Robust optimization for an olive oil producer' operations

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Abstract

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This thesis examines how robust optimization can address the operational uncertainties faced by APIA, a major Moroccan olive oil producer. In an industry marked by fluctuating yields, shifting costs, and unpredictable market demand, robust optimization provides a practical approach to making decisions that remain reliable and nearly optimal, even when conditions change. The model developed in this research incorporates robust optimization techniques into APIA's operations, balancing strategic long-term planning with the need for real-time adaptability. By allowing for scenario-based flexibility, the model tailors the level of conservatism to reflect specific uncertainties, ensuring solutions are both practical and resilient, even under worst-case scenarios. Scalable and computationally efficient, the model developed is designed to support key operational decisions in procurement, production, transportation and storage. Through numerical experiments, this thesis demonstrates how the model helps optimize APIA's operations, reducing costs while maintaining operational stability. By managing variability effectively, this research offers APIA a strategic tool to navigate the complexities of the agro-industrial sector, improve profitability, and achieve sustainable growth.

Keywords: robust optimization, operational parameter uncertainty, olive oil supply chain.

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Acronyms

DHS	Dirhams
ECOCERT	Ecological and Organic Certification
IoT	Internet of Things
JIT	Just-in-time
MAD	Mean Absolute Deviation
ONSSA	Office National de Sécurité Sanitaire des Produits Alimentaires
RHS	right-hand side
RO	Robust Optimization
RRMAD	Resilience Mean Absolute Deviation
SOCP	Second-order cone programming
SD	Standard Deviation

Chapter 1: Introduction

1.1. Context: Importance of managing uncertainty in the olive oil industry

The olive oil industry faces many challenges that can harm both production and profits. These challenges come from unpredictable factors such as changing weather, global price fluctuations, new regulations, and shifting consumer preferences. To remain stable and competitive, producers must find ways to manage these risks effectively. In an increasingly competitive and complex market, the ability to address these uncertainties is critical to achieving long-term growth and resilience.

One of the biggest challenges for olive oil producers is the increasing number of extreme weather events. Climate change has disrupted olive farming, causing lower yields and making it harder to plan production. Long periods of drought and intense heat weaken olive trees and reduce their ability to produce fruit. At the same time, late spring frosts can damage flowers and prevent proper fruit development. These unpredictable weather patterns create unstable harvests, which make production planning more difficult.

According to data from the Spanish Ministry of Agriculture, olive oil output dropped from 1.48 million tons in 2021–2022 to just 660,000 tons in 2022–2023, a 55% decline (Bontemps, 2023). Whether due to prolonged droughts or increasing competition for water resources, limited access to this essential input has forced many producers to rely more heavily on irrigation, driving up costs (Food and Agriculture Organization, 2024). These combined challenges, extreme weather and water scarcity, underscore the urgent need for effective risk management strategies to safeguard the future of the olive oil industry.

1.2. Problem statement: Impact of variations in costs, yields, and capacities on economic performance

Olive oil production is a complex process and depends on many factors, including the availability of raw materials, production methods used for olive processing, energy, maintenance and labor. All these phases in the production process have their associated costs, which directly influence profitability. These costs are often driven by external factors such as shifting economic conditions, environmental challenges, and changing regulatory requirements (Zhao et al., 2018).

One other critical element is the efficient transportation of freshly harvested olives to processing facilities to minimize spoilage. Transportation expenses are largely determined by the distance between orchards and mills, and fuel prices which impact the overall cost structure. At the mill, olives go through several processes: cleaning, grinding, malaxation, and oil extraction. Some oil producers choose advanced techniques, such as cold pressing, which can enhance both the yield and quality of the oil. However, these methods require substantial investments in specialized equipment which adds to the overall cost (Aznar-Sánchez et al., 2019).

Energy expenses are another key cost driver. Traditional energy sources are more prone to price volatility, making costs unpredictable. On the other hand, renewable energy sources, while offering more stable pricing, require a substantial upfront financial commitment for installation and infrastructure. Similarly, labor costs are particularly high in areas where manual harvesting remains the dominant method. Automation offers a cost-effective alternative in the long term but demands considerable initial capital investment to implement (Ouyang & Lin, 2014).

Compliance with environmental regulations also adds to the overall costs of production. Producers must invest in measures such as water conservation, emission reduction technologies, and waste treatment facilities to meet regulatory requirements. However, opting for more sustainable practices aligns with the increasing consumer demand for eco-friendly and organic products, giving producers an opportunity to enhance their market value (Bai et al., 2015).

Adding to these factors, packaging and marketing costs can vary depending on the materials, design, and quality used. Additionally, maintaining appropriate storage conditions, such as controlling temperature and humidity, is essential for preserving the quality of olive oil, which increases operational costs (Aznar-Sánchez et al., 2019).

1.3. Company overview: Key role in the olive oil industry and operational challenges

In this setting, APIA, a major Moroccan olive oil producer, demonstrates both the challenges and the opportunities within the olive oil industry. Known for producing premium-quality olive oil and upholding Morocco's agricultural heritage, APIA combines traditional practices with modern innovations. APIA works closely with local farmers, and ensures a steady supply of olives, which are processed using methods like cold pressing to retain their natural flavor and nutritional benefits. These techniques highlight the distinct qualities of APIA's extra virgin olive oil, which are recognized both locally and internationally for their robust taste and health advantages.

