A New Approach to Dating the Canadian Reference Cycle

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Abstract

I evaluate the ability of the mixture multiple change-point model to establish Canadian business cycle turning points dates. Following the technique by Camacho, Gadea, and Loscos (2022) I asses the efficacy and feasibility of this dating method in achieving turning point dates by comparing it with the already announced dates by the C.D. Howe Institute Business Cycle Council. Using key monthly economic indicators spanning the last 34 years, I find that this methodology successfully identifies three business cycle dates for Canada, offering valuable insights into the peaks and recessions the Canadian economy has likely experienced in the past. Nevertheless, It yields significant disparities when compared with the chronology established by the Howe Institute.

Contents

List of Figures

List of Tables

1 Introduction

Various methods for identifying business cycle phases have been extensively studied over the years. Researchers have delved into understanding these phases and the transitions between them through numerous macroeconomic papers. For instance, Bodman and Crosby (2000) test the Markov-switching model of Hamilton (1989) to generate Canadian business cycle dates, arriving to the conclusion that, regime-switching models outperform linear models in fitting the data. Supported by Demers and Macdonald (2007) that show how nonlinear models better adapt Canadian business cycle features. Another research by Chauvet and Hamilton (2006) analyze a univariate parametric version of the Markov-switching model studied by Hamilton. Similarly, their findings indicate that this method efficiently obtains turning point dates using monthly data, surpassing the speed achieved with quarterly data. Going further, Harding and Pagan (2006) propose a non-parametric algorithm to extract common cycles, and contrast the National Bureau of Economic Research "NBER" business cycle turning point dates. Subsequently, it underwent a more profound analysis by Chauvet and Piger (2008) that compare the multivariate Marvok-switching model and the Harding and Pagan (2006) algorithm using a real-time data set. Stating a significant improvement by both methods over the "NBER" in the speed with which business cycles troughs were identified. Relating all these studies, Camacho, Gadea, and Loscos (2022) developed a new approach to date the reference cycle based on an extent of the univariate multiple change-point model of Chib (1998) to a multivariate framework. Finding tempting results in identifying U.S. business cycle dates in contrast to the "NBER".

This paper investigates how the multiple change-point model works in the Canadian context. Following the approach by Camacho, Gadea, and Loscos (2022), I evaluate the ability of this dating method to establish Canadian business cycle turning point dates. I apply this approach to ten relevant economic variables, distinct from those analyzed in Camacho, Gadea, and Loscos (2022): (1) trade balance, (2) exports-balance of payments, (3) real exchange rate, (4) monthly GDP, (5) industrial production, (6) manufacturing, (7) employment for hourly paid employees, (8) employment for salaried employees, (9) average weekly earnings for all employees, and (10) average of hourly paid and salaried employment. Thus, with this new economic environment I explore the changes in the Canadian business cycles by comparing with announcements made by the C.D. Howe Institute Business Cycle Council.

Estimating the model yields three clusters of peak and trough dates, showing regimes of high and low growth that each time series has experienced over the years. Concluding that, while the method by Camacho, Gadea, and Loscos (2022) it is a useful tool for analyzing the reference cycle, the resulting chronology differs from the dates provided by the C.D. Howe

Institute. For example, in clusters one and three, there are significant month-long disparities between the peaks and troughs identified by the multiple change-point model and the Howe approach.

In the next section, a more detailed explanation of the implementation of this method for U.S. data will be presented, along with additional relevant papers. Section 3 describes the data. Section 4 explains the performance of the approach to obtain business cycle turning point dates. The results are provided in Section 5. Section 6 concludes.

2 Relevant Literature

Recessions and Expansions are periods of the economic activity when it deviates from its long-run growth path. Several institutions, such as central banks have developed chronologies of turning point dates at which shifts between recessions and expansions occur. There are significant findings with respect to the performance of different models to establish turning point dates, which overcome some of the critics arose when announcing business cycle dates.

The appropriate monitoring of the Canadian business cycles allows the different agents involved in the economy to take actions in response to future economic expansions or recessions, and explore more accurately the structure of the Canadian GDP. Bodman and Crosby (2000) suggest using the markov-switching model of Hamilton (1989) to examine the phase structure of the Canadian GDP and possible non-linearities hidden in the data. They test its ability to generate business cycle chronologies and how well the model fits the data. They conclude that regime-switching models represent the data better than linear models. This result is supported by Demers and Macdonald (2007), who show how econometric rules can better explain the different metrics that involve Canadian business cycle properties. Through a Monte Carlo exercise, they test the ability of different models (linear, nonlinear, univariate, and multivariate) to replicate the features of actual Canadian data. Their findings for Canada are in line with those of Morley and Piger (2006) for U.S. data, but at odds with those of Engel, Haugh, and Pagan (2005). Nonlinear models do improve the matching of business cycle features.

The procedures used by the NBER to establish U.S. turning point dates have been criticized by public agents involved in the economic activity, arguing important delays in their announcements. These delays could lead to wrong assumptions made by government bodies, private firms, and individual households. Chauvet and Hamilton (2006) analyze the performance of the formal statistical methods studied by Hamilton (1989), hoping to obtain more useful results to the public in real time. First, they apply a univariate parametric version of the markov-switching approach to GDP data. Second, the authors focus on a multivariate inference of this method to different economic indicators such as (manufacturing and trade sales, personal income less transfer payments, employment, industrial production). The authors find that using monthly data rather than quarterly data significantly improve how fast business cycle troughs can be identified.

The synchronization of cycles have a great impact when comes to define monetary unions between countries, as well as the time of monitoring stock market indices in a single or across countries. For that reason, Harding and Pagan (2006) demonstrate how the degree of synchronization to growth and classical cycles can be measured based on turning point dates. In addition, by the development of a non-parametric algorithm, common cycles are extracted. These tests show weak evidence of synchronization in the cycles of industrial production and strong evidence of synchronization with respect to the stock prices, with a feasible algorithm to extract common cycles from a set of specific cycles making formal the informal methods used by the NBER. Following all these studies, Chauvet and Piger (2008) apply to a new-real time data set the multivariate marvok-switching model and the (MPH) algorithm to achieve possible upgrades in the establishment of U.S. business cycle turning points dates. Their findings demonstrate how both methods provide improvement in the timeliness with which they identify those phases along with a reasonable accuracy of the dates.

