Additive Word-Problem Solving in Children With Language Difficulties: A Descriptive Analysis of Strategies and Errors

Éloïse Achim

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By: Éloïse Achim

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Signed by the final examining committee:

______________________________ Chair
Dr. Saul Carliner

______________________________ Examiner
Dr. Diane Pesco

______________________________ Thesis Co-Supervisor
Dr. Anne Lafay

______________________________ Thesis Supervisor
Dr. Helena P. Osana

Approved by ______________________________
Dr. Walcir Cardoso, Graduate Program Director

____
Dr. Pascale Sicotte, Dean of Faculty of Arts and Sciences

Abstract

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Éloïse Achim

Children with Developmental Language Disorder (DLD), a neurodevelopmental disorder affecting linguistic abilities, can experience difficulties throughout their schooling, such as in mathematics. Solving word problems, a language-dependent task, requires children to understand the text, and identify the semantic relationships between the problem's quantities to solve them. Therefore, gaining insights on the effects of DLD on word-problem solving can help support the learning of children with DLD. The present study compares the word-problem solving abilities of typically-developing (TD) children (n = 28) and children with DLD (n = 16). Children were recruited in schools in Montreal, Quebec City, and Sherbrooke, or in private speech-language pathology' clinics. During two videorecorded sessions, students were invited to solve additive word problems created by the research team. The groups were compared on accuracy, the appropriateness of their strategies, and error types. Also, a strategy profile was assigned to each child based on the most frequent strategy used to explore potential differences among the groups. The findings of this study highlight significant differences between the groups on accuracy, strategy appropriateness, and the frequency and types of errors produced. DLD appears to affect the way children understand the text, identify relevant information, abstract the problem structure, use a strategy aligned with the problem structure, and compute answers. In contrast, the distribution of the strategy profiles is similar in each group: They tend to use the standard algorithm (e.g., formal procedure taught in class) even if they make mistakes. Moreover, they still rely on manipulatives to solve word problems.

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Chapter 1: Statement of Purpose

Solving word problems is an important skill to develop as a student. It prepares children to find solutions to problems encountered in daily life, anticipate events, and create new ways to achieve goals (Riccomini et al., 2016). Word problems are mathematical stories designed to represent arithmetic relationships (Riley et al., 1983). Before formal instruction in school, young children are able to solve some additive word problems (Verschaffel & De Corte, 1993). In other words, they can conceptualize certain semantic relations presented in a word problem without translating the problem into an arithmetic equation: For example, kindergarteners can use concrete materials to solve simple additive word problems (Riley et al., 1983). This informal knowledge then provides a basis for teacher practice at the elementary level to extend children's solving abilities (Carpenter et al., 1996).

Since arithmetic knowledge is not solely responsible for children's ability to solve word problems, research has identified factors contributing to their successful (or unsuccessful) word-problem solving (i.e., Riley et al, 1983; Verschaffel & De Corte, 1993). The first factor identified is the possibility for children to correctly identify the problem structure. When a word problem is presented, children interpret the problem's sentences to form the text base and the problem model, which lead to the identification of a problem structure (Kintsch & Greeno, 1985), a representation of the quantitative relationships presented in a problem (Riley et al., 1983). Children who incorrectly process the information are at risk of identifying an erroneous problem structure (Verschaffel & De Corte, 1993). Also, more complex semantic structures, not yet internalized by children, lead to errors in their representation (Riley et al., 1983). For example, young children are unable to solve problems where an undefined quantity must be represented (i.e., Rita has some chocolates; Riley & Greeno, 1988).

The second factor to consider is the problem's features. Some problems are constructed in ways that are harder for children to understand, such as those that incorporate irrelevant information (Ng et al., 2017; Wang et al., 2016) and contain inconsistent vocabulary (e.g., having "less than" in a word problem that requires a "join" strategy; Verschaffel, 1994). When children fail to interpret the information, they can identify an incorrect problem structure leading to an incorrect answer (Verschaffel & De Corte, 1993).

The third factor to consider are the strategies children use to solve word problems. As previously mentioned, children can use diverse methods to obtain a result (e.g., materials: using blocks; verbal: counting sequence; mental: recall of number facts; Verschaffel & De Corte, 1993). The use of more or less developmentally mature strategies depends on their age (Carpenter & Moser, 1985), availability of materials (Carpenter & Moser, 1985; Riley et al., 1983), and the degree to which they are able to internalize their manipulation of the quantities (Verschaffel & De Corte, 1993). Children select a strategy that can be either aligned or not aligned with the problem structure (Carpenter et al., 2014). When a strategy is aligned to the problem structure, children usually find the correct answer. Conversely, a strategy not aligned with the problem structure is one that will lead to incorrect solutions. In summary, children need to adequately process the available information in the word problem to determine a problem structure that prompts a strategy. Accurate problem structures often lead to misaligned strategies that lead to errors and incorrect solutions.

Moreover, some of the skills necessary to appropriately solve word problems can be classified as either verbal or non-verbal. Non-verbal skills correspond to tasks involving magnitude comparison of numbers (Koponen et al., 2006). In contrast, verbal skills refer to the need to use language to complete tasks, such as counting, number transcoding, and calculation

(Koponen et al., 2006). Word problems are language-based tasks because children need to understand the sentences of the text to construct a problem structure and identify the missing information to solve them (Wang et al., 2016).

Most of previous research has investigated typically-developing (TD) children, leaving questions concerning the processing of word problems of neurodivergent children, including those with Developmental Language Disorder (DLD). Children with DLD have specific struggles in processing linguistic information, such as vocabulary and syntax (Bishop et al., 2017). They can also have deficits in mathematical tasks such as counting, transcoding, and calculating (Lafay & Raimbault, 2022). To our knowledge, only two studies (Cowan et al., 2005; Jordan et al., 1995) have focused on the word-problem solving abilities in children with DLD, revealing lower performance (i.e., accuracy) for children with DLD compared to TD children. Regardless, not much is known about the potential reasons for the difference in their performance.

Moreover, inclusivity of students with special needs in classrooms is a common practice in Quebec (Ministère de l'Éducation, 1999). Therefore, it is, in part, the teacher's role to help such students. Children who struggle in mathematics can receive extra help from teachers, tutors, or practitioners offering specific interventions (e.g., Jitendra et al., 2016), unlike children with language difficulties who are usually helped by professionals such as speech-language pathologists (SLP; Bishop et al., 2017). However, students who have difficulties with both mathematical concepts and language commonly fall through the education system's cracks because of a lack of knowledge on their specific challenges and on how to support their learning (Tsakirakis, 2024). Therefore, research on the impacts of language difficulties in word-problem solving is important. Since not much is known about difficulties of children with DLD, this study will contribute to the literature by describing the word-problem solving abilities of children with DLD, going further than comparing their accuracy to that of TD children.

The primary objectives of the current study are to identify the types of strategies that TD children and children with DLD use when solving additive word problems and to document their errors. The findings will provide avenues for practitioners to support word-problem solving in children with language difficulties. More specifically, it will inform teachers, SLPs, and other practitioners on the types of strategies children with DLD spontaneously use to solve additive word problems. With this knowledge, practitioners will be better equipped to accompany children with DLD in their learning of word-problem solving. Teaching programs rarely take into account the previous knowledge of children (Verschaffel & De Corte, 1993). In general, the results of this study will ease the creation of suitable instruction for children with DLD. The findings on problem-solving differences between children with DLD and TD children will allow me to generate more clear objectives for SLPs and more specific guidelines for teacher intervention in the classroom. Moreover, the analysis of the errors made by children with DLD will shed light on the factors that contribute to their challenges in word-problem solving, which will in turn guide specific in-class or pull-out interventions for children with language difficulties.

Chapter 2: Literature Review

Developmental Language Disorder

Developmental Language Disorder (DLD) is a neurodevelopmental condition affecting receptive and/or expressive abilities significantly limiting children in their daily activities (Breault et al., 2019). To determine whether a child has DLD, speech-language pathologists (SLP) must gather information about their language abilities from various sources, such as teachers' and parents' questionnaires, observations of the child, and standardized assessments (Bishop et al., 2016). However, some children may not have access to these services in schools because of limited professional resources, time constraints associated with completing assessments, and the SLP's workload (Christopulos & Redmond, 2023). Therefore, children with language difficulties may not be professionally identified, but would still experience problems in school, including in reading, writing, and mathematics (Ziegenfusz et al., 2022). The importance does not lie in the labeling of children with language difficulties, but rather on its impact on learning. In fact, the Ministère de l'Éducation (1999) emphasized the necessity to intervene as soon as students present signs of any difficulties in order to adapt teaching to better suit their needs. For the purpose of this research, children experiencing language difficulties, confirmed by a complete SLP evaluation or observations of parents and teachers, will be considered and referred to as DLD.

DLD can affect several linguistic components, including phonology, syntax, semantics, pragmatics, discourse, and verbal memory (Bishop et al., 2017). Syntax difficulties involve deficits in comprehension or production of certain grammatical units or sentence structures (Hsu & Bishop, 2014). Studies investigating French-speaking DLD children and adolescents show that they experience difficulties processing transitive verbs (identifying the correct object of the verb;

Pizzioli & Schelstraete, 2011), subject-verb agreement (accepting agrammatical sentences containing a mismatch between the subject and the verb as grammatical; Courteau et al., 2019), and wh-questions with inversion (Prévost et al., 2017). The erroneous judgment of grammatical structures shows syntax impairments that restrain the ability to understand the information conveyed by verbs. In other words, children and adolescents with DLD have more difficulties attributing the agent and the object of verbs (i.e., who did what on what), causing misunderstandings of spoken and written sentences and questions.

Adolescents with DLD show significant differences, when compared with TD adolescents, in the magnitude and the depth of their semantic networks; they have difficulties associating spoken words to the appropriate picture and recognizing semantically-related words (Courteau et al., 2023). Also, the processing of high and low frequency words in the context of sentences is impaired in children with DLD (Leclercq et al., 2014). Both these studies suggest that children and adolescents with DLD have problems extracting the meaning of words presented in isolation or in sentences. This finding may be explained by limited access to semantic knowledge (e.g., characteristics of objects, relation between words) that is overloaded by the amount of information that needs to be treated (Leclercq et al., 2014).

Discourse comprehension regroups different discourse types (e.g., narrative or expository texts, word problems; Fuchs et al., 2015), which can be evaluated via questions. Literal questions target information provided in the story, or story grammar (i.e., parts of the narrative structure), and inferential questions require children to deduce new information from the one presented in the story. Children with DLD exhibit poorer performance in literal and inferential questions about stories than TD controls (Bishop & Adams, 1992; Gough Kenyon et al., 2018). However, Dodwell and Bavin (2008) found that DLD children perform similarly to age-matched and younger language-matched controls on literal questions when the story is supported by illustrations. This finding suggests that the representation of the story may support the processing of narratives in children with DLD. Moreover, children with DLD struggle to correctly answer questions necessitating the identification of the story grammar (Merritt & Liles, 1987).

Verbal short-term memory refers to the ability to store verbal input momentarily to repeat it, whereas working memory refers to the ability to manipulate this information to achieve tasks such as planning, remembering, and reasoning (Henry & Botting, 2017). Verbal and working memory are both required for oral and written language comprehension (Delage et al., 2021; Henry & Botting, 2017). Children with DLD struggle to repeat forward and backward sequences of numbers, non-words, and sentences in comparison to TD children (Delage et al., 2021). This was also found in older students with DLD exhibiting more difficulties to adequately recall complex sentences read out loud, suggesting weaker verbal memory skills (Courteau et al., 2023). Verbal and working memory deficits affect DLD children's capacity to retain information in order to perform a language-based task, such as processing every component of a sentence.

