^M)^2$$

$$= \frac{\alpha^2}{2(1+\gamma)} + \frac{2\alpha(\theta + \lambda - \Omega) d_{NS}^M}{2(1+\gamma)} - \left[\frac{2(1+\gamma) - (\theta + \lambda - \Omega)^2}{2(1+\gamma)}\right](d_{NS}^M)^2$$

$$= \frac{\alpha^2}{2(1+\gamma)} + \frac{2\alpha(\theta + \lambda - \Omega) d_{NS}^M}{2(1+\gamma)} - \left[\frac{\alpha(\theta + \lambda - \Omega)}{2(1+\gamma) d_{NS}^M}\right](d_{NS}^M)^2$$

$$= \frac{\alpha^2}{2(1+\gamma)} + \frac{2\alpha(\theta + \lambda - \Omega) d_{NS}^M}{2(1+\gamma)} - \left[\frac{\alpha(\theta + \lambda - \Omega)}{2(1+\gamma) d_{NS}^M}\right](\, d^M)^2$$

$$= \frac{\alpha^2}{2(1+\gamma)} + \frac{\alpha(\theta + \lambda - \Omega)}{2(1+\gamma)} d_{NS}^M$$

$$= \frac{\alpha^2}{2(1+\gamma)} + \frac{\alpha(\theta+\lambda-\Omega)}{2(1+\gamma)}\left[\frac{\alpha(\theta+\lambda-\Omega)}{2(1+\gamma)-(\theta+\lambda-\Omega)^2}\right]$$

$$= \frac{\alpha^2[2(1+\gamma)-(\theta+\lambda-\Omega)^2] + \alpha^2(\theta+\lambda-\Omega)^2}{2(1+\gamma)[2(1+\gamma)-(\theta+\lambda-\Omega)^2]}$$

$$= \frac{\alpha^2}{[2(1+\gamma)-(\theta+\lambda-\Omega)^2]}.$$

$$\pi_{NS}^M = \frac{\alpha^2}{2[2(1+\gamma)-(\theta+\lambda-\Omega)^2]}.$$

$$\pi_{NS}^M - \pi_{NS}^C = \frac{\alpha^2}{2[2(1+\gamma)-(\theta+\lambda-\Omega)^2]} - \frac{\alpha^2}{[(2+\gamma)^2-2(\theta+\lambda-\Omega)^2]}$$

$$= \frac{\alpha^2\left[[(2+\gamma)^2-2(\theta+\lambda-\Omega)^2] - 2[2(1+\gamma)-(\theta+\lambda-\Omega)^2]\right]}{2[2(1+\gamma)-(\theta+\lambda-\Omega)^2][(2+\gamma)^2-2(\theta+\lambda-\Omega)^2]}.$$

Numerator of $(\pi_{NS}^M - \pi_{NS}^C) = \alpha^2\begin{bmatrix}(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2 \\ -4(1+\gamma) + 2(\theta+\lambda-\Omega)^2\end{bmatrix} = \alpha^2\gamma^2 > 0$ for all θ and for all $\gamma \in (0,1]$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions (for $s = 0$):

$2(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0$ $(\boldsymbol{VI})$ and $(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2 > 0$. $(\boldsymbol{IV})$

Therefore $\pi_{NS}^M > \pi_{NS}^C$ for all θ and for all $\gamma \in (0,1]$. $(\mathbf{17})$

From $(\mathbf{16})$ and $(\mathbf{17})$, I conclude that $\pi_{NS}^M > \pi_{NS}^C > \pi_{NS}^{NC}$ for all $\theta \neq \frac{\gamma(\lambda-\Omega)}{2}$.

Proof of Proposition 9:

- **I will show that $\boldsymbol{s^{NC} > s^C > s^M}$ for all $\boldsymbol{\theta > \frac{\gamma(\lambda-\Omega)}{2}}$:**

$$s^{NC} - s^C = \left[1 - \frac{2[2(\lambda-\Omega)-\theta\gamma]}{(3+\gamma)(2-\gamma)(\theta+\lambda-\Omega)}\right] - \left[1 - \frac{2}{(3+\gamma)}\right]$$

$$= \frac{2}{(3+\gamma)} - \frac{2[2(\lambda-\Omega)-\theta\gamma]}{(3+\gamma)(2-\gamma)(\theta+\lambda-\Omega)}$$

$$= \frac{2(2-\gamma)(\theta+\lambda-\Omega) - 4(\lambda-\Omega) + 2\theta\gamma}{(3+\gamma)(2-\gamma)(\theta+\lambda-\Omega)}$$

$$= \frac{4\theta + 4(\lambda-\Omega) - 2\gamma\theta - 2\gamma(\lambda-\Omega) - 4(\lambda-\Omega) + 2\theta\gamma}{(3+\gamma)(2-\gamma)(\theta+\lambda-\Omega)}$$

$$= \frac{2[2\theta - \gamma(\lambda - \Omega)]}{(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)}.$$

$$s^{NC} > s^{C} \text{ if } 2\theta > \gamma(\lambda - \Omega) \Leftrightarrow \theta > \frac{\gamma(\lambda - \Omega)}{2}. \qquad \textbf{(18)}$$

$$s^{C} - s^{M} = 1 - \frac{2}{(3+\gamma)} - 1 + \frac{2}{3} = \frac{2(3+\gamma) - 6}{3(3+\gamma)} = \frac{2\gamma}{3(3+\gamma)} > 0 \text{ for all } \theta \text{ and } \gamma \qquad \textbf{(19)}$$

such that $\gamma \in (0,1]$.

From conditions **(19)** and **(20),** I conclude that:

$$s^{NC} > s^{C} > s^{M} \text{ for all } \theta > \frac{\gamma(\lambda - \Omega)}{2}.$$

- **I will show that $s^{C} > s^{NC} > s^{M}$ for all $\frac{\gamma(1+\gamma)(\lambda-\Omega)}{(6+2\gamma-\gamma^2)} < \theta < \frac{\gamma(\lambda-\Omega)}{2}$:**

$$s^{NC} - s^{M} = (s^{NC} - s^{C}) + (s^{C} - s^{M}) = \frac{2[2\theta - \gamma(\lambda - \Omega)]}{(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)} + \frac{2\gamma}{3(3+\gamma)}$$

$$= \frac{6[2\theta - \gamma(\lambda - \Omega)] + 2\gamma(2-\gamma)(\theta + \lambda - \Omega)}{3(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)}$$

$$= \frac{2\theta[6 + \gamma(2-\gamma)] - 2\gamma(\lambda - \Omega)[3 - (2-\gamma)]}{3(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)}$$

$$s^{NC} - s^{M} = \frac{2\theta[6 + \gamma(2-\gamma)] - 2\gamma(\lambda - \Omega)[1+\gamma]}{3(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)}.$$

$$s^{NC} > s^{M} \text{for all } \theta > \frac{\gamma(\lambda - \Omega)[1+\gamma]}{[6 + 2\gamma - \gamma^2]}. \qquad \textbf{(20)}$$

Now, I will show that $\frac{\gamma(\lambda-\Omega)}{2} > \frac{\gamma(1+\gamma)(\lambda-\Omega)}{(6+2\gamma-\gamma^2)}$:

$$\frac{\gamma(\lambda - \Omega)}{2} - \frac{\gamma(1+\gamma)(\lambda - \Omega)}{(6 + 2\gamma - \gamma^2)} = \gamma(\lambda - \Omega)\left[\frac{1}{2} - \frac{(1+\gamma)}{(6 + 2\gamma - \gamma^2)}\right]$$

$$= \gamma(\lambda - \Omega)\left[\frac{(6 + 2\gamma - \gamma^2) - 2 - 2\gamma}{2(6 + 2\gamma - \gamma^2)}\right]$$

$$= \gamma(\lambda - \Omega)\left[\frac{(4 - \gamma^2)}{2(6 + 2\gamma - \gamma^2)}\right]$$

$$= \gamma(\lambda - \Omega)\left[\frac{(2-\gamma)(2+\gamma)}{2\big(6 + \gamma(2-\gamma)\big)}\right] > 0. \qquad \textbf{(21)}$$

From **(18)**, $s^{NC} < s^{C}$ if $2\theta < \gamma(\lambda - \Omega) \Leftrightarrow \theta < \frac{\gamma(\lambda-\Omega)}{2}$.

Given conditions **(18)**, **(20)** and **(21),** I conclude that:

$s^C > s^{NC} > s^M$ for all $\dfrac{\gamma(1+\gamma)(\lambda-\Omega)}{(6+2\gamma-\gamma^2)} < \theta < \dfrac{\gamma(\lambda-\Omega)}{2}$.

Proof of Proposition 10:

- **I will show that $d_S^{NC} = d_S^C > d_S^M$ for $s \neq 0$:**

$$d_S^{NC} = \frac{2\alpha[2(\lambda-\Omega)-\theta\gamma]}{(2-\gamma)(2+\gamma)^2(1-s^{NC})-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]}$$

$$= \frac{2\alpha[2(\lambda-\Omega)-\theta\gamma]}{(2-\gamma)(2+\gamma)^2\left(\dfrac{2[2(\lambda-\Omega)-\theta\gamma]}{(3+\gamma)(2-\gamma)(\theta+\lambda-\Omega)}\right)-2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]}$$

$$= \frac{\alpha}{\dfrac{(2+\gamma)^2}{(3+\gamma)(\theta+\lambda-\Omega)}-(\theta+\lambda-\Omega)} = \frac{\alpha(3+\gamma)(\theta+\lambda-\Omega)}{(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2}.$$

$$d_S^C = \frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2(1-s^C)-2(\theta+\lambda-\Omega)^2} = \frac{2\alpha(\theta+\lambda-\Omega)}{\dfrac{2(2+\gamma)^2}{(3+\gamma)}-2(\theta+\lambda-\Omega)^2}$$

$$= \frac{\alpha(\theta+\lambda-\Omega)(3+\gamma)}{(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2}.$$

Therefore, $d_S^{NC} = d_S^C$ for $s \neq 0$.

$$d_S^M = \frac{\alpha(\theta+\lambda-\Omega)}{2(1-s^M)(1+\gamma)-(\theta+\lambda-\Omega)^2} = \frac{\alpha(\theta+\lambda-\Omega)}{2(\frac{2}{3})(1+\gamma)-(\theta+\lambda-\Omega)^2}$$

$$= \frac{3\alpha(\theta+\lambda-\Omega)}{4(1+\gamma)-3(\theta+\lambda-\Omega)^2}.$$

$$d_S^C - d_S^M = \frac{\alpha(\theta+\lambda-\Omega)(3+\gamma)}{(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2} - \frac{3\alpha(\theta+\lambda-\Omega)}{4(1+\gamma)-3(\theta+\lambda-\Omega)^2}$$

$$= \frac{\begin{bmatrix}\alpha(\theta+\lambda-\Omega)(3+\gamma)[4(1+\gamma)-3(\theta+\lambda-\Omega)^2] \\ -3\alpha(\theta+\lambda-\Omega)[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2]\end{bmatrix}}{[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2][4(1+\gamma)-3(\theta+\lambda-\Omega)^2]}.$$

Numerator of $(d_S^C - d_S^M) = \begin{bmatrix}4\alpha(1+\gamma)(\theta+\lambda-\Omega)(3+\gamma)-3\alpha(3+\gamma)(\theta+\lambda-\Omega)^3 \\ -3\alpha(2+\gamma)^2(\theta+\lambda-\Omega)+3\alpha(3+\gamma)(\theta+\lambda-\Omega)^3\end{bmatrix}$

$$= \alpha(\theta+\lambda-\Omega)\begin{bmatrix}4(1+\gamma)(3+\gamma) \\ -3(2+\gamma)^2\end{bmatrix}$$

$$= \alpha(\theta+\lambda-\Omega)\begin{bmatrix}12+4\gamma+12\gamma+4\gamma^2 \\ -12-12\gamma-3\gamma^2\end{bmatrix}$$

$= \alpha(\theta+\lambda-\Omega)(4\gamma+\gamma^2) > 0$ for all $\theta, (\lambda-\Omega) > 0$ and

$\gamma \in (0,1]$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega] > 0 \quad (\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0,$

$\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]$

$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0$ and

$2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0 \quad (\boldsymbol{VI})$

$\Leftrightarrow 3[2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2] = 4(1+\gamma) - 3(\theta+\lambda-\Omega)^2 > 0 \; (s^M = \frac{1}{3}).$

Therefore $d_S^C - d_S^M > 0$ for all $\theta, (\lambda-\Omega) > 0$ and $\gamma \in (0,1]$.

$$\frac{\partial d_S^{NC}}{\partial \lambda} = \frac{\alpha(3+\gamma)[(2+\gamma)^2 + (3+\gamma)(\theta+\lambda-\Omega)^2]}{[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]^2},$$

$$\frac{\partial d_S^M}{\partial \lambda} = \frac{3\alpha[4(1+\gamma) + 3(\theta+\lambda-\Omega)^2]}{[4(1+\gamma) - 3(\theta+\lambda-\Omega)^2]^2}.$$

$$\frac{\partial d_S^{NC}}{\partial \lambda} - \frac{\partial d_S^M}{\partial \lambda} = \frac{\begin{bmatrix}\alpha(3+\gamma)[(2+\gamma)^2 + (3+\gamma)(\theta+\lambda-\Omega)^2][4(1+\gamma) - 3(\theta+\lambda-\Omega)^2]^2 \\ -3\alpha[4(1+\gamma) - 3(\theta+\lambda-\Omega)^2][(2+\gamma)^2 + (3+\gamma)(\theta+\lambda-\Omega)^2]^2\end{bmatrix}}{[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]^2[4(1+\gamma) - 3(\theta+\lambda-\Omega)^2]^2}.$$

After calculation, the numerator of $\left[\frac{\partial d_S^{NC}}{\partial \lambda} - \frac{\partial d_S^M}{\partial \lambda}\right]$ is equal to:

$$\alpha\begin{bmatrix}\gamma(4+\gamma)[4(1+\gamma)(2+\gamma)^2 - 9(3+\gamma)(\theta+\lambda-\Omega)^4] \\ +(\theta+\lambda-\Omega)^2[16(1+\gamma)^2(3+\gamma)^2 - 9(2+\gamma)^4]\end{bmatrix}.$$

If $\gamma = 1, \theta = 1, (\lambda-\Omega)$ close to zero, numerator of $\left[\frac{\partial d_S^{NC}}{\partial \lambda} - \frac{\partial d_S^M}{\partial \lambda}\right] > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega] > 0 \quad (\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0,$

$\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]$

$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0$ and

$2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0 \quad (\boldsymbol{VI})$

$\Leftrightarrow 3[2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2] = 4(1+\gamma) - 3(\theta+\lambda-\Omega)^2 > 0.$

Therefore $\left[\frac{\partial d_S^{NC}}{\partial \lambda} - \frac{\partial d_S^{M}}{\partial \lambda}\right] > 0$ if $\gamma = 1, \theta = 1, (\lambda - \Omega)$ close to zero.

When there is a very small net effect of quality improvement $[(\lambda - \Omega)$ close to zero$]$, full competition and very large demand spillover $(\theta = 1)$, the impact of quality sensitivity on the non-cooperative R&D quality investment is greater than its impact on the full cooperative R&D quality investment.

- **For s $\neq$ 0, I will determine the sign of ($\boldsymbol{\pi_S^C - \pi_S^M}$):**

$$\pi_S^C = (q_S^C)^2 - \frac{1}{2}(d_S^C)^2 = \left(\frac{\alpha + (\theta + \lambda - \Omega)d_S^C}{(2+\gamma)}\right)^2 - \frac{1}{2}(1 - s^C)(d_S^C)^2$$

$$= \frac{\alpha^2 + (\theta + \lambda - \Omega)^2 (d_S^C)^2 + 2\alpha(\theta + \lambda - \Omega)d_S^C}{(2+\gamma)^2} - \frac{1}{2}(1 - s^C)(d_S^C)^2$$

$$= \frac{\alpha^2}{(2+\gamma)^2} + \frac{2\alpha(\theta + \lambda - \Omega)d_S^C}{(2+\gamma)^2} - \left[\frac{(1 - s^C)(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2}{2(2+\gamma)^2}\right](d_S^C)^2$$

$$= \frac{\alpha^2}{(2+\gamma)^2} + \frac{2\alpha(\theta + \lambda - \Omega)d_S^C}{(2+\gamma)^2} - \left[\frac{2\alpha(\theta + \lambda - \Omega)}{2(2+\gamma)^2 d_S^C}\right](d_S^C)^2$$

$$= \frac{\alpha^2}{(2+\gamma)^2} + \frac{\alpha(\theta + \lambda - \Omega)d_S^C}{(2+\gamma)^2}$$

$$= \frac{\alpha^2}{(2+\gamma)^2} + \frac{2\alpha^2(\theta + \lambda - \Omega)^2}{(2+\gamma)^2[(1 - s^C)(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2]}$$

$$= \frac{\alpha^2[(1 - s^C)(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2] + 2\alpha^2(\theta + \lambda - \Omega)^2}{(2+\gamma)^2[(1 - s^C)(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2]}$$

$$= \frac{\alpha^2(1 - s^C)}{[(1 - s^C)(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2]} = \frac{\alpha^2\left(\frac{2}{3+\gamma}\right)}{\left[\left(\frac{2}{3+\gamma}\right)(2+\gamma)^2 - 2(\theta + \lambda - \Omega)^2\right]}.$$

$$\pi_S^C = \frac{\alpha^2}{[(2+\gamma)^2 - (3+\gamma)(\theta + \lambda - \Omega)^2]}.$$

$$\pi_S^M = (1+\gamma)(q_S^M)^2 - \frac{1}{2}(1 - s^M)(d_S^M)^2$$

$$= (1+\gamma)\left[\frac{\alpha + (\theta + \lambda - \Omega)d_S^M}{2(1+\gamma)}\right]^2 - \frac{1}{2}(1 - s^M)(d_S^M)^2$$

$$= (1+\gamma)\left[\frac{\alpha^2 + (\theta + \lambda - \Omega)^2(d_S^M)^2 + 2\alpha(\theta + \lambda - \Omega)d_S^M}{4(1+\gamma)^2}\right] - \frac{1}{2}(1 - s^M)(d_S^M)^2$$

$$= (1+\gamma)\left[\frac{\alpha^2 + (\theta + \lambda - \Omega)^2(d_S^M)^2 + 2\alpha(\theta + \lambda - \Omega)d_S^M}{4(1+\gamma)^2}\right] - \frac{1}{2}(1 - s^M)(d_S^M)^2$$

$$= \frac{\alpha^2}{4(1+\gamma)} + \frac{2\alpha(\theta+\lambda-\Omega)d_S^M}{4(1+\gamma)} - \left[\frac{2(1-s^M)(1+\gamma)-(\theta+\lambda-\Omega)^2}{4(1+\gamma)}\right](d_S^M)^2$$

$$= \frac{\alpha^2}{4(1+\gamma)} + \frac{2\alpha(\theta+\lambda-\Omega)d_S^M}{4(1+\gamma)} - \left[\frac{\alpha(\theta+\lambda-\Omega)}{4(1+\gamma)d_S^M}\right](d_S^M)^2$$

$$= \frac{\alpha^2}{4(1+\gamma)} + \frac{\alpha(\theta+\lambda-\Omega)}{4(1+\gamma)}d_S^M$$

$$= \frac{\alpha^2}{4(1+\gamma)} + \frac{\alpha(\theta+\lambda-\Omega)}{4(1+\gamma)}\left[\frac{\alpha(\theta+\lambda-\Omega)}{2(1-s^M)(1+\gamma)-(\theta+\lambda-\Omega)^2}\right]$$

$$= \frac{\alpha^2[2(1-s^M)(1+\gamma)-(\theta+\lambda-\Omega)^2]+\alpha^2(\theta+\lambda-\Omega)^2}{4(1+\gamma)[2(1-s^M)(1+\gamma)-(\theta+\lambda-\Omega)^2]}$$

$$= \frac{\alpha^2(1-s^M)}{2[2(1-s^M)(1+\gamma)-(\theta+\lambda-\Omega)^2]} = \frac{\alpha^2\left(\frac{2}{3}\right)}{2\left[2\left(\frac{2}{3}\right)(1+\gamma)-(\theta+\lambda-\Omega)^2\right]}$$

$$= \frac{\alpha^2}{[4(1+\gamma)-3(\theta+\lambda-\Omega)^2]}.$$

$$\pi_S^C - \pi_S^M = \frac{\alpha^2}{[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2]} - \frac{\alpha^2}{[4(1+\gamma)-3(\theta+\lambda-\Omega)^2]}$$

$$= \frac{\alpha^2[4(1+\gamma)-3(\theta+\lambda-\Omega)^2]-\alpha^2[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2]}{[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2][4(1+\gamma)-3(\theta+\lambda-\Omega)^2]}.$$

Numerator of $(\pi_S^C - \pi_S^M) = \alpha^2[-\gamma^2+\gamma(\theta+\lambda-\Omega)^2] = \alpha^2\gamma[(\theta+\lambda-\Omega)^2-\gamma] > 0$for $\theta > \sqrt{\gamma}-(\lambda-\Omega)$ such that $\gamma > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega] > 0$ $\quad(\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\big[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big] > 0,$

$\big[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big]$

$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0$ and

$2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0$ $\quad(\boldsymbol{VI})$

$\Leftrightarrow 3[2(1-s^M)(1+\gamma)-(\theta+\lambda-\Omega)^2] = 4(1+\gamma) - 3(\theta+\lambda-\Omega)^2 > 0.$

Therefore $\pi_S^C > \pi_S^M$ for $\theta > \sqrt{\gamma}-(\lambda-\Omega)$.

If $\sqrt{\gamma}-(\lambda-\Omega) < 0$, $\sqrt{\gamma}-(\lambda-\Omega)-\theta < 0$ since $\theta > 0$ which is equivalent to:

$-\theta < 0$ therefore $(\lambda-\Omega)+\theta-\sqrt{\gamma} > 0 \Leftrightarrow (\lambda-\Omega)+\theta > \sqrt{\gamma} \Leftrightarrow [(\theta+\lambda-\Omega)^2 > \gamma]$

$\Leftrightarrow \pi_S^C > \pi_S^M$.

If $\sqrt{\gamma} - (\lambda - \Omega) > 0$, two possibilities occur:

- ❖ $\sqrt{\gamma} - (\lambda - \Omega) - \theta < 0 \Leftrightarrow (\lambda - \Omega) + \theta - \sqrt{\gamma} > 0 \Leftrightarrow (\lambda - \Omega) + \theta > \sqrt{\gamma}$
  $\Leftrightarrow [(\theta + \lambda - \Omega)^2 > \gamma] \Leftrightarrow \pi_S^C > \pi_S^M$.
- ❖ $\sqrt{\gamma} - (\lambda - \Omega) - \theta > 0 \Leftrightarrow (\lambda - \Omega) + \theta - \sqrt{\gamma} < 0 \Leftrightarrow (\lambda - \Omega) + \theta < \sqrt{\gamma}$
  $\Leftrightarrow [(\theta + \lambda - \Omega)^2 < \gamma] \Leftrightarrow \pi_S^C < \pi_S^M$.

- **I will show that, for s $\neq 0$, $\boldsymbol{\pi_S^C > \pi_S^{NC}}$ for all $\boldsymbol{\theta < \frac{\gamma(\lambda-\Omega)}{2}}$:**

$$\pi_S^{NC} = (q_S^{NC})^2 - \frac{1}{2}(1 - s^{NC})(d_S^{NC})^2$$
$$= \frac{\alpha^2[(2-\gamma)(2+\gamma)^2 - (3+\gamma)[2(\lambda-\Omega) - \theta\gamma](\theta+\lambda-\Omega)]}{(2-\gamma)[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]^2}$$

such that $d_S^{NC} = \frac{\alpha(3+\gamma)(\theta+\lambda-\Omega)}{(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2}$, $q_S^{NC} = \frac{\alpha}{(2+\gamma)} + \frac{(\theta+\lambda-\Omega)}{(2+\gamma)} d_S^{NC} = \frac{\alpha(2+\gamma)}{(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2}$

and $s^{NC} = 1 - \frac{2[2(\lambda-\Omega)-\theta\gamma]}{(3+\gamma)(\theta+\lambda-\Omega)(2-\gamma)}$.

$$\pi_S^C - \pi_S^{NC} = (q_S^C - q_S^{NC})(q_S^C + q_S^{NC}) - \frac{1}{2}[(1-s^C)(d_S^C)^2 - (1-s^{NC})(d_S^{NC})^2],$$

$(q_S^C - q_S^{NC}) = \dfrac{(\theta+\lambda-\Omega)}{(2+\gamma)}[d_S^C - d_S^{NC}]$ or $d_S^C = d_S^{NC}$ for $s \neq 0 \Rightarrow$

$$\pi_S^C - \pi_S^{NC} = \frac{(d_S^C)^2}{2}[(s^C - s^{NC})]$$
$$= -\frac{\alpha^2(\theta+\lambda-\Omega)^2(3+\gamma)^2[2\theta - \gamma(\lambda-\Omega)]}{[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]^2[(3+\gamma)(2-\gamma)(\theta+\lambda-\Omega)]}.$$

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following condition:

$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega) - \theta\gamma][\theta+\lambda-\Omega] > 0$ $(\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0$,

$\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]$

$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0$.

Therefore $\pi_S^C > (<)\ \pi_S^{NC}$ for all $\theta < (>) \frac{\gamma(\lambda-\Omega)}{2}$.

- **I will show that, for s $\neq 0$, $\boldsymbol{SW_S^{NC} = SW_S^C > SW_S^M}$ for all $\boldsymbol{\theta, (\lambda-\Omega)}$ and $\boldsymbol{\gamma > 0}$:**

$SW_S^{NC} = (3+\gamma)(q_S^{NC})^2 - (d_S^{NC})^2$,

$$q_S^{NC} = \frac{\alpha + (\theta + \lambda - \Omega)d_S^{NC}}{(2+\gamma)}$$

$$= \frac{\alpha}{(2+\gamma)} + \frac{(\theta + \lambda - \Omega)}{(2+\gamma)}\left[\frac{2\alpha[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]}{(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta + \lambda - \Omega)[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]}\right]$$

$$= \frac{\alpha(2-\gamma)(2+\gamma)(1-s^{NC})}{(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta + \lambda - \Omega)[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]}.$$

$$SW_S^{NC} = \frac{\alpha^2(2-\gamma)^2(2+\gamma)^2(1-s^{NC})^2(3+\gamma) - 4\alpha^2[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]^2}{\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta + \lambda - \Omega)[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]\right]^2}.$$

Given

$$1 - s^{NC} = \frac{2[2(\lambda - \Omega) - \theta\gamma]}{(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)},$$

$$SW_S^{NC} = \frac{4\alpha^2(2-\gamma)^2(2+\gamma)^2\left(\frac{[2(\lambda - \Omega) - \theta\gamma]}{(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)}\right)^2(3+\gamma) - 4\alpha^2[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]^2}{\left[2(2-\gamma)(2+\gamma)^2(\frac{[2(\lambda - \Omega) - \theta\gamma]}{(3+\gamma)(2-\gamma)(\theta + \lambda - \Omega)}) - 2(\theta + \lambda - \Omega)[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]\right]^2}$$

$$= \frac{4\alpha^2(2+\gamma)^2\frac{[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]^2}{(3+\gamma)[\theta + \lambda - \Omega]^2} - 4\alpha^2[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]^2}{\left[2(2+\gamma)^2(\frac{[2(\lambda - \Omega) - \theta\gamma]}{(3+\gamma)(\theta + \lambda - \Omega)}) - 2(\theta + \lambda - \Omega)[2(\boldsymbol{\lambda} - \Omega) - \theta\gamma]\right]^2}$$

$$= \frac{\frac{4\alpha^2(2+\gamma)^2}{(3+\gamma)[\theta + \lambda - \Omega]^2} - 4\alpha^2}{\left[\frac{2(2+\gamma)^2}{(3+\gamma)(\theta + \lambda - \Omega)}) - 2(\theta + \lambda - \Omega)\right]^2}$$

$$= \frac{4\alpha^2(2+\gamma)^2(3+\gamma) - 4\alpha^2(3+\gamma)^2[\theta + \lambda - \Omega]^2}{[2(2+\gamma)^2 - 2(\theta + \lambda - \Omega)(3+\gamma)(\theta + \lambda - \Omega)]^2}$$

$$= \frac{\alpha^2(3+\gamma)[(2+\gamma)^2 - (3+\gamma)[\theta + \lambda - \Omega]^2]}{[(2+\gamma)^2 - (3+\gamma)[\theta + \lambda - \Omega]^2]^2}.$$

$$SW_S^{NC} = \frac{\alpha^2(3+\gamma)}{[(2+\gamma)^2 - (3+\gamma)[\theta + \lambda - \Omega]^2]}. \quad \mathbf{(22)}$$

$$SW_S^C = (3+\gamma)(q_S^C)^2 - (d_S^C)^{\mathbf{2}}.$$

$$q_S^C = \frac{\alpha + (\theta + \lambda - \Omega)d_S^C}{(2+\gamma)}$$

$$= \frac{\alpha}{(2+\gamma)} + \frac{(\theta + \lambda - \Omega)}{(2+\gamma)}\left[\frac{2\alpha(\theta + \lambda - \Omega)}{(2+\gamma)^2(1-s^C) - 2(\theta + \lambda - \Omega)^2}\right],$$

$$q_S^C = \frac{\alpha[(2+\gamma)^2(1-s^C) - 2(\theta+\lambda-\Omega)^2] + 2\alpha(\theta+\lambda-\Omega)^2}{(2+\gamma)[(2+\gamma)^2(1-s^C) - 2(\theta+\lambda-\Omega)^2]}$$

$$= \frac{\alpha(2+\gamma)(1-s^C)}{[(2+\gamma)^2(1-s^C) - 2(\theta+\lambda-\Omega)^2]}.$$

$$SW_S^C = (3+\gamma)\left[\frac{\alpha(2+\gamma)(1-s^C)}{[(2+\gamma)^2(1-s^C) - 2(\theta+\lambda-\Omega)^2]}\right]^2 - \left[\frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2(1-s^C) - 2(\theta+\lambda-\Omega)^2}\right]^2$$

$$= \frac{(3+\gamma)\alpha^2(2+\gamma)^2(1-s^C)^2 - 4\alpha^2[\theta+\lambda-\Omega]^2}{[(2+\gamma)^2(1-s^C) - 2(\theta+\lambda-\Omega)^2]^2}$$

$$= \frac{(3+\gamma)\alpha^2(2+\gamma)^2\left(\frac{2}{3+\gamma}\right)^2 - 4\alpha^2[\theta+\lambda-\Omega]^2}{\left[(2+\gamma)^2(\frac{2}{3+\gamma}) - 2(\theta+\lambda-\Omega)^2\right]^2}$$

$$= \frac{\alpha^2(2+\gamma)^2\frac{4}{3+\gamma} - 4\alpha^2[\theta+\lambda-\Omega]^2}{\left[(2+\gamma)^2(\frac{2}{3+\gamma}) - 2(\theta+\lambda-\Omega)^2\right]^2} = \frac{\alpha^2(3+\gamma)}{(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2}. \quad \textbf{(23)}$$

$$SW_S^M = 3(1+\gamma)(q_S^M)^2 - (d_S^M)^2,$$

$$q_S^M = \frac{\alpha + (\theta+\lambda-\Omega)d_S^M}{2(1+\gamma)} = \frac{\alpha}{2(1+\gamma)} + \frac{(\theta+\lambda-\Omega)}{2(1+\gamma)}\left[\frac{\alpha(\theta+\lambda-\Omega)}{2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2}\right]$$

$$= \frac{\alpha(1-s^M)}{2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2}.$$

$$SW_S^M = \frac{3(1+\gamma)\alpha^2(1-s^M)^2 - \alpha^2(\theta+\lambda-\Omega)^2}{[2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2]^2} = \frac{3(1+\gamma)\alpha^2\left(1-\frac{1}{3}\right)^2 - \alpha^2(\theta+\lambda-\Omega)^2}{\left[2(1-\frac{1}{3})(1+\gamma) - (\theta+\lambda-\Omega)^2\right]^2}$$

$$= \frac{\frac{4}{3}(1+\gamma)\alpha^2 - \alpha^2(\theta+\lambda-\Omega)^2}{\left[\frac{4}{3}(1+\gamma) - (\theta+\lambda-\Omega)^2\right]^2} = \frac{\alpha^2}{\frac{4}{3}(1+\gamma) - (\theta+\lambda-\Omega)^2}.$$

$$SW_S^C - SW_S^M = \frac{\alpha^2(3+\gamma)}{(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2} - \frac{\alpha^2}{\frac{4}{3}(1+\gamma) - (\theta+\lambda-\Omega)^2}$$

$$= \frac{\alpha^2(3+\gamma)\left[\frac{4}{3}(1+\gamma) - (\theta+\lambda-\Omega)^2\right] - \alpha^2[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]}{[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]\left[\frac{4}{3}(1+\gamma) - (\theta+\lambda-\Omega)^2\right]}.$$

Numerator of $(SW_S^C - SW_S^M)$

$$= \frac{4}{3}\alpha^2(3+\gamma)(1+\gamma) - \alpha^2(3+\gamma)(\theta+\lambda-\Omega)^2 + \alpha^2(3+\gamma)(\theta+\lambda-\Omega)^2 - \alpha^2(2+\gamma)^2$$

$$= \frac{4}{3}\alpha^2(3+\gamma)(1+\gamma) - \alpha^2(2+\gamma)^2 = \alpha^2\left[\frac{4}{3}(3+\gamma)(1+\gamma) - (2+\gamma)^2\right]$$

$= \alpha^2 \frac{\gamma}{3}(\gamma+4) > 0$ **(24)** such that $\gamma > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega] > 0$ $\quad(\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0,$

$\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]$

$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0$ and

$2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0.$ $\quad(\boldsymbol{VI})$

Since $s^M = \frac{1}{3}$, $\frac{4}{3}(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0$.