The challenges and limitations when applying or replicating business cycles dating methods have significantly improved in recent years. Camacho, Gadea, and Loscos (2022) came up with a precise and feasible approach that supplements existing methods for establishing business cycles dates. This innovative framework requires minimal assumptions and effectively addresses the gaps in existing literature. The approach pursued in this paper to date the reference cycle is based on aggregating specific turning point dates from ten coincident economic variables viewed as population concepts as in Stock and Watson (2010). Both the number and dates of the turning points are estimated in a single step. In addition, the method allows to make statistical inference about the reference cycle turning points. where the Chib (1998) multiple-change point model is applied. Concluding that this method identifies the dates of the NBER-reference cycle very accurately with a significant speed.

The establishment of business cycle turning point dates have been analyzed by several studies, which conclude that formal rules improve the speed and accuracy of these points. There are still some interrogatives when we talk about business cycle announcements, such as models feasibility, since there is not much information about that. The following study will investigate and test the results provided by Camacho, Gadea, and Loscos (2022) in the Canadian context. It will allow us to explore a bit more how well these non-linear models perform at the time of setting up these business cycle phases.

3 Data

In this section I present the Canadian data compiled for the research. The data encompasses ten key economic variables such as trade balance, exports-balance of payments, real exchange rate, GDP, industrial production, manufacturing, employment for hourly paid employees, employment for salaried employees, average weekly earnings for all employees, and average of hourly paid and salaried employment. I analyse monthly data from January 1988 to September 2022, since 2022 is the most recent data for the ten variables. For simplicity, the time series are sampled at monthly frequencies to avoid discontinuity challenges. I obtain exports-balance of payments, GDP, industrial production, manufacturing, employment for hourly paid employees, employment for salaried employees, and average weekly earnings for all employees from the Statistics Canada database. Real exchange rate is collected from (FRED), that is, the Federal Reserve Bank of St. Louis database. Data pertaining to trade balance is calculated from the subtraction of exports-balance of payments and imports-balance of payments.

One problem in the time series is that some indicators are not available at the beginning of the sample. In this context, that is not a major issue since it only implies that the probability distribution of turning points at early dates must be estimated from a relatively lower number of observations due to unavailable data.

4 Methodology

This section explains in detail how the methodological process by Camacho, Gadea, and Loscos (2022) works. The following approach derives turning point dates from a Markov mixture distribution, identifying them as outcomes from different Gaussian distributions. These realizations divide the sample into reference business cycle phases, whose transitions from one regime to another are limited as in multiple change-point model.

4.1 Baseline Model - Finite Markov Mixture Distribution

The reference cycle of the entire economy over a specific period of time is defined by a chronology of K pairs of peaks and troughs dates, that is, expansions and recessions. These realizations can break the sample into reference business cycle phases, with each phase containing a single pair of reference turning point dates, $u_k = (u_k^P, u_k^T)$ for all $k = 1, ..., K$,. Since these episodes are unknown, we can initially infer the reference turning point dates from different economic indicators R. Then, it is possible to extract specific pairs of turning point dates from these time series, r, in sets of size n_r, with $r = 1, ..., R$. This procedure gives a total of $N = n_1 + ... + n_R$ individual pairs of turning point dates collected in groups of peaks and troughs as in $\tau = (\tau_i^P, \tau_i^T)$, for each of the individual pair of turning points $i = 1, ..., N$, produced previously. According to Burns and Mitchell (1946), specific turning point dates are concentrated around the reference cycle K mentioned above. Similarly here, the distribution of turning points dates is heterogeneous across and homogeneous within the reference turning points.

Therefore, the resulting turning points arise at each episode k of expansions and recessions determined from the same bivariate Gaussian distributions. These turning points give insights into how the peaks and troughs dates are distributed across the time span. For that reason, the means of each distribution are taken as the reference turning points, u_k, with their respective covariance matrices denoted as $\sum_k$.

When the dates of the episodes are unknown the parameters of the distribution are defined as $\theta = (\theta_1, ..., \theta_K)$, where $\theta_k = (u_k, \sum_k)$ are the distribution parameters in group of peaks and troughs k. The reference cycle turning points is assumed to be labelled through an unobservable state variable s taking values in the set $\{1, ..., K\}$, which are collected in all the realizations $S = (s_1, ..., s_N)$. Therefore, $s_i = k$ indicates that the pair of turning points τ_i is drawn from a bivariate Gaussian distribution with parameters θ_k. The state variable s is modeled as a first-order K-state Markov-chain, which implies that the probability of a change in regime depends on the past only through the value of the most recent regime:

$$Pr(s_i = k|s_{i-1} = l, ..., s_1 = w, \tau^{i-1}) = Pr(s_i = k|s_{i-1} = l) = p_{lk}, \tag{1}$$

where $\tau^i = (\tau^1, ..., \tau^i)$. In order words, the economy's next state, whether recession or peak, will depend on its most recent state. The stochastic properties of this process are described by a (KxK) transition matrix, P, whose rows sum to one. Following Chib (1998), the unobserved state variable exhibits the following transition probability matrix.

$$P = \begin{pmatrix} p_{11} & 1-p_{11} & 0 & \dots & 0 \\ 0 & p_{22} & 1-p_{22} & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & 0 & p_{K-1K-1} & 1-p_{K-1K-1} \\ 0 & \dots & & 0 & 1 \end{pmatrix}. \tag{2}$$

Which basically says, when the Markov chain reaches a reference cycle date k for the first time, the process remains there with probability p_{kk} until it reaches the reference cycle date $k+1$ with probability $1-p_{kk}$, for $k = 1, ..., K-1$. Once the Markov chain reaches the latest reference cycle date, it remains there with probability one. In this case, P depends upon the set of probabilities p_{kk}, with $k = 1, ..., K-1$, collected in the vector π. In synthesis, the Markov chain procedure tracks changes from one pair of turning point dates to the next, always using the previous pair as a reference. This procedure repeats until it reaches the latest date, working as a multiple change-point model.