In summary, the profile of DLD children extends from difficulties in syntax processing, semantic knowledge, discourse and inferential comprehension, to verbal and working memory deficits that persistently impact school achievements. Preschoolers identified as having language difficulties still experience language difficulties up to 5 years after the initial report, while also performing below average in reading, writing, and mathematics (Aram & Nation, 1980). Furthermore, school-aged DLD children experience several mathematics difficulties during their schooling (Lafay & Raimbault, 2022).

The Role of Language in Mathematics Learning

In a meta-analysis of 344 studies, Peng et al. (2020) examined the presence of a significant relation between language and mathematics. The authors found that overall, language skills and mathematics were moderately correlated. More precisely, higher-level language skills, oral comprehension, and vocabulary were significantly related to performance on word problems. Inversely, word problems were more strongly related to language skills than numerical knowledge, calculation, fractions, and algebra. This suggests that the importance conferred to language abilities to achieve a mathematical task is inherently dependent on the task. Complex tasks such as word problems require higher language processing demands, namely the capacity to infer information and understand relations between the problem's quantities.

Additionally, language and mathematics seem to be more intertwined than previously thought. Indeed, language proficiency at a younger age predicts later mathematics achievement and the other way around after controlling for initial mathematics level and language abilities, respectively (Peng et al., 2020). Therefore, it is necessary to question the impact of DLD in mathematics in general and more specifically, in language-dependent tasks.

Mathematical Abilities of Children with DLD

Children with DLD as young as 4 years old present mathematical difficulties even before the introduction of complex tasks such as word problems (Lafay & Raimbault, 2022). Cross et al. (2019) suggested that the mathematical difficulties originate from struggles in memorizing, accessing, and using verbal representations of numbers (e.g., number names, arithmetic facts). These difficulties in turn affect language-dependent tasks, including counting, transcoding, mental calculations, and word-problem solving. In the following section, each of these language-dependent tasks will be described in the context of the mathematics performance of children with DLD.

Counting is the ability to count forward starting from 0 or from a different start number to as far as they can or a target number, and backward from a start number to 0 or a number target (Koponen et al., 2006). Counting can occur by ones or by leaps (Fazio, 1996). Storage and processing in phonological memory is essential to recall the number sequence, which makes counting highly dependent on language abilities (Fazio, 1996). Children with DLD and younger language-level controls do not differ in terms of counting skills (Cowan et al. 2005; Koponen et al., 2006). At the same time, when compared to age controls, a significant difference is observed in counting performances of children with DLD (Cowan et al. 2005). Indeed, difficulties reciting the counting sequence and enumerating are present in 4-5 years olds with DLD (Fazio, 1994) and are still present at 6-7 years old (Fazio, 1996). These findings indicate that children with DLD experience persistent difficulties in counting, and their counting skills are comparable to those of younger children with similar language abilities.

Transcoding corresponds to the act of translating numbers from one code into another (Cowan et al., 2005). The triple-code model, developed by Dehaene and Cohen (1995), distinguishes number mental representations in three categories: visual arabic number form represented by symbols (e.g., 4), verbal words corresponding to phonological and graphemic forms (e.g., “four”), and analogical magnitude representation expressed as a quantity (e.g., ∷). Transcoding is considered language-dependent because children need to access the phonological form of arabic numbers or analogical representations to convert numbers (e.g., ∷ and 4 trigger “four”; Dehaene & Cohen, 1995). Children with DLD show significantly poorer performance in transcoding tasks that require verbal answers, such as reading numbers (Rodríguez Rodríguez et al., 2020). Their performance resembles that of younger children matched on language-level, whereas the two groups' performances are significantly different from those of age controls (Cowan et al., 2005).

Calculation abilities are first based on counting strategies. Then, from successful single-digit calculation, children build representations called number facts that can be directly retrieved from long-term memory without computation (Koponen et al., 2006). Since the importance of language in counting and transcoding tasks has already been stated, it can be understood how calculation abilities are thus dependent on language abilities. Children need to translate arabic numbers to verbal mental representations to subsequently activate the answer in their long-term memory before formulating it in oral or written form (e.g., 2 x 4 → "two times four" which triggers the answer "eight" → 8; Dehaene & Cohen, 1995; Fazio, 1996). Difficulties in counting abilities and deficits in effective retrieval of number facts affect DLD children's performance in arithmetic compared to TD children (Cross et al., 2019). Interestingly, Jordan et al. (1995) found no significant differences between TD children and children with low language levels, in kindergarten and first grade, on single-digit operations. However, children with low level language relied more on fingers than TD children to perform the calculations, which suggest the necessity for children who struggle with language to use their fingers to perform similarly to their peers. This finding was also replicated by Cowan et al. (2005), who found that children with DLD use more counting strategies and fingers than TD children in single- and multi-digit addition and subtraction.

Word problems are stories that children need to understand in order to solve arithmetic operations (Riley et al., 1983). As previously mentioned, children with DLD struggle with numerous language-based tasks (e.g., transcoding, counting, calculations) that are necessary to solve problems (Cross et al., 2019). In other words, the difficulties experienced by children with DLD when solving word problems come, in part, from deficits found in basic mathematical skills previously described and the additional linguistic demands associated with the presentation of the problem in a text (Cross et al., 2019; Cowan et al., 2005). Specifically, children with DLD show significantly inferior word-problem solving abilities when compared to TD children (Cowan et al., 2005; Jordan et al., 1995). Having less adequate counting and number-facts retrieval strategies may explain why children with DLD struggle to solve word problems (Fazio, 1996). However, these are only hypotheses, considering that no study has focused on presenting potential explanations for the poor performance of children with DLD in word-problem solving.

Successful Word-Problem Solving

Kintsch and Greeno (1985) proposed a model for understanding and solving word problems that contains two distinct components: the text base and the problem model. In a back-and-forth process, children interpret information from the text to create meaning and identify a problem structure. The problem structure is the constructed representation made by children of the problem's relevant and missing information into a coherent network specific to a problem type (i.e., change problem; Riley et al., 1983). In the subsequent sections, the following "change" problem will be analyzed to describe the mechanisms implicated in the process of constructing a mental model together with difficulties children may encounter doing so. "This morning, before going to school, Lea read 16 books and she watched 2 videos. Before going to bed, Lea read 8 more books. How many books did she read that day?"

Understanding the Text

As indicated in the previous section, word problems consist of arithmetic operations presented in the form of stories (Riley et al., 1983). This definition seems to imply that similar processes are implicated in both word problems and story understanding. Accordingly, Fuchs et al. (2015) suggested that since oral language comprehension supports both word-problem and text comprehension, word problems are considered a form of text comprehension. Moreover, interpreting word problems as stories involving a main character, goals, intentions, and plans can lead to an easier understanding than considering physical states or events (van Dijk & Kintsch, 1983).

The first step of text analysis consists of reorganizing information into propositions representing the microstructure (Kintsch & Greeno, 1985). Table 1 presents an example of the creation of propositions with the sample problem. To do so, children identify the object, quantity, specification, and role in each sentence of the problem. The object represents the problem's common nouns, whereas quantity refers to the precise numbers or quantifiers accompanying the nouns such as "some" or "how many." Specification refers to information required to differentiate the sets of objects, specifically the owners of sets, time, and location. Furthermore, the role describes the relation between the different sets mentioned in the problem. In a change problem, the roles are the initial state, the change, and the final state (Riley et al., 1983). When gradually exposed to word-problems, children learn to pay attention to the numbers and relations between the quantities, thus supporting the creation of propositions (Kintsch & Greeno, 1985).

Table 1

Formulation of Propositions (Microstructure) of the Change Problem

Problem Sentences	Object	Quantity	Specification(s)	Role
This morning, before going to school, Lea read 16 books …	books	16	Lea; this morning	Initial State
… and she watched 2 videos	videos	2	she; this morning	Initial State
Before going to bed, Lea read 8 more books.	books	8	Lea; before going to bed	Change
How many books did she read that day?	books	how many	she; that day	Final State

Note. The first sentence in the problem is presented in two rows to simplify the presentation of the proposition's formulations.

Table 2 presents a syntactical analysis of the problem's sentences. Sentences are constructed using a predetermined structure called the basic sentence. The constituents of the basic sentence are the subject and predicate. Complements are optional because they are not indispensable to understanding the basic sentence, but they add information relative to time and place (Jones, 1996). To adequately create propositions, children need to process syntactic structures and semantic information (Kintsch & Greeno, 1985). In change problems, the aspect of time is crucial because the sets differ from start to end. Consequently, the temporal information relayed by the subject, predicate, and complements are essential to correctly attributing the role in the propositions (i.e., initial state, change, final state; Kintsch & Greeno, 1985). It is interesting to note that the complements carry information necessary to assign the roles presented in Figure 1 (i.e., initial state, change, final state).

Table 2

Syntactic Analysis of the Change Problem

Problem Sentences	Subject	Predicate	Complement
This morning, before going to school, Lea read 16 books …	Lea	read 16 books	this morning; before going to school
… and she watched 2 videos.	she	watched 2 videos	this morning; before going to school
Before going to bed, Lea read 8 more books.	Lea	read 8 books	before going to bed
How many books did she read that day?	she	did read how many books	that day

Note. The problem's first sentence is divided into two rows to simplify the presentation of the syntactic analysis.

Figure 1 presents the processes implicated in the vocabulary comprehension in problems. Since word-problem solving produces brain activity in regions dedicated to semantic processing, compared with arithmetic computations, the problem's vocabulary can have a critical impact on children's abilities to understand the text (Zhou et al., 2018). This indicates that semantic information carried by words is also relevant to the formation of propositions. Indeed, children need to categorize elements to differentiate sets (e.g., books and videos), but also mentally represent the words' meaning (Verschaffel & De Corte, 1993). Additionally, having two sets with comparable objects may challenge children's cognitive and linguistic abilities to a greater extent in order to distinguish them (green apples vs. red apples; Ng et al., 2017). Moreover, some common words need to be understood in a mathematical context, relevant to problem solving, for example, the only feature required to be processed for the word "give" in a word problem is the transfer of objects; children can ignore that the character that gave the marbles did not get anything in return (Kintsch & Greeno, 1985). However, word-problem comprehension differs from text comprehension because mathematical language processing (e.g., fewer, more than) predicts word-problem solving abilities, but not text comprehension (Fuchs et al., 2015). This suggests the importance for children to trigger adequate quantitative relations when faced with specific mathematical vocabulary (i.e., altogether, less than) for an adequate word-problem solving (Kintsch & Greeno, 1985).

Figure 1

Vocabulary Comprehension in the Change Problem

This morning, before going to school, Lea read 16 books and she watched 2 videos. Before going to bed, Lea read 8 more books. How many books did she read that day?