Therefore $SW_S^C > SW_S^M$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

From **(22)**, **(23)** and **(24)**, I conclude that $SW_S^{NC} = SW_S^C > SW_S^M$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

<u>Proof of Proposition 11:</u>

$$(d_S^{NC} - d_{NS}^{NC}) = \frac{\alpha(3+\gamma)(\theta+\lambda-\Omega)}{(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2} - \frac{2[2(\lambda-\Omega)-\theta\gamma]\alpha}{(2-\gamma)(2+\gamma)^2 - 2[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega]}$$

$$= \frac{\begin{bmatrix}\alpha(3+\gamma)(\theta+\lambda-\Omega)\begin{bmatrix}(2-\gamma)(2+\gamma)^2 \\ -2[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega]\end{bmatrix} \\ -2[2(\lambda-\Omega)-\theta\gamma]\alpha[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2]\end{bmatrix}}{\begin{bmatrix}(2-\gamma)(2+\gamma)^2 \\ -2[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega]\end{bmatrix}[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2]}.$$

Numerator of $(d_S^{NC} - d_{NS}^{NC}) = \alpha(2+\gamma)^2[(\lambda-\Omega)(1-\gamma)(2+\gamma) + \theta(6-\gamma^2) + \theta\gamma] > 0$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega] > 0$ $\quad(\boldsymbol{II})$

$$\Leftrightarrow (2-\gamma)\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0,$$

$$\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]$$

$$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0 \text{ and for } s = 0,$$

$$(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma] > 0.$$

Therefore $d_S^{NC} > d_{NS}^{NC}$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

$$(d_S^C - d_{NS}^C) = \frac{\alpha(\theta+\lambda-\Omega)(3+\gamma)}{(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2} - \frac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2}$$

$$= \frac{\begin{bmatrix}\alpha(3+\gamma)(\theta+\lambda-\Omega)[(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2] \\ -2\alpha(\theta+\lambda-\Omega)[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]\end{bmatrix}}{[(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2][(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]}.$$

Numerator of $(d_S^C - d_{NS}^C) = \alpha(\theta+\lambda-\Omega)(1+\gamma)(2+\gamma)^2 > 0$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega] > 0 \quad (\boldsymbol{II})$$

$$\Leftrightarrow (2-\gamma)\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0,$$

$$\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]$$

$$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0 \text{ and for } s = 0,$$

$$(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2 > 0. \quad (\boldsymbol{IV})$$

Therefore $d_S^C > d_{NS}^C$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

$$(d_S^M - d_{NS}^M) = \frac{3\alpha(\theta+\lambda-\Omega)}{4(1+\gamma) - 3(\theta+\lambda-\Omega)^2} - \frac{\alpha(\theta+\lambda-\Omega)}{2(1+\gamma) - (\theta+\lambda-\Omega)^2}$$

$$= \frac{\begin{bmatrix}3\alpha(\theta+\lambda-\Omega)[2(1+\gamma) - (\theta+\lambda-\Omega)^2] \\ -\alpha(\theta+\lambda-\Omega)[4(1+\gamma) - 3(\theta+\lambda-\Omega)^2]\end{bmatrix}}{[4(1+\gamma) - 3(\theta+\lambda-\Omega)^2][2(1+\gamma) - (\theta+\lambda-\Omega)^2]}.$$

Numerator of $(d_S^M - d_{NS}^M) = 2\alpha(\theta+\lambda-\Omega)(1+\gamma) > 0$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$$2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0 \quad (\boldsymbol{VI})$$

$$\Leftrightarrow 3[2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2] = 4(1+\gamma) - 3(\theta+\lambda-\Omega)^2 > 0 \text{ and for } s = 0,$$

$$2(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0. \quad (\boldsymbol{VI})$$

Therefore $d_S^M > d_{NS}^M$ for all $\theta, (\lambda - \Omega)$ and $\gamma > 0$.

$$(\pi_S^C - \pi_{NS}^C) = \frac{\alpha^2}{[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2]} - \frac{\alpha^2}{[(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2]}$$

$$= \frac{\alpha^2 \begin{bmatrix} [(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2] \\ -[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2] \end{bmatrix}}{[(2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2][(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2]}.$$

Numerator of $(\pi_S^C - \pi_{NS}^C) = \alpha^2(\theta+\lambda-\Omega)^2(1+\gamma) > 0$ for all $\theta, (\lambda - \Omega)$ and $\gamma > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega) - \theta\gamma][\theta+\lambda-\Omega] > 0 \quad (\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\big[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big] > 0,$

$\big[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\big]$

$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0$ and for $s = 0$,

$(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2 > 0. \quad (\boldsymbol{IV})$

Therefore $\pi_S^C > \pi_{NS}^C$ for all $\theta, (\lambda - \Omega)$ and $\gamma > 0$.

$$(\pi_S^M - \pi_{NS}^M) = \frac{\alpha^2}{[4(1+\gamma) - 3(\theta+\lambda-\Omega)^2]} - \frac{\alpha^2}{2[2(1+\gamma) - (\theta+\lambda-\Omega)^2]}$$

$$= \frac{\alpha^2 \begin{bmatrix} 2[2(1+\gamma) - (\theta+\lambda-\Omega)^2] \\ -[4(1+\gamma) - 3(\theta+\lambda-\Omega)^2] \end{bmatrix}}{[4(1+\gamma) - 3(\theta+\lambda-\Omega)^2]\big[2[2(1+\gamma) - (\theta+\lambda-\Omega)^2]\big]}.$$

Numerator of $(\pi_S^M - \pi_{NS}^M) = \alpha^2(\theta+\lambda-\Omega)^2 > 0$ for all $\theta, (\lambda - \Omega)$ and $\gamma > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0 \quad (\boldsymbol{VI})$

$\Leftrightarrow 3[2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2] = 4(1+\gamma) - 3(\theta+\lambda-\Omega)^2 > 0$ and for $s = 0$,

$2(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0. \quad (\boldsymbol{VI})$

Therefore $\pi_S^M > \pi_{NS}^M$ for all $\theta, (\lambda - \Omega)$ and $\gamma > 0$.

- I will determine the sign of $(\pi_S^{NC} - \pi_{NS}^{NC})$:

*$\pi_S^{NC} > \pi_S^C$ for all $\theta > \frac{\gamma(\lambda-\Omega)}{2}$, $\quad (\boldsymbol{i})$

*$\pi_S^C > \pi_{NS}^C$ for all $\theta, (\lambda - \Omega)$ and $\gamma > 0$, $\quad (\boldsymbol{ii})$

*$\pi_{NS}^M > \pi_{NS}^C > \pi_{NS}^{NC}$ for all $\theta \neq \frac{\gamma(\lambda-\Omega)}{2}$. $\quad (\boldsymbol{iii})$

From $(\boldsymbol{i}), (\boldsymbol{ii})$ and $(\boldsymbol{iii})$, I conclude that for $\theta > \frac{\gamma(\lambda-\Omega)}{2}, \pi_S^{NC} > \pi_S^C > \pi_{NS}^C > \pi_{NS}^{NC}$.

Therefore, $(\pi_S^{NC} - \pi_{NS}^{NC}) > 0$ for $\theta > \frac{\gamma(\lambda-\Omega)}{2}$.

This result is true even with numerical simulations.

$$(SW_S^{NC} - SW_{NS}^{NC}) = (3+\gamma)(q_S^{NC} - q_{NS}^{NC})(q_S^{NC} + q_{NS}^{NC}) - [(d_S^{NC})^2 - (d_{NS}^{NC})^2],$$

$$(q_S^{NC} - q_{NS}^{NC}) = \frac{(\theta+\lambda-\Omega)}{(2+\gamma)}[d_S^{NC} - d_{NS}^{NC}],$$

$$(q_S^{NC} + q_{NS}^{NC}) = \frac{2\alpha}{(2+\gamma)} + \frac{(\theta+\lambda-\Omega)}{(2+\gamma)}[d_S^{NC} + d_{NS}^{NC}],$$

$$(SW_S^{NC} - SW_{NS}^{NC}) = \frac{(3+\gamma)(\theta+\lambda-\Omega)}{(2+\gamma)}[d_S^{NC} - d_{NS}^{NC}]\left[\frac{2\alpha}{(2+\gamma)} + \frac{(\theta+\lambda-\Omega)}{(2+\gamma)}[d_S^{NC} + d_{NS}^{NC}]\right]$$
$$-[d_S^{NC} - d_{NS}^{NC}][d_S^{NC} + d_{NS}^{NC}],$$

$$(SW_S^{NC} - SW_{NS}^{NC})$$
$$= [d_S^{NC} - d_{NS}^{NC}]\left[\frac{2\alpha(\theta+\lambda-\Omega)(3+\gamma)}{(2+\gamma)^2} + \left[\frac{(3+\gamma)(\theta+\lambda-\Omega)^2 - (2+\gamma)^2}{(2+\gamma)^2}\right][d_S^{NC} + d_{NS}^{NC}]\right]$$
$$= [d_S^{NC} - d_{NS}^{NC}]\begin{bmatrix} \dfrac{2\alpha(\theta+\lambda-\Omega)}{(2+\gamma)^2} \\ -\left[\dfrac{(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2}{2(2+\gamma)^2}\right][d_S^{NC} + d_{NS}^{NC}] \end{bmatrix}.$$

By substituting d_S^{NC} and d_{NS}^{NC} by their expressions, I found that:

$$(SW_S^{NC} - SW_{NS}^{NC})$$
$$= \frac{[d_S^{NC} - d_{NS}^{NC}]}{(2+\gamma)^2\begin{bmatrix}(2-\gamma)(2+\gamma)^2 \\ -2(2(\lambda-\Omega)-\theta\gamma)(\theta+\lambda-\Omega)\end{bmatrix}}\begin{bmatrix}2\alpha(3+\gamma)(\theta+\lambda-\Omega)^2(2(\lambda-\Omega)-\theta\gamma) \\ +\alpha(\theta+\lambda-\Omega)(3+\gamma)(2-\gamma)(2+\gamma)^2 \\ -2\alpha(2+\gamma)^2(2(\lambda-\Omega)-\theta\gamma)\end{bmatrix}$$
$$= \frac{[d_S^{NC} - d_{NS}^{NC}]}{(2+\gamma)^2\begin{bmatrix}(2-\gamma)(2+\gamma)^2 \\ -2(2(\lambda-\Omega)-\theta\gamma)(\theta+\lambda-\Omega)\end{bmatrix}}\begin{bmatrix}2\alpha(3+\gamma)(\theta+\lambda-\Omega)^2(2(\lambda-\Omega)-\theta\gamma) \\ +\alpha(2+\gamma)^2\theta(6-\gamma-\gamma^2) \\ +\alpha(2+\gamma)^2(\lambda-\Omega)(1-\gamma)(2+\gamma)\end{bmatrix} > 0$$

since $\theta > 0, (\lambda-\Omega) > 0, \gamma \in (0,1], d_S^{NC} > d_{NS}^{NC}$ and

$(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma] > 0$ (given the concavity condition of the profit) $\Leftrightarrow$

$(4-\gamma^2)^2 - 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega] > 0$ $\quad(\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\left[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0$

$\Leftrightarrow \left[(2-\gamma)(2+\gamma)^2 - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0.$

$$(SW_S^C - SW_{NS}^C) = \frac{\alpha^2(3+\gamma)}{(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2} - \frac{\alpha^2[(2+\gamma)^2(3+\gamma)-4(\theta+\lambda-\Omega)^2]}{[(2+\gamma)^2-2(\theta+\lambda-\Omega)^2]^2}$$

$$= \frac{\alpha^2\begin{bmatrix}(3+\gamma)[(2+\gamma)^2-2(\theta+\lambda-\Omega)^2]^2 \\ -\begin{bmatrix}(2+\gamma)^2(3+\gamma) \\ -4(\theta+\lambda-\Omega)^2\end{bmatrix}[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2]\end{bmatrix}}{[(2+\gamma)^2-(3+\gamma)(\theta+\lambda-\Omega)^2][[(2+\gamma)^2-2(\theta+\lambda-\Omega)^2]^2]}.$$

Numerator of $(SW_S^C - SW_{NS}^C) = \alpha^2(2+\gamma)^2(\theta+\lambda-\Omega)^2[1+\gamma+\gamma^2] > 0$

for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$(4-\gamma^2)^2(1-s^{NC}) - 2(2-\gamma)[2(\lambda-\Omega)-\theta\gamma][\theta+\lambda-\Omega] > 0 \quad (\boldsymbol{II})$

$\Leftrightarrow (2-\gamma)\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right] > 0,$

$\left[(2-\gamma)(2+\gamma)^2(1-s^{NC}) - 2(\theta+\lambda-\Omega)[2(\lambda-\Omega)-\theta\gamma]\right]$

$= (2+\gamma)^2 - (3+\gamma)(\theta+\lambda-\Omega)^2 > 0$ and for $s = 0$,

$(2+\gamma)^2 - 2(\theta+\lambda-\Omega)^2 > 0. \quad (\boldsymbol{IV})$

Therefore $SW_S^C > SW_{NS}^C$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

$$(SW_S^M - SW_{NS}^M) = \frac{3\alpha^2}{4(1+\gamma)-3(\theta+\lambda-\Omega)^2} - \frac{\alpha^2[3(1+\gamma)-(\theta+\lambda-\Omega)^2]}{[2(1+\gamma)-(\theta+\lambda-\Omega)^2]^2}$$

$$= \frac{\alpha^2\begin{bmatrix}3[2(1+\gamma)-(\theta+\lambda-\Omega)^2]^2 \\ -[3(1+\gamma)-(\theta+\lambda-\Omega)^2][4(1+\gamma)-3(\theta+\lambda-\Omega)^2]\end{bmatrix}}{[4(1+\gamma)-3(\theta+\lambda-\Omega)^2][[2(1+\gamma)-(\theta+\lambda-\Omega)^2]^2]}.$$

Numerator of $(SW_S^M - SW_{NS}^M) = \alpha^2(1+\gamma)(\theta+\lambda-\Omega)^2 > 0.$

Based on the concavity of the profit with respect to d_i when $d_i = d_j$, I already found the following conditions:

$2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0 \quad (\boldsymbol{VI})$

$\Leftrightarrow 3[2(1-s^M)(1+\gamma) - (\theta+\lambda-\Omega)^2] = 4(1+\gamma) - 3(\theta+\lambda-\Omega)^2 > 0$ and for $s = 0$,

$2(1+\gamma) - (\theta+\lambda-\Omega)^2 > 0. \quad (\boldsymbol{VI})$

Therefore $SW_S^M > SW_{NS}^M$ for all $\theta, (\lambda-\Omega)$ and $\gamma > 0$.

4 R&D Investment under Symmetric and Asymmetric Absorptive Capacity

4.1 Introduction

Absorptive capacities play a crucial role in technological advancement as they facilitate the dissemination and the adoption of new ideas and technologies. They can accelerate the pace of innovation by allowing entities to build upon existing knowledge and leverage the advancements made by others. They can also promote competition and collaboration. Entities with strong absorptive capacities, such as Huawei and Apple, are well-positioned to identify and understand the knowledge that is being spilled over in the smartphone industry. By investing in R&D, these companies actively seek to generate new technologies. Additionally, participating in conferences and engaging in online exchanges of ideas allow their scientists and engineers to interact with peers, share knowledge, and potentially benefit from spillovers. In today's competitive landscape, firms could not successfully achieve their innovation process while being isolated from the external environment given that the basis of competition has shifted more and more to technological knowledge assimilation and absorption of external information. Interestingly, firms can internalize such externalities by cooperating on R&D efforts and can take advantage of higher profits compared to non-cooperation. According to Mukherjee and Ramani (2011), the R&D cooperation has many advantages such as sharing cost and knowledge, enhancing innovation input and output and internalizing technological externalities. However, despite these advantages, some members of the cooperative venture may not have fulfilled their obligations and may choose to defect and consequently, the cooperation agreement may end up in a failure. The opportunistic behavior represents the main reason for this failure (Walter, Kellermanns, and Lechner, 2012). This kind of behavior occurs when firm does not respect the cooperative engagement and chooses to maximize its own profit by assuming that its partner is respecting the agreement. In such situation, the opportunistic firm benefits unilaterally from the R&D effort of its partner.

A real-life example can be seen in the field of technology development. For instance, in 2010, Toyota and Tesla, two leading players in the automotive industry, announced a joint research project to develop a new electric vehicle platform.[12] The project aimed to combine Toyota's expertise in manufacturing and electric powertrains with Tesla's know-how in electric motor design and battery technology. The partnership was expected to reduce costs, enhance innovation, and accelerate the development of electric vehicles. Initially, the cooperation seemed promising. Both companies shared knowledge and resources, and the project was expected to generate significant cost savings and technological advancements. However, as the project progressed, tensions arose between the two partners. Tesla accused Toyota of not contributing sufficiently to the project, while Toyota felt that Tesla was not living up to its obligations. Despite the initial agreement, both companies began to prioritize their own interests over the joint project. Tesla, for instance, was focused on developing its own electric vehicle platform and continued to develop its own hybrid technology. In 2014, Toyota announced that it would no longer be pursuing the joint project with Tesla.[13] The project ultimately failed to deliver on its promises, and both companies went their separate ways. In this case, despite the initial benefits of cooperation, the partnership ultimately failed due to lack of commitment and sharing of knowledge and resources by both parties. This example illustrates how even partnerships with significant advantages can fail if members do not fulfill their obligations and prioritize their own interests over the joint venture.

In this paper, I consider the opportunistic behavior and I focus on a two-stage strategic R&D model in a duopoly market. In the first stage, both firms simultaneously decide whether to cooperate or not by choosing the level of R&D investment. The purpose of this investment is to reduce the production cost in the second stage. In the model, I include two possible R&D outcomes such that the original cost can be maintained or can be reduced with a certain probability. In addition to that, I incorporate the basic technological spillovers as well as the R&D absorptive capacity that may result from the uncertain R&D outcome. These technological externalities have the potential to benefit another firm by reducing its cost. In the second stage, after knowing the R&D outcome, both firms simultaneously choose the quantity to produce as in a traditional Cournot model. In this study, I consider an absorptive capacity parametrized between 0 and 1 and I examine three scenarios such that the unilateral defection, the non-cooperation (or mutual defection) and the mutual cooperation. For the last scenario and according to Amir, Evstigneev, and Wooders (2003),

---

[12] https://www.tesla.com/en_ca/blog/tesla-motors-and-toyota-motor-corporation-intend-work-jointly-ev-development-tm.

[13] https://www.thetorquereport.com/toyota-ends-partnership-with-tesla/

firms might coordinate their R&D spending in order to maximize their joint profits in a Cournot market while conducting R&D separately in their own labs (with a spillover parametrized between 0 and 1).[14]

By considering both deterministic and stochastic R&D technologies, I am broadly concerned about how the optimal R&D investments under the different scenarios are affected by the absorptive capacity as well as the probability of success and whether the firms end up in a prisoner's dilemma situation.

In the symmetric absorptive capacity case, no matter it is low or high, I show that, as the probability of success increases, the absolute value of the gap between the optimal R&D expenditures under defection and those under cooperation (while assuming the other firm is always cooperating) increases as well. In the asymmetric absorptive capacity case, the R&D investment of the defecting firm is larger (smaller) compared to the cooperative scenario when the absorptive capacity of the other firm is low (high).

Regardless of whether the absorptive capacity is low or high, symmetric or asymmetric, the results indicate that all the cases are in line with the prisoner's dilemma situation, where it is more efficient when both firms cooperate but there's a temptation for each to defect for individual gains. However, there is a nuanced perspective into the traditional prisoner's dilemma, where defection always leads to free-riding. Instead, I find that the decision to defect can result in a strategic choice that considers the potential benefits of leveraging the efforts of the other firm or investing more in R&D to gain a competitive advantage. Interestingly, the level of the absorptive capacity determines whether it results in a lower or higher investment when the firm defects. For low absorptive capacity, the defecting firm chooses to provide more investment, which leads to a higher payoff than when respecting the cooperation agreement. However, for high absorptive capacity, the firm uses the opportunity of defection to invest less in R&D by relying on the R&D efforts of the cooperating firm, essentially by free-riding on their innovation.

The remainder of the paper is organized as follows: Section 4.2 discusses the related literature. Section 4.3 presents the theoretical model, Section 4.4 and Section 4.5 derive the R&D expenditures and the results under the symmetric and the asymmetric absorptive capacity, respectively. Section 4.6 concludes. In the appendix, I present the theoretical proofs.

[14] Amir, Evstigneev, and Wooders (2003) also consider the case that firms may decide (in a joint lab) to mutually set the spillover effect to its maximum value of 1, enabling complete sharing of their R&D activities.

4.2 Related literature

The related literature on R&D investment can be characterized by two streams. The first one examines the idea that in order to take advantage of the competitor's R&D efforts, a company must possess absorptive capacity. In the R&D literature, this concept was firstly introduced by Cohen and Levinthal (1990). They demonstrated that investing in R&D enhances a firm's capability to recognize, assimilate, and leverage knowledge from the external environment. Following Grünfeld (2003), the absorptive capacity effect of R&D pertains to the notion that increased levels of R&D investment improve the ability to acquire knowledge from rival firms. Zahra and George (2002) identify four distinct dimensions to this absorption capacity which are the acquisition, the assimilation, the transformation and the exploitation. This absorptive capacity may be of two kinds whether symmetric or asymmetric. The majority of theoretical studies on R&D investment consider the symmetric case such that the technological externalities are identical between firms but they do not consider the opportunistic behavior and the stochastic R&D technology (D'Aspremont and Jacquemin, 1988; Henriques, 1990; Qiu, 1997; Symeonidis, 2003; El Ouardighi, Shnaiderman and Pasin, 2014). However, it is also worthwhile to consider the asymmetric case that is due to different aspects such that the product differentiation, the R&D efficiency and the cost levels.

The second stream of literature emerges from the prevalent theoretical framework used to explore the strategic R&D decisions in Cournot competition, specifically the two-stage model with simultaneous moves. As a seminal paper, D'Aspremont and Jacquemin (1988) examine the model where firms engage simultaneously in R&D investment during the first stage and compete based on quantities in the second stage. They investigate the effect of cooperation on the R&D effort and indicate that the R&D decisions of the firms are affected by the free technological spillovers from rivals. Specifically, they show that if the technological externalities are high, the R&D expenditures increase when there is a total or partial cooperation between firms. Their work has been widely extended by various studies. For instance, Ben Youssef et al. (2013) built upon the model pioneered by D'Aspremont and Jacquemin and consider the case where firms can engage cooperatively or non-cooperatively in both absorptive and innovative investment. They find that the investment in absorptive R&D is always lower than the investment in innovative R&D regardless of whether there is a cooperation or a non-cooperation. Compared to these studies, I consider the opportunistic behavior, the stochastic R&D technology and the absorptive capacity that is exogeneous instead of being endogenous.

Cabon-Dhersin and Ramani (2005) extend the framework of D'Aspremont and Jacquemin and Kamien et al (1992) by introducing incomplete information and heterogeneous firms of two types: opportunist and non-opportunist. They show that the trust requirements for initiating R&D cooperation depend on the degree of symmetric spillovers and that the decision to engage in cooperative agreement depends on the trust of firms in one another. In this paper, I do not include the belief of trust configurations under which the R&D cooperation is initiated, but I do include the opportunistic behavior under the stochastic environment with symmetric absorptive capacity as well as the opportunistic behavior under the deterministic environment with asymmetric absorptive capacity, which Cabon-Dhersin and Ramani (2005) didn't investigate.

Without considering the possible occurrence of the opportunistic behavior, Ishikawa and Shibata (2021) extend the work of D'Aspremont and Jacquemin (1988) and Shibata (2014) and examine the asymmetric spillovers in an oligopoly market with the incorporation of the degree of market competitiveness. According to their findings, an increase in market competitiveness is expected to result in a greater social preference for R&D competition rather than R&D cooperation. In addition to that, they show that as the spillover asymmetry between firms increases, the difference in aggregate R&D investment between asymmetric and symmetric spillovers increases as well. However, in this paper, I incorporate a stochastic R&D technology but I focus on a duopoly market without including the intensity of competition into the model.

By considering and extending the study of D'Aspremont and Jacquemin (1988), Slivko and Theilen (2014) investigate a Cournot duopoly model in which each firm has a choice between two R&D strategies, innovation and imitation. They take into consideration a two-stage game in which firms first choose simultaneously their R&D strategies, and then compete in quantities. In addition to that, they include the knowledge spillover in their model that they define as the level of intellectual property rights (IPR) protection. According to their findings, excessive competitive pressure negatively affects the incentives of firms to engage in R&D, while the efficiency of IPR protection has a positive effect. Nevertheless, smaller firms have less chance to be innovators in markets where the products are homogenous and the knowledge spillovers are high. By endogenizing the absorptive capacity (as a function of the R&D investment) and by including it into the d'Aspremont and Jacquemin (1988) model, Grünfeld (2003) shows that the spillovers drive up the incentive for R&D investment when the market size is small.

The absorptive capacity has been termed as a spillover parameter by Amir, Evstigneev, and Wooders (2003). These authors present a general version of a two-stage model of R&D in which firms set cooperatively their R&D investments as well as the strength of the spillover. In their model, the cooperating firms strategically select the optimal level of the spillover parameter by considering their cooperative decision-making process. For many sectors, R&D cooperation may have a negative effect on the appropriability strategies of firms. In fact, firms may not have the ability to monitor their partner's R&D effort which leads to a moral hazard problem (Silipo, 2008). The major consequence of moral risk can be considered to be problems of free riding. More precisely, a firm is not encouraged to keep its promises given that its behaviour is not observable by its partner. Indeed, since each partner cannot verify ex ante the real commitment of the other, a classic problem of asymmetric information emerges, opening the way to opportunistic behaviors. While the existing literature has investigated the spillover effect, none of the studies has included the opportunistic behavior in a Cournot duopoly market under the stochastic and deterministic R&D technology with symmetric and asymmetric absorptive capacity.

4.3 Model

I consider a Cournot duopoly game with two stages, similar to the one studied in D'Aspremont et Jacquemin (1988), but with a number of modifications. In line with their model, I consider two firms i and j and an inverse demand function $P(Q) = \delta - bQ = \delta - bq_i - bq_j$ where $\delta > 0$ and b is a positive constant representing the slope of the inverse demand function. It is assumed that $0 < b \leq 1$ and $0 < Q \leq \delta/b$. In the first stage of the model, both firms simultaneously make decisions regarding their R&D investments, denoted as x_i and x_j. They have the option to invest cooperatively or non-cooperatively, with the objective of reducing their production costs. In the second stage, the firms independently choose theHir production levels, q_i and q_j, respectively, with the aim of maximizing their own profits.

Following the work of Ben Youssef et al. (2013), I assume that there are free technological spillover and R&D absorptive capacity between the two firms such that the R&D activities can generate technological benefits for the other firm, resulting in cost reduction. The effective spillover of firm

$k, (k = i, j)$ is denoted as $\gamma(a_k) = \beta + la_k$ $(\beta < \gamma(a_k) < 1, l > 0)$. It is determined by the basic technological spillovers "β", the R&D absorptive capacity "a_k" and the learning or the absorptive parameter "l". The latter represents the extent of learning from the rival's R&D efforts.

At the beginning of the game, each firm has an initial marginal cost of $c > 0$ such that $\delta > c$. Through their R&D investments, the marginal costs of firm i and firm j are adjusted to c_i and c_j, respectively. In contrast to the work of D'Aspremont and Jacquemin (1988) and Ben Youssef et al. (2013), my assumption incorporates a stochastic R&D technology. Specifically, the cost reduction only takes effect with a certain probability of success, denoted as ρ $(0 < \rho < 1)$. Consequently, the linear cost function is contingent upon the outcome of the R&D project, whether it is successful or unsuccessful. In other words, there are two possible outcomes for each firm: whether a success or a failure. If the R&D projects of both firms fail, which occurs with a probability of $(1-\rho)^2$, both firms will maintain their original cost c. On the other hand, if the R&D project of firm i succeeds, firm i's cost is reduced by x_i and firm j's cost reduction by $\gamma(a_j)\, x_i$ due to the spillover effect. Similarly, if firm j's R&D project succeeds, firm j's cost is reduced by x_j, which also leads to firm i's cost reduced by $\gamma(a_i)\, x_j$. Following the literature, I also assume the effect of cost reduction by both firms are linearly additive, so if both firms' R&D projects are successful, the final marginal costs of firm i and firm j are adjusted to $c_i = c - x_i - \gamma(a_i)\, x_j$ and $c_j = c - x_j - \gamma(a_j)\, x_i$, respectively with the conditions $c_i, c_j \geq 0$ as it is assumed in Ben Youssef et al. (2013).

In this study, I assume that the outcome of the R&D technology of each firm is independent of one another, which means that the outcome of one firm's R&D project is not affected by the success or failure of the other firm. Under the stochastic innovation, the following table indicates the different possible outcomes regarding the marginal cost of the firms after undertaking the research project.

Table 4.1: The different possible marginal costs of each firm based on the project's outcome

Firm i \ Firm j		Two possible outcomes: Success	Two possible outcomes: Failure
Two possible outcomes	Success	$c_i = c - (x_i + \gamma(a_i)x_j)$ $c_j = c - (x_j + \gamma(a_j)x_i)$	$c_i = c - x_i$ $c_j = c - \gamma(a_j)x_i$
	Failure	$c_i = c - \gamma(a_i)x_j$ $c_j = c - x_j$	$c_i = c$ $c_j = c$

The firm i's expected profit is expressed as follows:

$$E\pi_i = \rho[\pi_i^S] + (1-\rho)[\pi_i^F] - IC_i,$$

such that

$$\pi_i^S = \rho[\pi_i^{SS}] + (1-\rho)[\pi_i^{SF}],$$
$$\pi_i^F = \rho[\pi_i^{FS}] + (1-\rho)[\pi_i^{FF}],$$

where π_i^{SS} is the profit of firm i when both firms succeed in their R&D project, π_i^{SF} is the profit of firm i when firm i succeeds while firm j fails, π_i^{FS} is the profit of firm i when firm i fails while firm j succeeds, and π_i^{FF} is the profit of firm i when both firms fail. Consequently, firm i's profit π_i^S is the weighted average of $\pi_i^{SS}$and π_i^{SF}, when firm i succeeds. Similar definition applies to π_i^F. Therefore, the R&D outcome for each firm can be either a success (denoted by S) or a failure (denoted by F).[15] For the R&D cost, it is assumed to be of quadratic form (as in D'Aspremont and Jacquemin, 1988; Ben Youssef et al., 2013). It is denoted by IC_i, which is equal to $\frac{1}{2}\lambda x_i^2$ where λ is a positive parameter which is inversely related to the efficiency of the innovation process. Therefore, the expected profit can be defined as follows:

$$E\pi_i = \rho\left[\rho[\pi_i^{SS}] + (1-\rho)[\pi_i^{SF}]\right] + (1-\rho)\left[\rho[\pi_i^{FS}] + (1-\rho)[\pi_i^{FF}]\right] - \frac{1}{2}\lambda x_i^2.$$

Lemma 1: At the equilibrium of the second stage, the profit of firm i can be expressed as $\pi_i^*(q_i^*) = (P(Q^*) - c_i)q_i^* = b(q_i^*)^2$ such that $q_i^* = \frac{\delta + c_j - 2c_i}{3b}$ and $P(Q^*) = \frac{\delta + c_j + c_i}{3}$.
The proofs are provided in the Appendix.

The equilibrium quantity q_i^* and the equilibrium price $P(c_i, c_j)$ are determined by applying backward induction. Based on Table 4.1, below are the different optimal quantities relative to the outcome of R&D investment, where $\delta > c$ ensures $q_i^{FF} > 0$.

$$q_i^{SS} = \frac{\delta + c - x_j - \gamma(a_j)x_i - 2c + 2x_i + 2\gamma(a_i)x_j}{3b} = \frac{\delta - c + [2\gamma(a_i) - 1]x_j + [2 - \gamma(a_j)]x_i}{3b},$$

$$q_i^{FS} = \frac{\delta + c - x_j - 2c + 2\gamma(a_i)x_j}{3b} = \frac{\delta - c + [2\gamma(a_i) - 1]x_j}{3b},$$

[15] These notations are going to be used for the optimal quantities as well.

$$q_i^{SF} = \frac{\delta + c - \gamma(a_j)x_i - 2c + 2x_i}{3b} = \frac{\delta - c + [2 - \gamma(a_j)]x_i}{3b},$$

$$q_i^{FF} = \frac{\delta - c}{3b}.$$

Since $\pi_i^*(q_i^*) = b(q_i^*)^2$ as it is stated in Lemma 1, the firm $i's$ expected profit can be written as follows:

$$E\pi_i^* = \rho\rho[\pi_i^{SS}] + \rho[1-\rho][\pi_i^{SF}] + (1-\rho)\rho[\pi_i^{FS}] + (1-\rho)(1-\rho)[\pi_i^{FF}] - IC_i$$
$$= \rho\rho[b(q_i^{SS})^2] + \rho[1-\rho][b(q_i^{SF})^2] + (1-\rho)\rho[b(q_i^{FS})^2] + (1-\rho)(1-\rho)[b(q_i^{FF})^2]$$
$$-IC_i.$$

The firms' profits at the second stage are conditional on the R&D investments (x_i, x_j) that were undertaken at the first stage. Before solving for the optimal R&D expenditures and the profit by backward induction, I consider the model where each firm can cooperate or defect.

Defection occurs when a cheating firm chooses its R&D investments to maximize its own profits rather than maximizing the joint profits while cooperation occurs when the two firms decide to maximize their joint profits. Theoretically speaking, it is shown in the literature that there is always a form of cooperation for non-oppoxrtunistic firms that makes it possible to obtain a greater profit than the profit of non-cooperation. However, a few studies integrate the possibility for firms to act as opportunists within the R&D cooperative agreement. In fact, most of the articles focus mainly on whether the R&D cooperation is socially beneficial. For that reason, I consider the situation that each firm can either cooperate or defect. When one firm interacts with another firm, there are three possible outcomes: 1) both defect, denoted by (D,D), where each firm maximizes its own non-cooperative R&D profit, 2) both cooperate, denoted by (C,C), where the absorptive capacities are internalised within a cooperative structure, representing a joint benefit for the two firms, 3) one firm cooperates and the other defects, (C,D) or (D,C). Under each possibility, the optimal investment is denoted by x_i with two superscripts: the first superscript is firm i's action, and the second subscript is firm j's action.