In addition to olive oil, APIA's product range includes honey, jams, table olives, tapenades, and cosmetics, showcasing the diversity of Morocco's agricultural sector. Despite APIA's success, the company faces several challenges that were identified following the meeting and discussion with the CEO and management team. These challenges affect the company's ability to optimize production, maintain high quality, and ensure profitability. The key challenges include:

- Supply chain risks and price volatility: The cost of raw olives can fluctuate significantly due to harvest quality, disruptions in supply chains, and trends in global markets. Poor harvests and increasing raw material prices often decrease profit margins (Zhao et al., 2018).

- Yield variability and harvest timing: Olive yields vary due to factors such as weather conditions, farming methods, and tree health. In Morocco, the harvest season occurs from October to December, making the timing crucial, any adverse conditions during these months can impact the quality and olive harvest.

- Quality control and consistency: APIA's reputation depends on delivering high-quality olive oil, but variations in olive quality, harvesting methods, and processing techniques can influence the final product's flavor and nutritional value.

- Market competition and pricing pressure: The premium olive oil market is becoming increasingly competitive, with growing global demand encouraging new players. APIA must carefully balance competitive pricing with maintaining profitability, especially during periods of fluctuating global prices.

- Logistics and distribution: Managing a complex supply chain adds to these challenges. From procurement and production to packaging and delivery, logistics require careful coordination. Weather-related disruptions or delays during peak demand periods can affect product availability and customer satisfaction.

- Sustainability and environmental impact: Consumer awareness and regulatory focus on sustainability require APIA to continuously invest in renewable energy, eco-friendly packaging, and environmentally responsible practices. While these efforts strengthen the brand's value, they also raise operational costs (Bai et al., 2015).

- Regulatory compliance and certifications: Meeting international standards, including ISO 22716, ISO 22000, ONSSA, and ECOCERT, is vital for market access. However, complying with diverse global regulations adds complexity and costs.

By addressing these issues, APIA aims to strengthen its leadership in the olive oil sector while advancing sustainable practices. The company's dedication to producing high-quality products reflects the evolving expectations of consumers worldwide, ensuring its continued growth and contribution to Morocco's agricultural legacy.

Chapter 2: Literature review

2.1. Overview of classical and robust optimization concepts

Over the past few decades, the fields of supply chain risk management and production planning have received extensive attention from researchers. This chapter explores the challenges inherent to food supply chains, with a particular focus on the olive oil industry. It also reviews strategies and planning methodologies used to address these risks, drawing insights from the Moroccan olive oil producer APIA.

The olive oil sector plays a critical role in the global economy, generating over $15.11 billion in revenue in 2022, with projections to reach $19.77 billion by 2030 (Fortune Business Insights, 2025). The economic importance of this industry highlights the pressing need for effective strategies to manage risks and improve operational efficiency.

Historically, classical optimization has been a key tool for solving supply chain problems. This approach focuses on maximizing or minimizing an objective function while adhering to a set of deterministic constraints. Although useful under ideal conditions, it assumes that all input data is deterministic and known, which limits its applicability in real-world scenarios where uncertainty is a constant factor. If the uncertain data are random and obey a known probability distribution, then the problem can be handled using one of the Stochastic Optimization (SO) techniques (Ben-Tal et al., 2009). "The SO approach is less conservative than the worst-case-oriented RO approach" (Ben-Tal et al., 2009) and the quality of SO based decisions depends upon the accuracy of probabilistic distribution.

To address these limitations, robust optimization has emerged as an effective alternative. Robust Optimization (RO) is a methodology that complements stochastic programming and sensitivity analysis, focusing on solutions that maintain acceptable performance across various realizations of uncertain inputs. Unlike probabilistic methods, RO typically does not assume a known distribution for uncertain parameters, making it adequate for situations where estimation errors exist, hard constraints must always be met, or solutions are highly sensitive to change. It is a conservative, worst-case-oriented approach, often used in scenarios where low-probability, high-magnitude risks cannot be tolerated, such as in critical infrastructure design (Pinar, 2005). This approach incorporates data uncertainties directly into the decision-making process, ensuring that solutions remain feasible and practical even under variable conditions. By focusing on resilience, robust optimization allows supply chain models to withstand disruptions and maintain performance in dynamic environments.

Melvyn Sim (2004), in his thesis Robust Optimization, underscores the importance of accounting for uncertainty in optimization models. His work illustrates how robust optimization enhances decision-making by ensuring that strategies are both computationally efficient and resilient against variability. Organizations applying this approach can achieve greater reliability and flexibility in their operations.

2.2. Robust stochastic optimization in Agri-Food production

Stochastic optimization has become increasingly important in the agri-food sector, where unpredictable factors like yield fluctuations, market demand shifts, and resource availability create significant challenges for production planning. To tackle these uncertainties, Lotfi et al. (2022) developed a robust stochastic optimization model aimed at minimizing the Robust and Resilience Mean Absolute Deviation (RRMAD). This metric assesses two key aspects of production planning: robustness, measured by the Mean Absolute Deviation (MAD), which calculates average prediction errors across different scenarios, and resilience, captured by the Standard Deviation (SD), which quantifies variability in those prediction errors. The model integrates constraints that align projected outputs with actual production data, ensuring greater stability while minimizing fluctuations. By incorporating techniques like polynomial regression and resiliency coefficients, it enhances adaptability to randomness and potential disruptions. Validated through correlation coefficients (R^2), this approach has demonstrated its effectiveness in managing uncertainty in agricultural operations, particularly in olive oil production.