Semantic processing:

- videos are not books

Specific mathematical language processing:

- 8 more books → 8 books *more than* the first 16

Figure 2 presents the required inferences to form the macrostructure, which is a mental representation of the quantities and their relation to each other, specific to word problems (Kintsch & Greeno, 1985). However, texts usually leave the readers with implicit information to deduce (van Dijk & Kintsch, 1983). Therefore, students need to make inferences based on the available information in the text and use reasoning to create a coherent text base (Peng et al., 2020). Not surprisingly, word problems containing implicit information to infer are more difficult to solve than the ones that do not necessitate inference-making (Pongsakdi et al., 2020). In fact, the impact of logical reasoning on word-problem solving is mediated by reading comprehension, which means that children need to understand the problem for the reasoning skills to have a positive impact on word-problem solving (Derya, 2020). Reading comprehension skills are essential for children to solve more complex word problems (e.g., containing implicit information; Pongsakdi et al., 2020). Moreover, the use of strategies that target reasoning skills positively impacts word-problem solving abilities (Fuchs et al., 2015). Another important aspect children develop is the knowledge that all the information required to solve the problem is necessarily presented in the problem (Kintsch & Greeno, 1985). They only need to reason from the presented information in order to form the macrostructure. For example, in the change problem, Lea does not read books at school that day.

Figure 2

Inferences Made to Form the Macrostructure in the Change Problem

This morning, before going to school, Lea read 16 books and she watched 2 videos. Before going to bed, Lea read 8 more books. How many books did she read that day? **Inferences:** ● Before going to bed → evening ● The day → morning and evening ● 8 more books → 8 more books than the books previously read ● No more books were read than the ones presented in the problem

In summary, to understand word problems, children need to correctly analyze syntactic and semantic information in the sentences to create propositions containing the objects, quantities, specifications, and roles. Furthermore, they must infer missing information in order to arrange the propositions in a coherent macrostructure (Kintsch & Greeno, 1985). Difficulties in the comprehension of syntax, inference-making, and reasoning skills can directly prevent children from adequately processing the word problem, leading to partial formation of propositions, prompting an incomplete or erroneous macrostructure.

Identifying the Problem Structure

Additive word problems are classified based on the nature of the relations between the quantities, the action's direction, and the unknown (Riley et al., 1983; Verschaffel et al., 2020). For the purpose of this study, only change problems will be discussed. The different types of change problems are presented in Figure 3.

Figure 3

Types of Change Problems

Result Unknown *This morning, before going to school, Lea read 16 books and she watched 2 videos. Before going to bed, Lea read 8 more books. How many books did she read that day?* **Change Unknown** *At the farm, Martine had 8 pigs, 5 chickens and 24 chicks. The chicken coop remained open and some chicks escaped. When Martine returned in the evening, she counted 5 chickens and 6 chicks. How many chicks escaped from the coop?* **Start Unknown** *In his costume box, Samuel has wigs and hats. This afternoon, the teacher gave Samuel 5 wigs and 2 hats. Samuel now has 22 wigs. How many wigs did Samuel have?*

During the processing of the problem's sentences, children integrate the relevant information into a schema that fits the problem structure (i.e., attributing the known and unknown quantities to the initial state, change, and final state; Kintsch & Greeno, 1985). Word problems are constructed following a predetermined structure which can be explicit or implicit depending on their wording (Verschaffel & De Corte, 1993). With experience, students tend to recognize the pattern of information available in the problem to identify its structure (Wang et al., 2016). This suggests that children can identify the corresponding problem model from the text with more or less facility. In fact, children's inability to represent the problem structure is caused by misunderstandings of the relations between the quantities because of some of the problem's features (Verschaffel & De Corte, 1993). Therefore, knowing the arithmetic operations is not sufficient to correctly solve a given problem (Riley et al., 1983). For instance, the problem structure and the identity of the unknown inherently govern the difficulty of a problem, as profiles of performance differ from younger to older children where the latter perform better than the former (Riley et al., 1983).

Moreover, the presence of irrelevant information in the problem exerts additional demands on students' language and cognitive abilities, limiting the construction of an accurate problem structure (Wang et al., 2016). For example, the inclusion of numerical distractors can affect children's ability to adequately determine the problem model whereas irrelevant literal information has no effect on performance in TD children (Ng et al., 2017). The authors suggest that, as opposed to irrelevant numerical information, irrelevant literal information, even in the presence of an undetermined quantifier (i.e., some), cannot be computed. Therefore, children may be more at risk of using the keyword method when facing irrelevant information (Riccomini et al., 2016).

Another feature to consider is consistency of language, which refers to the agreement between the problem's vocabulary and the arithmetic operation to perform (Verschaffel, 1994). Some words, general or specific to mathematics, can trigger the recognition of an erroneous problem structure leading to inadequate solution (e.g., "less than," necessitating a subtraction, "give," necessitating an addition). By relying only on the presence of keywords without interpreting their meaning, students are at risk of making mistakes about the problem structure (Riccomini et al., 2016).

Additionally, the problem's structure influences the types of strategies children can use to solve it (Verschaffel & De Corte, 1993). Since the representation of a problem differs from one child to another, not all will use the same techniques to solve it, leading to a variety of strategies (Riley et al., 1983). Children can use material (i.e., blocks), use verbal strategies (i.e., counting sequence), or mental strategies (i.e., recalling number facts; Verschaffel & De Corte, 1993). Also, with the sophistication of counting procedures, children become able to solve a wider variety of problems with more complex semantic relationships (Riley et al., 1983).

Children's Strategies for Solving Additive Word Problems

Children enter school with existing knowledge about problem solving acquired through experience (Carpenter & Moser, 1984). From the early development of problem-solving skills involving small quantities, children construct strategies for two- and three-digit problems (Carpenter et al., 1996). Carpenter et al. (2014) proposed a developmental continuum of strategies that TD children produce to answer multi-digit word problems characterized by movement from modeling to invented algorithms, which in turn support their understanding of the standard algorithm learned in class. Modeling is characterized by the use of external representations (i.e., chips, drawings, fingers) of the quantities in the problem to act out the actions described in the problem text. Children can represent and count single units or groups of units in tens (or larger denominations). Through practice and by verbally describing the actions they make with the materials, they become able to move from modeling by tens to invented algorithms. An invented algorithm is a strategy consisting of modifying one or more of the problem's quantities to facilitate the computations. For example, when children use incrementing, they will separate one of the numbers to add or remove a smaller quantity to the other number to make the computation easier and then add or subtract the rest (e.g., $17 + 5$: $5 = 3 + 2 \rightarrow 17 + 3 = 20 \rightarrow 20 + 2 = 22 \rightarrow 17 + 5 = 22$). Children can compute those calculations in their head or on paper. With practice, children become so efficient at using invented algorithms that they can solve problems in their head without any external support. However, in schools, children are not encouraged to create invented algorithms. Instead, teachers expect children to use the standard algorithm taught in class, which is the standard way to add or subtract numbers in columns with regrouping symbols (e.g., $\begin{array}{r} {\scriptstyle 1} \\ 17 \\ +\ 5 \\ \hline 22 \end{array}$).

It is important to note that children may switch between more or less sophisticated strategies depending on the material available and the type of problem presented (Carpenter & Moser, 1984). In other words, the observed strategies can provide insight into their developmental level. However, teachers and researchers need to be careful not to interpret the demonstrated strategies as the inability to progress developmentally.

Present Study

As previously mentioned, the dependency of word problems on language leads to trouble for children with DLD (Cowan et al., 2005; Cross et al., 2019; Jordan et al. 1995). Several processes are implicated in successful word-problem solving such as reading and understanding the text in order to identify the adequate problem structure, which, in turn, influences the strategies used (Carpenter et al., 2014; Kintsch & Greeno, 1985). It can be assumed that children with DLD will struggle at every step of that process because of underlying linguistic deficits leading to difficulty with verbal memory and in comprehending syntax, semantics, and discourse (Bishop et al., 2017). Given their difficulties in interpreting semantic and syntactic structures of the sentences in word probles, children with DLD can create erroneous micro- and macrostructures of the problem, leading to an incorrect identification of the problem structure. The features of word problems (e.g., irrelevant information, consistency of language) can also affect children's representation of the problem structure (Kintsch & Greeno, 1985; Riley et al., 1983; Verschaffel & De Corte, 1993). Lastly, word-problem solving abilities are based on other mathematical tasks (e.g., counting, transcoding, calculations) that are impaired in children with

DLD, thus possibly affecting the appropriateness of the strategies they use to solve word problems.

Previous studies have described mathematical solving abilities in TD children and compared their performance to children with DLD showing lower performance for the latter (Cowan et al., 2005; Jordan et al., 1995). However, there is a lack of information on how language difficulties impact children's abilities to solve word problems. The purpose of this study is to explore the word-problem solving of children with DLD compared to TD children to find differences in their respective skills in understanding the text, extracting the problem structure, and finding the correct solution. The research questions guiding this study are the following:

1. How does problem-solving accuracy differ between the DLD and TD groups?
2. How does strategy appropriateness differ between the DLD and TD groups?
3. How do the strategy profiles of children in both language groups differ?
4. How do the types of error produced by the children in both groups differ?

To address the above questions, children with DLD and typically-developing (TD) children were provided with six change problems where the unknown varied from the initial and final state or the change. They had access to pens, chips, and their fingers to answer the problems. During the solution process, they were asked to describe their reasoning.

With regards to problem-solving accuracy (RQ 1), I predict that the results will replicate those of Cowan et al. (2005) and Jordan et al. (1995), namely that children with DLD will have lower problem-solving accuracy than TD children. Concerning the appropriateness of the strategies used (RQ 2), I predict that the strategies used by children with DLD will be less frequently aligned to the actual problem structure than those of TD children. In other words, their strategies will less likely permit them to obtain the right answer. Concerning the types of strategies (RQ 3), I will assign a strategy profile to every child in the groups. I predict that children with DLD will tend to use more modeling strategies, relying on external representations (i.e., fingers, chips) to act on the relations in the problems. Therefore, Modeling by Ones and Modeling by Tens will be assigned to a majority of these children. Conversely, TD children will present more developmentally-mature strategies than their DLD counterparts, such as using Invented Algorithms. Having fewer difficulties processing numbers, TD children will be farther ahead on the developmental continuum than children with DLD, for whom the counting sequence may still present a challenge (Fazio, 1996).

With regards to the types of errors produced (RQ 4), I predict that children with DLD will present more errors related to difficulties in identifying the problem structure. Because of difficulties processing the linguistic information of the text, they will have difficulties interpreting the semantic relationships between the quantities in the problem, leading to more frequent misidentification of the problem structure. Also, they will be more susceptible to using irrelevant literal and numerical distractors in their solution, leading to an inaccurate answer. Lastly, they will commit more computational errors. These errors are associated with core deficits in the storing, accessing, and manipulating of the verbal representations of numbers in children with DLD (Cross et al., 2019). A descriptive analysis of the distribution of the error types and subtypes will also help to answer the fourth research question.

The answers to my research questions are crucial to provide children with DLD with specific teacher and SLP interventions related to the difficulties they encounter during word-problem solving. By having few to no studies describing the impact of language difficulties on word-problem solving abilities, it is difficult to help children with DLD, which occurs in almost

7% of the population, using evidence-based practices (Bishop et al., 2017). The findings of the present study promise to impact children with DLD in regular and specialized classrooms by providing valuable information to teachers, speech-and-language therapists, and administrators. With this information, the curriculum could be adapted to fit the needs of children with DLD in order for them to meet expectations. Moreover, by knowing the strategies that children with language difficulties use naturally, it could be possible to support their learning by capitalizing on their strengths and supporting the development of more mature strategies. Finally, the description of the types of errors they produce will help to create specific activities to target the difficulties encountered by children with DLD when problem solving.