In the following two sections, the optimal R&D investments are found by backward induction. In Section 4.4, I consider the symmetric absorptive capacity (i.e., $a_i = a_j = a$, or equivalently, $\gamma(a_i) = \gamma(a_j) = \gamma(a)$) under the stochastic R&D technology for all $0 < \rho < 1$ where there is an uncertainty with respect to the success of R&D projects and an expected cost reduction. In Section 4.5, I consider the asymmetric absorptive capacity (such that $(\gamma(a_i) \neq \gamma(a_j))$ under the deterministic R&D technology (i.e. $\rho = 1$), where any engaging firm in R&D innovates with certainty and has a perfect information about the extent of the cost reduction.

4.4 The symmetric absorptive capacity

4.4.1 Optimal R&D expenditures

Under a stochastic R&D technology, I assume that $\gamma(a_i) = \gamma(a_j) = \gamma(a)$ and determine the optimal R&D investments of both firms by backward induction. I provide more detailed procedure for the optimal R&D investments in the following three cases in Lemma A.1 in the appendix.

<u>Case 1</u> (Mutual defection): (Firm *i*'s decision, Firm *j*'s decision) = (D, D)

$$\frac{\partial E\pi_i^*}{\partial x_i} = 0 \Longrightarrow x_i^{DD} = \frac{\rho(\delta - c)\big(2 - \gamma(a)\big) + \rho^2(2\gamma(a) - 1)\big(2 - \gamma(a)\big)x_j^{DD}}{4.5b\lambda - \rho^2\,(2 - \gamma(a))^2 - \rho(1 - \rho)\,(2 - \gamma(a))^2},$$

$$\frac{\partial E\pi_j^*}{\partial x_j} = 0 \Longrightarrow x_j^{DD} = \frac{\rho(\delta - c)\big(2 - \gamma(a)\big) + \rho^2(2\gamma(a) - 1)\big(2 - \gamma(a)\big)x_i^{DD}}{4.5b\lambda - \rho^2\,(2 - \gamma(a))^2 - \rho(1 - \rho)\,(2 - \gamma(a))^2}.$$

From the above two expressions, I determine the optimal R&D expenditure of firm i under mutual defection as follows:

$$x_i^{DD} = x_j^{DD} = \frac{\rho(\delta - c)(2 - \gamma(a))}{4.5b\lambda - \rho^2(\gamma(a) + 1)\big(2 - \gamma(a)\big) - \rho(1 - \rho)\,(2 - \gamma(a))^2}.$$

$4.5b\lambda > \rho^2(\gamma(a) + 1)\big(2 - \gamma(a)\big) + \rho(1 - \rho)\,(2 - \gamma(a))^2$to ensure the concavity of $E\pi_i^*$ with respect to x_i^{DD}.

<u>Case 2</u> (Mutual cooperation): (Firm *i*'s decision, Firm *j*'s decision) = (C, C)

$$\frac{\partial E\big(\pi_i^* + \pi_j^*\big)}{\partial x_i} = \frac{\partial\big(E\pi_i^* + E\pi_j^*\big)}{\partial x_i} = \frac{\partial E\big(\pi_i^* + \pi_j^*\big)}{\partial x_j} = \frac{\partial\big(E\pi_i^* + E\pi_j^*\big)}{\partial x_j} = 0,$$

implies that

$$x_i^{CC} = \frac{\rho(\delta - c)(\gamma(a) + 1) + \rho^2[(2\gamma(a) - 1)\big(2 - \gamma(a)\big) + (2\gamma(a) - 1)\big(2 - \gamma(a)\big)]x_j^{CC}}{4.5b\lambda - \rho\big(2 - \gamma(a)\big)^2 - \rho(2\gamma(a) - 1)^2}.$$

$$x_j^{CC} = \frac{\rho(\delta - c)(\gamma(a) + 1) + \rho^2[(2\gamma(a) - 1)\big(2 - \gamma(a)\big) + (2\gamma(a) - 1)\big(2 - \gamma(a)\big)]x_i^{CC}}{4.5b\lambda - \rho\big(2 - \gamma(a)\big)^2 - \rho(2\gamma(a) - 1)^2}.$$

Therefore, the optimal cooperative R&D expenditure of firm i is expressed as follows:

$$x_i^{CC} = x_j^{CC} = \frac{\rho(\delta - c)(\gamma(a) + 1)}{4.5b\lambda - \rho\big(2 - \gamma(a)\big)^2 - \rho(2\gamma(a) - 1)^2 - 2\rho^2(2\gamma(a) - 1)\big(2 - \gamma(a)\big)}.$$

$4.5b\lambda > \boldsymbol{P} = \rho\big(2 - \gamma(a)\big)^2 + \rho(2\gamma(a) - 1)^2 + 2\rho^2(2\gamma(a) - 1)\big(2 - \gamma(a)\big)$ to ensure the concavity of $E(\pi_i^* + \pi_j^*)$ with respect to x_i^{CC}.

<u>Case 3</u> (Unilateral defection): (Firm *i*'s decision, Firm *j*'s decision) = (D, C)

In the R&D stage, one firm decides to cooperate and expects that the other firm cooperates while the latter does not. The firm that deviates assumes that the other firm will not change its behavior. I assume that firm i chooses to maximize its own profit while taking advantage of the cooperative behavior of the other firm ($x_j^{CD} = x_j^{CC}$).

$$\frac{\partial E\pi_i^*\big(x_i^{DC}, x_j^{CD}\big)}{\partial x_i^{DC}} = 0$$

$$\Rightarrow x_i^{DC} = \frac{\big(2 - \gamma(a)\big)\rho\big[(\delta - c) + x_j^{CD}\rho(2\gamma(a) - 1)\big]}{4.5b\lambda - \rho\big(2 - \gamma(a)\big)^2}$$

$$\Rightarrow x_i^{DC} = \frac{\rho(\delta - c)\big(2 - \gamma(a)\big)[\rho^2(2\gamma(a) - 1)(\gamma(a) + 1) + A]}{AB}$$

such that

$$A = 4.5b\lambda - \rho^2(\gamma(a) + 1)^2 - \rho\Big(1 - \rho)\ ((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2\Big) = 4.5b\lambda - \boldsymbol{P} > 0,$$

$$B = 4.5b\lambda - \rho\big(2 - \gamma(a)\big)^2.$$

$4.5b\lambda > \boldsymbol{Q} = \rho\,\big(2 - \gamma(a)\big)^2$ to ensure the concavity of $E\pi_i^*\big(x_i^{DC}, x_j^{CD}\big)$ with respect to x_i^{DC}.

$\rho^2(2\gamma(a) - 1)(\gamma(a) + 1) + A > 0 \Leftrightarrow 4.5b\lambda > \boldsymbol{R_2}$ to ensure that $x_i^{DC} > 0$ such that

$\boldsymbol{R_2} = \rho^2(\gamma(a) + 1)\big(2 - \gamma(a)\big) + \rho(1 - \rho)\ ((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2).$

4.4.2 Results under the symmetric case

In the following propositions, I present the properties of symmetric firms' R&D investment functions and expected profit functions.

Proposition 1: Under the following conditions on the absorptive capacity and the probability of success, the order of the R&D investments under mutual defection, mutual cooperation and unilateral defection is as follows:[16]

$$\left\{\gamma(a) > \frac{1}{2} \text{ and } \rho > 0\right\} \Rightarrow \left\{x_i^{CC}(a) > x_i^{DC}(a) > x_i^{DD}(a)\right\},$$

$$\left\{\gamma(a) < \frac{1}{2} \text{and } \rho \geq \frac{\left(1 - 2\gamma(a)\right)}{3\left(1 - \gamma(a)\right)}\right\} \Rightarrow \left\{x_i^{DC}(a) > x_i^{DD}(a) > x_i^{CC}(a)\right\}.$$

The proofs are provided in the Appendix.

The conditions in Proposition 1 are determined in a way to ensure that all the concavity conditions under each scenario are satisfied and that the R&D investment under the unilateral defection is positive. Interestingly, Proposition 1 indicates that, for any absorptive capacity different from the critical threshold ($\gamma(a) \neq \frac{1}{2}$) and for $\rho \geq \frac{(1-2\gamma(a))}{3(1-\gamma(a))}$, the R&D investment of the defecting firm, conditional on other firm is cooperating, is larger than that when both firms choose not to cooperate. However, compared to x_i^{CC}, the R&D investment of the defecting firm depends on whether the probability of success ρ and the absorptive capacity are low or high. Conditional on the range of ρ such that $\rho \geq \frac{(1-2\gamma(a))}{3(1-\gamma(a))}$, the order of the R&D investments shows that the cooperative R&D is the highest (the lowest) for high (low) absorptive capacity. Furthermore, a lower absorptive capacity, combined with a higher probability of success, leads to a higher return for the R&D investment and consequently a higher non-cooperative effort compared to the cooperative effort.

Furthermore, I show in the mathematical proofs of Proposition 1 that when the probability of success converges to zero, x_i^{CC}, x_i^{DD} and x_i^{DC} becomes equal. However, when ρ converges to 1, $x_i^{CC}(a) > x_i^{DC}(a) > x_i^{DD}(a)$ for $\gamma(a) > \frac{1}{2}$ and $x_i^{DC}(a) > x_i^{DD}(a) > x_i^{CC}(a)$ for $\gamma(a) < \frac{1}{2}$. These results are consistent with Proposition 1.

Under the different probabilities of success, Figure 4.1 shows the distance between each pair of R&D investments (the left panel for x_i^{DC} and x_i^{CC}, the right panel for $x_i^{CC}$and x_i^{DD}, and the bottom panel for $x_i^{DC}$and x_i^{DD}) as a function of the absorptive capacity. For any absorptive capacity other

[16] The ranking of the optimal investments cannot be determined when $\gamma(a) < \frac{1}{2}$ and $\rho < \frac{(1-2\gamma(a))}{3(1-\gamma(a))}$.

than the critical threshold (reached at $a = 0.25$), the left panel in Figure 4.1 indicates that, the larger the probability of success, the greater the difference in optimal R&D spending between defecting and cooperating firms (x_i^{DC} and x_i^{CC}). The right and the bottom panels in Figure 4.1 also show this trend. They indicate that, given any absorptive capacity except the critical threshold, the difference in R&D spending between cooperating and non-cooperating firms ($x_i^{CC}$and x_i^{DD}) as well as between defecting and non-cooperating firms ($x_i^{DC}$and x_i^{DD}) becomes larger as the probability of success increases.

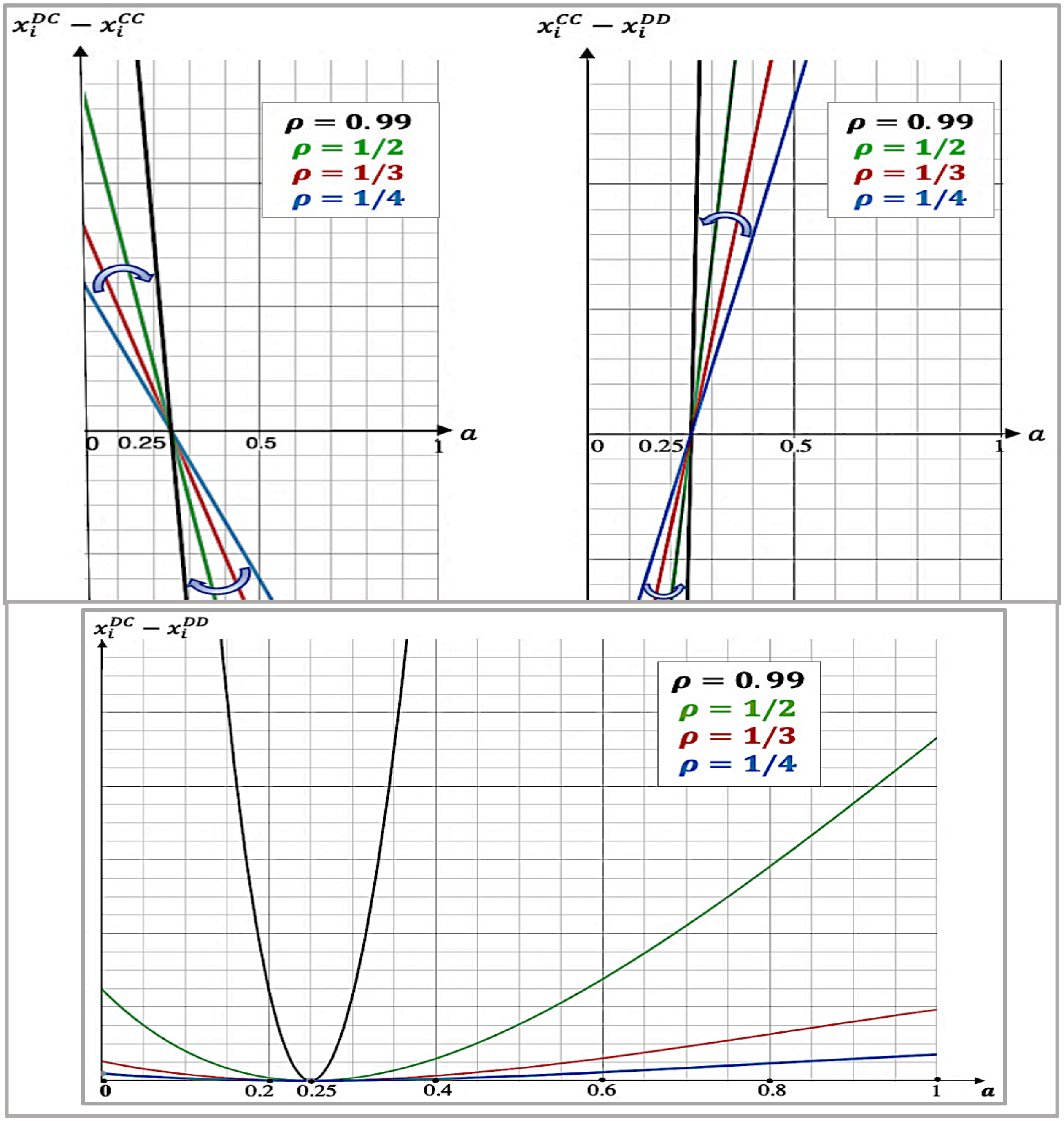
<u>**Proposition 2:**</u> For any value of absorptive capacity $a\backslash\{\frac{1-2\beta}{2l}\}$ (or equivalently $\gamma(a) \neq 1/2$) and for a probability of success $\rho > 0$,

- <u>**P 2.1.**</u> Defecting unilaterally always yields a higher expected payoff than respecting the cooperative agreement: $E\pi_i^*(x_i^{DC}, x_j^{CD}) > E\pi_i^*(x_i^{CC}, x_j^{CC})$,
- <u>**P 2.2.**</u> Cooperation yields a higher expected payoff than mutual defection: $E\pi_i^*(x_i^{CC}, x_j^{CC}) > E\pi_i^*(x_i^{DD}, x_j^{DD})$.

The proofs are provided in the Appendix.

Figure 4.2 shows the result given by Proposition 2.1. For each probability of success, the expected profit $E\pi_i^*(x_i^{DC}, x_j^{CD})$ is always greater than $E\pi_i^*(x_i^{CC}, x_j^{CC})$ for any a except the critical value. Furthermore, for any "a" different from the critical threshold, the larger the probability of success, the greater is the difference between $E\pi_i^*(x_i^{DC}, x_j^{CD})$ and $E\pi_i^*(x_i^{CC}, x_j^{CC})$, which implies that, for any absorptive capacity other than the critical threshold, the firm has more incentive to defect as the probability of success becomes higher.

Figure 4.1: Difference between the R&D investments as a function of the absorptive capacity under different probabilities of success



Notes: $l = 1, \beta = 0.25, \delta = 60, c = 20, \lambda = 0.5, b = 2,$

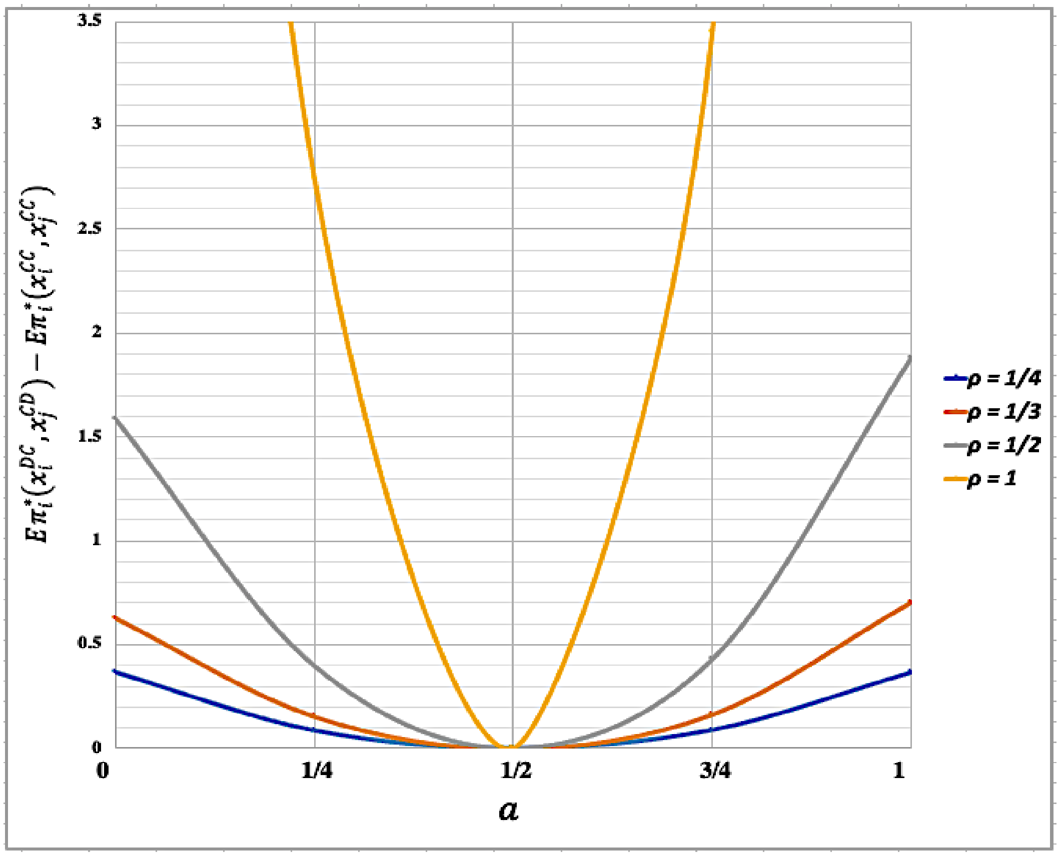
$$x_i^{DC} - x_i^{CC} = \frac{40\rho(0.5 - 2a)[4.5 - 3\rho(1-\rho)\,(1.75 - a)(0.75 - a)]}{\boldsymbol{AB}},$$

$$x_i^{CC} - x_i^{DD} = \frac{40\rho(2a - 0.5)[4.5 - 3\rho(1-\rho)(1.75 - a)(0.75 - a)]}{\boldsymbol{AH}},$$

$$x_i^{DC} - x_i^{DD} = 40(1.75 - a)\left[\frac{\rho^3(2a - 0.5)^2[4.5 - 3\rho(1-\rho)(1.75 - a)(0.75 - a)]}{\boldsymbol{ABH}}\right].$$

$\boldsymbol{A} = 4.5 - \rho^2(a + 1.25)^2 - \rho(1-\rho)(\,(2a - 0.5)^2 + \,(1.75 - a)^2), \boldsymbol{B} = 4.5 - \rho(1.75 - a)^2,$
$\boldsymbol{H} = 4.5 - \rho^2(a + 1.25)(1.75 - a) - \rho(1-\rho)\,(0.75 - a)^2.$

Figure 4.2: Expected profit comparison: Unilateral defection vs. mutual cooperation



Notes: $l = 1, \beta = 0, \delta = 60, c = 20, \lambda = 0.5, b = 4.$

Interestingly, as shown in Proposition 2.1, for a low a_i and for $\rho \geq \frac{(1-2\gamma(a))}{3(1-\gamma(a))}$, the defecting firm i is reducing its cost of production by providing more investment ($x_i^{DC}(a) > x_i^{CC}(a)$), which leads to earning higher payoff than when respecting the cooperative agreement, while for a high a_i and for $\rho > 0$, the defecting firm deviates to a smaller investment ($x_i^{DC}(a) < x_i^{CC}(a)$) and still achieves a higher expected profit $E\pi_i^*(x_i^{DC}, x_j^{CD}) > E\pi_i^*(x_i^{CC}, x_j^{CC})$, indicating that the defecting firm is managing to bring down its production cost by free riding on the R&D effort of the cooperative agreement respecting partner.

Proposition 2.2 indicates that the R&D cooperation without defecting always leads to a higher expected payoff than non-cooperation (mutual defection), which means that the cooperative agreement is of an economic interest to both firms even when the absorptive capacity is low.

Concerning the difference between $E\pi_i^*(x_i^{DD}, x_j^{DD})$ and $E\pi_i^*(x_i^{CD}, x_j^{DC})$, partial mathematical proofs are provided in the appendix on page 150, within the proof of Proposition 2. Due to the complexity of the mathematical proofs, I relied on the numerical simulations to demonstrate that $E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) > E\pi_i^*\left(x_i^{CD}, x_j^{DC}\right)$ for any $a\backslash\{\frac{1-2\beta}{2l}\}$ and $\rho > 0$.[17]

Using the mathematical proofs of Proposition 2 and the numerical simulations, I find that the results regarding the difference between each expected profit under each possible outcome (DD, CC, CD or DC) are in line with the prisoner's dilemma situation for any symmetric absorptive capacity different from the critical threshold and for any $\rho > 0$.

4.5 The asymmetric absorptive capacity:

In this section, I assume a large enough $b\lambda$ where b is the slope of the inverse demand function and λ is the efficiency measure of the R&D cost.

4.5.1 Optimal R&D expenditures

I investigate the optimal R&D investment by backward induction when the two firms have asymmetric absorptive capacity, $\gamma(a_i) \neq \gamma(a_j)$. Meanwhile, to obtain an analytical solution, I assume a deterministic R&D technology ($\rho = 1$) so that

$$\pi_i^*\left(x_i, x_j\right) = b(q_i^*)^2 - \frac{1}{2}\lambda x_i^2 = b\left[\frac{\delta - c + [2\gamma(a_i)-1]x_j + [2-\gamma(a_j)]x_i}{3b}\right]^2 - \frac{1}{2}\lambda x_i^2.$$

I provide more detailed procedure for the optimal R&D investments in the following three cases in Lemma A.2 in the appendix.

<u>Case 1</u> **(Mutual defection): (Firm *i*'s decision, Firm *j*'s decision) = (D, D)**

Firm i maximizes its own non-cooperative R&D profit under the case where it defects and the other firm defects as well.

$$\frac{\partial \pi_i^*\left(x_i^{DD}, x_j^{DD}\right)}{\partial x_i^{DD}} = 0$$

$$\Rightarrow\ x_i^{DD} = \frac{[\delta - c]\left[2-\gamma(a_j)\right] + [2\gamma(a_i) - 1]\left[2-\gamma(a_j)\right]x_j^{DD}}{\left[4.5b\lambda - \left[2-\gamma(a_j)\right]^2\right]},$$

which is the best response function.

[17] In the numerical simulations, I considered all the concavity conditions of the expected profit functions.

For all $\gamma(a_i) \neq \gamma(a_j)$, the non-cooperative equilibrium is (x_i^{DD}, x_j^{DD}) such that:

$$x_i^{DD} = \frac{(\delta - c)\left(2-\gamma(a_j)\right)[4.5b\lambda - \left(2-\gamma(a_i)\right)^2 + (2\gamma(a_i)-1)(2-\gamma(a_i))]}{\left[4.5b\lambda - \left(2-\gamma(a_j)\right)^2\right]\left[4.5b\lambda - \left(2-\gamma(a_i)\right)^2\right] - (2\gamma(a_i)-1)\left(2-\gamma(a_j)\right)(2-\gamma(a_i))(2\gamma(a_j)-1)},$$

$$x_j^{DD} = \frac{(\delta - c)(2-\gamma(a_i))[4.5b\lambda - \left(2-\gamma(a_j)\right)^2 + (2\gamma(a_j)-1)(2-\gamma(a_j))]}{\left[4.5b\lambda - \left(2-\gamma(a_i)\right)^2\right]\left[4.5b\lambda - \left(2-\gamma(a_j)\right)^2\right] - (2\gamma(a_j)-1)(2-\gamma(a_i))(2-\gamma(a_j))(2\gamma(a_i)-1)}.$$

Furthermore, it requires $4.5b\lambda - [2-\gamma(a_i)]^2 > 0$ and $4.5b\lambda - \left[2-\gamma(a_j)\right]^2 > 0$ to ensure the concavity of π_j^* with respect to x_j and the concavity of π_i^* with respect to x_i, respectively. Given large enough $b\lambda$, x_i^{DD} and x_j^{DD} are positive and both concavity conditions are satisfied.

<u>Case 2</u> (Mutual cooperation): (Firm *i*'s decision, Firm *j*'s decision) = (C, C)

The two firms cooperate in the R&D stage and compete in the production stage.

$$\frac{\partial(\pi_i^* + \pi_j^*)}{\partial x_i} = 0$$

$$\Rightarrow x_i^{CC} = \frac{(\delta - c)\left(\gamma(a_j)+1\right) + [(2\gamma(a_i)-1)\left(2-\gamma(a_j)\right) + \left(2\gamma(a_j)-1\right)(2-\gamma(a_i))]x_j^{CC}}{4.5b\lambda - \left(2-\gamma(a_j)\right)^2 - \left(2\gamma(a_j)-1\right)^2},$$

which is the best response function.

For all $\gamma(a_i) \neq \gamma(a_j)$, the cooperative equilibrium is (x_i^{CC}, x_j^{CC}) such that:

$$x_i^{CC} = \frac{(\delta - c)\left[E\left(\gamma(a_j)+1\right) + \begin{bmatrix}[2\gamma(a_i)-1][2-\gamma(a_j)] \\ +[2\gamma(a_j)-1][2-\gamma(a_i)]\end{bmatrix}(\gamma(a_i)+1)\right]}{EF - \left[[2\gamma(a_i)-1][2-\gamma(a_j)] + [2\gamma(a_j)-1][2-\gamma(a_i)]\right]^2}$$

$$x_j^{CC} = \frac{[\delta - c]\left[F[1+\gamma(a_i)] + [1+\gamma(a_j)]\begin{bmatrix}[2\gamma(a_j)-1][2-\gamma(a_i)] \\ +[2\gamma(a_i)-1][2-\gamma(a_j)]\end{bmatrix}\right]}{\left[EF - \left[[2\gamma(a_j)-1][2-\gamma(a_i)] + [2\gamma(a_i)-1][2-\gamma(a_j)]\right]^2\right]}.$$

Again for large enough $b\lambda$, it guarantees $E = 4.5b\lambda - [2-\gamma(a_i)]^2 - [2\gamma(a_i)-1]^2 > 0$ to ensure the concavity of $(\pi_i^* + \pi_j^*)$ with respect to $x_j, F = 4.5b\lambda - \left[2-\gamma(a_j)\right]^2 - \left[2\gamma(a_j) - 1\right]^2 > 0$ to ensure the concavity of $(\pi_i^* + \pi_j^*)$ with respect to x_i, and both x_i^{CC} and x_j^{CC} are positive.

<u>Case 3</u> (Unilateral defection): (Firm *i*'s decision, Firm *j*'s decision) = (D, C)

$$\frac{\partial \pi_i^*\left(x_i^{DC}, x_j^{CC}\right)}{\partial x_i^{DC}} = 0$$

$$\Rightarrow x_i^{DC} = \frac{[\delta - c]\left[2-\gamma(a_j)\right] + [2\gamma(a_i) - 1]\left[2-\gamma(a_j)\right]x_j^{CD}}{\left[4.5b\lambda - \left[2-\gamma(a_j)\right]^2\right]},$$

which is the best response function.

For all $\gamma(a_i) \neq \gamma(a_j)$, the equilibrium under the unilateral defection is (x_i^{DC}, x_j^{DC}) such that:

$$x_i^{DC} = \frac{\left(2-\gamma(a_j)\right)(\delta - c)}{4.5b\lambda - \left(2-\gamma(a_j)\right)^2}\left[\frac{F\left[E + (2\gamma(a_i)-1)[1+\gamma(a_i)]\right] - 3T\left(2\gamma(a_j)-1\right)\left(1-\gamma(a_i)\right)}{[EF - T^2]}\right],$$

$$x_j^{DC} = \frac{(2-\gamma(a_i))(\delta - c)}{4.5b\lambda - (2-\gamma(a_i))^2}\left[\frac{E\left[F + \left(2\gamma(a_j)-1\right)\left[1+\gamma(a_j)\right]\right] - 3T(2\gamma(a_i)-1)\left(1-\gamma(a_j)\right)}{[EF - T^2]}\right]$$

such that $T = \left[2\gamma(a_j) - 1\right][2-\gamma(a_i)] + [2\gamma(a_i)-1]\left[2-\gamma(a_j)\right]$. Given large enough $b\lambda$, x_i^{DC} and x_j^{DC} are positive.

4.5.2 Results under the asymmetric case

In the following propositions, I present the properties of asymmetric firms' optimal R&D investment and maximized profits which are verified using mathematical proofs and numerical simulations. In all the results in this subsection and the corresponding proofs, I assume that $\beta = 0$ and $l = 1$.

Proposition 3: For large enough $b\lambda$ and for $a_i \neq \frac{1}{2}$,

- $\{x_i^{DC} > x_i^{DD} > x_i^{CC}\}$ if $a_j < \frac{1}{2}$,
- $\{x_i^{CC} > x_i^{DC} > x_i^{DD}\}$ if $a_j > \frac{1}{2}$.

The proofs are provided in the Appendix.

Given Proposition 3, the comparison of optimal R&D investment is based essentially on the absorptive capacity of firm j. Regardless of whether firm i has low or high absorptive capacity, if firm j has low absorptive capacity, firm i, as a defecting firm, opts for a larger R&D investment compared to the cooperative agreement and increases its effort beyond the non-cooperative and cooperative efforts; however, if firm j has high absorptive capacity, firm i increases its cooperative R&D investment with firm j beyond x_i^{DC} and x_i^{DD}.

The intuition behind this is that when a firm i cooperates with another firm j that has high absorptive capacity, firm i can benefit from this by increasing their joint R&D investment. This is because firm j is able to effectively absorb and use the knowledge generated from the cooperation, leading to mutual gains for both firms. On the other hand, if firm j has low absorptive capacity, firm i may see less value in investing in cooperative R&D. In this case, firm i may choose to defect from the cooperative agreement and invest more in R&D independently. This decision is based on the belief that firm j would not be able to effectively utilize the knowledge from the cooperation, leading firm i to prioritize its own R&D efforts instead.

Proposition 4:

For $b\lambda$ and $(\delta - c)$ sufficiently large, if $a_i < a_j$, $x_i^{DD} < x_j^{DD}$, $x_i^{CC} > x_j^{CC}$ and $x_i^{DC} < x_j^{DC}$.

The proofs are provided in the Appendix.

When firm j has a higher absorptive capacity than does firm i $(a_j > a_i)$, Proposition 4 shows that, for $b\lambda$ and $(\delta - c)$ sufficiently large, the non-cooperative R&D of firm j is larger than that of firm i. This can be attributed to the higher absorptive capacity of firm j, allowing firm j to generate more innovative ideas and technologies on their own. However, when $a_j > a_i$, the cooperative R&D of firm j is smaller than that of firm i. This can be due to the lower absorptive capacity of firm i, allowing firm j to invest more in R&D when acting unilaterally compared to when cooperating with firm i. Based on Proposition 4, firm j's R&D investment under unilateral defection is larger than that of firm i ($x_j^{DC} > x_i^{DC}$). This indicates that when firm j decides to defect from the cooperative agreement and invest independently, they are likely to allocate more resources to R&D compared to firm i, which is driven by the fact that firm j can better utilize the knowledge and resources available to them. The results in Proposition 3 and 4 are summarized in Table 4.2.