2.3. Relevance to APIA's operations

For APIA, robust optimization provides a strategic tool for managing the uncertainties inherent in olive oil production. The company faces several challenges, including fluctuations in olive yields caused by weather conditions and farming practices, price volatility in raw materials, and logistical disruptions that impact transportation and delivery schedules. By integrating robust optimization techniques, APIA can develop strategies that strike a balance between cost efficiency and operational resilience.

Using tools like scenario analysis and adaptive decision-making, APIA can improve resource management and mitigate variability in its supply chain. Robust models can enable the company to determine optimal procurement quantities and production schedules that remain effective even when market conditions or environmental factors change unexpectedly. These models can also help minimize risks associated with unpredictable yields and price fluctuations, ensuring consistent product availability and quality.

Beyond operational benefits, robust optimization plays a crucial role in helping APIA maintain its competitive edge in both domestic and international markets. By leveraging advanced methods to align production with market demand and enhance operational stability, the company can meet customer expectations while navigating the complexities of a highly competitive and uncertain industry. This strategic approach ensures sustainable growth, optimized resource utilization, and the flexibility to adapt to evolving market conditions.

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Chapter 3: Robust optimization framework

3.1. Evolution of robust optimization: From conservative to practical approaches

Robust optimization has evolved significantly over the past decades, transitioning from highly conservative methods to more balanced and practical frameworks. The fundamental challenge remains the same: how to make optimal decisions under uncertainty without being too cautious or overly exposed to risk. This section examines the key developments in robust optimization, focusing on the contributions of Soyster (1973), Ben-Tal and Nemirovski (1998), and Bertsimas and Sim (2002), and how these models have shaped modern decision-making under uncertainty.

3.1.1. Conservative method

One of the earliest robust optimization models was introduced by Soyster (1973). His approach aimed to guarantee feasibility under all possible scenarios by assuming that every uncertain parameter simultaneously takes its worst-case value. This conservative assumption ensures that solutions remain feasible regardless of the uncertainty present in the system.

Mathematically, this method incorporates column-wise uncertainty, where each column of the constraint matrix is treated as part of a convex uncertainty set. To achieve robustness, it introduces protective buffers in the constraints, effectively expanding the feasible region to account for all worst-case possibilities. However, while this method eliminates the risk of infeasibility, it comes at a significant cost: the resulting solutions are often overly conservative and inefficient.

For example, in a supply chain context, this model may suggest over-purchasing inventory or securing excessive supplier contracts to mitigate risks. While this ensures that procurement remains viable in all cases, it greatly inflates costs and reduces operational efficiency. Critics, including Ben-Tal and Nemirovski (1998), argue that while Soyster's model guarantees feasibility, its excessive conservatism limits its applicability in real-world scenarios where a balance between robustness and efficiency is required.

3.1.2. Reducing conservatism

The Ben-Tal and Nemirovski (1998) model addresses uncertainty by using ellipsoidal uncertainty sets. An ellipsoid, in this context, is a convex region that bounds all possible variations of uncertain parameters around their nominal values. The center of the ellipsoid corresponds to the nominal values, while its size and shape are determined by the degree of uncertainty. This ellipsoidal representation allows for gradual variations in the uncertain parameters rather than assuming extreme worst-case deviations. The model guarantees that the solution will remain feasible even under the worst-case scenario, where uncertain parameters deviate from their nominal values within the ellipsoid.

However, Bertsimas and Sim (2002) introduced a simplified interpretation of the Ben-Tal and Nemirovski (1998) robust optimization model, providing a clearer understanding of the trade-off between protection against uncertainty and solution optimality.

Mathematical Formulation (Simplified Interpretation by Bertsimas and Sim, 2002)

The simplified robust counterpart of the uncertain constraint is expressed as:

$$\max c^T x$$

$$a'^T x + \Omega \left|\left|\hat{A}x\right|\right| \leq b$$

Where:

- $c^T x$ is the objective function to be maximized
- $a'^T x$ represents the deterministic (nominal) part of the constraint, where $\bar{a}'$are known coefficients.
- Ω is the robustness parameter, which controls the trade-off between the level of protection and solution optimality. A larger Ω increases conservatism by allowing for greater deviations but offers stronger guarantees of feasibility.
- $\hat{A}$ is a diagonal matrix representing the standard deviations (or maximum deviations) of the uncertain parameters from their nominal values. The term $\left|\left|\hat{A}x\right|\right|$ measures the cumulative impact of uncertainty across all parameters.
- b is the right-hand side of the constraint, ensuring feasibility under uncertainty.

According to Bertsimas and Sim (2002), this simplified representation of the Ben-Tal and Nemirovski model ensures probabilistic feasibility. Specifically, under the uncertainty set described, the probability that the constraint is violated is bounded above by:

$$\text{P(violation)} \leq e^{-\frac{\Omega^2}{2}}$$

This probabilistic interpretation introduces a meaningful trade-off. Decision-makers can set a predefined confidence level for feasibility, balancing protection against uncertainty with the desire for optimal performance.