Finally, each specific turning point needs to be assigned to a certain reference cycle turning point by making inference on the unobserved allocations S. In that sense, each pair of turning point τ_i is assumed to be a realization of a finite Markov mixture model of K components and, from the law of total probability, its marginal density is

$$p(\tau_i|\theta, \pi, \tau^{i-1}) = \sum_{k=1}^{K} Pr(s_i = k|, \theta, \pi, \tau^{i-1}) p(\tau_i|\theta_k, \tau^{i-1}). \tag{3}$$

Here, $Pr(s_i = k|, \theta, \pi, \tau^{i-1})$ is the filtered probability of observations from group k, laying between $0 \leq Pr(s_i = k|, \theta, \pi, \tau^{i-1}) \leq 1$, where $\sum_{k=1}^{K} Pr(s_i = k|, \theta, \pi, \tau^{i-1}) = 1$, which is governed by the transition probability matrix P. In this context, $p(\tau_i|\theta_k, \tau^{i-1})$ is the Gaussian density, integrated with its different means and covariance matrices $N(u_k, \sum_k)$.

4.2 Bayesian Estimation Using the Gibbs Sampler

Through a Markov chain Monte Carlo (MCMC) method the parameters collected in θ and π and the inference about S are estimated. The Gibbs sampler or MCMC algorithm is a useful tool that allows to estimate the subset of unknown parameters from the joint distribution rather than the entire set. It starts from some preliminary classification $S^{(0)} = (s_1^{(0)}, ..., s_N^{(0)})$, which reveals the number of observations assigned to each kth turning point, $N_k(S^{(0)})$, and its sample means $u_k^{(0)}$, with $k = 1, ..., K$. First step of the MCMC method is to assign values to the model parameters $\pi^{(m)}$ and $\theta^{(m)}$ conditional on the data and the classification $S^{(m-1)}$. When the transition probabilities π are assigned to values, the Beta distribution is the standard choice in the context of modeling time-series specifications that are subject to multiple-change points. Following the assumptions on Chib (1998), p_{kk} are initially independent, $p(p_{11}, ..., p_{k-1,k-1}) = p(p_{11})...p(p_{k-1k-1})$, that is, the joint probability of a change in all regimes is equal to each individual probability change. They follow Beta distributions as follows, $p_{kk} - Beta(e_{k1}, e_{k2})$. For estimating θ, the Gibbs sampler uses a prior which has u_k coefficients and $\sum_k$ covariances that are independent of one another.

Second, the classification $S^{(m)}$ of the state variables mentioned previously needs to be estimated, conditional on the parameters $\pi^{(m)}$ and $\theta^{(m)}$. As in Camacho, Gadea, and Loscos (2022), the multi-move Gibbs procedure is based on sampling the whole path S from its conditional posterior distribution, which, according to the properties of the first-order Markov chain, it can be viewed as:

$$Pr(S|\theta,\pi,\tau) = Pr(s_N|,\theta,\pi,\tau^N)\Pi_{i=1}^{N-1}Pr(s_i|,s_{i+1},\theta,\pi,\tau^i). \quad (4)$$

The joint probability of realizations from the collected parameters $\pi^{(m)}$ and $\theta^{(m)}$, whose parameters derive the different pairs of peaks and troughs dates is shown above. Referring to Chib (1996), who shows $Pr(s_i|,s_{i+1},\theta,\pi,\tau^i) \propto Pr(s_i|,\theta,\pi,\tau^i)\ Pr(s_{i+1}|,s_i)$, meaning, the joint distribution of the path S is proportional to each state and transition probability, then, equation 4 contains the filtered probabilities and the transition probabilities. Now, to sample $S^{(m)}$, requires computing the filtered probabilities of each state, $Pr(s_i = k|,\theta,\pi,\tau^i)$. Following Hamilton (1989), who analyses a filter being able to capture the effects of unobserved variables or states, the one-step ahead prediction with the information up to turning point $i-1$

$$Pr(s_i = k|\theta,\pi,\tau^{i-1}) = \sum_{l=1}^{K} p_{lk} Pr(s_{i-1} = l|,\theta,\pi,\tau^{i-1}) \quad (5)$$

is computed. Thus it gives inference about the states $s_{i-1} = l,...,s_1$ based on the turning points τ. Next, when the ith turning point is added, the filter probability is updated as follows

$$Pr(s_i = k|\theta,\pi,\tau^i) = \frac{Pr(s_i = k|\theta,\pi,\tau^{i-1})p(\tau_i|\theta_k,\tau^{i-1})}{p(\tau_i|\theta,\pi,\tau^{i-1})}, \quad (6)$$

where

$$p(\tau_i|\theta,\pi,\tau^{i-1}) = \sum_{k=1}^{K} Pr(s_i = k|,\theta,\pi,\tau^{i-1})p(\tau_i|\theta_k,\tau^{i-1}).$$

Using the the stored filtered probabilities, the states can be simulated from their join distribution 4 starting by sampling the state of the last observation s_N from the smoothed probability $Pr(s_N|\theta,\pi,\tau^N)$, which matches with the last filtered probability. Now, the conditional distribution of the sates, $Pr(s_i|s_{i+1},\theta,\pi,\tau^i)$, can be obtained by backward recursion. Since the MCMC method could have identifiability problems, the model requires an identifiability constraint that the draws must imply a segmentation of the time span into K non-overlapping episodes, that is, $u_k^{P(m)} \leq u_k^{T(m)} \leq u_{k+1}^{P(m)}$ for all $k = 1,...,K$.