Table 4.2: The ranking of the optimal R&D investments

$\boldsymbol{a_i}$	Absorptive capacity $\boldsymbol{a_i < a_j}$		
$(\boldsymbol{a_i, a_j})$	$(\boldsymbol{Low, Low})$	$(\boldsymbol{Low, High})$	$(\boldsymbol{High, High})$
	$\boldsymbol{a_i < a_j < \frac{1}{2}}$	$\boldsymbol{a_i < \frac{1}{2} < a_j}$	$\boldsymbol{\frac{1}{2} < a_i < a_j}$
Optimal R&D investments	$x_i^{DC} > x_i^{DD} > x_i^{CC}$ $x_j^{DC} > x_j^{DD} > x_j^{CC}$ $x_i^{CC} > x_j^{CC}$ $x_i^{DD} < x_j^{DD}$ $x_i^{DC} < x_j^{DC}$	$x_i^{CC} > x_i^{DC} > x_i^{DD}$ $x_j^{DC} > x_j^{DD} > x_j^{CC}$ $x_i^{CC} > x_j^{CC}$ $x_i^{DD} < x_j^{DD}$ $x_i^{DC} < x_j^{DC}$	$x_i^{CC} > x_i^{DC} > x_i^{DD}$ $x_j^{CC} > x_j^{DC} > x_j^{DD}$ $x_i^{CC} > x_j^{CC}$ $x_i^{DD} < x_j^{DD}$ $x_i^{DC} < x_j^{DC}$

In the following analysis, I examine the results from the perspective of strategic substitutes and strategic complements. As stated by Bulow et al. (1985) in the context of price (quantity) competition among firms, the strategic variables are strategic substitutes (complements) when they align with the concept of downward (upward) sloping reaction curves in the price (quantity). According to Miyagiwa and Ohno (1997), the mixed case of strategic complementarity and substitute might also occur when one best-response function is downward-sloping while the other is upward sloping. Recall when I derived the first order condition of firm i's profit, the best response function under the non-cooperative case is expressed as follows:

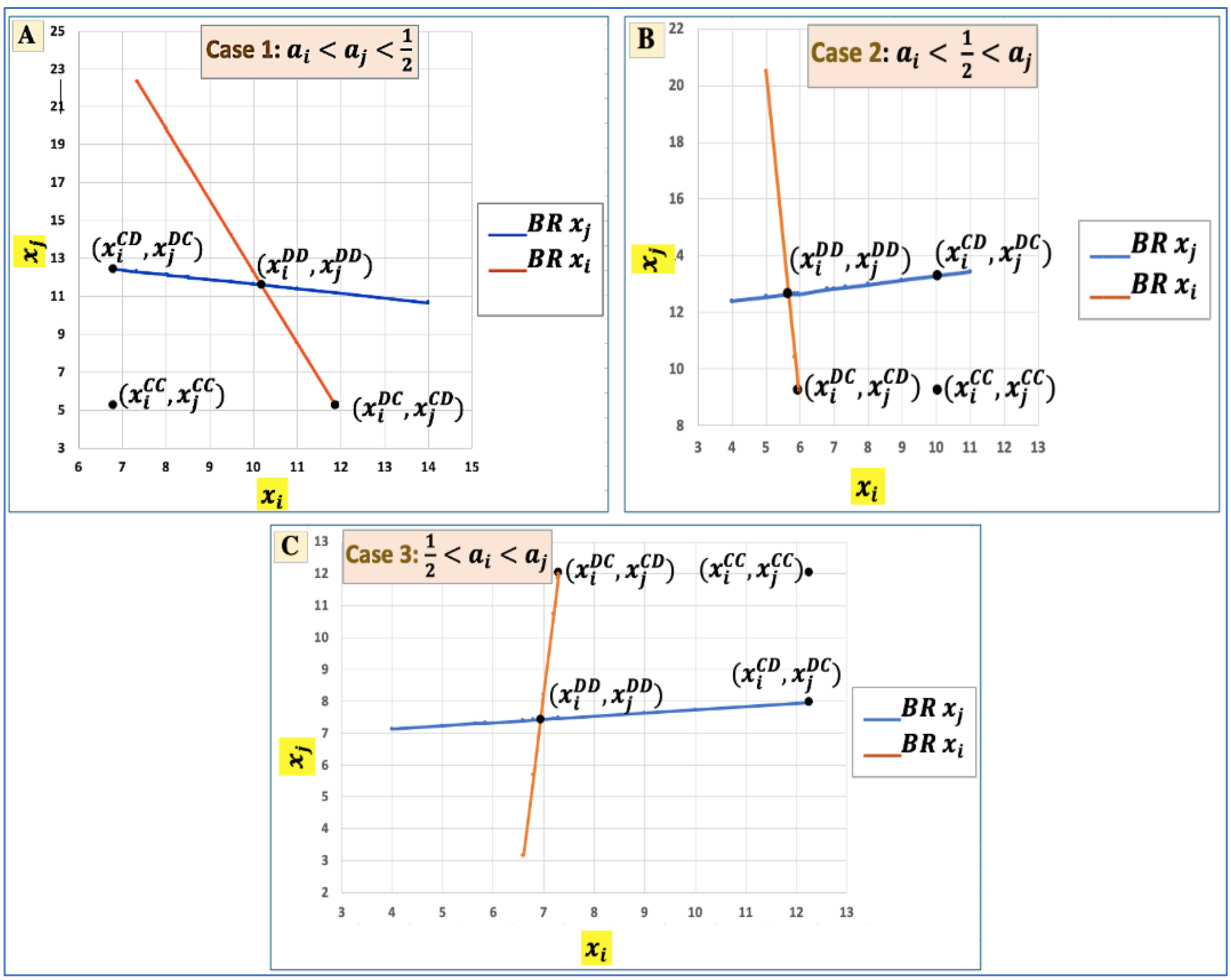
$$x_i = \frac{[\delta - c][2 - a_j] + [2a_i - 1][2 - a_j]x_j}{\left[4.5b\lambda - [2 - a_j]^2\right]}.$$

Similarly,

$$x_j = \frac{[\delta - c][2 - a_i] + [2a_j - 1][2 - a_i]x_i}{[4.5b\lambda - [2 - a_i]^2]}.$$

According to the definition of Bulow et al. (1985), x_i and x_j are referred to strategic complement (substitute) by the sign of $(2a_i - 1)$ and $(2a_j - 1)$ in the above expressions. Table 4.2 indicates that when the R&D expenditures are strategic substitutes for all $a_i < a_j < \frac{1}{2}$ or strategic complements for all $\frac{1}{2} < a_i < a_j$ or the mixed case of both for all $a_i < \frac{1}{2} < a_j$, the inequalities $\{x_i^{DD} < x_j^{DD},\ x_i^{CC} > x_j^{CC}$ and $x_i^{DC} < x_j^{DC}\}$ are always satisfied. I simulate the results in Table 4.2 and present in Figure 4.3 the best response curves with optimal investments.

Figure 4.3: Best-response functions



Notes: In all figures, $\delta = 60$, $c = 20$, $\lambda = 2$, $b = 1$. In Figure 4.3(**A**), $a_i = 0.1, a_j = 0.15$. In Figure 4.3(**B**), $a_i = 0.25, a_j = 0.75$. In Figure 4.3(**C**), $a_i = 0.75, a_j = 0.8$. ***BR***: Best Response.

Figure 4.3(A) indicates that $\{x_i^{DC} > x_i^{DD} > x_i^{CC}\}$ for all $a_i < a_j < \frac{1}{2}$ which is consistent with the results in Table 4.2. The consideration of low-high absorptive capacity in Figure 4.3(B) induces a mixed case of strategic complementarity and substitute. In this case, the slope reaction of one firm is negative while the other is positive and the firm that has low absorptive capacity invests more in cooperation or unilateral defection than non-cooperation given that the other firm has high absorptive capacity. Specifically, when $a_i < \frac{1}{2} < a_j, \{x_i^{CC} > x_i^{DC} > x_i^{DD}\}$ as it is shown in Table 4.2. When $\frac{1}{2} < a_i < a_j$, Figure 4.3(C) indicates that the R&D expenditures are strategic complements given that an investment increase of one firm increases the investment of its rival. Again, the results in Figure 4.3(C) are consistent with Table 4.2 since $\{x_i^{CC} > x_i^{DC} > x_i^{DD}\}$.

Table 4.3: Simultaneous move game

		Firm *j*	
		Defect	**Cooperate**
Firm *i*	**Defect**	$\pi_i^*(x_i^{DD}, x_j^{DD}), \pi_j^*(x_i^{DD}, x_j^{DD})$	$\pi_i^*(x_i^{DC}, x_j^{CD}), \pi_j^*(x_i^{DC}, x_j^{CD})$
	Cooperate	$\pi_i^*(x_i^{CD}, x_j^{DC}), \pi_j^*(x_i^{CD}, x_j^{DC})$	$\pi_i^*(x_i^{CC}, x_j^{CC}), \pi_j^*(x_i^{CC}, x_j^{CC})$

In the remaining analysis, I compare the profit of the firms conditional on whether they choose to defect or to cooperate and I investigate whether the results have the feature of the prisoner's dilemma game. Table 4.3 denotes the different profits under each possible outcome. In Proposition 5, I show that the results on the profit comparisons are in line with the prisoner's dilemma situation for all $a_j \neq \frac{1}{2}$ and $a_i = \frac{1}{2}$. Similarly, this consistency occurs for all $a_i \neq \frac{1}{2}$ and $a_j = \frac{1}{2}$.

<u>Proposition 5:</u> For all $a_j \neq \frac{1}{2}$, the profit functions are such that:

$\pi_i^*(x_i^{DC}, x_j^{CD}) > \pi_i^*(x_i^{CC}, x_j^{CC})$ for all a_i,

$\pi_i^*(x_i^{DD}, x_j^{DD}) > \pi_i^*(x_i^{CC}, x_j^{CC})$ for $a_i = \frac{1}{2}$,

$\pi_i^*(x_i^{DD}, x_j^{DD}) > \pi_i^*(x_i^{CD}, x_j^{DC})$ for $a_i = \frac{1}{2}$,

$\pi_j^*(x_i^{DD}, x_j^{DD}) > \pi_j^*(x_i^{DC}, x_j^{CD})$ for $a_i = \frac{1}{2}$,

$\pi_j^*(x_i^{CD}, x_j^{DC}) = \pi_j^*(x_i^{CC}, x_j^{CC})$ for $a_i = \frac{1}{2}$,

$\pi_i^*(x_i^{CC}, x_j^{CC}) + \pi_j^*(x_i^{CC}, x_j^{CC}) > \pi_i^*(x_i^{DD}, x_j^{DD}) + \pi_j^*(x_i^{DD}, x_j^{DD})$ for $a_i = \frac{1}{2}$.

The proofs are provided in the Appendix.

Due to the complexity of the mathematical proofs for all a_i and $a_j \neq \frac{1}{2}$, I rely on numerical simulations for results under general a_i and a_j and demonstrate that $\pi_i^*(x_i^{CC}, x_j^{CC}) + \pi_j^*(x_i^{CC}, x_j^{CC}) > \pi_i^*(x_i^{DD}, x_j^{DD}) + \pi_j^*(x_i^{DD}, x_j^{DD})$ and $\pi_i^*(x_i^{DD}, x_j^{DD}) > \pi_i^*(x_i^{CD}, x_j^{DC})$. Using the first result in Proposition 5 and the numerical simulations, I find that the payoff matrices are in line with the prisoner's dilemma situation, which are shown in Table 4.4 with the numerical examples for the cases where $a_i < a_j < \frac{1}{2}$, $a_i < \frac{1}{2} < a_j$, $\frac{1}{2} < a_i < a_j$ and $a_j < a_i = \frac{1}{2}$. For the entire space of (a_i, a_j), I provide, in Figure 4.4, the numerical results regarding the differences in the profits, $\pi_i^*(x_i^{DC}, x_j^{CD}) - \pi_i^*(x_i^{CC}, x_j^{CC})$ in Figure 4.4(A), $\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CD}, x_j^{DC})$ in Figure 4.4(B) and $[\pi_i^*(x_i^{CC}, x_j^{CC}) + \pi_j^*(x_i^{CC}, x_j^{CC})] - [\pi_i^*(x_i^{DD}, x_j^{DD}) + \pi_j^*(x_i^{DD}, x_j^{DD})]$ in Figure 4.4(C). Again, the results are consistent with the prisoner's dilemma situation for all a_i and $a_j \neq \frac{1}{2}$.

Table 4.4: Numerical examples

$a_i = 0.2$, $a_j = 0.4$		Firm j	
		Defect	**Cooperate**
Firm i	**Defect**	41.78, 63.9	49.57, 59.67
	Cooperate	41.56, 64.48	49.12, 60.58

$a_i = 0.6$, $a_j = 0.8$		Firm j	
		Defect	**Cooperate**
Firm i	**Defect**	57.64, 68.68	58.86, 67.87
	Cooperate	52.73, 78.39	53.72, 77.9

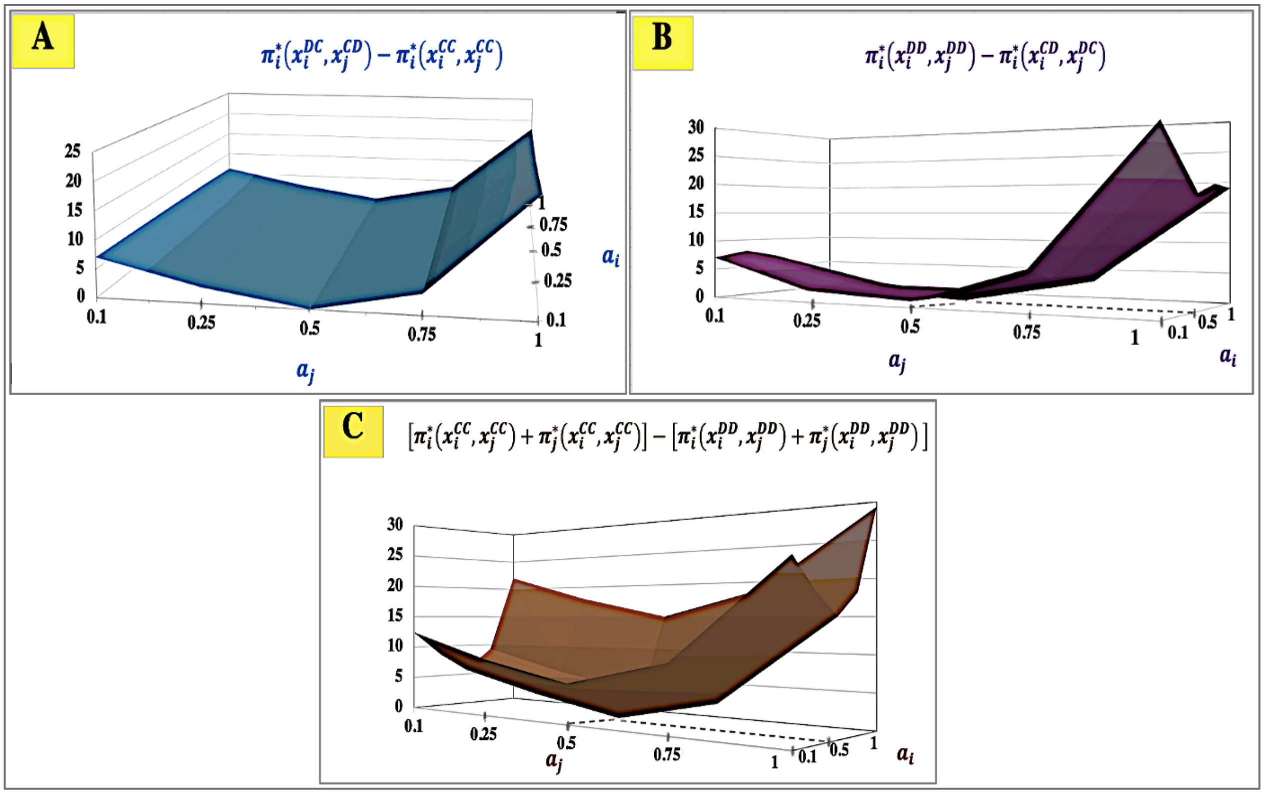
$a_i = 0.2$, $a_j = 0.8$		Firm j	
		Defect	**Cooperate**
Firm i	**Defect**	33.71, 80.4	38.79, 78.76
	Cooperate	26.07, 92.99	33.44, 89.19

$a_i = 0.5$, $a_j = 0.2$		Firm j	
		Defect	**Cooperate**
Firm i	**Defect**	69.44, 39.12	69.44, 39.04
	Cooperate	65.56, 46.56	65.56, 46.56

Figure 4.4(A), 4.4(B) and Proposition 3 show that firm i tends to free ride firm j's R&D effort when the absorptive capacity of firm j is high $(a_j > \frac{1}{2})$. For all a_i and $a_j \neq \frac{1}{2}$, Figure 4.4(C) indicates that mutual cooperation on R&D without defection always results in a higher profit compared to non-cooperation (mutual defection). This demonstrates that engaging in a cooperative

agreement is financially beneficial for both firms, even if their ability to absorb a new knowledge is limited.

Figure 4.4: Profit comparison with asymmetric absorptive capacities



Notes: $\delta = 60$, $c = 20$, $\lambda = 2$, $b = 1$.

4.6 Conclusion

In this paper, I considered a two-stage R&D model in a duopoly market. In the first stage, both firms make a simultaneous decision to either engage cooperatively or non-cooperatively in the R&D investment. The investment can reduce the production cost of the investing firm and also lower the cost of the rival via the absorptive capacity. After knowing the R&D outcome, both firms make a simultaneous choice in terms of the quantity to produce, similar to the traditional Cournot model.

I considered the case where the two firms have symmetric absorptive capacities with stochastic R&D technology and the case with asymmetric absorptive capacities combined with deterministic

R&D technology. Given this framework, I examined the outcomes under the unilateral defection, the mutual defection (or non-cooperation) and the mutual cooperation of the two firms.

In the case of symmetric absorptive capacities, no matter low or high, I showed that, the larger the probability of success, the greater the difference between the optimal R&D investments under defection and those under cooperation. For any positive probability of success and for any symmetric absorptive capacity other than the critical threshold, I found that the profit under unilateral defection is always greater than the profit under mutual cooperation and that firms end up in a prisoner's dilemma situation for their R&D decisions. By considering the asymmetric absorptive capacity, I showed that the defecting firm chooses to make a greater (lesser) investment than in the cooperative scenario when the other firm's absorptive capacity is low (high). No matter if the absorptive capacities of the two firms are equal or diverse, with the exception of the critical threshold, the R&D outcomes consistently reflect the prisoner's dilemma situation.

Under the asymmetric case, the model does not include the probability of success and only considers the deterministic environment. For future research, I am interested in investigating how the optimal R&D investments under the different scenarios are affected by the asymmetric absorptive capacity with stochastic R&D technology.

4.7 Appendix

<u>Proof of Lemma 1:</u>

Derivation the equilibrium quantity and the equilibrium price of firm i at the second stage:

$$\pi_i^*(q_i^*) = P(Q^*)q_i^* - c_i q_i^* = \left(\delta - bq_i^* - bq_j^*\right)q_i^* - c_i q_i^*.$$

$$\frac{\partial \pi_i^*(q_i^*)}{\partial q_i^*} = 0 \;\Rightarrow\; \delta - 2bq_i^* - bq_j^* - c_i = 0.$$

$$q_i^* = \frac{\delta - bq_j^* - c_i}{2b}, \qquad q_j^* = \frac{\delta - bq_i^* - c_j}{2b},$$

$$\Rightarrow q_i^* = \frac{\delta - b\left(\frac{\delta - bq_i^* - c_j}{2b}\right) - c_i}{2b} \quad \Rightarrow \quad q_i^* = \frac{\delta + c_j - 2c_i}{3b}.$$

$$P(Q^*) = \delta - bq_i^* - bq_j^* = \delta - bq_i^* - b\left(\frac{\delta - bq_i^* - c_j}{2b}\right) = \frac{\delta - bq_i^* + c_j}{2}$$

$$= \frac{\delta - b\left(\frac{\delta + c_j - 2c_i}{3b}\right) + c_j}{2} = \frac{\delta + c_j + c_i}{3}.$$

Derivation of firm i's profit:

$$\pi_i^*(q_i^*) = P(Q^*)q_i^* - c_i q_i^* = (P(Q^*) - c_i)q_i^* = (\delta - bq_i^* - bq_j^* - c_i)q_i^*$$

$$= [\delta - bq_i^* - b(\frac{\delta - bq_i^* - c_j}{2b}) - c_i]q_i^* = \left[\frac{\delta - bq_i^* - 2c_i + c_j}{2}\right] q_i^*$$

$$= \left[\frac{\delta - b\left(\frac{\delta + c_j - 2c_i}{3b}\right) - 2c_i + c_j}{2}\right] q_i^* = \left[\frac{\delta + c_j - 2c_i}{3}\right] q_i^* = b(q_i^*)^2.$$

__**Lemma A.1:**__

1) The optimal R&D expenditure of firm i under mutual defection is as follows:

$$x_i^{DD} = x_j^{DD} = \frac{\rho(\delta - c)(2 - \gamma(a))}{4.5b\lambda - \rho^2(\gamma(a) + 1)\big(2 - \gamma(a)\big) - \rho(1 - \rho)\,(2 - \gamma(a))^2}.$$

2) The optimal cooperative R&D expenditure of firm i is expressed as follows:

$$x_i^{CC} = x_j^{CC} = \frac{\rho(\delta - c)(\gamma(a) + 1)}{4.5b\lambda - \rho\big(2 - \gamma(a)\big)^2 - \rho(2\gamma(a) - 1)^2 - 2\rho^2(2\gamma(a) - 1)\big(2 - \gamma(a)\big)}.$$

3) The optimal R&D expenditure of firm i under deviation is as follows:

$$x_i^{DC} = x_j^{DC} = \frac{\rho(\delta - c)\big(2 - \gamma(a)\big)[\rho^2(2\gamma(a) - 1)(\gamma(a) + 1) + A]}{AB}$$

such that

$$A = 4.5b\lambda - \rho^2(\gamma(a) + 1)^2 - \rho(1 - \rho)\,((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2),$$

$$B = 4.5b\lambda - \rho\big(2 - \gamma(a)\big)^2.$$

__***Proof of Lemma A.1:***__

Derivation of x_i^{DD} under the symmetric absorptive capacity:

$$\text{Max } E\pi_i^* == \left(\frac{1}{9b}\right)\begin{bmatrix} \rho\rho\big[\delta - c + [2\gamma(a) - 1]x_j^{DD} + [2 - \gamma(a)]x_i^{DD}\big]^2 \\ + \rho[1 - \rho][\delta - c + [2 - \gamma(a)]x_i^{DD}]^2 \\ +\rho[1 - \rho]\big[\delta - c + [2\gamma(a) - 1]x_j^{DD}\big]^2 \\ +(1 - \rho)(1 - \rho)[\delta - c]^2 \end{bmatrix} - \frac{1}{2}\lambda(x_i^{DD})^2$$

subject to $x_i^{DD}, x_j^{DD}, E\pi_i^* \geq 0$.

$$\frac{\partial E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right)}{\partial x_i^{DD}} = 0$$

$$\Rightarrow \left(\frac{2}{9b}\right)\begin{bmatrix}\rho\rho\left[\delta - c + [2\gamma(a) - 1]x_j^{DD} + [2 - \gamma(a)]x_i^{DD}\right][2 - \gamma(a)] \\ +\rho[1-\rho][\delta - c + [2 - \gamma(a)]x_i^{DD}]\left[[2 - \gamma(a)]\right]\end{bmatrix} - \lambda x_i^{DD} = 0$$

$$\Rightarrow \ x_i^{DD} = \frac{\rho(\delta - c)\left(2 - \gamma(a)\right) + \rho^2(2\gamma(a) - 1)(2 - \gamma(a))x_j^{DD}}{4.5b\lambda - \rho\ (2 - \gamma(a))^2}. \qquad (\mathbf{1})$$

$$\frac{\partial E\pi_j^*\left(x_i^{DD}, x_j^{DD}\right)}{\partial x_j^{DD}} = \left(\frac{2}{9b}\right)\begin{bmatrix}\rho\rho\left[\delta - c + [2\gamma(a) - 1]x_i^{DD} + [2 - \gamma(a)]x_j^{DD}\right][2 - \gamma(a)] \\ +\rho[1-\rho]\left[\delta - c + [2 - \gamma(a)]x_j^{DD}\right]\left[[2 - \gamma(a)]\right]\end{bmatrix} - \lambda x_j^{DD} = 0$$

$$\Rightarrow \ x_j^{DD} = \frac{\rho(\delta - c)\left(2 - \gamma(a)\right) + \rho^2(2\gamma(a) - 1)(2 - \gamma(a))x_i^{DD}}{4.5b\lambda - \rho\ (2 - \gamma(a))^2}. \qquad (\mathbf{2})$$

$(\mathbf{2})$ in $(\mathbf{1}) \Rightarrow$

$$x_i^{DD} = \frac{\rho(\delta - c)(2 - \gamma(a))}{4.5\lambda b - \rho^2(\gamma(a) + 1)\left(2 - \gamma(a)\right) - \rho(1-\rho)\ (2 - \gamma(a))^2}.$$

$$\left.\frac{\partial^2 E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right)}{\partial (x_i^{DD})^2}\right|_{x_i^{DD} = x_j^{DD}} = \left(\frac{2}{9b}\right)\left[\rho^2(\gamma(a) + 1)\left(2 - \gamma(a)\right) + \rho(1-\rho)\ (2 - \gamma(a))^2 - \lambda\right] < 0$$

To ensure the concavity of $E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right)$ with respect to x_i^{DD}, I place the following restriction:

$$4.5b\lambda > \boldsymbol{R_1} = \rho^2(\gamma(a) + 1)\left(2 - \gamma(a)\right) + \rho(1-\rho)\ (2 - \gamma(a))^2 > 0 \quad (\mathbf{3}).$$

Derivation of $\mathbf{x_i^{CC}}$ under the symmetric absorptive capacity:

$$Max\ E(\pi_i^* + \pi_j^*) = E\pi_i^* + E\pi_j^* = \left(\frac{2}{9b}\right)\begin{bmatrix}\rho\rho\left[\delta - c + [2\gamma(a) - \gamma(a) + 1]x_i^{CC}\right]^2 \\ +\rho[1-\rho]\left[\delta - c + [2 - \gamma(a)]x_i^{CC}\right]^2 \\ +\rho[1-\rho]\left[\delta - c + [2\gamma(a) - 1]x_i^{CC}\right]^2 \\ +(1-\rho)(1-\rho)[\delta - c]^2\end{bmatrix}$$

$$-\tfrac{1}{2}\lambda(x_i^{CC})^2 - \tfrac{1}{2}\lambda(x_i^{CC})^2$$

subject to $x_i^{CC}, E(\pi_i^* + \pi_j^*) \geq 0$.

$$\frac{\partial E(\pi_i^* + \pi_j^*)}{\partial x_i^{CC}} = 0$$

$$\Rightarrow \left(\frac{2}{9b}\right)\begin{bmatrix}2\rho\rho\left[\delta - c + [\gamma(a) + 1]x_i^{CC}\right][1 + \gamma(a)] \\ +2\rho[1-\rho]\left[\delta - c + [2\gamma(a) - 1]x_i^{CC}\right]\left[[2\gamma(a) - 1]\right] \\ +2\rho[1-\rho]\left[\delta - c + [2 - \gamma(a)]x_i^{CC}\right]\left[[2 - \gamma(a)]\right]\end{bmatrix} - 2\lambda x_i^{CC} = 0 \ \ \Rightarrow$$

$$x_i^{CC} = \frac{\rho(\delta - c)(\gamma(a) + 1)}{4.5b\lambda - \rho^2\left(1 + \gamma(a)\right)^2 - \rho(1-\rho)\left((2\gamma(a) - 1)^2 + \left(2 - \gamma(a)\right)^2\right)}$$

$$= \frac{\rho(\delta - c)(\gamma(a) + 1)}{4.5b\lambda - \rho\left(2 - \gamma(a)\right)^2 - \rho(2\gamma(a) - 1)^2 - 2\rho^2(2\gamma(a) - 1)\left(2 - \gamma(a)\right)}.$$

$$\frac{\partial^2 E(\pi_i^* + \pi_j^*)}{\partial\left(x_i^{CC}\right)^2} = \left(\frac{2}{9b}\right)\left(\rho^2\left(1 + \gamma(a)\right)^2 + \rho(1-\rho)\left((2\gamma(a) - 1)^2 + \left(2 - \gamma(a)\right)^2\right)\right) - \lambda < 0.$$

To ensure the concavity of $E(\pi_i^* + \pi_j^*)$ with respect to x_i^{CC}, I place the following restriction:

$$4.5b\lambda > \boldsymbol{P} = \rho^2\left(1 + \gamma(a)\right)^2 + \rho(1-\rho)\left((2\gamma(a) - 1)^2 + \left(2 - \gamma(a)\right)^2\right) > 0 \qquad (\mathbf{4}).$$

Derivation of x_i^{DC} under the symmetric absorptive capacity:

$$Max\ E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right)$$

$$= \left(\frac{1}{9b}\right)\begin{bmatrix} \rho\rho\left[\delta - c + [2\gamma(a) - 1]x_j^{CD} + [2 - \gamma(a)]\, x_i^{DC}\right]^2 \\ + \rho[1-\rho]\left[\delta - c + [2 - \gamma(a)]\, x_i^{DC}\right]^2 \\ + \rho[1-\rho]\left[\delta - c + [2\gamma(a) - 1]x_j^{CD}\right]^2 \\ + (1-\rho)(1-\rho)[\delta - c]^2 \end{bmatrix} - \frac{1}{2}\lambda(x_i^{DC})^2$$

subject to $x_i^{DC}, x_j^{CD}, E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) \geq 0$.

$$\frac{\partial E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right)}{\partial x_i^{DC}} = \left(\frac{2}{9b}\right)\begin{bmatrix} \rho\rho\left[\delta - c + [2\gamma(a) - 1]x_j^{CD} + [2 - \gamma(a)]\, x_i^{DC}\right][2 - \gamma(a)] \\ +\rho[1-\rho]\left[\delta - c + [2 - \gamma(a)]\, x_i^{DC}\right]\left[[2 - \gamma(a)]\right] \end{bmatrix} - \lambda x_i^{DC}$$

$$= 0 \Rightarrow\ x_i^{DC} = \frac{\left(2 - \gamma(a)\right)\rho\left[(\delta - c) + x_j^{CD}\rho(2\gamma(a) - 1)\right]}{4.5b\lambda - \rho\left(2 - \gamma(a)\right)^2}.$$

Given $x_j^{CC} = x_j^{CD} = x_i^{CC}$ and the expression of the optimal investment x_i^{CC} is already found,

$$x_i^{DC} = \frac{\rho(\delta - c)\left(2 - \gamma(a)\right)[\rho^2(2\gamma(a) - 1)(\gamma(a) + 1) + A]}{AB}$$

such that $A = 4.5b\lambda - \rho^2(\gamma(a) + 1)^2 - \rho(1-\rho)\left((2\gamma(a) - 1)^2 + \left(2 - \gamma(a)\right)^2\right)$,

$B = 4.5b\lambda - \rho\left(2 - \gamma(a)\right)^2$.

$$\frac{\partial^2 E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right)}{\partial(x_i^{DC})^2} = \left(\frac{2}{9b}\right)\rho\ (2 - \gamma(a))^2 - \lambda < 0.$$

To ensure the concavity of $E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right)$ with respect to x_i^{DC}, I place the following restriction:

$$4.5b\lambda > \boldsymbol{Q} = \rho\left(2 - \gamma(a)\right)^2 \qquad (\mathbf{5}),$$

From the previous proofs, A and B are positive.

To ensure that $x_i^{DC} > 0$, I place the following restriction:

$$\rho^2(2\gamma(a)-1)(\gamma(a)+1)+A>0 \Leftrightarrow 4.5b\lambda > R_2 \qquad \textbf{(6)}$$

such that $\boldsymbol{R_2} = \rho^2(\gamma(a)+1)\big(2-\gamma(a)\big)+\rho(1-\rho)\,\big((2\gamma(a)-1)^2+\big(2-\gamma(a)\big)^2\big)$.

Proof of Proposition 1:

From conditions **(3), (4)**, **(5)** and **(6),** $4.5b\lambda$ should be greater than $Max\{P,Q,R_1,R_2\}$.

$Q-P=\big(1-2\,\gamma(a)\big)\rho\big[2\rho\big(2-\gamma(a)\big)-\big(1-2\gamma(a)\big)\big]>0$ for $\rho > \dfrac{(1-2\gamma(a))}{2\big(2-\gamma(a)\big)}$,

$P-R_2=\rho^2(2\gamma(a)-1)(\gamma(a)+1)>0$ if $\gamma(a)>1/2$,

$R_2-R_1=\rho(1-\rho)(2\gamma(a)-1)^2>0$,

$R_1-Q=\rho^2\big(2-\gamma(a)\big)(2\gamma(a)-1)>0$ if $\gamma(a)>1/2$,

$Q-R_2=\big(1-2\,\gamma(a)\big)\rho\big[3\rho\big(1-\gamma(a)\big)-\big(1-2\gamma(a)\big)\big]>0$ for $\rho > \dfrac{(1-2\gamma(a))}{3\big(1-\gamma(a)\big)}$,

$R_2-S=\big(1+\gamma(a)\big)\rho\big[3\rho\big(1-\gamma(a)\big)-\big(1-2\gamma(a)\big)\big]>0$ for $\rho > \dfrac{(1-2\gamma(a))}{3\big(1-\gamma(a)\big)}$,

$P-R_1=\big(1-2\,\gamma(a)\big)\rho\big[\big(1-2\gamma(a)\big)-\rho\big(2-\gamma(a)\big)\big]>0$ for $\rho < \dfrac{\big(1-2\gamma(a)\big)}{\big(2-\gamma(a)\big)}$ and

$\gamma(a) < \dfrac{1}{2}$,

$S-P=\big(1+\gamma(a)\big)\rho\big[\big(1-2\gamma(a)\big)-\rho\big(2-\gamma(a)\big)\big]>0$ for $\rho < \dfrac{\big(1-2\gamma(a)\big)}{\big(2-\gamma(a)\big)}$ and

$\gamma(a) < \frac{1}{2}$ because it is impossible to have ρ less than a negative value.

$S-Q=\big(2-\gamma(a)\big)\rho\big[\big(1-2\gamma(a)\big)-3\rho\big(1-\gamma(a)\big)\big]<0$ for $\rho > \dfrac{(1-2\gamma(a))}{3\big(1-\gamma(a)\big)}$.

$R_1-S=\big(2-\gamma(a)\big)\rho\big[(2\gamma(a)-1)+\rho\big(2-\gamma(a)\big)\big]>0$ for $\gamma(a)>1/2$

such that $S=3\rho(1-\rho)\,(2-\gamma(a))(1-\gamma(a))$.

Case 1: $\frac{1}{2} < \gamma(a) < 1 \Leftrightarrow \frac{1-2\beta}{2l} < a < \frac{1-\beta}{l}$,

$Max\{P,Q,R_1,R_2\}=P$ since $P>R_2>R_1>Q$ for all $\gamma(a)>1/2$.

$P-S=\big(1+\gamma(a)\big)\rho\big[(2\gamma(a)-1)+\rho\big(2-\gamma(a)\big)\big]>0$ for all $\rho>0$ and $\gamma(a)>\dfrac{1}{2}$.

Given that $4.5b\lambda > Max\{P,Q,R_1,R_2\}=P$, I have $4.5b\lambda > P > S \Rightarrow 4.5b\lambda > S$.

Case 2: $\gamma(a) = \frac{1}{2} \Longrightarrow Q = P = R_2 = R_1 = Q,\ P > S, Q > S, R_2 > S \Rightarrow 4.5b\lambda > P > S$.