Chapter 3: Method

Participants

Third graders were recruited for this study. Participants with Developmental Language Disorders (DLD) were recruited through private speech-and-language-therapy clinics or specialized schools in the metropolitan areas of Montreal, Quebec City, and Sherbrooke, Quebec, Canada. The study pamphlet was also posted on the social networks of the team members. Typically-developing (TD) children were recruited from public school boards in the previously-mentioned regions or through contacts in private schools. To be included in the study, participants had to be able to communicate in French and speak it at home.

The total sample (N = 45) consisted of TD children (n = 28) and children with DLD (n = 17). There was one exclusion from the DLD group because the recordings had no sound. The final DLD group was composed of 16 children. The mean age of the children in the TD group was 9.18 years old (SD = 0.27) and in the DLD group, the mean age was 10.07 years old (SD = 0.91). Eleven of the children in the DLD group were confirmed as having language difficulties by a speech-and-language-therapy evaluation and six were classified as children at risk for DLD based on school professional or parental report of language difficulties and/or a performance below the 25th percentile at Sentence Repetition of the CELF CDN-F (Secord et al., 2009). Children had to speak French to reduce the effects of second-language acquisition. Of the total sample (N = 45), 13% spoke another language at home in addition to French (English: n = 3; Arabic: n = 1; Japanese: n = 1; Portuguese: n = 1). Children with intellectual disabilities, behavioral, psychological, or attentional difficulties preventing them from completing the mathematical problems were not included.

Design and Procedure

The present study is part of a larger research project with the goal of identifying the difficulties experienced by children with DLD in word-problem solving and the instructional support that can increase their performance. To document children's problem-solving performance, six change and six compare problems were offered to students. Only the change problems will be described in this section because the compare problems will not be analyzed as part of the present study. More specifically, this study focused on describing and comparing the performance of children with DLD and TD children in terms of accuracy, appropriateness of the strategies they used in relation to the problem structure, the types of strategies they used, and the types of errors they generated.

After the consent of parents was obtained, the children gave their assent to participate in the activities. The research assistants gave the children the opportunity to stop the activities whenever they wanted during a session or for the whole experiment without any negative consequences. Children were individually tested by a research assistant during two meetings, each lasting 30 minutes. The testing took place in a quiet room at the school or in a private clinic. Children were video recorded to allow for subsequent data coding. The camera was oriented to show only the table and the children's hands.

During the first session, the CELF CDN-F Sentence Repetition test (Secord et al., 2009) was administered. Then, up to six mathematical word problems were offered to the child. Before reading the problem, the research assistant explained to the child that mathematical stories with a question would be presented where they had to find the answer. Participants had access to 20 plastic green chips and 20 blue chips, as well as green- and blue-colored pencils and a lead pencil in case the children wished to solve the problems using written marks on paper. They were instructed to use the materials as they wished. The session ended when the child had finished solving all six problems or when the allotted time (i.e., 30 minutes) was up.

In the second session, the children continued to solve the remaining problems with access to the same materials. As in the first meeting, the session ended when the child had completed all the problems or when the allotted time (i.e., 30 minutes) was over. Throughout the solution process, children were encouraged to verbalize their thinking for fine-grained data coding and analysis of their actions and reasoning. At the end of the two meetings, the children received a participation diploma to thank them for their participation. A few weeks after data collection, parents received a $10 gift card by email.

Measures

CELF CDN-F Sentence Repetition Test

The CELF CDN-F Sentence Repetition test (Secord et al., 2009) was administered to measure participants' language skills and to classify them in the two language groups. During the test, the children had to repeat sentences increasing in length and morphosyntactic complexity read out loud by the research assistant. The number of errors (change, omission, addition, inversion of words) was used to give a score for each sentence (no error: 3 points; presence of one error: 2 points; presence of two or three errors: 1 point; presence of 4 or more errors: 0 points). The score was calculated by adding the total number of points, with a minimum of 0 and a maximum of 96 points. The test ended when the child achieved a score of 0 points for five consecutive sentences.

Word-Problem Solving

Six change problems (see Appendix A) were presented to children. Two problems had an unknown initial state, two had an unknown final state, and two had the change as the unknown.

The problems were developed by team members respecting the following criteria. The statements contained between 30 and 45 words in total where the nouns, adjectives, and verbs had a frequency greater than 49.00 (Q3 of the standard frequency index according to Manulex; Lété et al., 2004; Ortega & Lété, 2010) for the CE1 grade level (equivalent to second grade in North America). The problems did not contain concepts of measurement and contained only the personal subject pronouns “he” and “she.” For problems requiring addition, the sum was between 20 and 25. For problems requiring subtraction, the minuend was between 20 and 25.

Coding and Scoring of the Word-Problem-Solving Task

By reviewing the recordings of the Word-Problem-Solving task, children’s performance was scored and coded based on the following. Since children had an allotted time, some of them did not solve all problems. The unattempted problems were left out of the analysis.

Research Question 1: Accuracy

The accuracy of the answer was scored (correct: 1 point, not correct: 0 points) to compare the performance of both groups. The overall accuracy score was the proportion of correct answers out of the total number of attempted problems for each group.

Research Question 2: Strategy Appropriateness

The strategy appropriateness referred to whether or not the strategy used was aligned with the problem structure (aligned: 1, not aligned: 0), regardless of calculation errors. For the sample problem “This morning, before going to school, Lea read 16 books and she watched 2 videos. Before going to bed, Lea read 8 more books. How many books did she read that day?”, an example of an aligned strategy would be a child taking 16 chips, joining 8 chips, and counting the total, whereas an unaligned strategy would be taking 16 chips, removing 8 chips, and counting the total. The overall appropriateness score was the proportion of the aligned strategies out of the total number of attempted problems for each group.

Research Question 3: Strategy Profiles

The strategies that were coded were the strategies that lead to the final answer, right or wrong. More than one strategy could appear per problem because some children answered problems using multiple strategies (i.e., the child wrote a standard algorithm and counted chips). Table B1 in Appendix B presents the strategies used by children to solve word problems that were coded using the following categories: Direct Modeling by Ones, Direct Modeling by Tens, and Invented Algorithms (Carpenter et al., 2014). Some children also used the Standard Algorithm, while some used an "Other" strategy, or did not answer the problem.

Direct modeling describes the act of representing one or more of the problem's quantities or the problem's actions with chips, drawings, or fingers. Direct Modeling by Ones and Direct Modeling by Tens differ in the use of base ten. Children using Direct Modeling by Ones counted the materials one by one to solve a word problem, whereas children using Direct Modeling by Tens displayed two sets of objects for the units and the tens (e.g., a bar to represent one ten and six dots to represent six units). Invented Algorithms represented mental or written computations where children would break down one or more of the problem's quantities to facilitate the actual computation.

The formal written algorithms that children learn in school, where they write down the computations in a column before solving it, was coded as Standard Algorithms. The "Other" strategy code was used when children gave an answer without explaining their reasoning. Some problems were attempted by children but they did not give a final answer.

Research Question 4: Error Types

To better identify the potential obstacles to successful word-problem solving, the problems that were not solved correctly were coded according to the types of errors produced (i.e., when the problem was assigned 0 points for the answer's accuracy or 0 points for the appropriateness of the strategy). Table B2 in Appendix B presents the types of errors that were coded: (a) Use of Distractors, (b) Misidentification of the Problem Structure, and (c) Computational Errors. The use of irrelevant information in children's solutions referred to errors involving the Use of Distractors. In the problems, numerical and literal irrelevant information were incorporated. There were two types of errors involving distractors. When children integrated an irrelevant number in the problem, the error was classified as Numerical Distractor. Some children seemed to have difficulties in understanding the context of the problem because they attributed numbers to literal irrelevant information in the problem and used that number in their solution (e.g., counted the days in the week as well as the objects). This type of error was classified as a Literal Distractor error.

Misidentification of the Problem Structure is characterized by the misidentification of the join or separate structure or the choice of a number from the problem as the answer. There were two types of errors that indicated difficulties with identifying the problem structure. The first is when children identified a join problem as a separate problem, or the reverse. This type of error was labeled as an Wrong Action error. The second type of error was when children chose a number in the problem as their answer. In this case, the error was labeled No Action error. Computational Errors were coded for problems with inaccurate answers. Different types of computation errors emerged from the data. Children who wrote a number that was different from the number of chips counted made a Transcoding error. Difficulties in reciting the counting sequence or errors in counting objects (e.g., counting the same chip two times) was classified as Counting error. Wrong answers generated after the children computed in their head were classified as Calculating errors. The Number Representation error was used when children did not align the units and tens correctly in the standard algorithm, resulting in an inaccurate answer.

Data Analysis

For the first research question concerning the accuracy, I compared the mean accuracy scores of the TD and DLD groups with an independent samples *t*-test. For the second research question on strategy appropriateness, I compared the mean strategy appropriateness scores of the TD and DLD groups with an independent samples *t*-test to test for a difference between their capacity to use a strategy that is aligned with the problem structure.

With regards to the third research question about strategy profile differences, the proportions of strategy profiles (i.e., the distribution of profiles) within each group were qualitatively described because the assumptions of the chi-square test analysis were not met (some expected counts were lower than five). More specifically, each child was assigned a strategy profile. The strategy profile was determined using a 50% cut-off; that is, a profile was assigned if a specific strategy was used half of the time or more out of all attempted problems. For example, a child who used a Modeling by Ones strategy for four out of six problems and a Standard Algorithm for the remaining part would be assigned the Modeling by Ones profile. If equivalent frequencies of two strategies were used by a child to solve the word problems (e.g., a child used three Standard Algorithms and three Modeling by Ones strategies), the more mature strategy determined the profile (Riley et al., 1983).

The fourth research question explored the differences in the types of errors made by the children in the two groups. For these analyses, the unit of analysis changed from child to item to provide a better understanding of all errors made instead of comparing each child. Frequencies of the major error types (i.e., Use of Distractors, Misidentification of the Problem Structure, and Computational Errors) as well as all error subtypes (i.e., Use of Distractors: Numerical and Literal Distractor Errors; Misidentification of the Problem Structure: Wrong Action and No Action Errors; Computational Errors: Transcoding, Counting, Calculating and Number Representation Errors) were computed for all attempted problems that received a score of 0 in accuracy or strategy appropriateness, or both.

First, for each student, I computed the proportion of all attempted items on which each major error type was made. I then compared the two groups on the mean proportion of each error type (i.e., between Use of Distractors, Misidentification of the Problem Structure, Computational) using three separate independent sample *t-tests*. Second, I compared the distribution of the major error types within and across groups to determine the most likely causes of the difficulties in word-problem solving for TD children and children with DLD. Lastly, the frequencies of the subtypes of errors within each major error category (i.e., Use of Distractors: Numerical and Literal Distractor Errors; Misidentification of the Problem Structure: Wrong Action and No Action Errors; Computational Errors: Transcoding, Counting, Calculating and Number Representation Errors) were computed to compare the distributions of the error subtypes across groups.

Chapter 4: Results

Research Question 1: Accuracy

The first research question explored the differences between the groups' problem-solving accuracy score. A significant difference was observed between the mean accuracy scores of the TD ($M = .69$; $SD = .23$) and DLD ($M = .33$; $SD = .34$) groups, $t(42) = 4.25$, $p < .001$, $d = .27$. In other words, TD children found the correct answer more frequently on average than the children with DLD.