4.3 Identifying the Number of Clusters of Turning Point Dates

The number of groups of specific turning point dates that are cohesive and form a distinct cluster separated from other clusters of specific business cycle turning point dates need to be inferred. In this context, the Poisson hierarchical approach by Koop and Potter (2007) is applied to determine a tentative number of clusters[1]. After rounding the estimated number of breaks to the nearest positive integer, the Bayes factor described above is used to determine the number of phases of the reference cycle K around this integer. Lastly, the estimates of the finite Markov mixture models for K will determine the single dates of the reference cycle turning points, breaking the desired time span into separated segments that determine the distinct business cycle phases.

5 Results

In this section, I apply the method from section 4 on my Canadian data to obtain business cycle dates. Firstly, I obtain individual turning points in each of the ten indicators by implementing the Bry and Boschan (1971) algorithm. Figure 1 shows the bivariate Gaussian distribution of the resulting turning points, suggesting various clusters of turning point dates around episodes of recoveries and declines throughout the collected time span. In contrast to Camacho, Gadea, and Loscos (2022), since the data is shorter, figure 1 presents in both axes shorter intervals, with 10-year vintages. This allow us to observe the distribution of peaks and recessions within these vintages from start to finish.

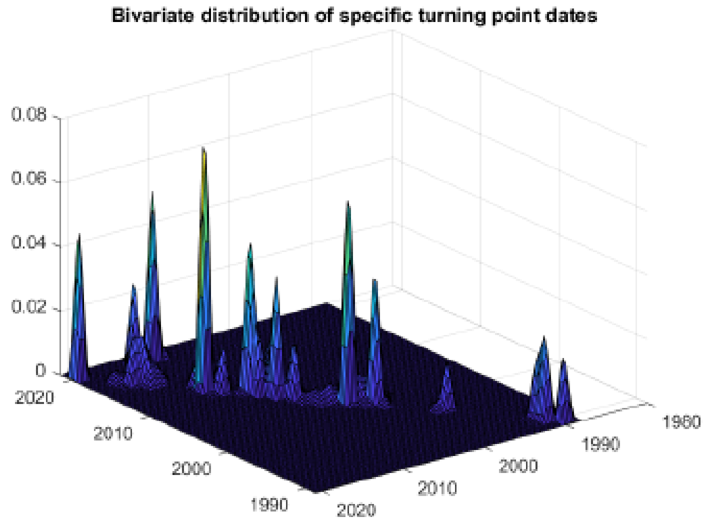
In order to determine the number of clusters formally, I compute sequentially (twice the log of) the Bayes factors that compare two model with $K-1$ and K for which an additional cluster does not increases the odds of the model with less clusters, giving us strong evidence that the complete cycles in the Canadian reference cycle from 1988 to 2022 is three. To support this result, the Poisson hierarchical approach suggested by Koop and Potter (2007) is applied to estimate the number of clusters. This technique suggests that the number of distinct cycles is three, which supports the Howe institute cycle dates and confirms the result of the Bayes factor comparison as we can see in table 1. Furthermore, the first column shows how the maximum likelihood increases uninterruptedly when the number of clusters increases from $K = 1$ to $K = 3$, and the values of AIC, BIC, and BIC-entropy reach their minimums of 390.29, 420.62, and 423.22, respectively, when $K = 3$.

Next, to examine the extent to which the mixture model provides a good partition of the time span, from the rejection sampler described in section 4, I use assigned observations that are useful to illustrate the shape of the posterior mixture distribution. For each of the three clusters, figure 2 shows the two-dimensional scatter plots of means and variances of the MCMC for (u_k^P, u_k^T) in Panel A and for $(u_k^P, \sum_{11,k})$ and $(u_k^T, \sum_{22,k})$ in Panel B.

The scatterplots show how the means differ drastically across the groups, as it is plotted

1. A more technical explanation of the Poisson hierarchical approach suggested by Koop and Potter (2007) is in the appendix B.1 for reference.

Figure 1: Gaussian Distributions



Note: Kernel density estimate of the bivariate distribution of specific business cycle turning point dates around different phases of recoveries and declines along the time span, with both axes representing 10-year vintages.

in panel A, where it also suggests that the clusters of turning points alternate sequentially. Panel B shows how the variances are similar for the first group of peaks and troughs but for second and third groups it shows high variances. The several representations or panels shown in figure 3 demonstrate the draws of the $K = 3$ different pair of clusters, u_k^P, u_k^T, log of the determinant of the covariance matrices, $log(\sum_k)$, and the transition probabilities, $p_{k,k}$ and $p_{k,k+1}$ respectively. The panels help us to identify convergences or any label switching problem between the clusters. As it is shown in the panels A and B for the peaks and troughs respectively, the clusters are well separated resulting in unique labeling, discarding switching concerns. This is illustrated by the number of posterior draws on the x-axis and the years of peaks and recessions on the y-axis. In addition, I can imply significant distortion within the means of each cluster, especially in the mean of the first cluster of the peak and trough. Where in the second and third group the behavior of the means are more cautions, showing a slightly increase in the distortion of the means of the peak compared to the trough. Going forward, table 2 exposes the results obtained by the mixture multiple change point model estimation, where I evaluate its capacity to determine the Howe Council turning point dates.

The column labeled as HOWE reports the business cycle dates announced by the Howe Council and the column labeled as MSMM reports the business cycles dates determined by

Table 1: Results of select K

K	LogLik	AIC	BIC	Entropy	BIC-Entropy	Bayes factor (k=i/k=i+1)
1	-497.61	1005.23	1014.15	-	-	-
2	-204.24	430.48	450.11	0.03	450.16	564.04
3	-178.14	390.29	420.62	1.30	423.22	29.49

Note: The first column indicates the marginal log-likelihoods; the second and third are the Bayesian AIC and BIC selection criteria; the fourth column refers to the entropy; the fifth column shows the BIC corrected for misclassification, and the last column places twice the log of the Bayes factor with $K = 2, ..., 3$.

the Markov-switching mixture model. The employed approach highlights significant discrepancies within the peaks and troughs reported by the Howe. According to table 2, in the first cluster the peak yield by the method is reported 122 months after the date announced by the Council, and the trough appears 110 months after the Howe dating. This indicates that the first recession detected by the MSMM model starts 122 months later and ends 110 months later than the first recession announced by the C.D. Howe institute. Same analogy for the second cluster, the peak is located by the model 7 months after the Howe announced date, where the trough is reported 14 months after the trough identified by the Howe Council. In the third cluster the model places a peak 66 months before the Howe, and a trough 22 months prior to the trough identified by the Howe approach. Notably, the MSMM turning point dates do not align with those announced by the Howe council, revealing notable disparities in the methodologies employed by this approach and the Council business cycle dating method.