A key innovation in this framework is the probabilistic feasibility guarantee. Unlike Soyster's (1973) worst-case method, which assumes all uncertainties simultaneously take their worst possible values, the ellipsoidal uncertainty model as interpreted by Bertsimas and Sim (2002) allows for a controlled and probabilistic level of protection. By adjusting Ω, decision-makers can calibrate the level of conservatism in the solution.

While this approach reduces conservatism and offers more adaptable and realistic solutions, it comes at the cost of increased computational complexity. The Ben-Tal and Nemirovski (1998) model is typically formulated as a second-order cone programming (SOCP) problem, requiring advanced optimization methods. Despite this, the model remains tractable and practical for many large-scale optimization problems in fields such as logistics, finance, and industrial planning.

3.1.3. Robust optimization framework

While the Ben-Tal and Nemirovski (1998) model provides a strong guarantee of feasibility under uncertainty, it can be too conservative and computationally expensive when dealing with large-scale problems. To address these issues, Bertsimas and Sim (2002) introduced an alternative robust optimization model that is designed to be more flexible and computationally efficient, while still ensuring feasibility under uncertainty.

A key element of the robust framework is the introduction of the parameter Ti, a value that adjusts the level of robustness for each constraint in the optimization model. Specifically, Ti determines how many uncertain coefficients within a constraint are assumed to deviate concurrently. Its value lies within the interval $[0, | \, Ji \, |]$, where $| \, Ji \, |$ represents the total number of coefficients subject to uncertainty in the i-th constraint.

The role of Ti can be summarized as follows:

- Whole Deviations: The integer part of Ti ($\lfloor Ti \rfloor$) represents the number of coefficients assumed to deviate fully to their worst-case values.

- Partial Deviation: The fractional part of Ti ($Ti - \lfloor Ti \rfloor$) allows one additional coefficient to deviate partially, scaled by the uncertainty range ($\hat{a}_{it}$). This feature fine-tunes the level of conservatism in the model.

It is unlikely that all uncertain coefficients in a constraint to change simultaneously. Instead, the Ti facilitates the protection against the most impactful subset of deviations, ensuring deterministic feasibility for up to Ti deviations. By restricting the nature of adverse changes, the model provides a robust solution that remains effective under realistic worst-case scenarios.

This robust approach aligns with the goal of achieving practical, reliable, and optimal solutions in the presence of uncertainty, making it an ideal framework for applications where data variability plays a significant role.

Before incorporating uncertainty, the standard (deterministic) linear optimization problem is formulated as:

$$\max c^T x$$

$$\sum_j a_{ij}\, x_j \le b_i \quad \forall i$$

where:

- c represents the objective function coefficients.
- x is the decision variable vector.
- a_{ij} represents the constraint coefficients, which may be subject to uncertainty.
- b_i is the right-hand side (RHS) limit of the constraint.

To incorporate uncertainty in the constraint coefficients, the Bertsimas & Sim robust formulation modifies the constraints as follows:

$$\sum_j a_{ij}\, x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{ij} \le b_i \quad \forall i$$

where:

- Γ_i is the robustness parameter that controls how many uncertain coefficients can simultaneously deviate from their nominal values.
- $z_i \Gamma_i$ represents the worst-case impact of uncertainty in constraint i, ensuring feasibility under uncertainty.
- p_{ij} are auxiliary variables used to adjust for individual deviations in uncertain parameters.
- Ji is the set of uncertain coefficients in constraint i.

The introduction of Γ_i allows decision-makers to adjust the level of conservatism in the model, ensuring constraints remain feasible without introducing excessive caution that could lead to suboptimal decisions. The following additional constraints are used to complete the model (based on the duality of the original problem, see Bertsimas & Sim (2002)):

$$z_i + p_{ij} \ge \hat{a}_{ij}\, y_j, \forall i\,, j \in J_i$$

where:

- $\hat{a}_{ij}$ represents the maximum possible deviation of coefficient a_{ij}

- z_i provides a buffer for worst-case deviations, ensuring constraints remain feasible under uncertainty

- y_j is an auxiliary decision variable that bounds parameter variations and maintains computational efficiency.

The key advantage of this formulation is that it allows decision-makers to fine-tune conservatism, instead of assuming that all uncertain coefficients reach their worst-case values at once. This makes the model more flexible and practical for real-world applications.

3.1.4. Comparison and evolution of robust optimization

The evolution of robust optimization reflects a shift from highly conservative methods, like Soyster's, to more balanced and scalable approaches, such as those proposed by Ben-Tal and Nemirovski (1998), and Bertsimas and Sim (2002). These advancements have transformed robust optimization into a critical tool for addressing uncertainty in decision-making. By balancing robustness, efficiency, and computational feasibility, modern robust optimization frameworks offer practical solutions for complex real-world challenges.

Table 1: Summary comparison of robust optimization model

Approach	Key concept	Advantages	Limitations
Soyster Model	Worst-case robustness: assumes all uncertain factors reach their worst values at once.	Guarantees feasibility in all cases.	Highly conservative, leads to inefficient and costly decisions.
Ben-Tal & Nemirovski Model	Probabilistic robustness: uncertainty modeled using ellipsoidal uncertainty sets.	Allows gradual variations, reducing conservatism.	Computationally complex due to reliance on second-order cone programming.
Bertsimas & Sim Model	Budgeted uncertainty: controls how many factors reach their worst values at once.	More practical and computationally efficient, maintains linearity.	Requires careful selection of the robustness budget (Γ) for optimal performance.