Case 3: $0 < \gamma(a) < \frac{1}{2} \Longleftrightarrow 0 < a < \frac{1-2\beta}{2l}$,

$Max\{P, Q, R_1, R_2\} = R_2$ since $R_2 > P \geq Q > R_1$ for all $0 < \rho \leq \frac{(1-2\gamma(a))}{2(2-\gamma(a))}$.

For all $\rho < \frac{(1-2\gamma(a))}{3(1-\gamma(a))}, R_2 <$ S.

Note that: $\frac{(1-2\gamma(a))}{2(2-\gamma(a))} < \frac{(1-2\gamma(a))}{3(1-\gamma(a))}$. Therefore, for all $\rho \leq \frac{(1-2\gamma(a))}{2(2-\gamma(a))}, 4.5b\lambda > R_2$ and $R_2 <$ S which means that $4.5b\lambda$ might be greater or lower than S.

For all $\frac{(1-2\gamma(a))}{2(2-\gamma(a))} \leq \rho < \frac{(1-2\gamma(a))}{3(1-\gamma(a))}, R_2 > Q \geq P > R_1$ and $R_2 <$ S, $4.5b\lambda > Max\{P, Q, R_1, R_2\} = R_2$.

For all $\frac{(1-2\gamma(a))}{3(1-\gamma(a))} \leq \rho \leq \frac{(1-2\gamma(a))}{(2-\gamma(a))}$,

$Q \geq R_2 > P \geq R_1$ and $Q \geq$ S, $4.5b\lambda > Max\{P, Q, R_1, R_2\} = Q \geq S$.

For all $\rho \geq \frac{(1-2\gamma(a))}{(2-\gamma(a))},\ Q > R_2 > R_1 \geq P$ and $Q >$ S, $4.5b\lambda > Max\{P, Q, R_1, R_2\} = Q > S$.

The above mathematical proofs show that:

- $\gamma(a) \geq \frac{1}{2} \Longrightarrow 4.5b\lambda > S$,
- $\gamma(a) < \frac{1}{2}$ and $\rho < \frac{(1-2\gamma(a))}{3(1-\gamma(a))} \Longrightarrow 4.5b\lambda$ might be greater or lower than S,
- $\gamma(a) < \frac{1}{2}$ and $\rho \geq \frac{(1-2\gamma(a))}{3(1-\gamma(a))} \Longrightarrow 4.5b\lambda > S$.

By relying on numerical simulations, I find that it is possible to have $4.5b\lambda < S$ for very small values for b and λ. Therefore, the condition such that:

$S > 4.5b\lambda > Max\{P, Q, R_1, R_2\}$ might occur.

Derivation of $(x_i^{CC} - x_i^{DD})$ under the symmetric absorptive capacity:

$$x_i^{CC} - x_i^{DD} = \frac{\rho(\delta - c)(\gamma(a) + 1)}{4.5b\lambda - \rho^2\big(1 + \gamma(a)\big)^2 - \rho(1-\rho)\big((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2\big)} - \frac{\rho(\delta - c)\big(2 - \gamma(a)\big)}{4.5b\lambda - \rho^2(\gamma(a) + 1)\big(2 - \gamma(a)\big) - \rho(1-\rho)\,(2 - \gamma(a))^2}.$$

Assume

$A = 4.5b\lambda - \rho^2\big(1 + \gamma(a)\big)^2 - \rho(1-\rho)\Big((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2\Big)$ and

$H = 4.5b\lambda - \rho^2(\gamma(a) + 1)\big(2 - \gamma(a)\big) - \rho(1-\rho)\,(2 - \gamma(a))^2$.

$x_i^{CC} - x_i^{DD}$

$$= \frac{\begin{matrix}\rho(\delta-c)(\gamma(a)+1)\left[4.5b\lambda-\rho^2(\gamma(a)+1)\big(2-\gamma(a)\big)-\rho(1-\rho)\,(2-\gamma(a))^2\right]\\ -\rho(\delta-c)(2-\gamma(a))\left[4.5b\lambda-\rho^2\big(1+\gamma(a)\big)^2-\rho(1-\rho)((2\gamma(a)-1)^2+\ \big(2-\gamma(a)\big)^2)\right]\end{matrix}}{AH}$$

$$= \frac{\rho(\delta-c)\begin{bmatrix}4.5b\lambda((\gamma(a)+1)-4.5b\lambda\big(2-\gamma(a)\big)-\rho^2(\gamma(a)+1)^2\big(2-\gamma(a)\big)\\ -\rho(1-\rho)\,(2-\gamma(a))^2(\gamma(a)+1)+\rho^2\big(1+\gamma(a)\big)^2\big(2-\gamma(a)\big)\\ +\rho(1-\rho)\big(2-\gamma(a)\big)((2\gamma(a)-1)^2+\ \big(2-\gamma(a)\big)^2)\end{bmatrix}}{AH}$$

$$= \frac{\rho(\delta-c)\begin{bmatrix}[4.5b\lambda((2\gamma(a)-1)]\\ -\rho(1-\rho)\big(2-\gamma(a)\big)\left[\big(2-\gamma(a)\big)(\gamma(a)+1)-((2\gamma(a)-1)^2-\ \big(2-\gamma(a)\big)^2)\right]\end{bmatrix}}{AH}$$

$$= \frac{\rho(\delta-c)\begin{bmatrix}[4.5b\lambda((2\gamma(a)-1)]\\ -\rho(1-\rho)\big(2-\gamma(a)\big)\left[\big(2-\gamma(a)\big)(2\gamma(a)-1)-((2\gamma(a)-1)^2)\right]\end{bmatrix}}{AH}$$

$$= \frac{\rho(\delta-c)\left[[4.5b\lambda((2\gamma(a)-1)]-\rho(1-\rho)\big(2-\gamma(a)\big)[3(2\gamma(a)-1)(1-\gamma(a))]\right]}{AH}$$

$$= \frac{\rho(\delta-c)(2\gamma(a)-1)\left[4.5b\lambda-3\rho(1-\rho)\big(2-\gamma(a)\big)(1-\gamma(a))\right]}{AH}.$$

$x_i^{CC} - x_i^{DD} > 0$ for all $a > \frac{1-2\beta}{2l}$ and $4.5b\lambda > 3\rho(1-\rho)\big(2-\gamma(a)\big)(1-\gamma(a))$.

$\lim_{\rho\to 0}\big(x_i^{CC} - x_i^{DD}\big) = 0$ while

$$\lim_{\rho\to 1}\big(x_i^{CC} - x_i^{DD}\big) = \frac{4.5b\lambda(\delta-c)(2\gamma(a)-1)}{\left[4.5b\lambda-\big(1+\gamma(a)\big)^2\right]\left[4.5b\lambda-(\gamma(a)+1)\big(2-\gamma(a)\big)\right]}.$$

Derivation of ($x_i^{CC} - x_i^{DC}$) under the symmetric absorptive capacity:

$x_i^{CC} - x_i^{DC}$

$$= \frac{\rho(\delta-c)(\gamma(a)+1)}{4.5b\lambda-\rho^2\big(1+\gamma(a)\big)^2-\rho(1-\rho)((2\gamma(a)-1)^2+\ \big(2-\gamma(a)\big)^2)}$$

$$- \frac{\rho(\delta-c)\big(2-\gamma(a)\big)\begin{bmatrix}\rho^2(2\gamma(a)-1)(\gamma(a)+1)+4.5b\lambda-\rho^2(\gamma(a)+1)^2\\ -\rho(1-\rho)\,((2\gamma(a)-1)^2+\big(2-\gamma(a)\big)^2)\end{bmatrix}}{\left[4.5b\lambda-\rho^2(\gamma(a)+1)^2-\rho(1-\rho)\,((2\gamma(a)-1)^2+\big(2-\gamma(a)\big)^2)\right]\left[4.5b\lambda-\rho\big(2-\gamma(a)\big)^2\right]}$$

$$= \frac{\rho(\delta - c)(\gamma(a) + 1)\left[4.5b\lambda - \rho\big(2 - \gamma(a)\big)^2\right]}{AB}$$

$$- \frac{\rho(\delta - c)\big(2 - \gamma(a)\big)\begin{bmatrix}\rho^2(2\gamma(a) - 1)(\gamma(a) + 1) + 4.5b\lambda - \rho^2(\gamma(a) + 1)^2 \\ - \rho(1 - \rho)\left((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2\right)\end{bmatrix}}{AB}$$

$$= \frac{\rho(\delta - c)\begin{bmatrix}4.5b\lambda(2\gamma(a) - 1) - \rho(\gamma(a) + 1)\big(2 - \gamma(a)\big)^2 - \rho^2(2\gamma(a) - 1)(\gamma(a) + 1)\big(2 - \gamma(a)\big) \\ +\rho^2(\gamma(a) + 1)^2\big(2 - \gamma(a)\big) + \rho(1 - \rho)\left((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2\right)\big(2 - \gamma(a)\big)\end{bmatrix}}{AB}$$

$$= \frac{\rho(\delta - c)\begin{bmatrix}4.5\lambda b\lambda(2\gamma(a) - 1) - \rho^2(2\gamma(a) - 1)(\gamma(a) + 1)\big(2 - \gamma(a)\big) \\ -\rho\big(2 - \gamma(a)\big)\begin{pmatrix}(\gamma(a) + 1)\big(2 - \gamma(a)\big) - \rho(\gamma(a) + 1)^2 \\ -(1 - \rho) \times \left((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2\right)\end{pmatrix}\end{bmatrix}}{AB}.$$

Assume $P_1 = (\gamma(a) + 1)\big(2 - \gamma(a)\big) - \rho(\gamma(a) + 1)^2 - (1 - \rho)\left((2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2\right)$,

$$P_1 = (\gamma(a) + 1)\big(2 - \gamma(a)\big) - (2\gamma(a) - 1)^2 - \big(2 - \gamma(a)\big)^2 + \rho\left[(2\gamma(a) - 1)^2 + \big(2 - \gamma(a)\big)^2 - (\gamma(a) + 1)^2\right]$$

$$= \rho\left[4\big(\gamma(a)\big)^2 - 10\gamma(a) + 4\right] - \left[6\big(\gamma(a)\big)^2 - 9\gamma(a) + 3\right]$$

$$= 2\rho[(\gamma(a) - 2)(2\gamma(a) - 1)] - 3[(\gamma(a) - 1)(2\gamma(a) - 1)].$$

$x_i^{CC} - x_i^{DC}$

$$= \frac{\rho(\delta - c)\begin{bmatrix}4.5b\lambda(2\gamma(a) - 1) - \rho^2(2\gamma(a) - 1)(\gamma(a) + 1)\big(2 - \gamma(a)\big) - \rho\big(2 - \gamma(a)\big) \\ \times [2\rho(\gamma(a) - 2)(2\gamma(a) - 1) - 3(\gamma(a) - 1)(2\gamma(a) - 1)]\end{bmatrix}}{AB}$$

$$= \frac{\rho(\delta - c)(2\gamma(a) - 1)\begin{bmatrix}4.5b\lambda - \rho^2(\gamma(a) + 1)\big(2 - \gamma(a)\big) - \rho\big(2 - \gamma(a)\big) \\ \times [2\rho(\gamma(a) - 2) - 3(\gamma(a) - 1)]\end{bmatrix}}{AB}$$

$$= \frac{\rho(\delta - c)(2\gamma(a) - 1)\left[4.5b\lambda - \big(2 - \gamma(a)\big)\rho[\rho(\gamma(a) + 1) + 2\rho(\gamma(a) - 2) - 3(\gamma(a) - 1)]\right]}{AB}$$

$$= \frac{\rho(\delta - c)(2\gamma(a) - 1)\left[4.5b\lambda - \big(2 - \gamma(a)\big)\rho[3\rho\gamma(a) - 3\rho - 3(\gamma(a) - 1)]\right]}{AB}$$

$$= \frac{\rho(\delta - c)(2\gamma(a) - 1)\left[4.5b\lambda - \left(2 - \gamma(a)\right)\rho[3\rho(\gamma(a) - 1) - 3(\gamma(a) - 1)]\right]}{AB}$$

$$= \frac{\rho(\delta - c)(2\gamma(a) - 1)\left[4.5b\lambda - 3\left(2 - \gamma(a)\right)\rho[(\gamma(a) - 1)(\rho - 1)]\right]}{AB},$$

$$x_i^{CC} - x_i^{DC} = \frac{\rho(\delta - c)(2\gamma(a) - 1)[4.5b\lambda - 3\rho(1 - \rho)\,(2 - \gamma(a))(1 - \gamma(a))]}{AB}.$$

$x_i^{CC} - x_i^{DC} > 0$ for all $a > \frac{1-2\beta}{2l}$ and $4.5b\lambda > 3\rho(1 - \rho)\left(2 - \gamma(a)\right)(1 - \gamma(a))$.

$\lim_{\rho \to 0}\left(x_i^{CC} - x_i^{DC}\right) = 0$ while

$$\lim_{\rho \to 1}\left(x_i^{CC} - x_i^{DC}\right) = \frac{4.5b\lambda(\delta - c)(2\gamma(a) - 1)}{\left[4.5b\lambda - \left(1 + \gamma(a)\right)^2\right]\left[4.5b\lambda - \left(2 - \gamma(a)\right)^2\right]}.$$

Derivation of ($x_i^{DC} - x_i^{DD}$) under the symmetric absorptive capacity:

$x_i^{DC} - x_i^{DD}$

$$= \frac{\rho(\delta - c)\left(2 - \gamma(a)\right)\begin{bmatrix}\rho^2(2\gamma(a) - 1)(\gamma(a) + 1) + 4.5b\lambda - \rho^2(\gamma(a) + 1)^2 \\ -\rho\left(1 - \rho\right)\left((2\gamma(a) - 1)^2 + \left(2 - \gamma(a)\right)^2\right)\end{bmatrix}}{\left[4.5b\lambda - \rho^2(\gamma(a) + 1)^2 - \rho(1 - \rho)\left((2\gamma(a) - 1)^2 + \left(2 - \gamma(a)\right)^2\right)\right]\left[4.5b\lambda - \rho\left(2 - \gamma(a)\right)^2\right]}$$

$$- \frac{\rho(\delta - c)\left(2 - \gamma(a)\right)}{4.5b\lambda - \rho^2(\gamma(a) + 1)\left(2 - \gamma(a)\right) - \rho(1 - \rho)\,(2 - \gamma(a))^2}.$$

Assume $H = 4.5b\lambda - \rho^2(\gamma(a) + 1)\left(2 - \gamma(a)\right) - \rho(1 - \rho)\,(2 - \gamma(a))^2$ and

$$G = \rho^2(2\gamma(a) - 1)(\gamma(a) + 1) + 4.5b\lambda - \rho^2(\gamma(a) + 1)^2 - \rho(1 - \rho)\,((2\gamma(a) - 1)^2 + + \left(2 - \gamma(a)\right)^2) = A + \rho^2(2\gamma(a) - 1)(\gamma(a) + 1),$$

$$x_i^{DC} - x_i^{DD} = \rho(\delta - c)\left(2 - \gamma(a)\right)\left[\frac{G}{AB} - \frac{1}{H}\right] = \rho(\delta - c)\left(2 - \gamma(a)\right)\left[\frac{GH - AB}{ABH}\right],$$

$$\begin{aligned} GH - AB &= [A + \rho^2(2\gamma(a) - 1)(\gamma(a) + 1)][B + \rho^2(2\gamma(a) - 1)(\gamma(a) - 2)] - AB \\ &= AB + A\rho^2(2\gamma(a) - 1)(\gamma(a) - 2) + B\rho^2(2\gamma(a) - 1)(\gamma(a) + 1) + \\ &\quad + \rho^4(2\gamma(a) - 1)^2(\gamma(a) - 2)(\gamma(a) + 1) - AB \\ &= \rho^2(2\gamma(a) - 1)\begin{bmatrix}B(\gamma(a) + 1) - A\left(2 - \gamma(a)\right) \\ -\rho^2\left(2 - \gamma(a)\right)(\gamma(a) + 1)(2\gamma(a) - 1)\end{bmatrix}. \end{aligned}$$

$GH - AB =$

$$\rho^2(2\gamma(a)-1)\begin{bmatrix}4.5b\lambda(\gamma(a)+1)-4.5b\lambda(2-\gamma(a))-\rho(2-\gamma(a))^2(\gamma(a)+1)\\+\rho^2(\gamma(a)+1)^2(2-\gamma(a))+\ \rho(1-\rho)(2\gamma(a)-1)^2(2-\gamma(a))\\+\rho(1-\rho)(2-\gamma(a))^3-\rho^2(\gamma(a)+1)(2-\gamma(a))(2\gamma(a)-1)\end{bmatrix},$$

$GH-AB=$

$$\rho^2(2\gamma(a)-1)\begin{bmatrix}4.5b\lambda(2\gamma(a)-1)+\rho(2-\gamma(a))\begin{bmatrix}(2-\gamma(a))^2+(2\gamma(a)-1)^2\\-(\gamma(a)+1)(2-\gamma(a))\end{bmatrix}\\+\rho^2(2-\gamma(a))\begin{bmatrix}(\gamma(a)+1)^2-(2\gamma(a)-1)^2-(2-\gamma(a))^2\\-(\gamma(a)+1)(2\gamma(a)-1)\end{bmatrix}\end{bmatrix},$$

$$GH-AB=\rho^2(2\gamma(a)-1)\begin{bmatrix}4.5b\lambda(2\gamma(a)-1)\\+\rho(2-\gamma(a))[3(2\gamma(a)-1)(\gamma(a)-1)]\\-\rho^2(2-\gamma(a))[[3(2\gamma(a)-1)(\gamma(a)-1)]]\end{bmatrix}$$

$$=\rho^2(2\gamma(a)-1)^2[4.5b\lambda-3\rho(1-\rho)(2-\gamma(a))(1-\gamma(a))],$$

$x_i^{DC}-x_i^{DD}>0$ for all $a\neq\frac{1-2\beta}{2l}$ and $4.5b\lambda>3\rho(1-\rho)(2-\gamma(a))(1-\gamma(a))$.

$\lim_{\rho\to 0}(x_i^{DC}-x_i^{DD})=0$ while

$$\lim_{\rho\to 1}(x_i^{DC}-x_i^{DD})=\frac{4.5b\lambda(\delta-c)(2-\gamma(a))(2\gamma(a)-1)^2}{\left[4.5b\lambda-(1+\gamma(a))^2\right]\left[4.5b\lambda-(\gamma(a)+1)(2-\gamma(a))\right]\left[4.5b\lambda-(2-\gamma(a))^2\right]}.$$

<u>Proof of Proposition 2:</u>

Derivation of the expected profit under cooperation, non-cooperation and defection with symmetric R&D absorptive capacity:

$$E\pi_i^*(x_i^{CC},x_j^{CC})\ =\frac{1}{9b}\begin{bmatrix}\rho\rho[\delta-c+[\gamma(a)+1]x_i^{CC}]^2\\+\rho[1-\rho][\delta-c+[2-\gamma(a)]x_i^{CC}]^2\\+\rho[1-\rho][\delta-c+[2\gamma(a)-1]x_i^{CC}]^2\\+(1-\rho)(1-\rho)[\delta-c]^2\end{bmatrix}-\frac{1}{2}\lambda(x_i^{CC})^2$$

$$=\frac{1}{9b}\begin{bmatrix}\begin{bmatrix}\rho^2(\gamma(a)+1)^2\\+\rho[1-\rho]\left[(2-\gamma(a))^2+(2\gamma(a)-1)^2\right]\end{bmatrix}(x_i^{CC})^2\\-4.5b\lambda(x_i^{CC})^2+2\rho(\delta-c)(\gamma(a)+1)x_i^{CC}+[\delta-c]^2\end{bmatrix}$$

$$=\frac{1}{9b}\begin{bmatrix}2\rho(\delta-c)(\gamma(a)+1)x_i^{CC}+[\delta-c]^2\\-\begin{bmatrix}4.5b\lambda-\rho^2(\gamma(a)+1)^2\\-\rho[1-\rho]\left[(2-\gamma(a))^2+(2\gamma(a)-1)^2\right]\end{bmatrix}(x_i^{CC})^2\end{bmatrix}\qquad(7)$$

$$= \frac{1}{9b}\begin{bmatrix} 2\rho(\delta - c)(\gamma(a)+1)x_i^{CC} + [\delta - c]^2 \\ -\left[\frac{\rho(\delta - c)(\gamma(a)+1)}{x_i^{CC}}\right](x_i^{CC})^2 \end{bmatrix}$$

$$= \frac{1}{9b}\begin{bmatrix} 2\rho(\delta - c)(\gamma(a)+1)x_i^{CC} + [\delta - c]^2 \\ -[\rho(\delta - c)(\gamma(a)+1)]x_i^{CC} \end{bmatrix}$$

$$= \frac{1}{9b}\left[\rho(\delta - c)(\gamma(a)+1)x_i^{CC} + [\delta - c]^2\right].$$

(7) $\Rightarrow E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right)$

$$= \frac{1}{9b}\begin{bmatrix} 2\rho(\delta - c)(\gamma(a)+1)x_i^{DD} + [\delta - c]^2 \\ -\left[4.5b\lambda - \rho^2(\gamma(a)+1)^2 - \rho[1-\rho]\left[\left(2-\gamma(a)\right)^2 + (2\gamma(a)-1)^2\right]\right](x_i^{DD})^2 \end{bmatrix}.$$

$E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right)$

$$= \frac{1}{9b}\begin{bmatrix} \rho(2\gamma(a)-1)^2(x_j^{CD})^2 + \rho\left(2-\gamma(a)\right)^2(x_i^{DC})^2 \\ +2\rho(\delta - c)\left(2-\gamma(a)\right)x_i^{DC} + 2\rho(\delta - c)(2\gamma(a)-1)x_j^{CD} \\ +2\rho^2(2\gamma(a)-1)\left(2-\gamma(a)\right)x_i^{DC}x_j^{CD} - 4.5b\lambda(x_i^{DC})^2 + [\delta - c]^2 \end{bmatrix}$$

such that

$$4.5b\lambda - \rho\left(2-\gamma(a)\right)^2 = \frac{\left(2-\gamma(a)\right)\rho\left[(\delta - c) + x_j^{CD}\rho(2\gamma(a)-1)\right]}{x_i^{DC}}.$$

Derivation $\left[\boldsymbol{E\pi_i^*\left(x_i^{CC}, x_j^{CC}\right) - E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right)}\right]$ ***under the symmetric absorptive capacity:***

$E\pi_i^*\left(x_i^{CC}, x_j^{CC}\right) - E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right)$

$$= \frac{1}{9b}\left[2\rho(\delta - c)(\gamma(a)+1)\left[x_i^{CC} - x_i^{DD}\right] - W_1\left[(x_i^{CC})^2 - (x_i^{DD})^2\right]\right]$$

such that $W_1 = 4.5b\lambda - \rho^2(\gamma(a)+1)^2 - \rho[1-\rho]\left[\left(2-\gamma(a)\right)^2 + (2\gamma(a)-1)^2\right]$

$$= \left[\frac{\rho(\delta - c)(\gamma(a)+1)}{x_i^{CC}}\right].$$

$E\pi_i^*\left(x_i^{CC}, x_j^{CC}\right) - E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right)$

$$= \frac{1}{9b}\begin{bmatrix} 2\rho(\delta - c)(\gamma(a)+1)\left[x_i^{CC} - x_i^{DD}\right] \\ -\left[\frac{\rho(\delta - c)(\gamma(a)+1)}{x_i^{CC}}\right]\left[(x_i^{CC})^2 - (x_i^{DD})^2\right] \end{bmatrix},$$

$$= \frac{1}{9b}\rho(\delta - c)(\gamma(a)+1)\left[x_i^{CC} - x_i^{DD}\right]\left[2 - \left[\frac{(x_i^{CC} + x_i^{DD}]}{x_i^{CC}}\right]\right]$$

$$= \frac{1}{9b}\rho(\delta - c)(\gamma(a)+1)\left[\frac{(x_i^{CC} - x_i^{DD})^2}{x_i^{CC}}\right] > 0 \text{ for all } \rho > 0, a \neq \frac{1-2\beta}{2l}.$$

From the proofs of Proposition 1,

$$x_i^{CC} - x_i^{DD} = \frac{\rho(\delta - c)(2\gamma(a)-1)\left[4.5b\lambda - 3\rho(1-\rho)\big(2-\gamma(a)\big)(1-\gamma(a))\right]}{AH}.$$

Derivation $E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) - E\pi_i^*\left(x_i^{CC}, x_j^{CC}\right)$ under the symmetric absorptive capacity:

$$E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) - E\pi_i^*\left(x_i^{CC}, x_j^{CC}\right)$$

$$= \left(\frac{1}{9b}\right)\begin{bmatrix} \rho\rho\left[\delta - c + [2\gamma(a)-1]x_j^{CD} + [2-\gamma(a)]\, x_i^{DC}\right]^2 \\ +\rho[1-\rho]\left[\delta - c + [2-\gamma(a)]\, x_i^{DC}\right]^2 \\ +\rho[1-\rho]\left[\delta - c + [2\gamma(a)-1]x_j^{CD}\right]^2 \\ +(1-\rho)(1-\rho)[\delta - c]^2 \end{bmatrix} - \frac{1}{2}\lambda(x_i^{DC})^2$$

$$-\left(\frac{1}{9b}\right)\begin{bmatrix} \rho\rho\left[\delta - c + [\gamma(a)+1]x_i^{CC}\right]^2 + \rho[1-\rho]\left[\delta - c + [2-\gamma(a)]x_i^{CC}\right]^2 \\ +\rho[1-\rho]\left[\delta - c + [2\gamma(a)-1]x_i^{CC}\right]^2 + (1-\rho)(1-\rho)[\delta - c]^2 \end{bmatrix} + \frac{1}{2}\lambda(x_i^{CC})^2$$

$$= \left(\frac{1}{9b}\right)\left[\rho^2\begin{bmatrix} 2(\delta - c) + 2[2\gamma(a)-1]x_j^{CD} \\ +[2-\gamma(a)]\,(x_i^{DC} + x_i^{CC}) \end{bmatrix}[2-\gamma(a)](x_i^{DC} - x_i^{CC})\right]$$

$$-4.5b\lambda(x_i^{DC} + x_i^{CC})(x_i^{DC} - x_i^{CC}) + \rho[1-\rho][2-\gamma(a)](x_i^{DC} - x_i^{CC})\begin{bmatrix} 2(\delta - c) \\ +[2-\gamma(a)]\,(x_i^{DC} + x_i^{CC}) \end{bmatrix},$$

$$E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) - E\pi_i^*\left(x_i^{CC}, x_j^{CC}\right)$$

$$= \left(\frac{1}{9b}\right)(x_i^{DC} - x_i^{CC})\begin{bmatrix} 2\rho^2(\delta - c)[2-\gamma(a)] + 2\rho^2[2\gamma(a)-1][2-\gamma(a)]x_j^{CD} \\ +\rho^2\big(2-\gamma(a)\big)^2(x_i^{DC} + x_i^{CC}) - 4.5b\lambda(x_i^{DC} + x_i^{CC}) \\ +2\rho(\delta - c)[2-\gamma(a)] + \rho\big(2-\gamma(a)\big)^2\,(x_i^{DC} + x_i^{CC}) \\ -2\rho^2(\delta - c)[2-\gamma(a)] - \rho^2\big(2-\gamma(a)\big)^2\,(x_i^{DC} + x_i^{CC}) \end{bmatrix}. \qquad \textbf{(8)}$$

Given

$$x_i^{DC} = \frac{\big(2-\gamma(a)\big)\rho\left[(\delta - c) + x_j^{CD}\rho(2\gamma(a)-1)\right]}{4.5b\lambda - \rho\big(2-\gamma(a)\big)^2} \Leftrightarrow$$

$$2\rho(\delta - c)[2-\gamma(a)] + 2\rho^2[2\gamma(a)-1][2-\gamma(a)]x_j^{CD}$$

$$= 2\left[4.5b\lambda - \rho\big(2-\gamma(a)\big)^2\right]x_i^{DC},$$

expression **(8)** becomes:

$$E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) - E\pi_i^*\left(x_i^{CC}, x_j^{CC}\right)$$

$$= \left(\frac{1}{9b}\right)(x_i^{DC} - x_i^{CC})\begin{bmatrix} 2\left[4.5b\lambda - \rho\left(2-\gamma(a)\right)^2\right]x_i^{DC} \\ -\left[4.5b\lambda - \rho\left(2-\gamma(a)\right)^2\right]\ (x_i^{DC} + x_i^{CC}) \end{bmatrix}$$

$$= \left(\frac{1}{9b}\right)(x_i^{DC} - x_i^{CC})\left[\left[4.5b\lambda - \rho\left(2-\gamma(a)\right)^2\right]\ (x_i^{DC} - x_i^{CC})\right]$$

$$= \left(\frac{1}{9b}\right)\left[4.5b\lambda - \rho\left(2-\gamma(a)\right)^2\right](x_i^{CC} - x_i^{DC})^2.$$

From the proofs of Proposition 1,

$$x_i^{CC} - x_i^{DC} = \frac{\rho(\delta - c)(2\gamma(a) - 1)\left[4.5b\lambda - 3\rho(1-\rho)\left(2-\gamma(a)\right)\left(1-\gamma(a)\right)\right]}{AB}.$$

$E\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) - E\pi_i^*\left(x_i^{CC}, x_j^{CC}\right) > 0$ for all $\rho > 0, a \neq \frac{1-2\beta}{2l}$.

Note that $4.5b\lambda - \rho[2-\gamma(a)]^2 > 0$ that is verified for which the second derivative of the expected profit is negative.

Derivation $E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) - E\pi_i^*\left(x_i^{CD}, x_j^{DC}\right)$ under the symmetric absorptive capacity:

$E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) - E\pi_i^*\left(x_i^{CD}, x_j^{DC}\right)$

$$= \left(\frac{1}{9b}\right)\begin{bmatrix} \rho\rho\begin{bmatrix} \delta - c + [2\gamma(a) - 1]x_j^{DD} \\ +[2-\gamma(a)]\ x_i^{DD} \end{bmatrix}^2 \\ +\rho[1-\rho][\delta - c + [2-\gamma(a)]\ x_i^{DD}]^2 \\ +\rho[1-\rho]\left[\delta - c + [2\gamma(a) - 1]x_j^{DD}\right]^2 \\ +(1-\rho)(1-\rho)[\delta - c]^2 \end{bmatrix} - \frac{1}{2}\lambda(x_i^{DD})^2$$

$$-\left(\frac{1}{9b}\right)\begin{bmatrix} \rho\rho\left[\delta - c + [2\gamma(a) - 1]x_j^{DC} + [2-\gamma(a)]\ x_i^{CD}\right]^2 \\ +\rho[1-\rho]\left[\delta - c + [2-\gamma(a)]x_i^{CD}\right]^2 \\ +\rho[1-\rho]\left[\delta - c + [2\gamma(a) - 1]x_j^{DC}\right]^2 \\ +(1-\rho)(1-\rho)[\delta - c]^2 \end{bmatrix} + \frac{1}{2}\lambda(x_i^{CD})^2$$

$$= \left(\frac{1}{9b}\right)\begin{bmatrix} \rho^2\begin{bmatrix} [2\gamma(a) - 1](x_j^{DD} - x_j^{DC}) \\ +[2-\gamma(a)](x_i^{DD} - x_i^{CD}) \end{bmatrix}\begin{bmatrix} 2(\delta - c) + [2\gamma(a) - 1](x_j^{DD} + x_j^{DC}) \\ +[2-\gamma(a)](x_i^{DD} + x_i^{CD}) \end{bmatrix} \\ +(1-\rho)\rho[2-\gamma(a)](x_i^{DD} - x_i^{CD})\left[2(\delta - c) + [2-\gamma(a)](x_i^{DD} + x_i^{CD})\right] \\ +(1-\rho)\rho[2\gamma(a) - 1](x_j^{DD} - x_j^{DC})\left[2(\delta - c) + [2\gamma(a) - 1](x_j^{DD} + x_j^{DC})\right] \\ +4.5b\lambda\left[(x_i^{CD})^2 - (x_i^{DD})^2\right] \end{bmatrix}$$

$$= \left(\frac{1}{9b}\right)\begin{bmatrix} \rho^2 \begin{bmatrix} 2(\delta - c)[2\gamma(a)-1](x_j^{DD} - x_j^{DC}) + (2\gamma(a)-1)^2\left[\left(x_j^{DD}\right)^2 - \left(x_j^{DC}\right)^2\right] \\ +(2\gamma(a)-1)\left(2-\gamma(a)\right)\left(x_j^{DD} - x_j^{DC}\right)(x_i^{CD} + x_i^{DD}) \\ +2(\delta - c)[2-\gamma(a)](x_i^{DD} - x_i^{CD}) + (2\gamma(a)-1)\left(2-\gamma(a)\right)\left(x_i^{DD} + x_i^{DC}\right)(x_i^{DD} - x_i^{CD}) \\ +\left(2-\gamma(a)\right)^2\left[(x_i^{DD})^2 - \left(x_i^{CD}\right)^2\right] \end{bmatrix} \\ +(1-\rho)\rho[2-\gamma(a)](x_i^{DD} - x_i^{CD})2(\delta - c) + (1-\rho)\rho\left(2-\gamma(a)\right)^2\left[(x_i^{DD})^2 - \left(x_i^{CD}\right)^2\right] \\ +(1-\rho)\rho[2\gamma(a)-1](x_i^{DD} - x_i^{DC})2(\delta - c) + (1-\rho)\rho\left[(2\gamma(a)-1)^2\left[(x_i^{DD})^2 - \left(x_i^{DC}\right)^2\right]\right] \\ +4.5b\lambda\left[(x_i^{CD})^2 - (x_i^{DD})^2\right] \end{bmatrix},$$

$$= \frac{1}{9b}\begin{bmatrix} 2\rho(\delta - c)[2\gamma(a)-1](x_i^{DD} - x_i^{DC}) + \rho(2\gamma(a)-1)^2\left[(x_i^{DD})^2 - \left(x_i^{DC}\right)^2\right] \\ +\rho^2(2\gamma(a)-1)\left(2-\gamma(a)\right)\left[\left(x_i^{DD} - x_i^{DC}\right)(x_i^{CC} + x_i^{DD}) + \left(x_i^{DD} + x_i^{DC}\right)(x_i^{DD} - x_i^{CD})\right] \\ +2\rho(\delta - c)[2-\gamma(a)](x_i^{DD} - x_i^{CD}) + \rho\left(2-\gamma(a)\right)^2\left[(x_i^{DD})^2 - \left(x_i^{CD}\right)^2\right] \\ +4.5b\lambda\left[(x_i^{CD})^2 - (x_i^{DD})^2\right] \end{bmatrix}.$$

I already found that:

$$x_i^{CC} - x_i^{DC} = \frac{\rho(\delta - c)(2\gamma(a)-1)\left[4.5b\lambda - 3\rho(1-\rho)\left(2-\gamma(a)\right)\left(1-\gamma(a)\right)\right]}{AB},$$

$\left(x_i^{CD} = x_i^{CC}\right)$.