Table 2: Results of empirical illustration (HOWE series)

HOWE		MSMM		Deviation (in months)	
Peaks	Troughs	Peaks	Troughs	Peaks	Troughs
1990.03	1992.05	2000.05	2001.07	-122.00	-110.00
2008.10	2009.05	2009.05	2010.07	-7.00	-14.00
2020.02	2020.04	2014.08	2018.06	66.00	22.00

Note: Columns 1 and 2 report turning point dates established by the Howe Council. Columns 3 and 4 exhibit turning point dates identified by the mixture multiple change point model. Deviations provide the difference in months between the Howe and the model estimated dates.

Figure 7 plots the posterior classification probabilities estimated by the model through the corresponding relative frequency of the retained state draws, $Pr(s_i = k|\theta)$, with $k = 1, ..., 3$, and $i = 1, ..., N$. The model assigns probabilities to each specific date, distributing them between 0 and 1, where they clearly don not agree with the cycles referenced by the Howe.

To conclude, I plot each time series with shaded recession periods as established by the C.D. Howe Institute, alongside those identified by the MSMM model. This allows for a clearer comparison of the two outputs in table 2 and highlights any differences as it is shown in figure 8 and figure 9.

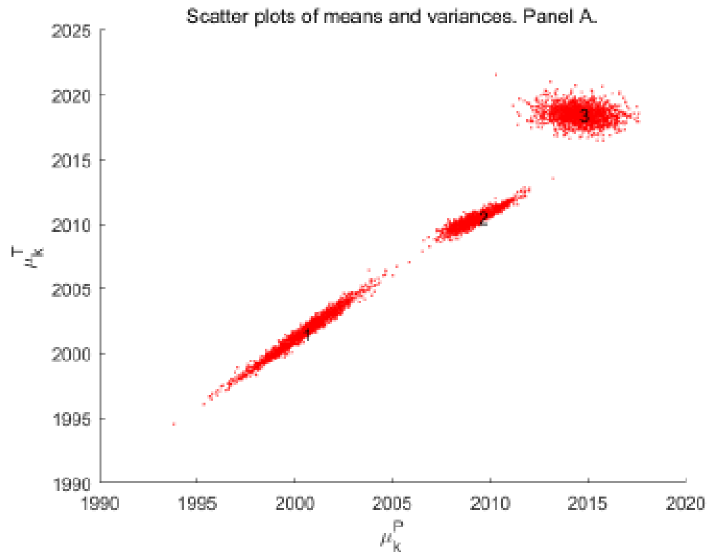
6 Conclusion

The research analyze how the mixture multiple change-point model can establish business cycle turning point dates for the Canadian Economy. Different models to obtain Business cycle dates have been analyzed over the years, since there has been doubts about the announced dates by central banks or institutions across countries. For instance, Bodman and Crosby (2000) suggest than non-linear models as the Markov-switching model better establish turning point dates than linear models. Supported by Demers and Macdonald (2007), that show how econometric rules better explain Canadian business cycle features. Figure 10 and 11 show the Canadian business cycles chronologies from the different methods analyzed by the papers respectively. Similar to the findings of Camacho, Gadea, and Loscos (2022), who came up with a new feasible approach showing accurate results in the establishment of U.S. business cycle turning point dates. Where I try to demonstrate the enhanced precision of the model they employ in announcing these dates using different economic variables. The proposed method effectively identifies turning point dates within the Canadian context. However, it is worth noting that the referenced cycles diverge from the expected patterns when compared to the Howe Council business cycles dates.

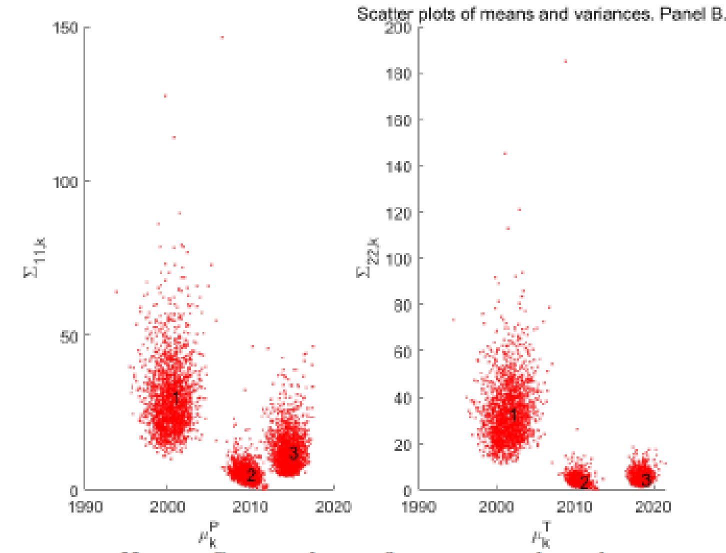
The model locates peaks and troughs, exhibiting a temporal disparity of several months compared to the dates pinpointed by the Howe Council. For instance, in the first cluster, the peak is located by the model 122 months after the date announced by the Howe Council, and the trough is reported 110 months after the Howe dating. Similar discrepancies are observed in the other two clusters. This divergence may stem from various factors, with one plausible explanation being the limited availability of the data. This paper relies on information spanning from 1988 to 2022, resulting in a relatively constrained number of recessions within this time frame. The introduction of a new international framework could also contribute to the disparity in business cycle dates, therefore, it may be necessary to re-run this analysis without international time series such as trade balance, balance of payments, real exchange rate, to observe any changes in the turning points. In conclusion, the proposed approach provides promising results, offering a valuable tool for the extend analysis of business cycles.