Chapter 4: Materials and methods

4.1. Data collection and sources

After discussions with the CEO, several key risks in the olive oil supply chain were identified, each carrying significant financial implications. To support our study, the company provided detailed financial reports covering the past six years (Table 2), offering valuable insights into trends and challenges affecting operations.

The data was collected through direct discussions with APIA's management and a thorough review of historical company records. It focuses on critical supply chain metrics and financial performance indicators that influence olive oil production, including harvest periods, production costs, market prices, and quality control parameters. These factors play a crucial role in maintaining efficiency and profitability.

In Morocco, olives for oil production are harvested within a specific timeframe, typically from late October to December, to ensure optimal quality and yield. The collected data provides a comprehensive understanding of the dynamics shaping APIA's supply chain, highlighting how risks such as yield variations, fluctuating production costs, and supply disruptions can directly impact the company's performance and profitability.

Table 2: APIA's olive oil production cost breakdown (2017–2023)

Time	Yield	Olive cost (dhs/ kg)	Olive oil sale price per liter	Processing cost/liter	Warehousing Cost + inventory carrying cost (dhs/liter)	Energy, labor, maintenance (dhs/liter)	packaging and label (dhs/liter)	Transportation cost per liter
Oct-17	0.15	3.72	40.00	1.50	0.30	1.25	0.80	1.00
Nov-17	0.17	3.72	40.00	2.50	0.30	1.25	0.80	1.00
Dec-17	0.15	3.72	40.00	3.00	0.30	1.25	0.80	1.00
Oct-18	0.15	6.26	55.00	2.30	0.50	1.25	0.90	1.00
Nov-18	0.15	6.26	55.00	3.30	0.50	1.25	0.90	1.00
Dec-18	0.20	6.26	55.00	0.34	0.50	1.25	0.90	1.00
Oct-19	0.20	5.42	60.00	0.37	0.60	1.25	0.90	1.00
Nov-19	0.20	5.42	60.00	1.70	0.60	1.50	0.90	1.00
Dec-19	0.20	5.42	60.00	1.70	0.60	1.50	0.90	1.00
Oct-20	0.19	6.78	70.00	2.00	0.80	1.50	0.95	1.25
Nov-20	0.19	6.78	70.00	2.60	0.80	1.50	0.95	1.25
Dec-20	0.20	6.78	70.00	2.70	0.80	1.50	0.95	1.25
Oct-21	0.19	6.90	70.00	1.70	0.80	1.75	0.95	1.50
Nov-21	0.18	6.90	70.00	2.00	0.80	1.75	0.95	1.50
Dec-21	0.17	6.90	70.00	3.00	0.80	1.75	0.95	1.50
Oct-22	0.16	11.23	90.00	3.00	1.20	1.75	1.00	1.60
Nov-22	0.15	11.23	90.00	3.00	1.20	1.75	1.00	1.60
Dec-22	0.15	11.23	90.00	2.30	1.20	1.75	1.00	1.60
Oct-23	0.15	12.00	127.00	2.00	2.00	2.24	1.30	1.75
Nov-23	0.15	12.00	127.00	2.50	2.00	2.24	1.30	1.75
Dec-23	0.14	12.00	127.00	3.62	2.00	2.24	1.30	1.75

4.2. Cost forecasts for 2024 based on inflation

Economic forecasts for Morocco in 2024 indicate that the average inflation rate is expected to decline to 2.4%, a significant drop from 6.1% in 2023. This decrease is largely driven by falling food prices and a general slowdown in price increases. However, some factors may continue to exert inflationary pressure in the coming months, including the planned rise in butane gas prices and the impact of drought-related reductions in agricultural output (Alby S., 2024).

Given these projections, cost forecasts for 2024 (Table 3) have been adjusted to reflect the 2.4% inflation rate, allowing for a more reliable assessment of potential economic fluctuations affecting olive oil production. To account for uncertainty, possible deviations were estimated by analyzing historical data standard deviations and incorporating insights from the company's past experience in the industry. This approach ensures a more reliable forecast, helping to navigate price volatility and optimize financial planning.

Table 3 : Cost forecast for olive oil production in Morocco (2024)

Time	Olive Cost (dhs/kg)	Olive Oil Sale Price (dhs/L)	Processing Cost (dhs/L)	Warehousing & Inventory Cost (dhs/L)	Energy, Labor, Maintenance (dhs/L)	Packaging & Label (dhs/L)	Transportation Cost (dhs/L)
23-Oct	12.00	127.00	2.00	2.00	2.24	1.30	1.75
23-Nov	12.00	127.00	2.50	2.00	2.24	1.30	1.75
23-Dec	12.00	127.00	3.62	2.00	2.24	1.30	1.75
Adjusted for Inflation 2.4% (2024)	12.29	130.05	3.73	2.05	2.29	1.34	1.79

4.3. Variable descriptions

To facilitate the understanding of the models used in this study, Table 4 provides a detailed breakdown of the decision variables, auxiliary variables, and robustness adjustment variables, while Table 5 outlines the parameters and their descriptions.