Research Question 2: Strategy Appropriateness

The second research question examined any discrepancy in strategy appropriateness between the two groups. The strategy appropriateness referred to the alignment of a strategy with the corresponding problem structure and was defined as any strategy would result in a correct answer, regardless of computational errors. The difference between the mean strategy appropriateness scores of the TD children ($M = .77$; $SD = .21$) and children with DLD ($M = .42$; $SD = .37$) was significant, $t(42) = 4.04$, $p < .001$, $d = .28$. More specifically, TD children used strategies that were aligned with the problem structure more frequently on average compared to children with DLD.

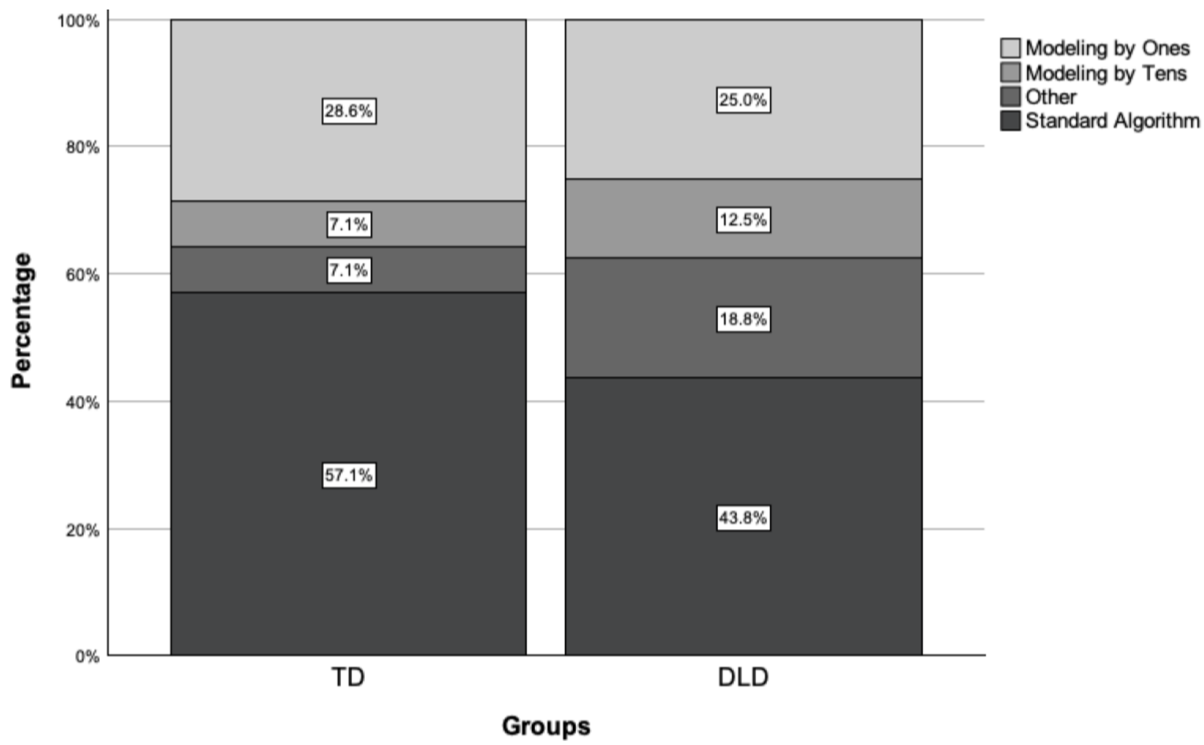
Research Question 3: Strategy Profiles

The strategy profiles in the TD group were distributed as follows, from the most to the least frequent strategy: Standard Algorithms ($n = 16$), Modeling by Ones ($n = 8$), Modeling by Tens ($n = 2$), Other ($n = 2$), and Invented Algorithms ($n = 0$). In the DLD group, the frequencies of strategy profiles followed a similar pattern: Standard Algorithms ($n = 7$), Modeling by Ones ($n = 4$), Other ($n = 3$), Modeling by Tens ($n = 2$), and Invented Algorithms ($n = 0$).

In Figure 4, the proportions of each strategy profile within each group (i.e., TD and DLD) are presented. The most frequent strategy profile in each group was the Standard Algorithm assigned to 57.1% of the TD children and 43.8% of the children with DLD. The next most frequent was Modeling by Ones representing 28.6% and 25% of the TD children and children with DLD, respectively. The Other category represented 7.1% of the children in the TD group and 18.8% of those in the DLD group. Also at 7.1% in the TD group, the Modeling by Tens profile was assigned to 12.5% of the DLD group. The Invented Algorithm profile was not assigned to any child, so it does not appear in Figure 4. The results suggest a similar pattern of strategy use where children in both groups used the Standard Algorithms more frequently than any other strategy, followed by the Modeling by Ones, and then by the Modeling by Tens. The Modeling by Ones and Modeling by Tens strategies were coded as such if they were carried out with the use of external representations (i.e. chips, fingers, or drawings). The DLD group's strategies were more frequently unidentifiable (classified in the Other category) compared to the TD group.

Figure 4

Proportions of Each Strategy Profile by Group



Research Question 4: Error Types

Major Error Types

The fourth research question investigated differences in the number of errors produced by TD children and children with DLD, and the distribution of the errors in each group. The major error types were Use of Distractors, Misidentification of the Problem Structure, and Computational Errors. A significant difference was found in the mean proportion of Use of Distractors Errors between the TD (M = .083; SD = .21) and DLD (M = .40; SD = .38) groups, $t(42) = -3.58$, p = <.001, d = .28, the mean proportion of Misidentification of Problem Structure Errors between the TD (M = .15; SD = .12) and DLD (M = .32; SD = .26) groups, $t(42) = -2.81$, *p* = .007, *d* =.19, and the mean proportion of Computational Errors between the TD (*M* = .056; *SD* = .095) and DLD (*M* = .27; *SD* = .38) groups, *t*(42) = -2.87, *p* = .006, *d* = .24. These findings show that the DLD group made more errors on average in every major error type than the TD group.

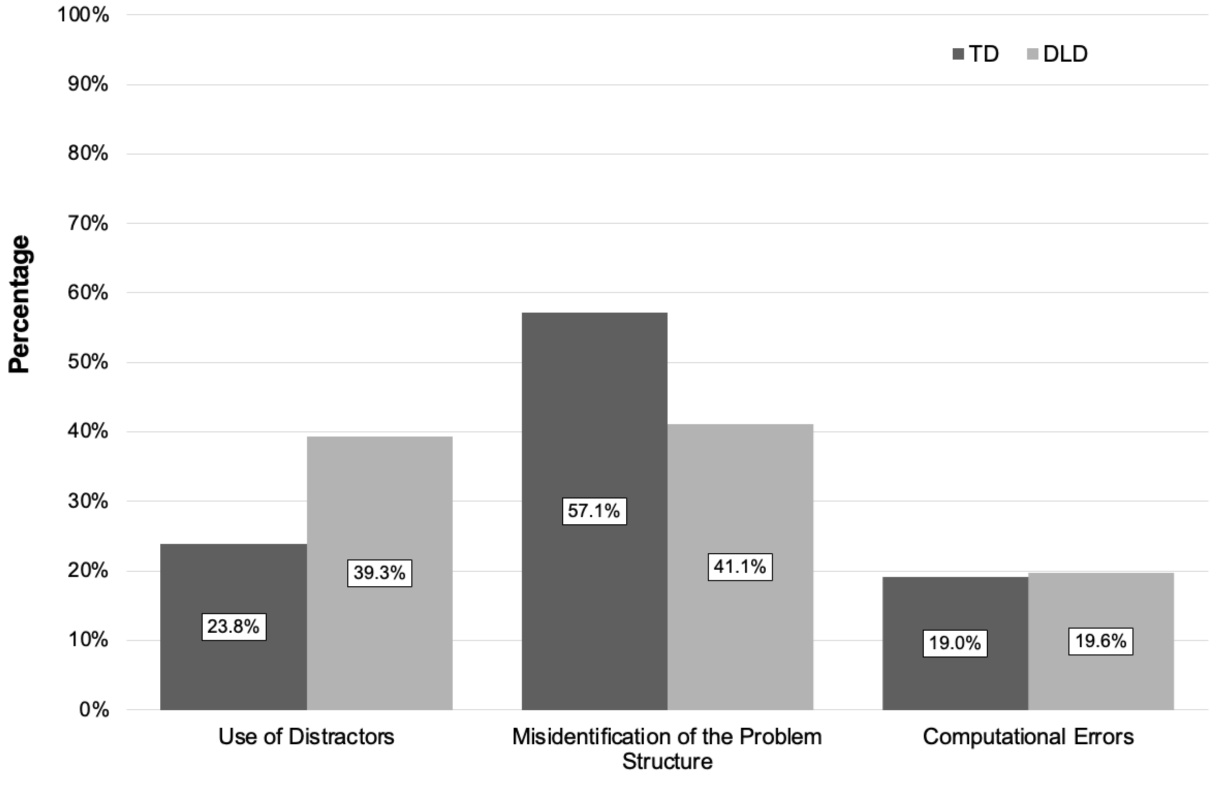
Concerning the distributions of the major error types in the two groups, the proportions of each major error type out of all the errors committed in each group were computed and are presented in Figure 5. The most frequent type of error for both groups was the Misidentification of the Problem Structure at 57.1% in the TD group and 41.1% in the DLD group. The second most frequent error was the Use of Distractors at 23.8% of the total number of errors for the TD group and at 39.3% for the DLD group. Lastly, the Computational Errors represented 19% of the TD group errors and 19.6% of the DLD group errors. In both groups, the identification of the problem structure seems to represent the main cause of error.

Error Subtypes

The fourth research question explored the distribution of the error subtypes in each group. In the broader distractor category, the two error subtypes were the use of Numerical Distractors and Literal Distractors. Of all Use of Distractors Errors produced in the TD group, none was a Literal Distractor Error, whereas in the DLD group, 9.5% of all Distractor Errors were Literal Errors. Therefore, all the Distractor Errors in the TD group were because of irrelevant numerical information in the problem compared to 90.5% in the DLD group.