A Figures

Figure 2: Scatterplots

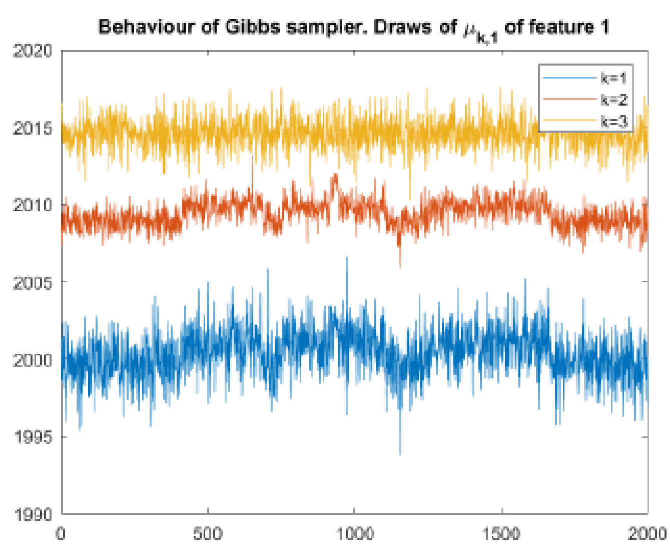


Note: Scatterplot of means and variances of the Markov chain Monte Carlo method, (u_k^P, u_k^T), for each of the $K = 3$ clusters.



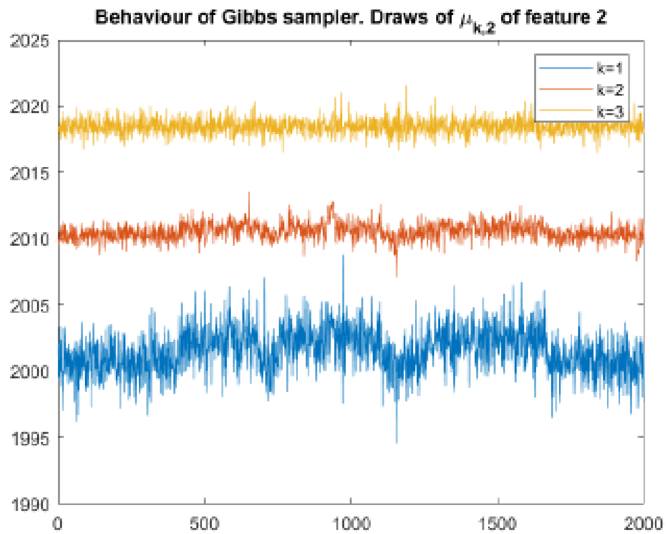
Note: Scatterplots of means and variances, $(u_k^P, \sum_{11,k})$, and $(u_k^T, \sum_{22,k})$, of the MCMC method for each of the $K = 3$ clusters.

Figure 3: Gibbs Sampler-Panel A



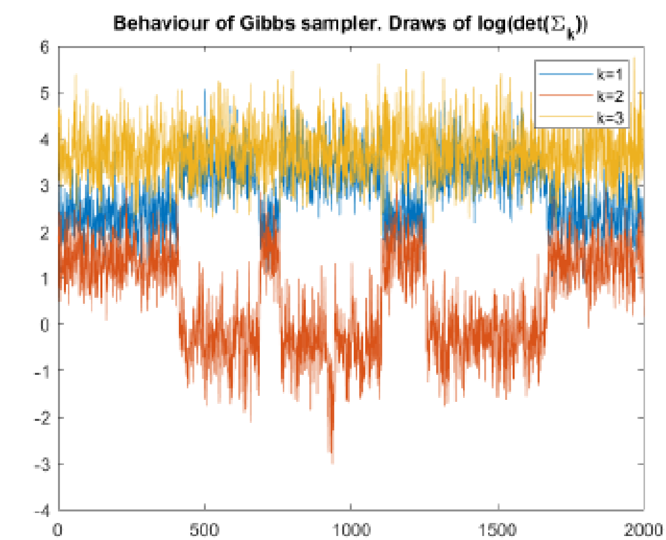
Note: Illustration of posterior draws for each cluster u_k^P.

Panel B



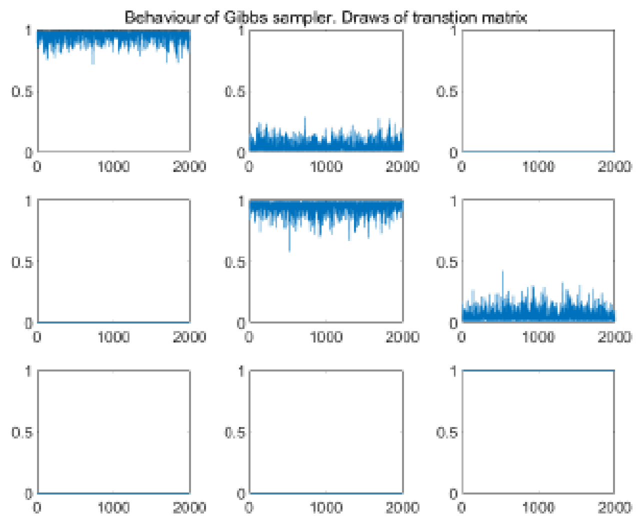
Note: Illustration of posterior draws for each cluster u_k^T.

Panel C



Note: Illustration of posterior draws in terms of log, $log(\sum_k)$, for all clusters $K = 3$ of peaks and troughs.