With these variables and parameters defined, the next section explores how robust optimization techniques are integrated into APIA's supply chain to handle uncertainty.

Table 4 : Variable description

Type	Variable	Description
Decision Variable	$Xs(i)$	Quantity of olives purchased from supplier i.
Decision Variable	Xo	Total quantity of olive oil produced.
Auxiliary Variable	$Ys(i)$	Auxiliary variable ensuring feasibility of procurement under uncertainty.
Auxiliary Variable	$Yo(i)$	Auxiliary variable ensuring feasibility of production under uncertainty.
Protection Variable	$Ps(i)$	Slack adjustment for purchasing cost uncertainty.
Protection Variable	$Po(i)$	Slack adjustment for yield uncertainty.
Protection Variable	$Ptps(i)$	Slack adjustment for transportation cost uncertainty.
Protection Variable	Ppc	Slack adjustment for processing cost uncertainty.
Protection Variable	Pel	Slack adjustment for energy cost uncertainty.
Protection Variable	Ppl	Slack adjustment for packaging cost uncertainty.
Protection Variable	Pwc	Slack adjustment for warehousing cost uncertainty.
Protection Variable	Pr	Slack adjustment for revenue uncertainty.
Robustness Adjustment Variable	Mr	Worst-case adjustment buffer for procurement cost uncertainty.
Robustness Adjustment Variable	Mo	Worst-case adjustment buffer for yield uncertainty.
Robustness Adjustment Variable	$Mtps$	Worst-case adjustment buffer for transportation cost uncertainty.
Robustness Adjustment Variable	Mpc	Worst-case adjustment buffer for processing cost uncertainty.
Robustness Adjustment Variable	Mel	Worst-case adjustment buffer for energy cost uncertainty.
Robustness Adjustment Variable	Mpl	Worst-case adjustment buffer for packaging cost uncertainty.
Robustness Adjustment Variable	Mwc	Worst-case adjustment buffer for warehousing cost uncertainty
Robustness Adjustment Variable	Mr	Worst-case adjustment buffer for revenue uncertainty.

Table 5 : Parameters

Parameter	Description
$As(i), Bs(i)$	Nominal purchasing cost and uncertainty per supplier i.
$Ao(i), Bo(i)$	Yield coefficient and uncertainty for supplier i.
$Atps(i), Btps(i)$	Transportation cost and uncertainty per supplier i.
Apc, Bpc	Nominal and uncertainty in processing costs per liter of oil.
Ael, Bel	Nominal and uncertainty in energy costs per liter of oil.
Apl, Bpl	Nominal and uncertainty in packaging costs per liter of oil.
Awc, Bwc	Nominal and uncertainty in warehousing costs per unit of olives.
Ar, Br	Nominal selling price per liter of oil and its uncertainty.
$Ts, To, Ttps, Tpc, Tel, Tpl, Twc, Tr$	Robustness parameters controlling the impact of uncertainty in procurement, yield, transportation, processing, energy, packaging, warehousing, and revenue.

Chapter 5: Experimentation

5.1. Deterministic optimization framework

To establish a foundation for optimizing APIA's supply chain operations, a deterministic optimization model was developed, assuming perfect knowledge of parameters. The primary objective of this model is to maximize APIA's profit (Z) under fixed and predictable conditions, where all costs, yields, and revenues remain constant. This approach provides valuable insights into operational efficiency and serves as a baseline for decision-making. However, it does not account for real-world uncertainties, such as supplier price fluctuations, processing efficiency variations, and demand volatility.

The deterministic formulation of the objective function is mathematically expressed in Equation:

$$Z = (Ar.Xo) - \left(\sum_{i=1}^{3} \big(As(i).Xs(i)\big) + \sum_{i=1}^{3} (Atps(i).Xs(i)) + (Apc.Xo) + (Ael.Xo) + (Apl.Xo) + \sum_{i=1}^{3} \big(Awc.Xs(i)\big) \right) \qquad \forall i \in \{1,2,3\}$$

Where:

- Total revenue is represented by $Ar \times Xo$ where Xo is the quantity of oil produced and sold and Ar is the unit sale price.
- Cost components include:
  1. Procurement costs of olives from each supplier $\sum_{i=1}^{3}\big(As(i).Xs(i)\big)$,
  2. Transportation costs of olives from each supplier $\sum_{i=1}^{3}\big(Atps(i).Xs(i)\big)$,
  3. Processing costs $(Apc.Xo)$,
  4. Energy, labor and maintenance costs $(Ael.Xo)$,
  5. Packaging and labeling costs $(Apl.Xo)$
  6. Warehousing costs $\sum_{i=1}^{3}\big(Awc.Xs(i)\big)$.

The deterministic model considers three suppliers instead of a single source to assess supplier diversification and its impact on risk mitigation. While computationally efficient, it does not account for uncertainty in supplier costs, processing efficiency, and market prices, necessitating a robust optimization approach.