Assume $K = [4.5b\lambda - 3\rho(1-\rho)\,(2-\gamma(a))(1-\gamma(a))]$ and $W = x_i^{CC} - x_i^{DC}$,

$$x_i^{CC} - x_i^{DC} = \frac{\rho(\delta - c)(2\gamma(a)-1)K}{AB} = W.$$

$$(x_i^{DD})^2 - \left(x_i^{DC}\right)^2 = (x_i^{DD})^2 - \left(x_i^{CC} - W\right)^2 = (x_i^{DD})^2 - \left((x_i^{CC})^2 - 2Wx_i^{CC} + W^2\right)$$

$$= (x_i^{DD})^2 - \left(x_i^{CC}\right)^2 + 2Wx_i^{CC} - W^2 = (x_i^{DD})^2 - \left(x_i^{CC}\right)^2 + W\left(2x_i^{CC} - W\right)$$

$$= (x_i^{DD})^2 - \left(x_i^{CC}\right)^2 + W\left(2x_i^{CC} - x_i^{CC} + x_i^{DC}\right) = (x_i^{DD})^2 - \left(x_i^{CC}\right)^2 + W\left(x_i^{CC} + x_i^{DC}\right).$$

$$E\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) - E\pi_i^*\left(x_i^{CD}, x_j^{DC}\right)$$

$$= \frac{1}{9b}\begin{bmatrix} 2\rho(\delta-c)\big[2\gamma(a)-1\big](x_i^{DD}-x_i^{CD}\big)+2\rho(\delta-c)(2\gamma(a)-1)W \\ +\rho(2\gamma(a)-1)^2\left[(x_i^{DD})^2-\left(x_i^{CD}\right)^2\right]+\rho(2\gamma(a)-1)^2W\left(x_i^{CD}+x_i^{DC}\right) \\ +2\rho^2(2\gamma(a)-1)\big(2-\gamma(a)\big)\left[(x_i^{DD})^2-\left(x_i^{CD}\right)^2\right] \\ +2\rho^2(2\gamma(a)-1)\big(2-\gamma(a)\big)Wx_i^{CD}+2\rho(\delta-c)\big[2-\gamma(a)\big](x_i^{DD}-x_i^{CD}) \\ +\rho\big(2-\gamma(a)\big)^2\left[(x_i^{DD})^2-\left(x_i^{CD}\right)^2\right]+4.5b\lambda\big[(x_i^{CD})^2-(x_i^{DD})^2\big] \end{bmatrix}$$

$$= \frac{1}{9b}\begin{bmatrix} 2\rho(\delta-c)\big[[\gamma(a)+1](x_i^{DD}-x_i^{CD}\big)+(2\gamma(a)-1)W\big] \\ +\rho(2\gamma(a)-1)^2W\left(x_i^{CD}+x_i^{DC}\right)+2\rho^2(2\gamma(a)-1)\big(2-\gamma(a)\big)Wx_i^{CD} \\ +\big[(x_i^{CD})^2-(x_i^{DD})^2\big]\left[4.5b\lambda-\rho(2\gamma(a)-1)^2-2\rho^2(2\gamma(a)-1)(2-a_i)-\rho\big(2-\gamma(a)\big)^2\right] \end{bmatrix}.$$

I have:

$$x_i^{CC}-x_i^{DD} = \frac{\rho(\delta-c)(2\gamma(a)-1)\big[4.5b\lambda-3\rho(1-\rho)\big(2-\gamma(a)\big)\big(1-\gamma(a)\big)\big]}{AH}$$

$$= \frac{\rho(\delta-c)(2\gamma(a)-1)K}{AH},$$

$$x_i^{CC} = \frac{\rho(\delta-c)(\gamma(a)+1)}{4.5b\lambda-\rho^2\big(1+\gamma(a)\big)^2-\rho(1-\rho)\ \big((2\gamma(a)-1)^2+\ \big(2-\gamma(a)\big)^2\big)}$$

$$= \frac{\rho(\delta-c)(\gamma(a)+1)}{4.5b\lambda-\rho(2\gamma(a)-1)^2-2\rho^2(2\gamma(a)-1)\big(2-\gamma(a)\big)-\rho\big(2-\gamma(a)\big)^2}$$

$$= \frac{\rho(\delta-c)(\gamma(a)+1)}{A},$$

$$x_i^{CC}+x_i^{DC} = \frac{\rho(\delta-c)(\gamma(a)+1)}{A}$$

$$+\frac{\rho(\delta-c)\big(2-\gamma(a)\big)[\rho^2(2\gamma(a)-1)(\gamma(a)+1)+A]}{AB}$$

$$= \frac{\rho(\delta-c)(\gamma(a)+1)B}{AB}$$

$$+\frac{\rho(\delta-c)\big(2-\gamma(a)\big)[\rho^2(2\gamma(a)-1)(\gamma(a)+1)+A]}{AB}$$

$$= \frac{\rho(\delta-c)}{AB}\Big[(\gamma(a)+1)B+\big(2-\gamma(a)\big)[\rho^2(2\gamma(a)-1)(\gamma(a)+1)+A]\Big],$$

$$x_i^{CC}+x_i^{DD} = \frac{\rho(\delta-c)(\gamma(a)+1)}{A}+\frac{\rho(\delta-c)\big(2-\gamma(a)\big)}{H}$$

$$= \frac{\rho(\delta - c)(\gamma(a)+1)H + \rho(\delta - c)\big(2-\gamma(a)\big)A}{AH}$$

$$= \frac{\rho(\delta - c)\big[(\gamma(a)+1)H + \big(2-\gamma(a)\big)A\big]}{AH},$$

$E\pi_i^*\big(x_i^{DD}, x_j^{DD}\big) - E\pi_i^*\big(x_i^{CD}, x_j^{DC}\big)$

$$= \frac{1}{9b}\begin{bmatrix} 2\rho(\delta - c)\big[[\gamma(a)+1](x_i^{DD} - x_i^{CD}) + (2\gamma(a)-1)W\big] \\ +\rho(2\gamma(a)-1)^2 W\big(x_i^{CD} + x_i^{DC}\big) + 2\rho^2(2\gamma(a)-1)\big(2-\gamma(a)\big)W x_i^{CD} \\ +\big[(x_i^{CD})^2 - (x_i^{DD})^2\big]\Big[4.5b\lambda - \rho(2\gamma(a)-1)^2 - 2\rho^2(2\gamma(a)-1)\big(2-\gamma(a)\big) - \rho\big(2-\gamma(a)\big)^2\Big] \end{bmatrix}.$$

Note that $x_i^{CD} = x_i^{CC}$.

By replacing W, x_i^{CD}, $\big(x_i^{CD} + x_i^{DC}\big), (x_i^{CD} + x_i^{DD}), (x_i^{CD} - x_i^{DD})$ by their expressions, I get the following results:

$E\pi_i^*\big(x_i^{DD}, x_j^{DD}\big) - E\pi_i^*\big(x_i^{CD}, x_j^{DC}\big)$

$$= \frac{1}{9b}\rho^2(\delta - c)^2(2\gamma(a)-1)\frac{K}{A}$$

$$\times \begin{bmatrix} \left(-\frac{2(\gamma(a)+1)}{H} + \frac{2(2\gamma(a)-1)}{B}\right) \\ +\left(\frac{\rho(2\gamma(a)-1)^2\Big[(\gamma(a)+1)B + \Big(\big(2-\gamma(a)\big)\rho^2(2\gamma(a)-1)(\gamma(a)+1)\Big) + \big(2-\gamma(a)\big)A\Big]}{AB^2}\right) \\ +\left(2(2\gamma(a)-1)\big(2-\gamma(a)\big)\rho^2\frac{(\gamma(a)+1)}{AB}\right) + \left(\frac{(\gamma(a)+1)}{H} + \frac{\big(2-\gamma(a)\big)A}{H^2}\right) \end{bmatrix} \quad \mathbf{(9)}$$

$$= \frac{1}{9b}\rho^2(\delta - \mathrm{c})^2(2\gamma(a)-1)\frac{K}{A}$$

$$\times \begin{bmatrix} \frac{2(2\gamma(a)-1)}{B} \\ +\left(\frac{\rho(2\gamma(a)-1)^2\Big[(\gamma(a)+1)B + \Big(\big(2-\gamma(a)\big)\rho^2(2\gamma(a)-1)(\gamma(a)+1)\Big) + \big(2-\gamma(a)\big)A\Big]}{AB^2}\right) \\ +\left(2(2\gamma(a)-1)\big(2-\gamma(a)\big)\rho^2\frac{(\gamma(a)+1)}{AB}\right) - \frac{(\gamma(a)+1)}{H} + \frac{\big(2-\gamma(a)\big)A}{H^2} \end{bmatrix},$$

$$\frac{2(2\gamma(a)-1)}{B} + 2(2\gamma(a)-1)\big(2-\gamma(a)\big)\rho^2\frac{(\gamma(a)+1)}{AB}$$

$$= \frac{\begin{bmatrix} 2(2\gamma(a)-1)A \\ +2(2\gamma(a)-1)\big(2-\gamma(a)\big)(\gamma(a)+1)\rho^2 \end{bmatrix}}{AB}$$

$$= \frac{2(2\gamma(a)-1)\left[A+\left(2-\gamma(a)\right)(\gamma(a)+1)\rho^2\right]}{AB},$$

$$-\frac{(\gamma(a)+1)}{H}+\frac{\left(2-\gamma(a)\right)A}{H^2}=\frac{-(\gamma(a)+1)H+\left(2-\gamma(a)\right)A}{H^2}, \qquad (\mathbf{10})$$

$$H-A=\rho(2\gamma(a)-1)\left[(2\gamma(a)-1)+\rho\left(2-\gamma(a)\right)\right] \Leftrightarrow$$

$$H=A+\rho(2\gamma(a)-1)\left[(2\gamma(a)-1)+\rho\left(2-\gamma(a)\right)\right]$$

Expression ($\mathbf{10}$) becomes:

$$\frac{-(\gamma(a)+1)H+\left(2-\gamma(a)\right)A}{H^2}$$

$$=\frac{-(\gamma(a)+1)\left(A+\rho(2\gamma(a)-1)\left[(2\gamma(a)-1)+\rho\left(2-\gamma(a)\right)\right]\right)+\left(2-\gamma(a)\right)A}{H^2}$$

$$=\frac{-(\gamma(a)+1)\left(\rho(2\gamma(a)-1)\left[(2\gamma(a)-1)+\rho\left(2-\gamma(a)\right)\right]\right)+\left(1-2\gamma(a)\right)A}{H^2}$$

$$=\frac{-(\gamma(a)+1)\rho(2\gamma(a)-1)\left[(2\gamma(a)-1)+\rho\left(2-\gamma(a)\right)\right]-(2\gamma(a)-1)A}{H^2}$$

$$=\frac{-(2\gamma(a)-1)\left[A+(\gamma(a)+1)\rho\left[(2\gamma(a)-1)+\rho\left(2-\gamma(a)\right)\right]\right]}{H^2}$$

$$=\frac{-(2\gamma(a)-1)[4.5b\lambda-3\rho(1-\rho)\,(2-\gamma(a))(1-\gamma(a))]}{H^2}=\frac{-(2\gamma(a)-1)K}{H^2}.$$

Expression ($\mathbf{9}$) becomes:

$$E\pi_i^*\left(x_i^{DD},x_j^{DD}\right)-E\pi_i^*\left(x_i^{CD},x_j^{DC}\right)=\frac{1}{9b}\rho^2(\delta-c)^2(2\gamma(a)-1)\frac{K}{A}$$

$$\times\left[\begin{array}{l}\left(\dfrac{2(2\gamma(a)-1)\left[A+\left(2-\gamma(a)\right)(\gamma(a)+1)\rho^2\right]}{AB}\right)\\ +\left(\dfrac{\rho(2\gamma(a)-1)^2\left[(\gamma(a)+1)B+\left(\left(2-\gamma(a)\right)\rho^2(2\gamma(a)-1)(\gamma(a)+1)\right)+\left(2-\gamma(a)\right)A\right]}{AB^2}\right)\\ -\dfrac{(2\gamma(a)-1)K}{H^2}\end{array}\right].(\mathbf{11})$$

Assume $\Delta=A+\left(2-\gamma(a)\right)(\gamma(a)+1)\rho^2$,

$Z=(\gamma(a)+1)B+\left(\left(2-\gamma(a)\right)\rho^2(2\gamma(a)-1)(\gamma(a)+1)\right)+\left(2-\gamma(a)\right)A.$

Expression ($\mathbf{11}$) becomes:

$$E\pi_i^*\left(x_i^{DD},x_j^{DD}\right)-E\pi_i^*\left(x_i^{CD},x_j^{DC}\right)$$

$$= \frac{1}{9b}\rho^2(\delta - c)^2(2\gamma(a) - 1)\frac{K}{A}\left[\begin{array}{c}\left(\frac{2(2\gamma(a) - 1)\Delta}{AB}\right) \\ + \left(\frac{\rho(2\gamma(a) - 1)^2 Z}{AB^2}\right) - \frac{(2\gamma(a) - 1)K}{H^2}\end{array}\right]$$

$$= \frac{1}{9b}\rho^2(\delta - c)^2(2\gamma(a) - 1)^2\frac{K}{A}\left[\left(\frac{2\Delta}{AB}\right) + \left(\frac{\rho(2\gamma(a) - 1)Z}{AB^2}\right) - \frac{K}{H^2}\right]$$

$$= \frac{1}{9b}\rho^2(\delta - c)^2(2\gamma(a) - 1)^2\frac{K}{A}\left(\frac{2\Delta BH^2 + \rho(2\gamma(a) - 1)ZH^2 - KAB^2}{AB^2H^2}\right) > 0$$

$$= \frac{1}{9b}\rho^2(\delta - c)^2(2\gamma(a) - 1)^2\frac{K^2}{A^2H^2}\left(\frac{2\Delta BH^2 + \rho(2\gamma(a) - 1)ZH^2 - KAB^2}{KB^2}\right) > 0$$

$$= \frac{1}{9b}\left(x_i^{CC} - x_i^{DD}\right)^2\left(\frac{2\Delta BH^2 + \rho(2\gamma(a) - 1)ZH^2 - KAB^2}{KB^2}\right) > 0$$

$U = \frac{2\Delta BH^2 + \rho(2\gamma(a) - 1)ZH^2 - KAB^2}{KB^2} > 0$ given the numerical simulations.

Lemma A.2: For all $\delta > c$ and for large enough $b\lambda$,

1) The optimal R&D expenditures of firm i and firm j under mutual defection are expressed as follows:

$$x_i^{DD} = \frac{(\delta - c)\left(2 - \gamma(a_j)\right)\left[4.5b\lambda - \left(2 - \gamma(a_i)\right)^2 + (2\gamma(a_i) - 1)(2 - \gamma(a_i))\right]}{\left[\begin{array}{c}\left[4.5b\lambda - \left(2 - \gamma(a_j)\right)^2\right]\left[4.5b\lambda - \left(2 - \gamma(a_i)\right)^2\right] \\ -(2\gamma(a_i) - 1)\left(2 - \gamma(a_j)\right)(2 - \gamma(a_i))(2\gamma(a_j) - 1)\end{array}\right]},$$

$$x_j^{DD} = \frac{(\delta - c)(2 - \gamma(a_i))\left[4.5b\lambda - \left(2 - \gamma(a_j)\right)^2 + (2\gamma(a_j) - 1)(2 - \gamma(a_j))\right]}{\left[\begin{array}{c}\left[4.5b\lambda - \left(2 - \gamma(a_j)\right)^2\right]\left[4.5b\lambda - \left(2 - \gamma(a_i)\right)^2\right] \\ -(2\gamma(a_i) - 1)\left(2 - \gamma(a_j)\right)(2 - \gamma(a_i))(2\gamma(a_j) - 1)\end{array}\right]}.$$

2) The optimal cooperative R&D expenditures of firm i and firm j under mutual cooperation are expressed as follows:

$$x_i^{CC} = \frac{(\delta - c)\left[E\left(\gamma(a_j) + 1\right) + \left[\begin{array}{c}[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] \\ + \left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\end{array}\right](\gamma(a_i) + 1)\right]}{EF - \left[[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] + \left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\right]^2},$$

$$x_j^{CC} = \frac{(\delta - c)\left[F(\gamma(a_i)+1) + \begin{bmatrix} [2\gamma(a_i)-1][2-\gamma(a_j)] \\ +[2\gamma(a_j)-1][2-\gamma(a_i)] \end{bmatrix}(\gamma(a_j)+1)\right]}{EF - \left[[2\gamma(a_i)-1][2-\gamma(a_j)] + [2\gamma(a_j)-1][2-\gamma(a_i)]\right]^2}.$$

3) The optimal R&D expenditures of firm i and firm j under unilateral defection are expressed as follows:

$$x_i^{DC} = \frac{\left(2-\gamma(a_j)\right)(\delta - c)}{4.5b\lambda - \left(2-\gamma(a_j)\right)^2}\left[\frac{\begin{matrix} F\left[E + (2\gamma(a_i)-1)[1+\gamma(a_i)]\right] \\ -3T\left(2\gamma(a_j)-1\right)\left(1-\gamma(a_i)\right)\end{matrix}}{[EF - T^2]}\right],$$

$$x_j^{DC} = \frac{\left(2-\gamma(a_i)\right)(\delta - c)}{4.5b\lambda - \left(2-\gamma(a_i)\right)^2}\left[\frac{\begin{matrix} E\left[F + \left(2\gamma(a_j)-1\right)\left[1+\gamma(a_j)\right]\right] \\ -3T(2\gamma(a_i)-1)\left(1-\gamma(a_j)\right)\end{matrix}}{[EF - T^2]}\right].$$

<u>Proof of Lemma A.2:</u>

Derivation of x_i^{DD} under the deterministic R&D technology with asymmetric absorptive capacity:

$$Max\ \pi_i^* = b(q_i^*)^2 - \frac{1}{2}\lambda x_i^2 = b\left[\frac{\delta - c + [2\gamma(a_i) - 1]x_j + [2-\gamma(a_j)]x_i}{3b}\right]^2 - \frac{1}{2}\lambda x_i^2$$

subject to x_i, x_j and $\pi_i^* \geq 0$.

$$\frac{\partial \pi_i^*}{\partial x_i} = 0 \Leftrightarrow \left[\delta - c + [2\gamma(a_i) - 1]x_j + [2-\gamma(a_j)]x_i\right]\left[2-\gamma(a_j)\right] - 4.5b\lambda x_i = 0$$

$$\Leftrightarrow [\delta - c]\left[2-\gamma(a_j)\right] + [2\gamma(a_i) - 1]\left[2-\gamma(a_j)\right]x_j = \left[4.5b\lambda - \left[2-\gamma(a_j)\right]^2\right]x_i$$

$$\Leftrightarrow x_i = \frac{[\delta - c]\left[2-\gamma(a_j)\right] + [2\gamma(a_i) - 1]\left[2-\gamma(a_j)\right]x_j}{\left[4.5b\lambda - \left[2-\gamma(a_j)\right]^2\right]}. \qquad (\mathbf{12})$$

Similarly, the optimal x_j for firm j is expressed as follows:

$$x_j = \frac{[\delta - c][2-\gamma(a_i)] + \left[2\gamma(a_j) - 1\right][2-\gamma(a_i)]x_i}{[4.5b\lambda - [2-\gamma(a_i)]^2]}. \qquad (\mathbf{13})$$

Given $(\mathbf{12})$ and $(\mathbf{13})$,

$$x_i = \frac{[\delta - c]\left[2-\gamma(a_j)\right]}{\left[4.5b\lambda - \left[2-\gamma(a_j)\right]^2\right]}$$

$$+\frac{[2\gamma(a_i)-1]\big[2-\gamma(a_j)\big]}{\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big]}\left[\frac{[\delta-c][2-\gamma(a_i)]+[2\gamma(a_j)-1][2-\gamma(a_i)]x_i}{[4.5b\lambda-[2-\gamma(a_i)]^2]}\right]$$

$$=\frac{[\delta-c]\big[2-\gamma(a_j)\big]}{\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big]}+\frac{[2\gamma(a_i)-1]\big[2-\gamma(a_j)\big][\delta-c][2-\gamma(a_i)]}{\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big][4.5b\lambda-[2-\gamma(a_i)]^2]}$$

$$+\left[\frac{[2\gamma(a_i)-1]\big[2-\gamma(a_j)\big][2\gamma(a_j)-1][2-\gamma(a_i)]x_i}{[4.5b\lambda-[2-\gamma(a_i)]^2]\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big]}\right]$$

$$\Leftrightarrow\left[\frac{[4.5b\lambda-[2-\gamma(a_i)]^2]\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big]-[2\gamma(a_i)-1]\big[2-\gamma(a_j)\big][2\gamma(a_j)-1][2-\gamma(a_i)]}{[4.5b\lambda-[2-\gamma(a_i)]^2]\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big]}\right]x_i$$

$$=\frac{[\delta-c]\big[2-\gamma(a_j)\big][4.5b\lambda-[2-\gamma(a_i)]^2]+[2\gamma(a_i)-1]\big[2-\gamma(a_j)\big][\delta-c][2-\gamma(a_i)]}{\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big][4.5b\lambda-[2-\gamma(a_i)]^2]},$$

$$x_i^{DD}$$

$$=\frac{[\delta-c]\big[2-\gamma(a_j)\big]\big[[4.5b\lambda-[2-\gamma(a_i)]^2]+[2\gamma(a_i)-1][2-\gamma(a_i)]\big]}{[4.5b\lambda-[2-\gamma(a_i)]^2]\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big]-[2\gamma(a_i)-1]\big[2-\gamma(a_j)\big][2\gamma(a_j)-1][2-\gamma(a_i)]}.$$

Similarly,

$$x_j^{DD}$$

$$=\frac{[\delta-c][2-\gamma(a_i)]\Big[\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big]+\big[2\gamma(a_j)-1\big]\big[2-\gamma(a_j)\big]\Big]}{[4.5b\lambda-[2-\gamma(a_i)]^2]\Big[4.5b\lambda-\big[2-\gamma(a_j)\big]^2\Big]-[2\gamma(a_i)-1]\big[2-\gamma(a_j)\big][2\gamma(a_j)-1][2-\gamma(a_i)]}.$$

$$\frac{\partial\pi_i^*}{\partial x_i}=\left(\frac{2}{9b}\right)\big[\delta-c+[2\gamma(a_i)-1]x_j+[2-\gamma(a_j)]x_i\big]\big[2-\gamma(a_j)\big]-\lambda x_i,$$

To ensure the concavity of π_i^* with respect to x_i,

$$\frac{\partial^2\pi_i^*}{\partial x_i^2}<0\Leftrightarrow 4.5b\lambda-[2-\gamma(a_i)]^2>0.$$

Similarly, $\frac{\partial^2\pi_j^*}{\partial x_j^2}<0\Leftrightarrow 4.5b\lambda-\big[2-\gamma(a_j)\big]^2>0.$

Derivation of x_i^{CC} under the deterministic R&D technology with asymmetric absorptive capacity:

$$\pi_i^*+\pi_j^*=\frac{1}{9b}\big[\delta-c+[2\gamma(a_i)-1]x_j+[2-\gamma(a_j)]x_i\big]^2-\frac{1}{2}\lambda x_i^2$$

$$+\frac{1}{9b}\big[\delta-c+[2\gamma(a_j)-1]x_i+[2-\gamma(a_i)]x_j\big]^2-\frac{1}{2}\lambda x_j^2,$$

$$\frac{\partial(\pi_i^* + \pi_j^*)}{\partial x_i} = 0$$

$$\Leftrightarrow \left[\delta - c + [2\gamma(a_i) - 1]x_j + [2 - \gamma(a_j)]x_i\right]\left[2 - \gamma(a_j)\right] - 4.5b\lambda x_i$$

$$+\left[\delta - c + [2\gamma(a_j) - 1]x_i + [2 - \gamma(a_i)]x_j\right]\left[2\gamma(a_j) - 1\right] = 0$$

$$\Leftrightarrow [\delta - c]\left[2 - \gamma(a_j)\right] + \left[[2\gamma(a_i) - 1]x_j\right]\left[2 - \gamma(a_j)\right] + \left[2 - \gamma(a_j)\right]^2 x_i$$

$$+[\delta - c]\left[2\gamma(a_j) - 1\right] + \left[2\gamma(a_j) - 1\right]^2 x_i + \left[[2\gamma(a_j) - 1]x_j\right]\left[2 - \gamma(a_i)\right] = 4.5b\lambda x_i$$

$$\Leftrightarrow [\delta - c]\left[1 + \gamma(a_j)\right] + \left[[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] + \left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\right] x_j$$

$$= \left[4.5b\lambda - \left[2 - \gamma(a_j)\right]^2 - \left[2\gamma(a_j) - 1\right]^2\right] x_i$$

$$\Leftrightarrow x_i^{CC} = \frac{[\delta - c]\left[1 + \gamma(a_j)\right] + \left[[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] + \left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\right] x_j^{CC}}{4.5b\lambda - \left[2 - \gamma(a_j)\right]^2 - \left[2\gamma(a_j) - 1\right]^2}.$$

Similarly,

$$x_j^{CC} = \frac{[\delta - c][1 + \gamma(a_i)] + \left[[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] + \left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\right] x_i^{CC}}{4.5b\lambda - [2 - \gamma(a_i)]^2 - [2\gamma(a_i) - 1]^2}.$$

Assume:

$E = 4.5b\lambda - [2 - \gamma(a_i)]^2 - [2\gamma(a_i) - 1]^2,$

$F = 4.5b\lambda - \left[2 - \gamma(a_j)\right]^2 - \left[2\gamma(a_j) - 1\right]^2,$

$$x_i^{CC} = \frac{[\delta - c]\left[1 + \gamma(a_j)\right] + \left[[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] + \left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\right] x_j^{CC}}{F}$$

$$= \frac{[\delta - c]E\left[1 + \gamma(a_j)\right]}{EF} + \frac{\left[[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] + \left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\right]}{F}$$

$$\times \frac{\left[[\delta - c][1 + \gamma(a_i)] + \begin{bmatrix}[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] \\ +\left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\end{bmatrix} x_i^{CC}\right]}{E}$$

$$\Leftrightarrow EFx_i^{CC} = [\delta - c]E\left[1 + \gamma(a_j)\right] + \left[[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] + \left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\right]$$

$$\times \left[[\delta - c][1 + \gamma(a_i)] + \begin{bmatrix}[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] \\ +\left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\end{bmatrix} x_i^{CC}\right]$$

$$\Leftrightarrow EFx_i^{CC} = [\delta - c]E\left[1 + \gamma(a_j)\right] + [\delta - c][1 + \gamma(a_i)]\begin{bmatrix}[2\gamma(a_i) - 1]\left[2 - \gamma(a_j)\right] \\ +\left[2\gamma(a_j) - 1\right][2 - \gamma(a_i)]\end{bmatrix}$$

$$+\left[\left[[2\gamma(a_i)-1]\left[2-\gamma(a_j)\right]+\left[2\gamma(a_j)-1\right][2-\gamma(a_i)]\right]\right]^2 x_i^{CC}$$

$$\Leftrightarrow \left[EF-\left[\left[[2\gamma(a_i)-1]\left[2-\gamma(a_j)\right]+\left[2\gamma(a_j)-1\right][2-\gamma(a_i)]\right]\right]^2\right] x_i^{CC}$$

$$=[\delta-c]\left[E\left[1+\gamma(a_j)\right]+[1+\gamma(a_i)]\begin{bmatrix}[2\gamma(a_i)-1]\left[2-\gamma(a_j)\right]\\+\left[2\gamma(a_j)-1\right][2-\gamma(a_i)]\end{bmatrix}\right]$$

$$\Leftrightarrow x_i^{CC}=\frac{[\delta-c]\left[E\left[1+\gamma(a_j)\right]+[1+\gamma(a_i)]\begin{bmatrix}[2\gamma(a_i)-1]\left[2-\gamma(a_j)\right]\\+\left[2\gamma(a_j)-1\right][2-\gamma(a_i)]\end{bmatrix}\right]}{\left[EF-\left[\left[[2\gamma(a_i)-1]\left[2-\gamma(a_j)\right]+\left[2\gamma(a_j)-1\right][2-\gamma(a_i)]\right]\right]^2\right]}.$$

Similarly,

$$x_j^{CC}=\frac{[\delta-c]\left[F[1+\gamma(a_i)]+\left[1+\gamma(a_j)\right]\begin{bmatrix}[2\gamma(a_i)-1]\left[2-\gamma(a_j)\right]\\+\left[2\gamma(a_j)-1\right][2-\gamma(a_i)]\end{bmatrix}\right]}{\left[EF-\left[\left[[2\gamma(a_i)-1]\left[2-\gamma(a_j)\right]+\left[2\gamma(a_j)-1\right][2-\gamma(a_i)]\right]\right]^2\right]}.$$

$$\frac{\partial(\pi_i^*+\pi_j^*)}{\partial x_i}=\left(\frac{2}{9b}\right)\left[\delta-c+[2\gamma(a_j)-1]x_i+[2-\gamma(a_i)]x_j\right]\left[2\gamma(a_j)-1\right]$$

$$+\left(\frac{2}{9b}\right)\left[\delta-c+[2\gamma(a_i)-1]x_j+\left[2-\gamma(a_j)\right]x_i\right]\left[2-\gamma(a_j)\right]-\lambda x_i.$$

To ensure the concavity of $\left(\pi_i^*+\pi_j^*\right)$ with respect to x_i, I place the following restriction: $\frac{\partial^2\left(\pi_i^*+\pi_j^*\right)}{\partial x_i^2}<0 \Leftrightarrow 4.5b\lambda-\left[2-\gamma(a_j)\right]^2-\left[2\gamma(a_j)-1\right]^2>0.$

Similarly,

$$\frac{\partial^2\left(\pi_i^*+\pi_j^*\right)}{\partial x_j^2}<0 \Leftrightarrow 4.5b\lambda-[2-\gamma(a_i)]^2-[2\gamma(a_i)-1]^2>0.$$

Derivation of x_i^{DC} under the deterministic R&D technology with asymmetric absorptive capacity:

$Max\ \pi_i^*\left(x_i^{DC},x_j^{CD}\right)$

$$=\left(\frac{1}{9b}\right)\left[\delta-c+\ [2\gamma(a_i)-1]x_j^{CD}+\left[2-\gamma(a_j)\right]x_i^{DC}\right]^2-\frac{1}{2}\lambda(x_i^{DC})^2$$

subject to $x_i^{DC},x_j^{CD},\pi_i^*\left(x_i^{DC},x_j^{CD}\right)\geq 0.$

$$\frac{\partial \pi_i^*\left(x_i^{DC}, x_j^{CD}\right)}{\partial x_i^{DC}} = \left(\frac{2}{9b}\right)\left[\delta - c + [2\gamma(a_i) - 1]x_j^{CD} + \left[2 - \gamma\left(a_j\right)\right] x_i^{DC}\right]\left[2 - \gamma\left(a_j\right)\right] - \lambda x_i^{DC} = 0$$

$$\Rightarrow x_i^{DC} = \frac{\left(2 - \gamma\left(a_j\right)\right)\left[(\delta - c) + x_j^{CD}(2\gamma(a_i) - 1)\right]}{4.5b\lambda - \left(2 - \gamma\left(a_j\right)\right)^2}$$

such that $x_j^{CD} = x_j^{CC}$.