Figure 5

Proportions of Major Error Types by Group



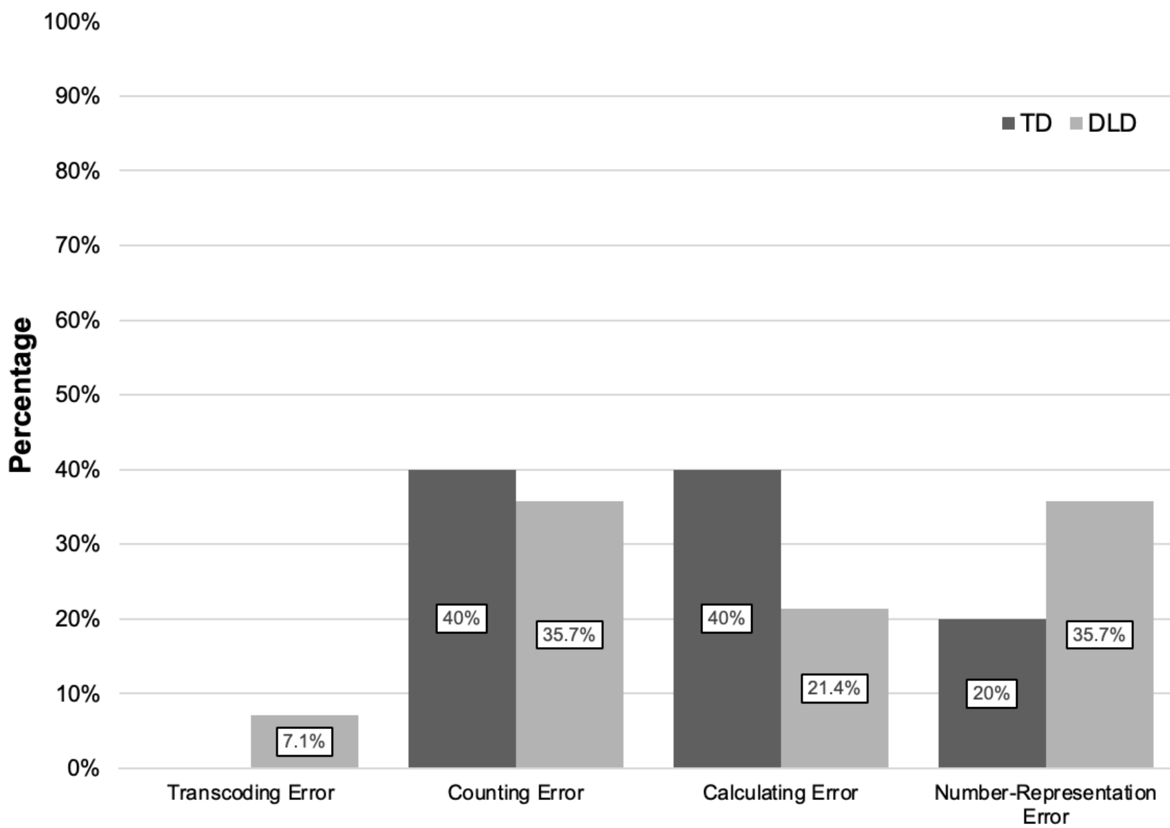
Note. The percentages represent the proportions of each major type of errors out of all the errors made by the children in each group.

With regards to the Misidentification of the Problem Structure, the error subtypes were the identification of the Wrong Action (e.g., join instead of separate) or No Action, where children did not perform a change to an identified set (i.e.., a child answered with a number from the problem). The proportions of the Wrong Action and No Action Errors out of all the Misidentification of the Problem Structure Errors were similar in the two groups: TD children made 87% Wrong Action Errors and 13% No Action Errors, while children with DLD committed 88% of Wrong Action Errors and 12% of No Action Errors.

Figure 6 illustrates the proportions of the four Computational Error subtypes out of all Computational Errors made by children in the TD and DLD groups. The error subtypes for the Computational Error category were mistake in Transcoding (i.e., a child took three chips instead of four), Counting (i.e., a child neglected to count a chip), Calculating (i.e., a child arrived at an incorrect answer in a mental computation), and Number Representation (i.e., a child misaligned the units and tens when performing the standard algorithm). The observed pattern of Computational error subtypes in each group was different. The DLD group made equal proportions of Counting and Number Representation Errors at 35.7% of all Computational Errors, followed by Calculating Errors at 21.4%, and Transcoding Errors at 7.1%. For the TD group, no Transcoding Errors were observed. The most frequent error subtypes in the TD group were the Calculating and Counting Errors, each at 40% of all Computational Errors, followed by the Number Representation Errors at 20%. In summary, the children in the DLD group committed Transcoding Errors while the TD group did not. Moreover, the Number Representation Error was (together with the Counting Errors) the main error subtype in the Computational category for children with DLD, whereas in the TD group, it was the least frequent computational error subtype.

Figure 6

Proportions of the Subtypes of Computational Error: Transcoding, Counting, Calculating and Number Representation Errors in the two Groups



Note. The percentages represent the proportions of each error subtype out of all the Computational Errors made by the children in each group.

Chapter 5: Discussion

The objectives of the present study were to compare the performances of the TD and DLD groups on accuracy and strategy appropriateness, to determine the types of strategies that TD children and children with DLD use during word-problem solving, and to describe their errors. To do so, we recruited TD children and children with DLD or at risk of DLD who solved six change word problems. During the task, children were asked to explain their reasoning, and later, the recordings were analyzed. The answers' accuracy and the appropriateness of their strategies, defined as the alignment of the strategies with the word-problem structure, were scored to compare the groups' performance. Furthermore, using the taxonomy presented by Carpenter et al. (2014), each child was assigned a strategy profile that represented the child's most predominant strategy. The groups' strategy profile composition was described. Finally, the errors leading to an incorrect answer, or an inappropriate strategy were coded to find out what most likely could explain the children's difficulties in solving word problems.

Research Question 1: Accuracy

The first research question examined the discrepancies between the groups' accuracy scores in change word problems, where the accuracy score represented the correctness of the answers. I predicted that the results would replicate existing literature on children with DLD (Cowan et al., 2005; Jordan et al., 1995), which has established that their problem-solving accuracy was lower than that of TD children. This prediction was supported by our data. Indeed, a significant difference was observed between the mean accuracy scores of TD children and children with DLD, where TD children outperformed children with DLD.

Since word-problem solving is a language-based task in which the multiple steps leading to an answer require oral comprehension (Peng et al., 2020), it is expected that children with language difficulties would obtain lower scores compared to their TD peers (Cross et al., 2019). According to Kintsch and Greeno (1985), problems presented in a verbal format require children to construct a conceptual representation (i.e., problem structure), via language-processing, from which they can solve it. The word-problem solving process demands strong language abilities (Fuchs et al., 2015). However, children with DLD have difficulties in semantics, syntax, discourse, inference-making, and weaker verbal memory skills (Bishop, 2017; Gough Kenyon et al., 2018). As a result, they may experience more difficulties to execute without mistakes the previously mentioned process. For example, they can fail to correctly formulate the propositions (i.e., identify the objects, quantities, specifications, roles; Kintsch & Greeno, 1985) which will ultimately lead to an incorrect problem structure and an incorrect answer.

Language difficulties affect not only the way children with DLD process word problems but can also impact their learning in classrooms. Mathematics instruction is primarily delivered using oral and written language which puts children with DLD at a greater disadvantage (Cross et al., 2019). They can be challenged when learning new mathematical vocabulary or understanding complex explanations about mathematical concepts and processes, leading to more difficulties in word-problem solving, resulting in lower problem-solving accuracy. Moreover, as suggested by Cowan et al. (2005), some children recruited from specialized classes may have been less exposed to the mathematics curriculum, thereby affecting their performance (e.g., if they are less exposed to a certain type of change problem, they are less likely to successfully solve it). However, more research needs to be conducted on that topic to establish relations between format and level of instruction and word-problems performance in children with DLD.

Research Question 2: Strategy Appropriateness

The second research question addressed groups' variations in strategy appropriateness, which corresponds to the strategy's alignment with the problem structure. I predicted that the strategies used by children with DLD would be less frequently aligned to the actual problem structure than those of TD children. Again, the results supported this hypothesis because I found a significant difference between the mean strategy appropriateness scores of the two groups with the TD group scoring higher than the DLD group. This finding suggests that children with DLD struggle to use a strategy aligned with the problem structure. The problem structure dictates the types of strategies children can use to solve the problem (Verschaffel & De Corte, 1993) because the specific combination of known and unknown elements that children incorporate into the schema (i.e., initial state, change, final state) triggers one or more strategies that are suited to solve the problem (Kintsch & Greeno, 1985). With this in mind, two potential causes can explain the struggles of the children with DLD in using a strategy that is aligned with the problem structure: Either they have difficulty correctly identifying the problem structure or they have a hard time selecting a strategy that is aligned with it. To my knowledge, there is no literature on the strategies used by children with DLD in word-problem solving, making it impossible to explain the data based on research about children with DLD. For this reason, the following sections use previous research conducted with TD children to speculate about the reasons the children with DLD struggled to use structure-appropriate strategies.

As described in the earlier section, children with DLD have specific difficulties in processing language which in turn affect the way they represent word problems. Difficulties in mentally representing the problem structure can affect the type of strategies they use. According to Kintsch and Greeno (1985), the high demands on working memory during the construction of the propositions and the problem structure's schema can affect the ability of TD children to add new information to the existing schema in order to update it. In an attempt to reduce the load on working memory in the present study, the problems were read aloud to all children and repeated as necessary. Nevertheless, having weak verbal working memory (Cowan et al., 2005; Henry & Botting, 2017), children with DLD may have greater difficulty when identifying the problem structure. Moreover, the problems presented in this study all contained irrelevant information, which further create cognitive and linguistic demands for children in general (Wang et al., 2016). Therefore, the language difficulties characterizing children with DLD can limit their ability to construct adequate problem structures. Another explanation for the difficulty to construct the problem structure in children with DLD comes from Carpenter and Moser (1985), who proposed three levels of skills to solve change problems. TD children in level 1 and 2 are able to represent numbers with external aids (e.g., chips or fingers) or in their head by counting. In level 3, they can fully form the problem structure before solving it. It is possible that children with DLD have not reached the level of abstraction needed in level 3 to construct the problem structure in its entirety. With only a partial problem structure, they may be less able to choose a strategy aligned with it.

The alternative cause of the difference in strategy appropriateness is that children with DLD successfully identified the problem structure but were unable to come up with a strategy reflecting their mental representation. Data from TD children suggest that the strategy's evolution follows a developmental continuum where children are capable of solving more complex word problems the older they get using more sophisticated strategies (e.g., understanding of the part-whole concept is necessary to solve compare and combine problems; Kintsch & Greeno, 1985; Riley et al., 1983). Change problems are the least complex type of additive word problems for TD children, with from 80 to 95% of third graders successfully solving initial state unknown change problems, and 100% of third graders successfully solving change unknown and final state unknown change problems (Riley, 1981, as cited in Riley et al., 1983). However, the developmental continuum of the strategies in children with DLD has not been previously studied. Interestingly, some children with DLD in the sample were not successful on any change problem (i.e., no problems accurately or appropriately solved). Therefore, it is possible that they will only reach their peers' performance after third grade, resulting in an inability, for some, to identify a strategy aligned appropriate for solving change problems at this age. Moreover, the high proportions of use of standard algorithms in the DLD group combined with their lower score on strategy appropriateness can reflect difficulties in using the taught procedures. Children with DLD may be susceptible to selecting standard algorithms to solve change problems in a way that does not directly reflect the problem structure. In other words, they will compute numbers without understanding how their computations represent the quantities' semantic relationship with the problem structure.

Since the strategies used by children during word-problem solving are interchangeable (Carpenter & Moser, 1985), it is also possible that children in the DLD group selected strategies that were less suited for the problem structure on the first try. In the current study, only the first attempt was coded and scored, so it is possible that with more time, they could have come up with an aligned strategy.

Research Question 3: Strategy Profiles

The third research question examined the strategy profile composition in each group. The assignment of the strategy profile was based on the most frequent strategy used to solve the word problems. I predicted a developmental discrepancy between the two groups, with the DLD group being at a less advanced level (i.e., Modeling by Ones, Modeling by Tens) and the TD group exposing more mature strategy profiles, such as Invented Algorithms. Data did not support my prediction because the most frequent strategy profile in both groups was the Standard Algorithm followed by Modeling by Ones. No Invented Algorithm profile was assigned to any child. The main difference between the groups was the proportion of unclassifiable strategy profiles in the DLD group (i.e., Other profile), which was assigned 2.5 more times in the DLD group than in the TD group. As noted earlier, no data exist in the literature on the strategies used by children with DLD. Consequently, evidence from research on TD children will be presented in the subsequent section to explain these findings.