Panel D

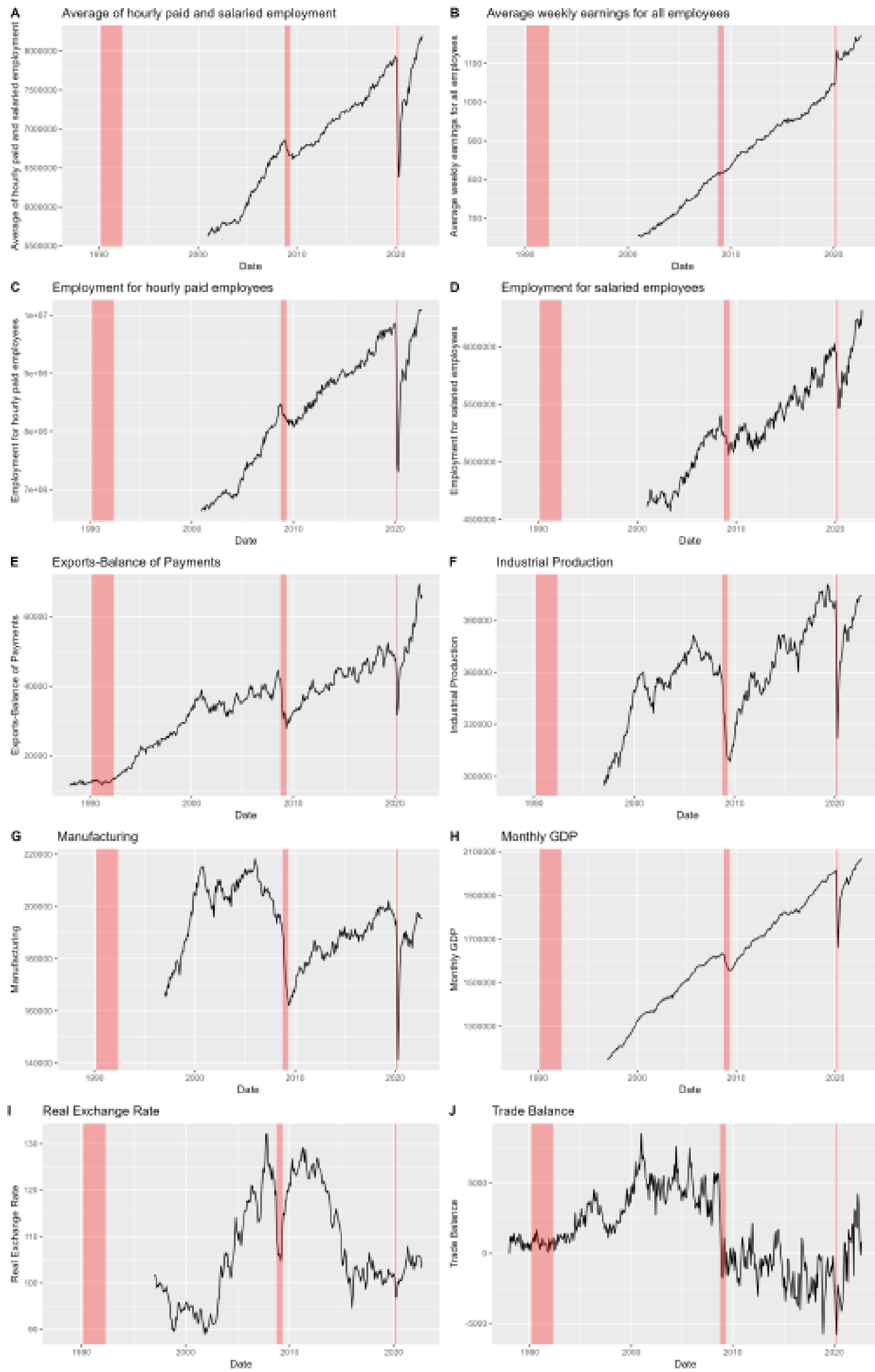


Note: Illustration of posterior draws of each transition probability, $p_{k,k}$ and $p_{k,k+1}$, for the different pairs $K = 3$ of peaks and troughs.

Figure 7: Posterior Classification Probabilities

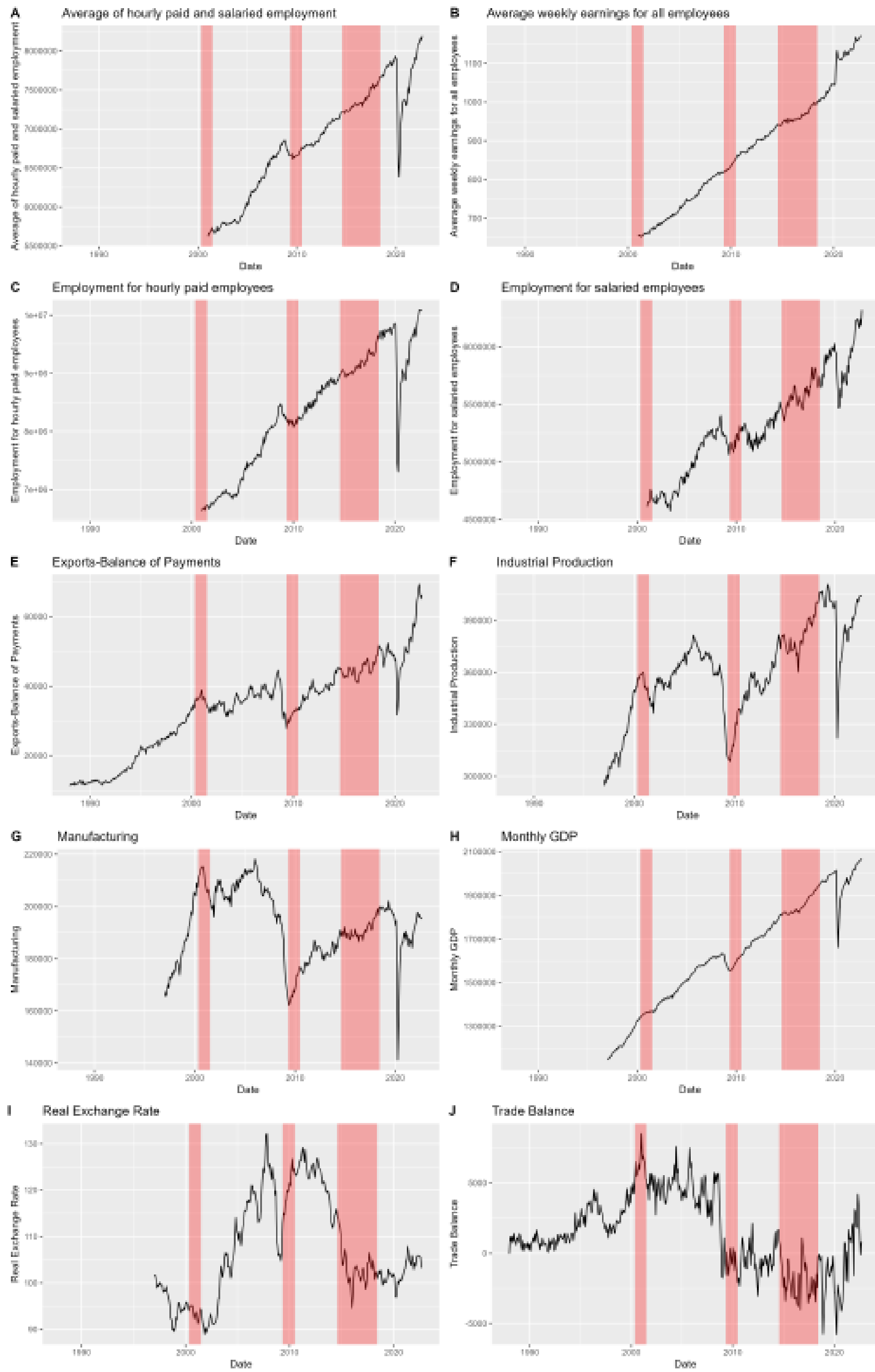
Note: The plot shows the estimated posterior classification probabilities $Pr(s_t = k|\theta)$ on a chronology from January 1988 to September 2022, assigned to each cluster $k = 1, ..., 3$, and observation $i = 1, ..., N$.