5.2. Bertsimas & Sim application to APIA's supply chain optimization

The Bertsimas & Sim (2002) approach is applied to APIA's supply chain optimization to maximize profitability while effectively managing uncertainty in key cost components. This framework employs a control parameter (T) to strike a balance between robustness and feasibility, ensuring that operational constraints remain practical without being overly conservative

Table 6 : Optimized constraints for APIA's supply chain using the Bertsimas & Sim robust model

Constraint	**Equation**	**Explanations**
Objective function	$Max\ Z = REV - C$	Maximizes total profit by optimizing revenue (REV) and minimizing costs (C) while managing uncertainty
Revenue Formulation	$Ar.Xo - Pr - Mr.Tr - Rev \geq 0$	Ensures revenue covers uncertainties in pricing. Pr captures fluctuations, Mr buffers deviations, and Tr controls exposure to risk
Revenue Risk	$Mr + Pr - Br.Yo(i) \geq 0 \quad \forall i \in \{1,2,3\}$	Limits revenue fluctuations by keeping deviations within controlled bounds.
Procurement Cost Constraint	$\sum_{i=1}^{3}\big(As(i).Xs(i) + Ps(i)\big) + Ms.Ts - C1 \leq 0 \quad \forall i \in \{1,2,3\}$	Ensures procurement costs remain stable under supplier price variations. $Ps(i)$ captures uncertainty, Ms buffers deviations, and Ts controls impact
Procurement Risk Constraint	$M_s + P_s(i) - B_s(i).Y_s(i) \geq 0 \quad \forall i \in \{1,2,3\}$	Limits procurement cost fluctuations to maintain predictability
Procurement Quantity Constraint	$Xs(i) - Ys(i) \geq 0 \quad \forall i \in\ Suppliers$	Ensures purchased quantity aligns with procurement limits under uncertainty.

Constraint	Equation	Explanations
Transportation Cost Constraint	$\sum_{i=1}^{3}\left(Atps(i).Xs(i) + Ptps(i)\right) + Mtps.Ttps - C2 \leq 0 \quad \forall i \in \{1,2,3\}$	Keeps transportation costs stable despite price fluctuations.
Transportation Risk Constraint	$Mtps + Ptps(i) - Btps(i).Ys(i) \geq 0 \quad \forall i \in \{1,2,3\}$	Prevents excessive transportation cost variations.
Processing Cost Constraint	$Apc.Xo + Ppc + Mpc.Tpc - C3 \leq 0$	Controls processing cost fluctuations by incorporating buffer terms.
Processing Risk Constraint	$Mpc + Ppc - Bpc.Yo(i) \geq 0 \quad \forall i \in \{1,2,3\}$	Maintains stability in processing costs.
Energy, Labor & Maintenance Cost Constraint	$Ael.Xo + Pel + Mel.Tel - C4 \leq 0$	Ensures operational costs remain stable under energy and labor fluctuations.
Energy, Labor & Maintenance Risk Constraint	$Mel + Pel - Bel.Yo(i) \geq 0 \quad \forall i \in \{1,2,3\}$	Limits cost deviations in energy, labor, and maintenance.
Packaging Cost Constraint	$Apl.Xo + Ppl + Mpl.Tpl - C5 \leq 0$	Keeps packaging costs within feasible limits despite material price variations.

Constraint	Equation	Explanations
Packaging Risk Constraint	$Mpl + Ppl - Bpl.Yo(i) \geq 0 \quad \forall i \in \{1,2,3\}$	Prevents excessive fluctuations in packaging costs.
Warehousing Cost Constraint	$\sum_{i=1}^{3}(Awc.Xs(i) + Pwc) + Mwc.Twc - C6 \leq 0$	Ensures storage costs remain controlled despite price fluctuations.
Warehousing Risk Constraint	$Mwc + Pwc - Bwc.Ys(i) \geq 0 \quad \forall i \in \{1,2,3\}$	Limits deviations in warehousing costs.
Yield Constraint	$\sum_{i=1}^{3}\left(A_o(i).Xs(i) - P_o(i)\right) - M_o.T_o - X_o \geq 0 \quad \forall i \in \{1,2,3\}$	Ensures feasible oil production despite uncertainties in yield.
Yield Risk Constraint	$Mo + P_o(i) - B_o(i).Y_o(i) \geq 0 \quad \forall i \in \{1,2,3\}$	Limits fluctuations in production efficiency.
Capacity Constraints	$\sum_{i=1}^{3} Xs(i) \leq$ WarehouseProdCapacity $Xo \leq$ MaxOilProcessingCapacity	Ensures procurement and production remain within operational limits.

This comprehensive framework offers a robust approach to optimizing profit in olive oil production while effectively addressing uncertainty. Decision variables Xj represent the key quantities to optimize, determining procurement, production, and sales levels, while auxiliary variables Yj adjust for uncertainty, ensuring that all constraints remain feasible under fluctuating conditions.

Protection variables Mi enhances the model's robustness by acting as buffers that account for worst-case deviations in up to Ti coefficients, thereby absorbing cost and yield variations without making the model excessively conservative. Adjustment variables pij capture the impact of uncertainty on constraints, enabling the model to react to fluctuations in supplier prices, yield rates, or other key parameters.

Key parameters, including nominal values Aij and uncertainty ranges Bij, define allowable variation, balancing risk management with profit optimization. The robustness parameter Ti introduces flexibility, enabling the model to accommodate different risk tolerance levels ranging from conservative strategies, which prepare for extreme scenarios, to profit-maximizing approaches, which assume only limited deviations in key parameters.