Carpenter et al. (1996) proposed the following developmental continuum for strategies in multi-digit word-problems: Modeling by Ones, Modeling by Tens, and Invented Algorithms. The data from the current study were consistent with this developmental sequence, namely that more students used less sophisticated strategies (i.e., Modeling by Ones), which was followed by Modeling by Tens at a substantially lower rate. A similar pattern was found in both groups. Invented Algorithms were so rarely used that no child was assigned to the Invented Algorithm profile. One possible reason that so few students used the Modeling by Tens strategy is a lack of understanding of base-ten concepts. In fact, to pass from Modeling by Ones, where children count ones, to Modeling by Tens, where they group units into tens and count them as if they were ones, children need to be able to represent 10 units as a whole without counting each of them individually (Carpenter et al., 2014). Therefore, the lower rate of the Modeling by Tens strategy and Invented Algorithms can be an indicator that some children in the study may have under-developed understanding of base-ten number concepts (Carpenter et al., 1998).

The absence of the Invented Algorithm profile in the groups may be explained by the children's preference to use Standard Algorithms taught in class. In fact, the timing of instruction of the Standard Algorithms seems to influence the use of Invented Algorithms. Children who are taught Standard Algorithms in school before having had the chance to manipulate numbers and create their own procedures (i.e., Invented Algorithms) tend to rely exclusively on standard procedures to solve multi-digit problems (Carpenter et al., 1998). Moreover, Verschaffel (2007) pointed out that after the introduction of the Standard Algorithm, TD children tend to abandon previous strategies even if they make mistakes in the Standard Algorithm procedure (as cited in Hickendorff et al., 2019). In other words, TD children and children with DLD may have considered Standard Algorithms as an effective way to solve word problems because they are a central part of the instruction they receive at school, even if they still do not master them. This speculation about the preference of Standard Algorithms over any other strategy can be linked to the lower accuracy and appropriateness in the DLD group. It is possible that children with DLD have more difficulties understanding the procedure taught in classrooms because of their language difficulties, but tend to use it anyway, resulting in more errors when problem solving (Carpenter et al., 1998; Verschaffel, 2007, as cited in Hickendorff et al., 2019).

Some of the explanations provided by the students were not clear enough to classify the strategy they used in another category. Such strategies were placed in the Other category. The higher proportion of children in the Other profile in the DLD group then the TD group is not surprising because their oral explanations of their strategies were more often unclear because of their expressive language difficulties. However, this category is surely an obstacle to creating a complete portrait of their strategy use because we would have placed several of their strategies in defined categories had they been able to articulate them more clearly. For example, a child may have used an Invented Algorithm in their head but was unable to clearly explain their process. Another example would be that some children in the DLD and TD groups who were efficient in mental calculations provided only the answer without describing their problem-solving process.

Research Question 4: Error Types

Major Error Types

The fourth research question concerned differences between the frequency of the groups' major error types, namely the Use of Distractors, Misidentification of the Problem Structure, and Computational Errors. My prediction that children with DLD would make more errors in each major type was supported by the data. Indeed, children with DLD made proportionally more errors in all three major categories than the TD children. This is consistent with the higher accuracy and more frequent use of structure-appropriate strategies observed in the TD group than the DLD group. Below, I discuss each major error type and provide potential explanations for the observed discrepancies between the groups.

In the current study, the mean proportion of Use of Distractors Errors made by the TD group was 8%, which is not elevated. For this reason, they will not be discussed. No data are available concerning children with DLD's responses to distractors in word-problem solving. Studies on TD children showed that the incorporation of distractors in word problems tends to decrease accuracy because of difficulties in identifying the relevant information in the problem (Ng et al., 2017; Wang et al., 2016). I hypothesized that with language difficulties, children with DLD will struggle even more than TD children to successfully solve word problems containing irrelevant information. Difficulties with semantic categorization of words and the processing of words in the context of sentences are known as being a defining characteristic of DLD (Courteau et al., 2023; Leclercq et al., 2014). Consequently, they can struggle to understand word meaning and differentiate words that are semantically related (e.g., books and videos) – in other words, they struggle to differentiate relevant and irrelevant information. The presence of irrelevant information in word problems therefore put children with DLD at risk of selecting distractors as being part of the solution.

In my sample, children with and without DLD made Misidentification of the Problem Structure errors. Similarly, problem structure identification has not been studied in children with DLD. Even TD children in the third grade sometimes have difficulty detecting the problem structure of change word problems where the initial state is unknown (Riley et al., 1983), which was also observed in the data in the present study. For the TD group, the majority of children answered all the change problems correctly, while a minority struggled only with initial unknown change problems. In contrast, in my DLD sample, some children did not answer any of the change problems correctly (i.e., including change and final state unknown), nor did they use an aligned strategy for these same problems. Accordingly, it is possible that the problem structure by itself can explain the difficulties of children in both groups but especially for children with DLD.

Nevertheless, the irrelevant information in a word problem can also prevent an adequate detection of the problem structure (Wang et al., 2016). Previous research with TD children has indicated that the presence of irrelevant information in word problems hinders their ability to link the problem structure to a schema they have already internalized, leading to inaccurate answers. In the present study, all the word problems contained irrelevant information, and they presented a similar challenge for children in both the DLD and TD groups. Therefore, it is possible that the low performance observed on the children's use of structure-related strategies was at least in part due to the irrelevant details in the problems.

Additionally, the consistency of language in a word problem can affect children's ability to effectively recognize the problem structure. Verschaffel (1994) concluded that word problems with inconsistent mathematical language (i.e., when the direction of the action does not match the usual meaning of a mathematical word, such as "more" necessitating a separate action) diminish the performance of TD children. Out of the six word problems presented to the children in the present study, four contained inconsistent language, which could have caused errors in identification of the problem structure in both groups. However, children with DLD may be in greater difficulty because of the mental manipulations required to invert the vocabulary from inconsistent into consistent (Verschaffel, 1994). These mental manipulations exert demands on working memory, which is impaired in children with DLD (Cowan et al., 2005; Henry & Botting, 2017). Simply put, children with DLD can be more challenged than TD children when identifying the problem structure because of the type of change problems I administered in the study, and the high linguistic demands caused by the processing of inconsistent vocabulary, and the distractors.

The mean proportion of Computational Errors made by TD children was low (6%). For this reason, only the difficulties observed in the DLD group will be discussed below. The language difficulties experienced by children with DLD not only affect oral and written language but also the mathematical abilities that are language dependent. The source of the difficulty is believed to reside in the memorizing, accessing, and using verbal representations of numbers (e.g., number names, arithmetic facts; Cross et al., 2019). The last step of word-problem solving is to actually compute an answer. To do so, children need to represent the numbers in their head and decide whether they want to mentally or physically manipulate them. Whichever way they decide leads to mental manipulations of numbers. For example, if children decide to use their fingers to compute 3 + 4, they need to mentally trigger a representation of three and four and store the numbers in the working memory for later use. Next, they transcode that representation to three fingers. At that step, there is a possibility that they make an error, and raise two fingers. Having difficulties accessing their working memory, they can make an additional mistake and represent three more fingers instead of four. When counting, they can make another mistake and skip a number leading to an incorrect answer of four. Another child in the same situation may be more familiar with number facts: When they read 3 + 4, an answer is immediately activated, because of difficulties in retrieving number facts, they can answer eight, which is close to seven, but incorrect. Computations are a language-based task which is impaired in children with DLD leading to more mistakes (Koponen et al., 2006).

The distribution of the major error types in the two groups was created in each group based on the proportions of each major error type out of all the errors committed. Interestingly, the distribution of errors follows the same pattern. In both groups, the most frequent error was the Misidentification of the Problem Structure, followed by the Use of Distractors, and finally Computational Errors. In the TD group, not many Use of Distractors and Computational Errors were made, which may suggest that the identification of the problem structure is the main difficulty for TD children in change word problems. However, for the DLD group, the proportions for Use of Distractors and Misidentification of the Problem Structure were comparable. This finding may suggest that both selecting relevant information and identifying the problem structure are significant challenges for children with DLD.

Errors Subtypes

The fourth research question also addressed the distribution of the error subtypes (i.e., Use of Distractors: Numerical and Literal Distractor Errors; Misidentification of the Problem

Structure: Wrong Action and No Action Errors; Computational Errors: Transcoding, Counting, Calculating and Number Representation Errors). The results of the study are consistent with the findings of Ng et al. (2017) on the effect of literal irrelevant information on word-problem solving in TD children. Indeed, no TD child in the present study used any literal irrelevant information to solve the word problems, whereas 9.5% of all Use of Distractors Errors in the DLD group were related to the use of Literal Distractors. The other 90.5% were attributed to Numerical Distractors.

Surprisingly, the distribution of the Misidentification of the Problem Structure subtypes was the same among the TD (Wrong Action Errors: 87%; No Action Errors: 13%) and DLD groups (Wrong Action Errors: 88%; No Action Errors: 12%). The No Action Error was coded when no transformation was made to an identified set (e.g., when a child selected a number from the problem as the answer), whereas the Wrong Action Error represented difficulties in identifying the direction of the change (i.e., a child does a join action instead of a separate). This finding may suggest that, even if children with DLD made more Misidentification of the Problem Structure Errors than TD children, most children in both groups understand that a change needs to be performed to the identified sets, and that the answer is not typically in the problem itself.

For the Computational Errors subtypes, the error patterns were distinct for each group. The TD group made Counting and Calculating Errors most frequently, followed by half as many Number Representation Errors, and finally, no Transcoding Errors. The DLD group made Counting and Number Representation Errors most frequently, followed by Calculating Errors, and Transcoding Errors the least frequently. Recall that Counting Errors were coded when children made errors in enumerating objects or written marks, whereas Calculating Errors were coded when the error followed a mental computation. The lower proportion of Calculating Errors in the DLD group may be explained by their tendency to use fingers when solving word problems (Jordan et al., 1995), or to use backup strategies when faced with complex mental calculations involving multi-digit numbers (i.e., counting instead of doing mental calculations; Cowan et al., 2005). Deciding to count more often than carry out mental calculations may be reflected in the higher proportions of Counting Errors compared to Calculating Errors.

Number Representation Errors represented errors in the tens and units' column alignment in standard algorithms or the mix of tens and units when counting with chips. This error subtype was present in both groups with a higher proportion in children with DLD. Above, I speculated that the higher proportion of the Standard Algorithm profiles in both groups could be due to the focus on the standard algorithm in their classroom instruction (Carpenter et al., 1998; Verschaffel, 2007, as cited in Hickendorff et al., 2019). When a strong emphasis is placed on standard algorithms, TD children tend to use more buggy procedures (i.e., making mistakes in the standard algorithm; Carpenter et al., 1998). In the present study, therefore, the reason for the Number Representation Errors in TD children is similar to the reason there were so few Invented Algorithms, namely because they held incomplete understandings of base-ten concepts (Carpenter et al., 1998). Moreover, children with DLD are known to experience difficulty understanding place value (Donlan et al., 2007; Lafay et al. 2023), which can accentuate the presence of Number Representation errors. Simply put, the errors in Number Representation in the DLD group can be explained, as for the TD group, by the emphasis on Standard Algorithms in school as well as the additional difficulties in understanding place value concepts in the DLD population. Transcoding Errors were errors when converting quantities from one code to the another (e.g., answering with "four" but writing "3"). Difficulties in Transcoding are typical for children with DLD (Cowan et al., 2005; Rodríguez Rodríguez et al., 2020), so their presence was not surprising. No Transcoding errors were observed for TD children, which was also expected.