Assume $T = [2\gamma(a_i) - 1]\left[2 - \gamma\left(a_j\right)\right] + \left[2\gamma\left(a_j\right) - 1\right][2 - \gamma(a_i)]$

By substituting x_j^{CC} in x_i^{DC}, I get

$$x_i^{DC} = \frac{\left(2 - \gamma\left(a_j\right)\right)(\delta - c)}{4.5b\lambda - \left(2 - \gamma\left(a_j\right)\right)^2}\left[1 + (2\gamma(a_i) - 1)\frac{\left[F[1 + \gamma(a_i)] + \left[1 + \gamma\left(a_j\right)\right]T\right]}{[EF - T^2]}\right]$$

$$= \frac{\left(2 - \gamma\left(a_j\right)\right)(\delta - c)}{4.5b\lambda - \left(2 - \gamma\left(a_j\right)\right)^2}\left[\frac{EF - T^2 + (2\gamma(a_i) - 1)\left[F[1 + \gamma(a_i)] + \left[1 + \gamma\left(a_j\right)\right]T\right]}{[EF - T^2]}\right]$$

$$= \frac{\left(2 - \gamma\left(a_j\right)\right)(\delta - c)}{4.5b\lambda - \left(2 - \gamma\left(a_j\right)\right)^2}\left[\frac{\begin{array}{c}EF - T^2 + F(2\gamma(a_i) - 1)[1 + \gamma(a_i)] \\ +(2\gamma(a_i) - 1)\left[1 + \gamma\left(a_j\right)\right]T\end{array}}{[EF - T^2]}\right]$$

$$= \frac{\left(2 - \gamma\left(a_j\right)\right)(\delta - c)}{4.5b\lambda - \left(2 - \gamma\left(a_j\right)\right)^2}\left[\frac{\begin{array}{c}F\left[E + (2\gamma(a_i) - 1)[1 + \gamma(a_i)]\right] \\ +T\left[\begin{array}{c}(2\gamma(a_i) - 1)\left(1 + \gamma\left(a_j\right)\right) - [2\gamma(a_i) - 1]\left[2 - \gamma\left(a_j\right)\right] \\ -\left[2\gamma\left(a_j\right) - 1\right][2 - \gamma(a_i)]\end{array}\right]\end{array}}{[EF - T^2]}\right]$$

$$= \frac{\left(2 - \gamma\left(a_j\right)\right)(\delta - c)}{4.5b\lambda - \left(2 - \gamma\left(a_j\right)\right)^2}\left[\frac{\begin{array}{c}F\left[E + (2\gamma(a_i) - 1)[1 + \gamma(a_i)]\right] \\ -3T\left(2\gamma\left(a_j\right) - 1\right)\left(1 - \gamma(a_i)\right)\end{array}}{[EF - T^2]}\right].$$

Similarly,

$$x_j^{DC} = \frac{\left(2-\gamma(a_i)\right)(\delta - c)}{4.5b\lambda - \left(2-\gamma(a_i)\right)^2}\left[\frac{E\left[F + \left(2\gamma(a_j)-1\right)\left[1+\gamma(a_j)\right]\right] - 3T(2\gamma(a_i)-1)\left(1-\gamma(a_j)\right)}{[EF - T^2]}\right].$$

To ensure the concavity of $\pi_i^*\left(x_i^{DC}, x_j^{CD}\right)$ with respect to x_i^{DC}, I place the following restriction:

$$\frac{\partial^2 \pi_i^*\left(x_i^{DC}, x_j^{CD}\right)}{\partial (x_i^{DC})^2} = \left(\frac{2}{9b}\right)\,(2-\gamma(a_j))^2 - \lambda < 0 \Leftrightarrow 4.5b\lambda > \left(2-\gamma(a_j)\right)^2.$$

Similarly,

$$\frac{\partial^2 \pi_j^*\left(x_i^{DC}, x_j^{CD}\right)}{\partial (x_j^{DC})^2} = \left(\frac{2}{9b}\right)\,(2-\gamma(a_i))^2 - \lambda < 0 \Leftrightarrow 4.5b\lambda > \left(2-\gamma(a_i)\right)^2.$$

<u>Proof for Proposition 3:</u>

Derivation of ($x_i^{DC} - x_i^{CC}$) under the asymmetric absorptive capacity for which $\beta = 0$ and $l = 1$:

$$x_i^{DC} - x_i^{CC} = \frac{[\delta - c][2-a_j] + [2a_i - 1][2-a_j]x_j^{CD}}{\left[4.5b\lambda - [2-a_j]^2\right]} - \frac{[\delta - c][1+a_j] + \left[[2a_i-1][2-a_j] + [2a_j-1][2-a_i]\right]x_j^{CC}}{4.5b\lambda - [2-a_j]^2 - [2a_j-1]^2}.$$

Assume $B_j = 4.5b\lambda - [2-a_j]^2$,

$$x_i^{DC} - x_i^{CC}$$

$$= \frac{[\delta - c][2-a_j] + [2a_i - 1][2-a_j]x_j^{CD}}{B_j} - \frac{[\delta - c][1+a_j] + \left[[2a_i-1][2-a_j] + [2a_j-1][2-a_i]\right]x_j^{CC}}{B_j - [2a_j-1]^2}$$

$$= \frac{\left[B_j - [2a_j-1]^2\right]\left[[\delta - c][2-a_j] + [2a_i-1][2-a_j]x_j^{CD}\right]}{B_j\left[B_j - [2a_j-1]^2\right]} - \frac{B_j[\delta - c][1+a_j] + B_j[2a_i-1][2-a_j]x_j^{CC} + B_j[2a_j-1][2-a_i]x_j^{CC}}{B_j\left[B_j - [2a_j-1]^2\right]}$$

$$= \frac{\begin{bmatrix} B_j[\delta - c][2 - a_j] - B_j[\delta - c][1 + a_j] + B_j[2a_i - 1][2 - a_j]x_j^{CC} \\ -[2a_j - 1]^2[\delta - c][2 - a_j] - [2a_j - 1]^2[2a_i - 1][2 - a_j]x_j^{CC} \\ -B_j[2a_i - 1][2 - a_j]x_j^{CC} - B_j[2a_j - 1][2 - a_i]x_j^{CC} \end{bmatrix}}{B_j\left[B_j - [2a_j - 1]^2\right]}$$

$$= \frac{\left[B_j[1 - 2a_j][(\delta - c) + (2 - a_i)x_j^{CC}] - [2a_j - 1]^2[2 - a_j][(\delta - c) + (2a_i - 1)x_j^{CC}]\right]}{B_j\left[B_j - [2a_j - 1]^2\right]}. \qquad \mathbf{(14)}$$

Expression $(\mathbf{14})$ can be written as follows:

$x_i^{DC} - x_i^{CC}$

$$= \frac{[1 - 2a_j]\left[B_j[(\delta - c) + (2 - a_i)x_j^{CC}] - [1 - 2a_j][2 - a_j][(\delta - c) + (2a_i - 1)x_j^{CC}]\right]}{B_j\left[B_j - [2a_j - 1]^2\right]}.$$

$B_j > [1 - 2a_j][2 - a_j]$ for large enough $b\lambda$,

I have $(\delta - c) + (2 - a_i)x_j^{CC} > (\delta - c) + (2a_i - 1)x_j^{CC}$.

$x_i^{DC} = \frac{[2-a_j]\left[[\delta-c]+[2a_i-1]x_j^{CC}\right]}{B_j} > 0$ for large enough $b\lambda, B_j > 0$ which implies

$[\delta - c] + [2a_i - 1]x_j^{CC} > 0$. Consequently, $(\delta - c) + (2 - a_i)x_j^{CC}$ should be positive.

Therefore, for large enough $b\lambda$,

$\left[B_j[(\delta - c) + (2 - a_i)x_j^{CC}] - [1 - 2a_j][2 - a_j][(\delta - c) + (2a_i - 1)x_j^{CC}]\right] > 0.$

Consequently, the sign of $(x_i^{DC} - x_i^{CC})$ depends on the sign of $[1 - 2a_j]$.

If $[1 - 2a_j] < 0$ $(a_j > \frac{1}{2})$, $x_i^{DC} < x_i^{CC}$ for all $a_i \in]0,1[$.

If $[1 - 2a_j] > 0$, $x_i^{DC} > x_i^{CC}$ for all $a_i \in]0,1[$.

Derivation of $(x_i^{CC} - x_i^{DD})$ under the asymmetric absorptive capacity for which $\beta = 0$ and $l = 1$:

$$x_i^{CC} - x_i^{DD} = \frac{[\delta - c]\left[E[1 + a_j] + [1 + a_i]\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]}{\left[EF - \left[\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]^2\right]}$$

$$- \frac{[\delta - c][2 - a_j][[4.5b\lambda - [2 - a_i]^2] + [2a_i - 1][2 - a_i]]}{[4.5b\lambda - [2 - a_i]^2]\left[4.5b\lambda - [2 - a_j]^2\right] - [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i]}.$$

$$E = 4.5b\lambda - [2 - a_i]^2 - [2a_i - 1]^2 = B_i - [2a_i - 1]^2,$$

$$F = 4.5b\lambda - [2 - a_j]^2 - [2a_j - 1]^2 = B_j - [2a_j - 1]^2,$$

$x_i^{CC} - x_i^{DD}$

$$= \frac{[\delta - c]\left[E[1 + a_j] + [1 + a_i]\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]}{\left[[B_i - [2a_i - 1]^2]\left[B_j - [2a_j - 1]^2\right] - \left[\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]^2\right]}$$

$$- \frac{[\delta - c][2 - a_j]\left[[4.5b\lambda - [2 - a_i]^2] + [2a_i - 1][2 - a_i]\right]}{B_iB_j - [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i]}.$$

Assume $N = [B_i - [2a_i - 1]^2]\left[B_j - [2a_j - 1]^2\right] - \left[\begin{bmatrix}[2a_i - 1][2 - a_j] \\ +[2a_j - 1][2 - a_i]\end{bmatrix}\right]^2 > 0$for large enough $b\lambda$.

$$N = B_iB_j - B_i[2a_j - 1]^2 - B_j[2a_i - 1]^2 + [2a_i - 1]^2[2a_j - 1]^2 - [2a_i - 1]^2[2 - a_j]^2$$
$$-[2a_j - 1]^2[2 - a_i]^2 - 2[2a_i - 1][2 - a_j][2a_j - 1][2 - a_i].$$

Assume $M = B_iB_j - [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i] > 0$ for large enough $b\lambda$ and

$$X = M - N$$
$$= B_i[2a_j - 1]^2 + B_j[2a_i - 1]^2 - [2a_i - 1]^2[2a_j - 1]^2 + [2a_i - 1]^2[2 - a_j]^2$$
$$+[2a_j - 1]^2[2 - a_i]^2 + [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i]$$
$$= [2a_j - 1]^2[B_i - [2a_i - 1]^2] + B_j[2a_i - 1]^2 + [2a_i - 1]^2[2 - a_j]^2$$
$$+[2a_j - 1]^2[2 - a_i]^2 + [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i]$$
$$= [2a_j - 1]^2E + 4.5b\lambda[2a_i - 1]^2 + [2a_j - 1]^2[2 - a_i]^2$$
$$+[2a_i - 1][2 - a_j][2a_j - 1][2 - a_i].$$

I already have

$$N = B_iB_j - B_i[2a_j - 1]^2 - B_j[2a_i - 1]^2 + [2a_i - 1]^2[2a_j - 1]^2 - [2a_i - 1]^2[2 - a_j]^2$$
$$-[2a_j - 1]^2[2 - a_i]^2 - 2[2a_i - 1][2 - a_j][2a_j - 1][2 - a_i].$$
$$= M - B_i[2a_j - 1]^2 - B_j[2a_i - 1]^2 + [2a_i - 1]^2\left[[2a_j - 1]^2 - [2 - a_j]^2\right]$$
$$-[2a_j - 1]^2[2 - a_i]^2 - [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i].$$
$$= M - B_i[2a_j - 1]^2 - B_j[2a_i - 1]^2 - 3[2a_i - 1]^2[1 - a_j][1 + a_j]$$

$$-[2a_j-1]^2[2-a_i]^2-[2a_i-1][2-a_j][2a_j-1][2-a_i].$$

Therefore $X=M-N>0$ for large enough $b\lambda$ and for all $(a_i,a_j)\neq\left(\frac{1}{2},\frac{1}{2}\right)$.

$X=M-N \Leftrightarrow N=M-X$.

$$x_i^{CC}-x_i^{DD}=\frac{[\delta-c]\left[E[1+a_j]+[1+a_i]\left[[2a_i-1][2-a_j]+[2a_j-1][2-a_i]\right]\right]}{M-X}$$

$$-\frac{[\delta-c][2-a_j]\left[[4.5b\lambda-[2-a_i]^2]+[2a_i-1][2-a_i]\right]}{M}$$

$$=\frac{[\delta-c]\left[E[1+a_j]+[1+a_i]\left[[2a_i-1][2-a_j]+[2a_j-1][2-a_i]\right]\right]}{M-X}$$

$$-\frac{[\delta-c][2-a_j]\left[B_i+[2a_i-1][2-a_i]\right]}{M}$$

$$=\frac{[\delta-c]\begin{bmatrix}MB_i[1+a_j]-[2a_i-1]^2M[1+a_j]+M[1+a_i][2a_i-1][2-a_j]\\+M[1+a_i][2a_j-1][2-a_i]-[2-a_j]B_iM+[2-a_j]B_iX\\-[2-a_j]M[2a_i-1][2-a_i]+[2-a_j]X[2a_i-1][2-a_i]\end{bmatrix}}{M(M-X)}$$

$$=\frac{[\delta-c]\begin{bmatrix}MB_i[2a_j-1]+[2a_i-1]^2Q[2-a_j]\\+[2-a_j]X\left[B_i+[2a_i-1][2-a_i]\right]\\+M\left[[1+a_i][2a_j-1][2-a_i]-[2a_i-1]^2[1+a_j]\right]\end{bmatrix}}{M(M-X)}$$

$$=\frac{[\delta-c]\begin{bmatrix}M\begin{bmatrix}B_i[2a_j-1]+[2a_i-1]^2[2-a_j]\\+[1+a_i][2a_j-1][2-a_i]-[2a_i-1]^2[1+a_j]\end{bmatrix}\\+[2-a_j]X\left[B_i+[2a_i-1][2-a_i]\right]\end{bmatrix}}{M(M-X)}$$

$$=\frac{[\delta-c]\begin{bmatrix}M\begin{bmatrix}B_i[2a_j-1]-[2a_i-1]^2[2a_j-1]\\+[1+a_i][2a_j-1][2-a_i]\end{bmatrix}\\+[2-a_j]X\left[B_i+[2a_i-1][2-a_i]\right]\end{bmatrix}}{M(M-X)}$$

$$=\frac{[\delta-c]\begin{bmatrix}M[2a_j-1]\left[B_i-[2a_i-1]^2+[1+a_i][2-a_i]\right]\\+[2-a_j]X\left[B_i+[2a_i-1][2-a_i]\right]\end{bmatrix}}{M(M-X)}$$

I already know that, $X>0, M>0$, $\left[B_i+[2a_i-1][2-a_i]\right]>0$ for large enough $b\lambda$ and

$B_i - [2a_i - 1]^2 + [1 + a_i][2 - a_i] = E + [1 + a_i][2 - a_i] > 0$ since $E > 0$. For all $a_j > 1/2$ and for all a_i, $x_i^{CC} > x_i^{DD}$. Similarly, for all $a_i > 1/2$ and for all a_j, $x_j^{CC} > x_j^{DD}$.

$\left[B_i - [2a_i - 1]^2 + [1 + a_i][2 - a_i]\right] - \left[B_i + [2a_i - 1][2 - a_i]\right]$

$= 3[a_i + 1][1 - a_i] > 0 \Leftrightarrow \left[B_i - [2a_i - 1]^2 + [1 + a_i][2 - a_i]\right] > \left[B_i + [2a_i - 1][2 - a_i]\right] > 0$

$$X = M - N < M \ (M > 0, N > 0)$$

Since $\left[B_i - [2a_i - 1]^2 + [1 + a_i][2 - a_i]\right] > \left[B_i + [2a_i - 1][2 - a_i]\right]$ and $M > X > 0$,

$$M\left[B_i - [2a_i - 1]^2 + [1 + a_i][2 - a_i]\right] > X\left[B_i + [2a_i - 1][2 - a_i]\right] > 0$$

If $a_j < 1/2$,

$$M\left[2a_j - 1\right]\left[B_i - [2a_i - 1]^2 + [1 + a_i][2 - a_i]\right]$$

$$< X\left[2a_j - 1\right]\left[B_i + [2a_i - 1][2 - a_i]\right] < X\left[2 - a_j\right]\left[B_i + [2a_i - 1][2 - a_i]\right]$$

since $3[1 - a_i] > 0$.

Therefore, if $a_j < \frac{1}{2}$, $x_i^{CC} < x_i^{DD}$ for all a_i. Similarly, if $a_i < \frac{1}{2}$, $x_j^{CC} < x_j^{DD}$ for all a_j.

Derivation of $(\boldsymbol{x_i^{DC} - x_i^{DD}})$ ***under the asymmetric absorptive capacity for which*** $\boldsymbol{\beta = 0}$ **and** $\boldsymbol{l = 1}$***:***

$$x_i^{DC} - x_i^{DD} = \frac{[\delta - c]\left[2 - a_j\right] + [2a_i - 1]\left[2 - a_j\right]x_j^{CD}}{B_j}$$

$$- \frac{[\delta - c]\left[2 - a_j\right] + [2a_i - 1]\left[2 - a_j\right]x_j^{DD}}{B_j}$$

$$= \frac{[2a_i - 1]\left[2 - a_j\right]\left[x_j^{CD} - x_j^{DD}\right]}{B_j}, \qquad x_j^{CD} = x_j^{CC}.$$

From the previous proofs, if $a_i < \frac{1}{2}$, $x_j^{CC} < x_j^{DD}$ for all a_j. Therefore, $x_i^{DC} > x_i^{DD}$ $(B_j > 0)$.

If $a_i > \frac{1}{2}$, $x_i^{DC} > x_i^{DD}$ for all a_j. If $a_j < \frac{1}{2}$, $x_i^{CC} < x_i^{DD}$ for all a_i. Therefore, $x_j^{DC} > x_j^{DD}$ $(B_i > 0)$.

If $a_j > \frac{1}{2}$, $x_j^{DC} > x_j^{DD}$ for all a_i.

Proof for Proposition 4:

Derivation of $(\boldsymbol{x_i^{DD} - x_j^{DD}})$ ***under the asymmetric absorptive capacity:***

$x_i^{DD} - x_j^{DD}$

$$= \frac{[\delta - c][2 - a_j][[4.5b\lambda - [2 - a_i]^2] + [2a_i - 1][2 - a_i]]}{[4.5b\lambda - [2 - a_i]^2]\left[4.5b\lambda - [2 - a_j]^2\right] - [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i]}$$

$$- \frac{[\delta - c][2 - a_i]\left[\left[4.5b\lambda - [2 - a_j]^2\right] + [2a_j - 1][2 - a_j]\right]}{\left[4.5b\lambda - [2 - a_j]^2\right][4.5b\lambda - [2 - a_i]^2] - [2a_j - 1][2 - a_i][2a_i - 1][2 - a_j]}$$

$$= \frac{[\delta - c]\begin{bmatrix}[2 - a_i]4.5b\lambda - [2 - a_j][2 - a_i]^2 + [2 - a_j][2 - a_i][2a_i - 1] - [2 - a_i]4.5b\lambda \\ +[2 - a_i][2 - a_j]^2 - [2 - a_i][2a_j - 1][2 - a_j]\end{bmatrix}}{[4.5b\lambda - [2 - a_i]^2]\left[4.5b\lambda - [2 - a_j]^2\right] - [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i]}$$

$$= \frac{[\delta - c]\begin{bmatrix}[a_i - a_j]4.5b\lambda + [2 - a_j][a_i - a_j][2 - a_i] \\ +2[2 - a_i][2 - a_j][a_i - a_j]\end{bmatrix}}{[4.5b\lambda - [2 - a_i]^2]\left[4.5b\lambda - [2 - a_j]^2\right] - [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i]}$$

$$= \frac{[\delta - c][a_i - a_j]\left[4.5b\lambda + 3[2 - a_j][2 - a_i]\right]}{[4.5b\lambda - [2 - a_i]^2]\left[4.5b\lambda - [2 - a_j]^2\right] - [2a_i - 1][2 - a_j][2a_j - 1][2 - a_i]}.$$

$$If\, a_i < a_j, \quad x_i^{DD} < x_j^{DD}.$$

Derivation of ($x_i^{CC} - x_j^{CC}$) under the asymmetric absorptive capacity:

$$x_i^{CC} - x_j^{CC} = \frac{[\delta - c]\left[E[1 + a_j] + [1 + a_i]\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]}{\left[EF - \left[\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]^2\right]}$$

$$- \frac{[\delta - c]\left[F[1 + a_i] + [1 + a_j]\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]}{\left[EF - \left[\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]^2\right]}.$$

$E = 4.5b\lambda - [2 - a_i]^2 - [2a_i - 1]^2$,

$F = 4.5b\lambda - [2 - a_j]^2 - [2a_j - 1]^2$,

Assume $J = [2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]$.

$$x_i^{CC} - x_j^{CC} = \frac{[\delta - c][(E - F) + Ea_j - Fa_i + J(a_i - a_j)]}{\left[EF - \left[\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]^2\right]}. \qquad \mathbf{(15)}$$

After calculation, I get the following results:

$E - F = (a_j - a_i)[5(a_i + a_j) - 8],$

$$Ea_j - Fa_i = 4.5b\lambda(a_j - a_i) + 4a_j^2 a_i + a_i - 4a_j a_i - 4a_j - a_i^2 a_j + 4a_j a_i - 4a_i^2 a_j - a_j + 4a_j a_i + 4a_i + a_j^2 a_i - 4a_j a_i,$$

$$J(a_i - a_j) = 5a_i^2 - 4a_i^2 a_j + 5a_j a_i - 4a_i - 5a_j a_i + 4a_j^2 a_i - 5a_j^2 + 4a_j.$$

After calculation, I get the following result for $(E - F) + Ea_j - Fa_i + J(a_i - a_j)$:
$(E - F) + Ea_j - Fa_i + J(a_i - a_j) = (a_j - a_i)[4.5b\lambda - 9(1 - a_i a_j)].$

Therefore, expression ($\mathbf{15}$) becomes:

$$x_i^{CC} - x_j^{CC} = \frac{[\delta - c](a_j - a_i)[4.5b\lambda - 9(1 - a_i a_j)]}{\left[EF - \left[\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]^2\right]}.$$

For large enough $b\lambda$, $[4.5b\lambda - 9(1 - a_i a_j)] = [4.5b\lambda - 9 + 9a_i a_j] > 0.$

If $a_i < a_j$, $x_i^{CC} > x_j^{CC}$.

Derivation of $(\boldsymbol{x_i^{DC}} - \boldsymbol{x_j^{DC}})$ ***under the asymmetric absorptive capacity:***

$$x_i^{DC} = \frac{[\delta - c][2 - a_j] + [2a_i - 1][2 - a_j]x_j^{CD}}{B_j},$$

$$x_j^{DC} = \frac{[\delta - c][2 - a_i] + [2a_j - 1][2 - a_i]x_i^{CD}}{B_i}.$$

$$x_i^{DC} - x_j^{DC} = \frac{B_i[\delta - c][2 - a_j] + B_i[2a_i - 1][2 - a_j]x_j^{CD} - B_j[\delta - c][2 - a_i] - B_j[2a_j - 1][2 - a_i]x_i^{CD}}{B_i B_j}.$$

Note that:

$$x_j^{CD} = x_j^{CC} = x_i^{CC} + \frac{[\delta - c](a_i - a_j)[4.5b\lambda - 9(1 - a_i a_j)]}{\left[EF - \left[\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]^2\right]}.$$

Assume $L = EF - \left[\left[[2a_i - 1][2 - a_j] + [2a_j - 1][2 - a_i]\right]\right]^2$,

$$x_j^{CC} = x_i^{CC} + \frac{[\delta - c](a_i - a_j)[4.5b\lambda - 9(1 - a_i a_j)]}{L}$$

$x_i^{DC} - x_j^{DC}$

$$= \frac{\begin{bmatrix} 4.5b\lambda[\delta - c][2 - a_j] - [\delta - c][2 - a_j][2 - a_i]^2 + B_i[2a_i - 1][2 - a_j]x_i^{CC} \\ +B_i[2a_i - 1][2 - a_j]\dfrac{[\delta - c](a_i - a_j)[4.5b\lambda - 9(1 - a_i a_j)]}{L} - 4.5b\lambda[\delta - c][2 - a_i] \\ +[\delta - c][2 - a_i][2 - a_j]^2 - 4.5b\lambda[2a_j - 1][2 - a_i]x_i^{CC} + [2 - a_j]^2[2a_j - 1][2 - a_i]x_i^{CC} \end{bmatrix}}{B_i B_j}$$

$$= \frac{\begin{bmatrix} 4.5b\lambda[\delta - c][a_i - a_j] - [\delta - c][2 - a_j][2 - a_i]^2 + B_i[2a_i - 1][2 - a_j]x_i^{CC} \\ +B_i[2a_i - 1][2 - a_j]\dfrac{[\delta - c](a_i - a_j)[4.5b\lambda - 9(1 - a_i a_j)]}{L} \\ +[\delta - c][2 - a_i][2 - a_j]^2 - 4.5b\lambda[2a_j - 1][2 - a_i]x_i^{CC} + [2 - a_j]^2[2a_j - 1][2 - a_i]x_i^{CC} \end{bmatrix}}{B_i B_j}$$

$$= \frac{\begin{bmatrix} 4.5b\lambda[\delta - c][a_i - a_j] + [\delta - c][2 - a_j][2 - a_i][a_i - a_j] \\ + \\ +4.5b\lambda(3)(a_i - a_j)x_i^{CC} - [2 - a_j][2 - a_i](a_i - a_j)[5 - 2(a_i + a_j)]x_i^{CC} \\ +B_i[2a_i - 1][2 - a_j]\dfrac{[\delta - c](a_i - a_j)[4.5b\lambda - 9(1 - a_i a_j)]}{L} \end{bmatrix}}{B_i B_j}$$

$$= [a_i - a_j]\frac{\begin{bmatrix} 4.5b\lambda[\delta - c] + [\delta - c][2 - a_j][2 - a_i] + 4.5b\lambda(3)x_i^{CC} \\ + \\ -[2 - a_j][2 - a_i][5 - 2(a_i + a_j)]x_i^{CC} \\ +B_i[2a_i - 1][2 - a_j]\dfrac{[\delta - c][4.5b\lambda - 9(1 - a_i a_j)]}{L} \end{bmatrix}}{B_i B_j}$$

$$= [a_i - a_j]\frac{\begin{bmatrix} [\delta - c]\left[4.5b\lambda + [2 - a_j][2 - a_i]\right] + B_i[2a_i - 1][2 - a_j]\dfrac{(x_i^{CC} - x_j^{CC})}{[a_i - a_j]} \\ + [4.5b\lambda(3) - [2 - a_j][2 - a_i][5 - 2(a_i + a_j)]]x_i^{CC} \end{bmatrix}}{B_i B_j}.$$

If $a_i < a_j$, $x_i^{CC} > x_j^{CC}$.

If $a_i < \frac{1}{2} < a_j$ and $a_i < a_j < \frac{1}{2}$, $x_i^{CC} > x_j^{CC}$.Therefore, $x_i^{DC} < x_j^{DC}$.

Note that $0 < a_i + a_j < 2 \Leftrightarrow 1 < 5 - 2(a_i + a_j) < 5$.

For large enough $b\lambda$, $[4.5b\lambda(3) - [2 - a_j][2 - a_i][5 - 2(a_i + a_j)]] > 0$.

If $\frac{1}{2} < a_i < a_j$ and $(\delta - c)$ is sufficiently large, $x_i^{DC} < x_j^{DC}$ for large enough $b\lambda$.

Using numerical simulations, I find that $x_i^{DC} < x_j^{DC}$ if $a_i < a_j$.

Proof of Proposition 5:

Derivation $\boldsymbol{\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) - \pi_i^*\left(x_i^{CC}, x_j^{CC}\right)}$ ***under the asymmetric absorptive capacity for*** $\boldsymbol{a_j \neq \frac{1}{2}}$ ***and for all*** $\boldsymbol{a_i}$***:***

$$\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) - \pi_i^*\left(x_i^{CC}, x_j^{CC}\right)$$

$$= \left(\frac{1}{9b}\right)\left[\delta - c + [2a_i - 1]x_j^{CD} + \left[2 - a_j\right] x_i^{DC}\right]^2 - \frac{1}{2}\lambda(x_i^{DC})^2$$

$$-\left(\frac{1}{9b}\right)\left[\delta - c + [2a_i - 1]x_j^{CC} + \left[2 - a_j\right] x_i^{CC}\right]^2 + \frac{1}{2}\lambda(x_i^{CC})^2$$

$$= \left(\frac{1}{9b}\right)\begin{bmatrix}\delta - c + [2a_i - 1]x_j^{CD} + \left[2 - a_j\right] x_i^{DC} \\ -(\delta - c) - [2a_i - 1]x_j^{CC} - \left[2 - a_j\right] x_i^{CC}\end{bmatrix}\begin{bmatrix}(\delta - c) + [2a_i - 1]x_j^{CD} \\ +\left[2 - a_j\right] x_i^{DC} + (\delta - c) \\ + [2a_i - 1]x_j^{CC} + \left[2 - a_j\right] x_i^{CC}\end{bmatrix}$$

$$- \frac{1}{2}\lambda(x_i^{DC})^2 + \frac{1}{2}\lambda(x_i^{CC})^2$$

$$= \left(\frac{1}{9b}\right)(2 - a_j)(x_i^{DC} - x_i^{CC}) \times \begin{bmatrix}2(2a_i - 1)x_j^{CC} \\ +(2 - a_j)\left(x_i^{DC} + x_i^{CC}\right) \\ +2(\delta - c)\end{bmatrix} - \frac{1}{2}\lambda\left(x_i^{DC} - x_i^{CC}\right)\left(x_i^{DC} + x_i^{CC}\right)$$

$$= \left(\frac{1}{9b}\right)\left(x_i^{DC} - x_i^{CC}\right)\begin{bmatrix}2\left[\left[2 - a_j\right]\left[(\delta - c) + (2a_i - 1)x_j^{CC}\right]\right] \\ +\left(2 - a_j\right)^2\left(x_i^{DC} + x_i^{CC}\right) - 4.5b\lambda\left(x_i^{DC} + x_i^{CC}\right)\end{bmatrix}$$

$$= \left(\frac{1}{9b}\right)\left(x_i^{DC} - x_i^{CC}\right)\left[2B_j x_i^{DC} - \left(4.5b\lambda - \left(2 - a_j\right)^2\right)\left(x_i^{DC} + x_i^{CC}\right)\right]$$

$$= \left(\frac{1}{9b}\right)\left(x_i^{DC} - x_i^{CC}\right)\left[2B_j x_i^{DC} - B_j\left(x_i^{DC} + x_i^{CC}\right)\right] = \left(\frac{1}{9b}\right)\left(x_i^{DC} - x_i^{CC}\right)\left[B_j x_i^{DC} - B_j x_i^{CC}\right]$$

$$= \left(\frac{1}{9b}\right)B_j\left(x_i^{DC} - x_i^{CC}\right)^2.$$

If $\left[1 - 2a_j\right] < 0$ $(a_j > \frac{1}{2})$, $x_i^{DC} < x_i^{CC}$ for all $a_i \in]0,1[$.

If $\left[1 - 2a_j\right] > 0$, $x_i^{DC} > x_i^{CC}$ for all $a_i \in]0,1[$.

If $a_j = \frac{1}{2}$, $x_i^{DC} = x_i^{CC}$ for all $a_i \in]0,1[$.

$\Rightarrow \pi_i^*\left(x_i^{DC}, x_j^{CD}\right) - \pi_i^*\left(x_i^{CC}, x_j^{CC}\right) > 0$ for $a_j \neq \frac{1}{2}$ and for all a_i .

Similarly, $\pi_j^*\left(x_i^{CD}, x_j^{DC}\right) - \pi_j^*\left(x_i^{CC}, x_j^{CC}\right) > 0$ for $a_i \neq \frac{1}{2}$ and for all a_j .

Derivation $\boldsymbol{\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_i^*\left(x_i^{CC}, x_j^{CC}\right)}$ ***and*** $\boldsymbol{\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_i^*\left(x_i^{CD}, x_j^{DC}\right)}$ ***under the asymmetric absorptive capacity such that*** $\boldsymbol{a_i = \frac{1}{2}}$ ***and*** $\boldsymbol{a_j \neq \frac{1}{2}}$***:***

$\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CD}, x_j^{DC})$

$= \left(\frac{1}{9b}\right)\left[\delta - c + [2a_i - 1]x_j^{DD} + [2 - a_j]\, x_i^{DD}\right]^2$

$-\frac{1}{2}\lambda(x_i^{DD})^2 - \left(\frac{1}{9b}\right)\left[\delta - c + [2a_i - 1]x_j^{DC} + [2 - a_j]\, x_i^{CD}\right]^2 - \frac{1}{2}\lambda(x_i^{CD})^2,$

$\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC}) = \left(\frac{1}{9b}\right)\left[\delta - c + [2a_i - 1]x_j^{DD} + [2 - a_j]\, x_i^{DD}\right]^2$

$-\frac{1}{2}\lambda(x_i^{DD})^2 - \left(\frac{1}{9b}\right)\left[\delta - c + [2a_i - 1]x_j^{CC} + [2 - a_j]\, x_i^{CC}\right]^2 - \frac{1}{2}\lambda(x_i^{CC})^2$

$$= \left(\frac{1}{9b}\right)\begin{bmatrix}\delta - c + [2a_i - 1]x_j^{DD} + [2 - a_j]\, x_i^{DD} \\ -(\delta - c) - [2a_i - 1]x_j^{CC} - [2 - a_j]\, x_i^{CC}\end{bmatrix}\begin{bmatrix}(\delta - c) + [2a_i - 1]x_j^{DD} + [2 - a_j]\, x_i^{DD} \\ +(\delta - c) + [2a_i - 1]x_j^{CC} + [2 - a_j]\, x_i^{CC}\end{bmatrix}$$

$-\frac{1}{2}\lambda(x_i^{DD})^2 + \frac{1}{2}\lambda(x_i^{CC})^2.$

- For $a_i = \frac{1}{2}$ and for all a_j, I have:

$\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC}) = \pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CD}, x_j^{DC})$

$$= \left(\frac{1}{9b}\right)(2 - a_j)(x_i^{DD} - x_i^{CC})\begin{bmatrix}(2 - a_j)(x_i^{DD} + x_i^{CC}) \\ +2(\delta - c)\end{bmatrix} - \frac{1}{2}\lambda(x_i^{DD} - x_i^{CC})(x_i^{DD} + x_i^{CC})$$

$= \left(\frac{1}{9b}\right)(x_i^{DD} - x_i^{CC})\left[(2 - a_j)^2(x_i^{DD} + x_i^{CC}) + 2(\delta - c)(2 - a_j) - 4.5b\lambda(x_i^{DD} + x_i^{CC})\right]$

$= \left(\frac{1}{9b}\right)(x_i^{DD} - x_i^{CC})\left[2(\delta - c)(2 - a_j) - B_j(x_i^{DD} + x_i^{CC})\right].$

Given that, for $a_i = \frac{1}{2}$ and for all a_j, I have:

$$x_i^{DD} = \frac{(\delta - c)[2 - a_j]}{\left[4.5b\lambda - [2 - a_j]^2\right]} \Longrightarrow 2(\delta - c)[2 - a_j] = 2x_i^{DD}B_j,$$

$\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC}) = \pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CD}, x_j^{DC})$

$= \left(\frac{1}{9b}\right)(x_i^{DD} - x_i^{CC})\left[2x_i^{DD}B_j - B_j(x_i^{DD} + x_i^{CC})\right] = \left(\frac{1}{9b}\right)(x_i^{DD} - x_i^{CC})\left[x_i^{DD}B_j - B_j x_i^{CC}\right]$

$= \left(\frac{1}{9b}\right)B_j(x_i^{DD} - x_i^{CC})^2$

For all a_i, if $a_j > (<)\frac{1}{2}$, $x_i^{CC} > (<) x_i^{DD}$.