Figure 8: Time Series During Each Recession Announced by the C.D. Howe Institute



Note: Subfigures of each economic variable indicating each recession period established by the C.D. Howe Institute Business Cycle Council.

Figure 9: Time Series During Each Recession Identified by the MSMM model



Note: Subfigures of each economic variable indicating each recession period established by the Markov-switching mixture model.

Figure 10: Canadian Business Cycle Chronology (Bodman and Crosby (2000))

Business cycle chronologies for Canada

Cross: Recession dates for GDP.		Economic cycle research institute dates		'Two quarters of negative growth' rule		MS2 model		Growth during (MS2) contractions	Recovery duration to previous peak
Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough		
1947:3	1948:2								
1951:2	1951:4			1950:4	1951:2	1950:4	1951:3	−2.884	1 quarter
1954:1	1954:2	5/53	6/54			1953:3	1954:1	−7.091	3 quarters
1957:1	1957:4	10/56	2/58	1956:3	1957:1				
				1957:2	1957:4	1957:2	1957:4	−2.071	1 quarter
1960:2	1961:1								
1970:2	1970:2	7/74	1/75	1969:3	1970:1	1969:3	1970:1	−1.329	1 quarter
1980:2	1980:2			1979:4	1980:2	1979:4	1980:2	−1.204	1 quarter
1981:3	1982:4	4/81	11/82	1981:1	1982:3	1981:1	1982:3	−4.359	3 quarters
1990:2	1991:1	3/90	3/92	1989:4	1990:4	1989:4	1990:4	−3.512	9 quarters

SOURCES: The Cross dates are from P. Cross (1996), who uses quarterly and six-monthly GDP growth to date recessions. The ECRI dates follow the methodology of the NBER in dating U.S. business cycles. This chronology uses a large number of economic variables to construct business cycle dates. ECRI dates and details on their construction can be found in Moore and Zarnowitz (1986). Dates for the most recent cycle are available on the ECRI website, www.businesscycle.com.

Figure 11: Canadian Business Cycle Dates

Peak		Trough	
ECRI	BBQ	ECRI	BBQ
—	1980Q1	—	1980Q3
1981 Q1	1981Q2	1982Q4	1982Q4
1990 Q1	1990Q1	1992Q1	1991Q1

Note: Business cycle chronology comparison of the Economic Cycle Research Institute (ECRI) with the BBQ algorithm analyzed in Demers and Macdonald (2007).

B Equations

B.1 the Poisson hierarchical approach

Koop and Potter (2007) model the duration of a business cycle phase k, d_k, using a Poisson distribution with mean λ_k, $d_k - 1 \sim P_0(\lambda_k)$. The non-constant transition probability from phase k to $k+1$ depends on the current duration of the regime. Now, assuming that regime 1 starts with the initial observation, the transition probabilities are follows

$$Pr[s_{t+1} = k+1 | s_t = k, d_k] = \frac{exp(-\lambda_k)\lambda_k^{d_k-1}}{(d_k-1)!\left(1 - \sum_{j=0}^{d_k-2} \frac{exp(-\lambda_k)\lambda_k^j}{-j!}\right)},$$

where the sum in the denominator is defined to be 0 when $d_k = 1$. The authors propose a hierarchical prior for λ_k such that $p(\lambda_1, ..., \lambda_T) = p(\lambda_1)...p(\lambda_T)$ with the prior

$$\lambda_k | B_\lambda \sim G\left(\alpha_\lambda, B_\lambda\right),$$

where B_λ, which reflects the degree of dissimilarity of the durations, is an unknown parameter, with prior

$$B_\lambda^{-1} \sim G\left(\xi_1, \frac{1}{\xi_2}\right).$$

Then, the conditional posterior for λ_k is

$$\lambda_k | B_\lambda \sim G(\alpha_k, B_k),$$

where $\alpha_k = \alpha_\lambda + d_k$ and $B_k = \left(B_\lambda^{-1} + 1\right)^{-1}$.

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