Contributions to the Literature

This study contributes new data about word-problem solving in children with DLD. Only their performance was assessed in previous research, situating their performance as lower than those of TD children (Cowan et al., 2005; Jordan et al., 1995). Therefore, the current study extends the literature by shedding light on strategy appropriateness, strategy profiles, and error types. Children with DLD struggle to use a strategy that is aligned with the problem structure, either because they struggle to correctly identify the problem structure, or they have a hard time selecting a strategy that is aligned with their mental representation of the problem. Concerning strategy profiles, data from this study revealed a similar pattern among the TD and DLD groups, with the most frequent strategy being the use of Standard Algorithms and Modeling by Ones. This outcome extends the work of Carpenter et al. (1998) on the children's preference to use the standard algorithm after instruction, which seems to also affect children with DLD.

The descriptive analysis of error types also contributes to literature. Children with DLD make more Use of Distractors, Misidentification of the Problem Structure, and Computational errors, which is consistent with and explains their lower accuracy. The findings point out that differentiating relevant from irrelevant information is one obstacle that children with DLD face when problem solving. Problem structure identification seems to be more challenging for children with DLD than TD children. Moreover, the presence of difficulties associated with verbal mathematical abilities (i.e., counting, calculating, transcoding, and representing numbers) is well documented in the literature on children with DLD (Cowan et al. 2005; Cross et al., 2019;

Lafay & Raimbault, 2022), and this study indicates that these difficulties can affect their performance on word problems as well.

Strengths and Limitations

The major strength of this study was to compare the TD children with children with DLD in third grade on several problem-solving processes and outcomes: accuracy, strategy appropriateness, strategy profile, and error types. Therefore, the scope of this study extended previous research because the two populations, namely TD children and children with DLD, could be directly compared within the same educational and cultural environments. This design allowed for a meaningful comparison of these two francophone populations in the Quebec educational context, on which there were no existing data on the mathematics learning of children with language difficulties. Moreover, this study was conducted in French, which contributes to current research on the problem-solving of French-speaking children with DLD.

As in research on clinical populations, the sample size was small. The reality in Quebec schools is that not every child who needs a professional evaluation can obtain one. Therefore, the DLD group was composed of professionally diagnosed children with DLD and children at risk for DLD, which can affect the uniformity of the group. Therefore, the conclusions drawn from my data are to be taken with caution. Additionally, the novelty of the study's scope solicits more research to consolidate what are undoubtedly preliminary findings.

The attribution of strategy profiles did not consider the variety of strategies used by the children on different types of change problems (i.e., initial state, change, final state unknown), or the extent to which children used more than one strategy to solve the word problems (Carpenter & Moser, 1985). The problems provided to children were selected such that there were two with the initial state was unknown, two where the change was unknown, and two where the final state was unknown. Therefore, the attribution of one strategy profile did not reflect the range of strategies children used. In addition, some of the children did not have the time to solve all the change word problems because the data came from a larger research project that included both change and compare word problems, which then left only one or two word problems on which to base the profile analysis. Furthermore, when children used more than one strategy in equal proportions, the more mature strategy profile was attributed (e.g., if a child systematically used Standard Algorithms and Modeling by Ones, they were assigned the Standard Algorithm profile). This decision masked the extent to which the children needed to rely on less developmentally-mature strategies to answer a word problem. Also, the effectiveness of the strategies used was not analyzed. As such, it is possible that even though a more mature strategy was attributed to some students, they may not have been able to successfully solve the word problems.

Lastly, the methodology chosen for this research required children with DLD to verbally explain their thought processes. Children with DLD have expressive language difficulties (Bishop et al., 2017), which affect oral description and explanation. This could have limited our understanding of the strategies they used, which may have augmented the frequency of observed Other profiles in the DLD group. By increasing the amount of time given to children with DLD to complete the problems and to express themselves, and by supporting their ability to articulate their thinking with visual aids (e.g., asking the child to show by a drawing), we could have had access to a better understanding of their reasoning.

Educational Implications

This study informs teachers, special education teachers, and speech-and-language therapists about the difficulties experienced by children with DLD in word-problem solving.

Children with DLD struggle at every step in the process of word-problem solving. They need help to understand the text, identify the problem structure, choose a strategy that is aligned with the problem structure, and compute the answers. More time may be spent in class on explicit instruction to ensure that children with DLD develop the skills and knowledge required to perform each of these steps individually. By creating teaching sequences specifically for each step, children's attention and effort can be focused on one part of word-problem solving at a time. For example, to work on understanding the text, children can be invited to focus on reading the problem, identifying the meaning of the words in the context of the problem, and underlining the information needed to solve the problem (e.g., objects, quantities, time, place, set owners, roles [initial state, change, final state]). A group discussion can follow where children compare their answers, and the teacher justifies their choices.

The strategy profile distribution in the groups showed a preference for children with and without DLD to use standard algorithms and modeling strategies. By offering the possibility of using manipulatives, teachers could support children at their actual developmental level. The use of manipulatives can also be used to teach base-ten numbers concepts, which form a central part of the mathematics curriculum. The absence of Invented Algorithms in both groups suggests limited opportunities for children to manipulate numbers in the classroom. Stepping back from the standard algorithms could permit children to create their own algorithms to solve word problems, resulting in a better understanding of base-ten concepts (Carpenter et al., 1998).

Furthermore, the analysis of the error types revealed that children with DLD have substantial difficulties selecting the relevant information in a word problem. The inclusion of numerical and literal distractors in word problems needs to be considered with a view to the instructional objective. For instance, if the teacher’s intention is to help students select relevant information, then including distractors and focusing on which ones are essential and which ones are not should be the purpose of instruction. On the other hand, if a teacher wishes to work on the identification of the problem structure, then the inclusion of distractors does not serve that purpose because children with DLD will struggle to differentiate the irrelevant information from the relevant.

Concerning the observed difficulties in identifying the problem structure, children with DLD seem to follow a different developmental continuum than the TD children. The sample for this study was composed of third graders. At this age, most TD children are able to successfully solve change word problems (Riley et al., 1983), which was not the case for children with DLD. Teachers in higher elementary grades can assess the level of development of children with DLD to offer extra help for those who have not yet mastered change problems.

Finally, the Computational Errors were more frequent in the DLD group than in the TD group. Activities specifically targeting counting, transcoding, mental and written calculations, and place value could be offered through special education activities and speech-and-language therapies to children with DLD to help them to improve. Difficulties in these mathematical abilities have a considerable impact on word-problem solving for children with DLD.

Conclusion

This study explored the difficulties of children with DLD when solving change word problems. The findings suggest lower performance in children with DLD compared to their TD peers, which can be explained by a variety of difficulties related to language and mathematics. Because of their language difficulties, children with DLD have more difficulty understanding the text, identifying relevant information, constructing an adequate problem structure, selecting a strategy that is aligned with the problem structure, and computing the answers. Their developmental continuum seems to be different from TD children because, in the third grade, some still struggle to successfully solve change problems, which are typically mastered at this age. The strategy profiles of children with and without DLD were similar; both groups preferred to use standard algorithms and modeling strategies. Children with DLD are at risk of greater problem-solving challenges because of language and mathematical difficulties, both being inherently part of their diagnosis.

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Appendix A

Change Problems Presented to Children

Unknown	Problem
Final state	This morning, before going to school, Lea read 16 books and she watched 2 videos. Before going to bed, Lea read 8 more books. How many books did she read that day?
Final state	Elijah and Zoe are at school. Elijah took 23 paintbrushes and 5 pencils to draw in his notebook. He gave 8 paintbrushes to his friend Zoé. How many paintbrushes does Elijah have?
Initial state	In his costume box, Samuel has wigs and hats. This afternoon, the teacher gave Samuel 5 wigs and 2 hats. Samuel now has 22 wigs. How many wigs did Samuel have?
Initial state	For his little brother's birthday, Jayden gave 1 pair of roller skates, 5 large stuffed animals and 2 construction sets. Before the party, Jayden had some stuffed animals. He now has 17 stuffed animals. How many stuffed animals did Jayden have before the party?
Change	Noémie had 4 stars in her notebook. Then she took and glued stars to complete the design. At the end, there are 21 stars, 2 planets and a moon in her notebook. How many stars did Noémie take for her notebook?
Change	At the farm, Martine had 8 pigs, 5 hens and 24 chicks. The henhouse was left open and some chicks escaped. When Martine returned in the evening, she counted 5 hens and 6 chicks. How many chicks escaped from the henhouse?

Note. The problems were originally presented in French.

Appendix B

Coding Rubrics

Consider the following problem to understand the examples of strategies provided in Table B1: This morning, before going to school, Lea read 16 books and she watched 2 videos. Before going to bed, Lea read 8 more books. How many books did she read that day?

Table B1

Coding Rubric of Types of Strategies

Code name	Description	Example
Direct Modeling with Ones	The child uses objects, fingers, etc., to count one by one the operands (no use of base-ten knowledge).	The child counts 16 chips one by one then adds eight chips. He counts the sum of 24 chips one by one.
Direct Modeling with Tens	The child uses objects, fingers, etc., to represent operands using base 10. He represents the units and tens separately.	The child takes one chip to represent the tens and six chips to represent the units. He counts eight more chips. To form the tens, he makes a pile of 10 unit chips from the 14 chips. He exchanges the pile for one ten unit chip and adds it to the first ten. He counts the sum: two tens and four units.
Invented Algorithms	The child invents an algorithm by modifying one or more quantities in the problem. The invented algorithms can be written or computed mentally.	On paper, the child writes 16 + 4 = 20. Then, he writes 20 + 4 = 24. He explains that he separated the eight in two fours to facilitate the computation.
Standard Algorithms	These are the algorithms taught and used in class. Standard algorithms were coded as so even when the child did not align the units and tens and when the child did not write the regrouping mark (i.e., the little "1").	The child writes: $\begin{array}{r} 16 \\ +\ 8 \\ \hline 24 \end{array}$
Other	The strategy cannot be classified because the child uses a strategy not listed in the previous strategies or the child computes in his head and does not explain the process.	The child answers 24 but is unable to explain his process. The child uses a number in the problem as the answer.

Consider the following problem to understand the examples of errors presented in Table B2: This morning, before going to school, Lea read 16 books and she watched 2 videos. Before going to bed, Lea read 8 more books. How many books did she read that day?

Table B2

Coding Rubric for Error Types

Code name	Description	Example
Use of Distractors		
Numerical Distractor	The child uses a numerical distractor to solve the problem.	The child selects a numerical distractor in the problem. He adds 16 + 2 + 8 = 26
Literal Distractor	The child uses a literal distractor to solve the problem.	The child adds 16 + 8 + 1 = 25. He explains that he also counted the day.
Misidentification of the Problem Structure		
Wrong Action Error	The child does not identify the correct problem structure - join or separate.	The child subtracts 8 from 16 instead of adding. He uses a separate structure instead of a join.
No Action Error	The child does not transform the set.	The child answers 8.
Computational Errors		
Transcoding Error	The child makes an error in converting written numbers in verbal numbers or the reverse.	The child answers 24 and writes 23.
Counting Error	The child makes an error when reciting the counting sequence or when tagging objects or drawings.	The child omits one chip when he is counting a collection.
Calculating Error	The child makes an error in a mental computation.	The child answers 22 as the sum of 16 + 8.
Number Representation Error	The child does not align the units and tens correctly in a standard algorithm.	The child writes: $\begin{array}{r} 16 \\ +8\ \ \\ \hline 96 \end{array}$