$\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC}) = \pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CD}, x_j^{DC}) > 0$ for $a_i = \frac{1}{2}$ and $a_j \neq \frac{1}{2}$.

Similarly, $\pi_j^*(x_i^{DD}, x_j^{DD}) - \pi_j^*(x_i^{DC}, x_j^{CD}) > 0$ for $a_j = \frac{1}{2}$ and $a_i \neq \frac{1}{2}$.

Derivation $\boldsymbol{\pi_j^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_j^*\left(x_i^{DC}, x_j^{CD}\right)}$ ***under the asymmetric absorptive capacity such that*** $\boldsymbol{a_i = \frac{1}{2}}$ **and** $\boldsymbol{a_j \neq \frac{1}{2}}$***:***

$$\pi_j^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_j^*\left(x_i^{DC}, x_j^{CD}\right)$$

$$= \left(\frac{1}{9b}\right)\left[\delta - c + \left[2a_j - 1\right]x_i^{DD} + \left[2 - a_i\right] x_j^{DD}\right]^2 - \frac{1}{2}\lambda(x_j^{DD})^2$$

$$-\left(\frac{1}{9b}\right)\left[\delta - c + \left[2a_j - 1\right]x_i^{DC} + \left[2 - a_i\right] x_j^{CC}\right]^2 - \frac{1}{2}\lambda(x_j^{CC})^2$$

$$= \left(\frac{1}{9b}\right)\left(\begin{array}{l} \begin{bmatrix} \left[2a_j - 1\right]\left(x_i^{DD} - x_i^{DC}\right) \\ +\left[2 - a_i\right]\left(x_j^{DD} - x_j^{CC}\right) \end{bmatrix} \begin{bmatrix} 2(\delta - c) + \left[2a_j - 1\right](x_i^{DD} + x_i^{DC}) \\ +\left[2 - a_i\right]\left(x_j^{DD} + x_j^{CC}\right) \end{bmatrix} \\ -4.5b\lambda\left(x_j^{DD} - x_j^{CC}\right)\left(x_j^{DD} + x_j^{CC}\right) \end{array}\right)$$

$$= \left(\frac{1}{9b}\right)\left(\begin{array}{l} \left[\left[2a_j - 1\right]\left(x_i^{DD} - x_i^{DC}\right)\right] \begin{bmatrix} 2(\delta - c) + \left[2a_j - 1\right](x_i^{DD} + x_i^{DC}) \\ +\left[2 - a_i\right]\left(x_j^{DD} + x_j^{CC}\right) \end{bmatrix} \\ +\left(x_j^{DD} - x_j^{CC}\right) \begin{bmatrix} 2(\delta - c)(2 - a_i) \\ +(2 - a_i)\left[2a_j - 1\right](x_i^{DD} + x_i^{DC}) \\ -B_i\left(x_j^{DD} + x_j^{CC}\right) \end{bmatrix} \end{array}\right).$$

If $a_i = \frac{1}{2}, x_i^{DD} = x_i^{DC}$

$$\pi_j^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_j^*\left(x_i^{DC}, x_j^{CD}\right) = \left(\frac{1}{9b}\right)\left(x_j^{DD} - x_j^{CC}\right)\begin{bmatrix} 2(\delta - c)(2 - a_i) \\ +(2 - a_i)\left[2a_j - 1\right](x_i^{DD} + x_i^{DC}) \\ -B_i\left(x_j^{DD} + x_j^{CC}\right) \end{bmatrix}.$$

For $a_i = \frac{1}{2}$,

$$x_j^{CC} = \frac{[\delta - c][2 - a_i] + \left[2a_j - 1\right][2 - a_i]x_i^{CC}}{B_i}.$$

$$x_j^{DD} = \frac{[\delta - c][2 - a_i] + \left[2a_j - 1\right][2 - a_i]x_i^{DD}}{B_i}.$$

From the above two expressions, I conclude that:

$$-B_i\left(x_j^{DD} + x_j^{CC}\right) = -2[\delta - c][2 - a_i] - \left[2a_j - 1\right][2 - a_i](x_i^{CC} + x_i^{DD}) \Rightarrow$$

$$\pi_j^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_j^*\left(x_i^{DC}, x_j^{CD}\right) = \left(\frac{1}{9b}\right)\left(x_j^{DD} - x_j^{CC}\right)\begin{bmatrix} (2 - a_i)\left[2a_j - 1\right](x_i^{DD} + x_i^{DC}) \\ -\left[2a_j - 1\right][2 - a_i](x_i^{CC} + x_i^{DD}) \end{bmatrix}$$

$$= \left(\frac{1}{9b}\right)(2 - a_i)\left[2a_j - 1\right]\left(x_j^{DD} - x_j^{CC}\right)(x_i^{DC} - x_i^{CC}).$$

If $a_j < \frac{1}{2}, x_i^{DC} > x_i^{CC}$. If $a_i = \frac{1}{2}, x_j^{DD} < x_j^{CC}$.

Therefore $\pi_j^*\left(x_i^{DD}, x_j^{DD}\right) > \pi_j^*\left(x_i^{DC}, x_j^{CD}\right)$ for $a_i = \frac{1}{2}$ and $a_j \neq \frac{1}{2}$.

Similarly, $\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) > \pi_i^*\left(x_i^{CD}, x_j^{DC}\right)$ for $a_j = \frac{1}{2}$ and $a_i \neq \frac{1}{2}$.

Derivation $\boldsymbol{\pi_j^*\left(x_i^{CD}, x_j^{DC}\right) - \pi_j^*\left(x_i^{CC}, x_j^{CC}\right)}$ ***under the asymmetric absorptive capacity such that*** $\boldsymbol{a_i = \frac{1}{2}}$ **and** $\boldsymbol{a_j \neq \frac{1}{2}}$**:**

$$\pi_j^*\left(x_i^{CD}, x_j^{DC}\right) - \pi_j^*\left(x_i^{CC}, x_j^{CC}\right)$$

$$= \left(\frac{1}{9b}\right)\left[\delta - c + \left[2a_j - 1\right]x_i^{CC} + [2 - a_i]\, x_j^{DC}\right]^2 - \frac{1}{2}\lambda(x_j^{DC})^2$$

$$-\left(\frac{1}{9b}\right)\left[\delta - c + \left[2a_j - 1\right]x_i^{CC} + [2 - a_i]\, x_j^{CC}\right]^2 - \frac{1}{2}\lambda(x_j^{CC})^2$$

$$= \left(\frac{1}{9b}\right)\left(\begin{array}{l} [2 - a_i]\left(x_j^{DC} - x_j^{CC}\right)\left[\begin{array}{c} 2(\delta - c) + 2\left[2a_j - 1\right]x_i^{CC} \\ +[2 - a_i]\left(x_j^{DC} + x_j^{CC}\right) \end{array}\right] \\ \quad -4.5b\lambda\left(x_j^{DC} - x_j^{CC}\right)\left(x_j^{DC} + x_j^{CC}\right) \end{array}\right)$$

$$= \left(\frac{1}{9b}\right)\left(x_j^{DD} - x_j^{CC}\right)\left[2(\delta - c)(2 - a_i) + 2(2 - a_i)\left[2a_j - 1\right]x_i^{CC} - B_i\left(x_j^{DC} + x_j^{CC}\right)\right].$$

For $a_i = \frac{1}{2}$,

$$x_j^{CC} = \frac{[\delta - c][2 - a_i] + \left[2a_j - 1\right][2 - a_i]x_i^{CC}}{B_i}.$$

$$x_j^{DC} = \frac{[\delta - c][2 - a_i] + \left[2a_j - 1\right][2 - a_i]x_i^{CC}}{B_i}.$$

From the above two expressions, I conclude that:

$$-B_i\left(x_j^{DC} + x_j^{CC}\right) = -2[\delta - c][2 - a_i] - 2\left[2a_j - 1\right][2 - a_i]x_i^{CC} \Rightarrow$$

$\pi_j^*\left(x_i^{CD}, x_j^{DC}\right) = \pi_j^*\left(x_i^{CC}, x_j^{CC}\right)$ for $a_i = \frac{1}{2}$ and $a_j \neq \frac{1}{2}$.

Similarly, $\pi_i^*\left(x_i^{DC}, x_j^{CD}\right) = \pi_i^*\left(x_i^{CC}, x_j^{CC}\right)$ for $a_j = \frac{1}{2}$ and $a_i \neq \frac{1}{2}$.

I will show that $\boldsymbol{\pi_i^*\left(x_i^{CC}, x_j^{CC}\right) + \pi_j^*\left(x_i^{CC}, x_j^{CC}\right) > \pi_i^*\left(x_i^{DD}, x_j^{DD}\right) + \pi_j^*\left(x_i^{DD}, x_j^{DD}\right)}$ **for** $\boldsymbol{a_i = \frac{1}{2}}$ **and** $\boldsymbol{a_j \neq \frac{1}{2}}$:

$$\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) + \pi_j^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_i^*\left(x_i^{CC}, x_j^{CC}\right) - \pi_j^*\left(x_i^{CC}, x_j^{CC}\right)$$

$$= \left[\pi_i^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_i^*\left(x_i^{CC}, x_j^{CC}\right)\right] + \left[\pi_j^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_j^*\left(x_i^{CC}, x_j^{CC}\right)\right].$$

I already showed that $\pi_j^*\left(x_i^{CC}, x_j^{CC}\right) = \pi_j^*\left(x_i^{CD}, x_j^{DC}\right)$,

$$\pi_j^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_j^*\left(x_i^{CC}, x_j^{CC}\right) = \pi_j^*\left(x_i^{DD}, x_j^{DD}\right) - \pi_j^*\left(x_i^{CD}, x_j^{DC}\right),$$

$$\pi_j^*(x_i^{DD}, x_j^{DD}) - \pi_j^*(x_i^{CD}, x_j^{DC})$$

$$= \left(\frac{1}{9b}\right)\left[\delta - c + \left[2a_j - 1\right]x_i^{DD} + [2 - a_i]\, x_j^{DD}\right]^2 - \frac{1}{2}\lambda(x_j^{DD})^2$$

$$-\left(\frac{1}{9b}\right)\left[\delta - c + \left[2a_j - 1\right]x_i^{CC} + [2 - a_i]\, x_j^{DC}\right]^2 - \frac{1}{2}\lambda(x_j^{DC})^2$$

$$= \left(\frac{1}{9b}\right)\begin{pmatrix} \begin{bmatrix} \left[2a_j - 1\right](x_i^{DD} - x_i^{CC}) \\ +[2 - a_i]\,(x_j^{DD} - x_j^{DC}) \end{bmatrix} \begin{bmatrix} 2(\delta - c) + \left[2a_j - 1\right](x_i^{DD} + x_i^{CC}) \\ +[2 - a_i](x_j^{DD} + x_j^{DC}) \end{bmatrix} \\ -4.5b\lambda(x_j^{DD} - x_j^{DC})(x_j^{DD} + x_j^{DC}) \end{pmatrix}$$

$$= \left(\frac{1}{9b}\right)\begin{pmatrix} \left[\left[2a_j - 1\right](x_i^{DD} - x_i^{CC})\right] \begin{bmatrix} 2(\delta - c) + \left[2a_j - 1\right](x_i^{DD} + x_i^{CC}) \\ +[2 - a_i](x_j^{DD} + x_j^{DC}) \end{bmatrix} \\ +(x_j^{DD} - x_j^{DC}) \begin{bmatrix} 2(\delta - c)(2 - a_i) \\ +(2 - a_i)\left[2a_j - 1\right](x_i^{DD} + x_i^{CC}) \\ -B_i(x_j^{DD} + x_j^{DC}) \end{bmatrix} \end{pmatrix}.$$

For $a_i = \frac{1}{2}$,

$$x_j^{DC} = \frac{[\delta - c][2 - a_i] + \left[2a_j - 1\right][2 - a_i]x_i^{CC}}{B_i}.$$

$$x_j^{DD} = \frac{[\delta - c][2 - a_i] + \left[2a_j - 1\right][2 - a_i]x_i^{DD}}{B_i}.$$

From the above two expressions, I conclude that:

$$-B_i(x_j^{DD} + x_j^{DC}) = -2[\delta - c][2 - a_i] - \left[2a_j - 1\right][2 - a_i](x_i^{CC} + x_i^{DD}) \Rightarrow$$

$$\pi_j^*(x_i^{DD}, x_j^{DD}) - \pi_j^*(x_i^{CD}, x_j^{DC})$$

$$= \left(\frac{1}{9b}\right)\left[\left[2a_j - 1\right](x_i^{DD} - x_i^{CC})\right] \begin{bmatrix} 2(\delta - c) + \left[2a_j - 1\right](x_i^{DD} + x_i^{CC}) \\ +[2 - a_i](x_j^{DD} + x_j^{DC}) \end{bmatrix} \qquad \mathbf{(16)}$$

$$= \left(\frac{1}{9b}\right)\left[\frac{\left[2a_j - 1\right]}{[2 - a_i]}(x_i^{DD} - x_i^{CC})\right] \begin{bmatrix} 2(\delta - c)[2 - a_i] + \left[2a_j - 1\right][2 - a_i](x_i^{DD} + x_i^{CC}) \\ +[2 - a_i]^2(x_j^{DD} + x_j^{DC}) \end{bmatrix}$$

$$= \left(\frac{1}{9b}\right)\left[\frac{\left[2a_j - 1\right]}{[2 - a_i]}(x_i^{DD} - x_i^{CC})\right]\left[B_i(x_j^{DD} + x_j^{DC}) + 4.5b\lambda(x_j^{DD} + x_j^{DC}) - B_i(x_j^{DD} + x_j^{DC})\right]$$

$$= \left(\frac{\lambda}{2}\right)\frac{\left[2a_j - 1\right]}{[2 - a_i]}(x_i^{DD} - x_i^{CC})(x_j^{DD} + x_j^{DC}).$$

I already showed that $[\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC}) = \left(\frac{1}{9b}\right) B_j (x_i^{DD} - x_i^{CC})^2$ for $a_i = \frac{1}{2}$ and $a_j \neq \frac{1}{2}$.

$$\pi_i^*(x_i^{DD}, x_j^{DD}) + \pi_j^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC}) - \pi_j^*(x_i^{CC}, x_j^{CC})$$

$$= [\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC})] + [\pi_j^*(x_i^{DD}, x_j^{DD}) - \pi_j^*(x_i^{CC}, x_j^{CC})]$$

$$= [\pi_i^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC})] + [\pi_j^*(x_i^{DD}, x_j^{DD}) - \pi_j^*(x_i^{CD}, x_j^{DC})]$$

$$= \left(\frac{1}{9b}\right) B_j (x_i^{DD} - x_i^{CC})^2 + \left(\frac{\lambda}{2}\right) \frac{[2a_j - 1]}{[2 - a_i]} (x_i^{DD} - x_i^{CC})(x_j^{DD} + x_j^{DC})$$

$$= \left(\frac{1}{9b}\right) (x_i^{DD} - x_i^{CC}) \left[B_j (x_i^{DD} - x_i^{CC}) + \frac{4.5b\lambda[2a_j - 1]}{[2 - a_i]} (x_j^{DD} + x_j^{DC}) \right]. \quad (\mathbf{17})$$

For $a_i = \frac{1}{2}$,

$$x_i^{CC} = \frac{[\delta - c][1 + a_j] + [2a_j - 1][2 - a_i]x_j^{CC}}{B_j - [2a_i - 1]^2}$$

$$\Leftrightarrow B_j x_i^{CC} = [2a_i - 1]^2 x_i^{CC} + [\delta - c][1 + a_j] + [2a_j - 1][2 - a_i]x_j^{CC},$$

$$x_i^{DD} = \frac{[\delta - c][2 - a_j] + [2a_i - 1][2 - a_j]x_j^{DD}}{B_j} = \frac{[\delta - c][2 - a_j]}{B_j} \Leftrightarrow B_j x_i^{DD} = [\delta - c][2 - a_j].$$

Therefore, $B_j(x_i^{DD} - x_i^{CC}) = B_j x_i^{DD} - B_j x_i^{CC}$

$$= [\delta - c][2 - a_j] - [2a_i - 1]^2 x_i^{CC} - [\delta - c][1 + a_j] - [2a_j - 1][2 - a_i]x_j^{CC}$$

$$= [2a_j - 1][-(\delta - c) - [2a_j - 1]x_i^{CC} - [2 - a_i]x_j^{CC}]. \quad (\mathbf{18})$$

Given **(16)** and **(18)**, Expression **(17)** can be written as follows:

$$\left(\frac{1}{9b}\right) (x_i^{DD} - x_i^{CC})[2a_j - 1] \begin{bmatrix} -(\delta - c) - [2a_j - 1]x_i^{CC} - [2 - a_i]x_j^{CC} + 2(\delta - c) \\ + [2a_j - 1](x_i^{DD} + x_i^{CC}) + [2 - a_i](x_j^{DD} + x_j^{DC}) \end{bmatrix}.$$

For $a_i = \frac{1}{2}$,

$$x_j^{DC} = \frac{[\delta - c][2 - a_i] + [2a_j - 1][2 - a_i]x_i^{CC}}{B_i} = x_j^{CC}.$$

$$\pi_i^*(x_i^{DD}, x_j^{DD}) + \pi_j^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC}) - \pi_j^*(x_i^{CC}, x_j^{CC})$$

$$= \left(\frac{1}{9b}\right) (x_i^{DD} - x_i^{CC})[2a_j - 1][(\delta - c) + [2a_j - 1](x_i^{DD}) + [2 - a_i](x_j^{DD})]$$

$$= \frac{[2a_j - 1]}{9b[2 - a_i]} (x_i^{DD} - x_i^{CC}) \begin{bmatrix} (\delta - c)[2 - a_i] + [2a_j - 1][2 - a_i](x_i^{DD}) \\ +[2 - a_i]^2(x_j^{DD}) \end{bmatrix}.$$

Given $B_i x_j^{DD} = [\delta - c][2 - a_i] + [2a_j - 1][2 - a_i]x_i^{DD}$,

$\pi_i^*(x_i^{DD}, x_j^{DD}) + \pi_j^*(x_i^{DD}, x_j^{DD}) - \pi_i^*(x_i^{CC}, x_j^{CC}) - \pi_j^*(x_i^{CC}, x_j^{CC})$

$$= \frac{[2a_j - 1]}{9b[2 - a_i]}(x_i^{DD} - x_i^{CC})[B_i x_j^{DD} + [2 - a_i]^2(x_j^{DD})]$$

$$= \frac{[2a_j - 1]}{9b[2 - a_i]}(x_i^{DD} - x_i^{CC})[4.5b\lambda x_j^{DD}] = \frac{\lambda[2a_j - 1]}{2[2 - a_i]}(x_i^{DD} - x_i^{CC})x_j^{DD} < 0$$

because if $a_j > \frac{1}{2}, x_i^{CC} > x_i^{DD}$. Therefore,

$\pi_i^*(x_i^{CC}, x_j^{CC}) + \pi_j^*(x_i^{CC}, x_j^{CC}) > \pi_i^*(x_i^{DD}, x_j^{DD}) + \pi_j^*(x_i^{DD}, x_j^{DD})$.

References

Abbink, K., Brandts, J., Herrmann, B., and Orzen, H. (2010). "Inter-group conflict and intra-group punishment in an experimental contest game", *American Economic Review*, 100(1), 420-447.

Aghion, P., Bechtold, S., Cassar L., Herz, H. (2018). "The Causal Effects of Competition on Innovation: Experimental Evidence, *The Journal of Law, Economics, and Organization*, 34 (2), 162-195.

Agyekum, C.K., Haifeng H., Agyeiwaa, A. (2015). "Consumer perception of product quality", *Microeconomics and Macroeconomics*, 3(2), 25-29.

Amir, R. (2000). "Modelling imperfectly appropriable R&D via spillovers", *International Journal of Industrial Organization,* 18(7), 1013-1032.

Amir, R., Wooders, J. (2000). "One-way spillovers, endogenous innovator/imitator roles, and research joint ventures", *Games and Economic Behavior*, 31(1), 1-25.

Amir, R., Evstigneev, I., Wooders, J. (2003). "Noncooperative versus cooperative R&D with endogenous spillover rates", *Games and Economic Behavior*, 42(2), 183-207.

Aoki, R. (1991). "Competition for product innovation: an endless race", *American Economic Review*, 81(2), 252-256.

Ben Youssef, S., Breton, M., Zaccour, G. (2013). "Cooperating and non-cooperating firms in inventive and absorptive research", *Journal of Optimization Theory and Applications*, 157(1), 229-251.

Board, S., Meyer, M. (2013). "Reputation for quality", *Econometrica*, 81(6), 2381-2462.

Breitmoser, Y., Tan, J.H.W., Zizzo, D.J. (2010). "Understanding perpetual R&D races", *Economic Theory*, 44, 445-467.

Brekke, K.R., Siciliani, L., Straume, O.R. (2010). "Price and quality in spatial competition", *Regional Science and Urban Economics*, 40(6), 471-480.

Brüggemann, J., Bizer, K. (2016). "Laboratory experiments in innovation research: a methodological overview and a review of the current literature", *Journal of Innovation and Entrepreneurship*, 5(1), 1-13.

Brüggemann, J. , Meub, L., (2017). "Experimental evidence on the effects of innovation contests," *Information Economics and Policy*, 39(C), 72-83.

Bulow, J.I., Geanakoplos, J.D., Klemperer, P.D. (1985). "Multimarket oligopoly: strategic substitutes and complements", *Journal of Political Economy*, 93(3), 488-511.

Burr, C., Knauff, M., Stepanova, A. (2013). "On the prisoner's dilemma in R&D with input spillovers and incentives for R&D cooperation", *Mathematical Social Sciences*, 66(3), 254-261.

Cabon-Dhersin, M-L. (2008). "R&D cooperation and collusion : the case of joint labs", *Manchester School*, 76(4), 424-435.

Cabon-Dhersin, M-L., Gibert, R. (2020). "R&D cooperation, proximity and distribution of public funding between public and private research sectors", *Manchester School*, 88(6), 773-800.

Cabon-Dhersin, M.L., Ramani, S.V. (2005). "Does trust matter for R&D cooperation? a game theoretic examination", *Theory and Decision*, 57(2), 143-180.

Cellini, R., Siciliani, L., Straume, O.R. (2018). "A dynamic model of quality competition with endogenous prices", *Journal of Economic Dynamics and Control*, 94(C), 190-206.

Chen, J., Lee, S.H. (2023). "Cournot–Bertrand comparisons under R&D competition: output versus R&D subsidies", *International Journal of Economic Theory*, 19(1), 77-100.

Cohen, W.M., Levinthal, D.A. (1989). "Innovation and learning: the two faces of R&D", *The Economic Journal*, 99(397), 569-596.

Cohen, W. M., Levinthal, D.A. (1990). "Absorptive capacity: a new perspective on learning and innovation", *Administrative Science Quarterly*, 35(1), 128-152.

D'Aspremont, C., Jacquemin, A. (1988). "Cooperative and noncooperative R&D in duopoly with spillovers", *The American Economic Review*, 78(5), 1133-1137.

Das, T.K., Rahman, N. (2010). "Determinants of partner opportunism in strategic alliances: a conceptual framework", *Journal of Business and Psychology*, 25(1), 55-74.

Dasgupta, P., Stiglitz, J. (1980). "Industrial structure and the nature of innovative activity", *The Economic Journal*, 90(358), 266-293.

Dechenaux, E., Kovenock, D., Sheremeta, R.M. (2015). "A survey of experimental research on contests, all-pay auctions and tournaments", *Experimental Economics*, 18(4), 609-669.

Diduch, A.M. (2010). "Patents and R&D: a classroom experiment", *International Review of Economic Education*, 9(2), 67-83.

Doraszelski, U. (2003). "An R&D race with knowledge accumulation", *RAND Journal of Economics*, 34(1), 20-42.

Duan, W., Adnan, K., Abdur, R., Adrian, C.C. (2022). "Government subsidies' influence on corporate social responsibility of private firms in a competitive environment", *Journal of Innovation & Knowledge*, 7(2), 1-10.

Eaton, B., Eswaran, M. (1997). "Technology-trading coalitions in supergames", *RAND Journal of Economics*, 28(1), 135-149.

El Ouardighi, F., Shnaiderman, M., Pasin, F. (2014). "Research and Development with stock-dependent spillovers and price competition in a duopoly", *Journal of Optimization Theory and Applications,* 161(2), 626-647.

Erkal, N., Piccinin, D. (2010). "Cooperative R&D under uncertainty with free entry", *International Journal of Industrial Organization*, 28(1), 74-85.

Fudenberg, D., Gilbert, R., Stiglitz, J., Tirole, J. (1983). "Preemption, leapfrogging and competition in patent races", *European Economic Review*, 22(1), 3-31.

Gil-Molto, M.J., Poyago-Theotoky, J., Zikos, V. (2011). "R&D subsidies, spillovers and privatization in mixed markets", *Southern Economic Journal*, 78 (1), 233-255.

Goto, A., Odagiri, H. (1997). "Innovation in Japan", 1st ed, *Japan Business and Economics Series,* Clarendon Press.

Grossman, G.M., Shapiro, C. (1987). "Dynamic R&D programs", *The Economic Journal*, 97, 372-387.

Grünfeld, L.A. (2003). "Meet me halfway but don't rush: absorptive capacity and strategic R&D investment revisited", *International Journal of Industrial Organization*, 21(8), 1091-1109.

Grünfeld, L.A. (2006). "Multinational production, absorptive capacity and endogenous R&D spillovers", *Review of International Economics*, 14(5), 922-940.

Halmenschlager, C. (2006). "Spillovers and absorptive capacity in a patent race", *The Manchester School*, 74(1), 85-102.

Hammerschmidt, A. (2009). "No pain, no gain: an R&D model with endogenous absorptive capacity", *Journal of Institutional and Theoretical Economics*, 165(3), 418-437.

Harris, C., Vickers, J. (1987). "Racing with uncertainty", *The Review of Economic Studies*, 54(1), 1-21.

Harris, C., Vickers, J. (1985). "Perfect equilibrium in a model of a race", *Review of Economic Studies*, 52(2), 193-209.

Heggedal, T. (2015). "Knowledge spillovers and R&D subsidies to new, emerging technologies", *Economics of Innovation and New Technology*, 24(7), 710-733.

Henriques, I. (1990). "Cooperative and noncooperative R&D in duopoly with spillovers: comment", *American Economic Review*, 80(3), 638-640.

Hörner, J. (2002). "Reputation and competition", *American Economic Review*, 92(3), 644-663.

Hörner, J. (2004). "A perpetual race to stay ahead", *Review of Economic Studies*, 71(4), 1065-1088.

Isaac, R.M., Reynolds, S. (1992). "Stochastic innovation and product market organization", *Economic theory*, 2(4), 525-545.

Ishikawa, N., Shibata, T. (2021). "R&D competition and cooperation with asymmetric spillovers in an oligopoly market", *International Review of Economics and Finance*, 72(8), 624-642.

Kabiraj, A. (2018). "R&D cooperation in a three-firm Cournot industry", *Economics Bulletin*, 38(3), 1541-1546.

Kamien, M. I., Schwartz, N.L. (1982). "Market structure and innovation", Cambridge University Press.

Kamien, M., Muller, E., Zang, I. (1992). "Research joint ventures and R&D cartels", *American Economic Review*, 82, 1293-1306.

Kesavayuth, D., Zikos, V. (2013). "R&D versus output subsidies in mixed markets", *Economics Letters*, 118(2), 293-296.

Kesteloot, K., Voet, N. (1998). "Incentives for cooperation in quality improvement among hospitals--the impact of the reimbursement system", *Journal of Health Economics*, 17(6), 701-728.

Klette, T. J., Moen, J., Griliches, Z. (2000). "Do subsidies to commercial R&D reduce market failures? microeconomic evaluation studies", *Research Policy*, 29, 471-495.

Kunz, W., Schmitt, B., Meyer, A. (2011). "How does perceived firm innovativeness affect the consumer?", *Journal of Business Research*, 64(8), 816-822.

Lahiri, S., Ono, Y. (1999). "R&D subsidies under asymmetric duopoly: a note", *Japanese Economic Review,* 50(1), 104-111.

Leahy, D., Neary, J.P. (1997). "Public policy towards R&D in oligopolistic industries", *American Economic Review*, 87(4), 642-662.

Leahy, D., Neary, J.P. (2007). "Absorptive capacity, R&D spillovers, and public policy", *International Journal of Industrial Organization*, 25(5), 1089-1108.

Lee, S-H., Muminov, T., Tomaru, Y. (2017). "Partial privatization and subsidization in a mixed duopoly: R&D versus output subsidies", *Hitotsubashi Journal of Economics*, 58(2), 163-177.

Lee, T., Wilde, L. (1980). "Market structure and innovation: a reformulation", *Quarterly Journal of Economics*, 94(2), 429-436.

Loury, G.C. (1979). "Market structure and innovation", *Quarterly Journal of Economics*, 93(3), 395-410.

Luo, Y. (2006). "Opportunism in inter-firm exchanges in emerging markets", *Management and Organization Review*, 2(1), 121-147.

Miyagiwa, K., Ohno, Y. (2002). "Uncertainty, spillovers, and cooperative R&D", *International Journal of Industrial Organization*, 20(6), 855-876.

Miyagiwa, K., Ohno, Y. (1997). "Strategic R&D policy and appropriability", *Journal of International Economics*, 42(1-2), 125-148.

Mukherjee, V., Ramani, S.V. (2011). "R&D cooperation in emerging industries, asymmetric innovative capabilities and rationale for technology parks", *Theory and Decision*, 71(3), 373-394.

Ono, Y., Sekozawa, T. (2016). "Evaluation of patent race between three firms using a game-theoretic real options approach", *Electronics and Communications in Japan*, 99(7), 64-76.

Qiu, L.D. (1997). "On the dynamic efficiency of Bertrand and Cournot equilibria", *Journal of Economic Theory*, 75(1), 213-229.

Reinganum, J.F. (1981). "Dynamic games of innovation", *Journal of Economic theory*, 25(1), 21-41.

Reinganum, J.F. (1984). "Practical implications of game theoretic models of R&D", *American Economic Review*, 74, 61-66.

Richard, J.G. (2019). "Competition, mergers, and R&D diversity", *Review of Industrial Organization*, 54(3), 465-484.

Sah, R., Stiglitz, J. (1987). “The invariance of market innovation to the number of firms”, *The RAND Journal of Economics*, 18(1), 98-108.

Sbriglia, P., Hey, J.D. (1994). “Experiments in multi-Stage R&D competition”, *Empirical Economics*, 19, 291-316.

Sheremeta, RM. (2010). "Experimental comparison of multi-stage and one-stage contests", *Games and Economic Behavior*, 68(2), 731-747.

Sheshinski, E. (1976). “Price, quality and quantity regulation in monopoly situations”, *Economica*, 43(170), 127-137.

Shibata, T. (2014). “Market structure and R&D investment spillovers”, *Economic Modelling*, 43, 321-329.

Shih-Wei, C. (2005). “Knowledge creation: absorptive capacity, organizational mechanisms, and knowledge storage/retrieval capabilities”, *Journal of Information Science*, 31(6), 453-465.

Silipo, D.B. (2005). “The evolution of cooperation in patent races: theory and experimental evidence”, *Journal of Economics*, 85 (1), 1-38.

Silipo, D.B. (2008). “Incentives and forms of cooperation in research and development”, *Research in Economics*, 62, 101-119.

Singh, N., Vives, X. (1984). “Price and quantity competition in a differentiated duopoly”, *RAND Journal of Economics*, 15(4), 546-554.

Slivko, O., Theilen, B. (2014). “Innovation or imitation? the effect of spillovers and competitive pressure on firms’ R&D strategy choice”, *Journal of Economics*, 112(3), 253-282.

Spence, A. M. (1975). “Monopoly, quality and regulation”, *Bell Journal of Economics*, 6(2), 417-429.

Stuebs, M.T., Sun, L. (2009). “Corporate reputation and technical efficiency: evidence from the chemical and business services industries”, *Journal of Applied Business Research*, 25(5), 21-29.

Symeonidis, G. (2003). “Comparing Cournot and Bertrand equilibria in a differentiated duopoly with product R&D”, *International Journal of Industrial Organization*, 21(1), 39-55.

Toshimitsu, T. (2003). “Optimal R&D policy and endogenous quality choice”, *International Journal of Industrial Organization*, 21(8), 1159-1178.

Toshimitsu, T. (2014). “Strategic product R&D investment policy under international rivalry in the presence of demand spillover effects”, *International Economic Journal*, 28(2), 293-309.

Veugelers, R., Kesteloot, K. (1994). “On the design of stable joint ventures”, *European Economic Review*, 38(9), 1799-1815.

Villena, V.H., Craighead, C.W. (2017). “On the same page? How asymmetric buyer-supplier relationships affect opportunism and performance”, *Production and Operations Management*, 26(3), 491-508.

Walter, J., Kellermanns, F.W., Lechner C. (2012). “Decision making within and between organizations: rationality, politics, and alliance performance”, *Journal of Management*, 38(5), 1582-1610.

Wu, C.H. (2021). “Production-quality policy for a make-from-stock remanufacturing system”, *Flexible Services and Manufacturing Journal*, Springer, 33(2), 425-456.

Yang, Y.C. (2014). “Government preference and the optimal choice of R&D subsidy policy: innovation subsidy or product subsidy?”, *Journal of Applied Mathematics*, 2014, 1-9.

Zahra, S.A., George, G. (2002). “Absorptive capacity: a review, reconceptualization, and extension”, *The Academy of Management review*, 27(2), 185-204.

Zizzo, D.J. (2002). “Racing with uncertainty: a patent race experiment”, *International Journal of Industrial Organization*, 20(6), 877-902.