By integrating intermediate variables, protective adjustments, and cardinality-controlled uncertainty through Ti, the model supports reliable, adaptable, and practical decision-making across procurement, processing, and revenue generation. Ultimately, it achieves an effective balance between maximizing performance and managing risk, ensuring that the supply chain remains stable and profitable despite external fluctuations.

5.3. Comparative model: Ellipsoid approach

In the context of APIA's supply chain operations, the ellipsoid model is implemented to enhance decision-making under uncertainty by introducing a probabilistic feasibility framework. This method replaces rigid worst-case constraint buffers with ellipsoidal protection terms (ELP), ensuring feasibility at a predefined confidence level while allowing for more adaptive risk management.

The key modifications to the constraints include:

- Ellipsoidal Uncertainty Representation:
  Instead of explicitly modeling a set number of worst-case deviations, each constraint integrates an ellipsoidal protection term (ELP):

$$\Sigma \cdot ELP, \text{ where } \quad ELP^2 = B^2 Z^2$$

This probabilistic adjustment applies consistently across procurement, transportation, processing, energy, packaging, and warehousing constraints, capturing correlated uncertainties rather than independent worst-case deviations.

- Probabilistic Feasibility Guarantee:
  The model ensures feasibility at a predefined confidence level, providing a more flexible and adaptive treatment of uncertainty compared to rigid worst-case assumptions.

By replacing the deterministic worst-case robustness framework with an adaptive, probability-driven structure, the ellipsoid model enables a smoother response to uncertainty, balancing robustness with computational efficiency while maintaining a high level of feasibility.

5.4. Model solving and implementation

Both robust optimization models are implemented and solved using LINGO software, with the primary analysis and results focusing on the primal model (Bertsimas & Sim), while the Ellipsoid Model serves as a comparative benchmark.

The integer programming formulation is specifically designed to address the optimization problem efficiently, leveraging LINGO's capabilities for both linear and nonlinear problem-solving. The Bertsimas & Sim (2002) robust optimization framework is implemented first, with its LINGO code detailed in Table A1 and the corresponding mathematical formulation in Table A2. Each run was completed in under a second, ensuring computational efficiency. Since LINGO do not provide the guaranteed optimal solution for non-linear problems, the objective function value for the Ellipsoid Model may not be the best one.

Additionally, the Ellipsoid Model is implemented as an alternative approach, with its LINGO code provided in Table A5 and its formulation in Table A6. A comparative analysis of both models, presented in Section 7.6, evaluates their trade-offs in terms of computational efficiency and solution conservatism, highlighting the practical advantages of each approach.

Chapter 6: Discussion on scenario design and modeling decisions

6.1. Scenario design: Evaluating procurement and yield uncertainty

To evaluate the primal model's performance under varying conditions, we established different sets of scenarios.

The robustness parameter for revenue (Tr) was fixed at its best-case value across all scenarios. This decision aligns with the company's strategic control over pricing, which allows for stabilization of revenue regardless of other uncertainties. By maintaining Tr at its optimal value, the model ensures that revenue remains a stable factor, enabling a clearer analysis of the impact of procurement and yield uncertainties.

Fixing $Ttps$, Tpc, Tel, Tpl, Twc as either all best-case or all worst-case acknowledges the correlation among operational costs. These parameters represent interconnected processes, such as transportation, processing, and storage. By grouping them into coherent sets, the scenarios provide a comprehensive view of how operational cost variability affects overall performance, isolating the effects of procurement and yield uncertainties for clarity.

6.2. Set of scenarios 1: Operational costs at best case

In this set of scenarios, the robustness parameters for operational costs ($Ttps$, Tpc, Tel, Tpl, Twc) including transportation, processing, energy, packaging, and storage were fixed at their best-case values. This assumption reflects an optimistic scenario, where operational cost uncertainties are minimal. The robustness parameters for procurement (Ts) and yield (To) were changed to evaluate their isolated impacts under stable operational conditions.

6.3. Set of scenarios 2: Operational costs at worst case

In this scenario $Ttps$, Tpc, Tel, Tpl, Twc were set to their worst-case values, representing a more conservative outlook where operational costs are highly variable and correlated. This set highlights the compounded effect of operational cost uncertainties on the model's performance. Similarly to the first set of scenarios, Ts and To were varied to evaluate their contributions under challenging operational conditions.

This thesis employs a robust optimization framework to improve decision-making in the olive oil production process, accounting for uncertainties in procurement, yield, transportation, and operational costs. While the model provides a structured approach to handling uncertainty, this discussion examines further the rationale behind key modeling decisions specifically, the choice not to use midpoint values for robustness parameters (T), the exclusion of statistical analysis and the implications of using LINGO's deterministic approach.

6.4. Rationale for not using mid-point values for T

The decision to exclude mid-point values for T is based on the price of robustness which is the balance between maximizing profit and managing uncertainty. T determines how many uncertainties, can fluctuate simultaneously. When T is set to mid-point values (1 or 2), risk is distributed too evenly, making the model less effective in both stable and uncertain conditions.

Mid-point values fail to fully account for disruptions, leading to poor inventory and cost planning. For instance, if two out of three suppliers experience lower yields but the model assumes only one will be affected, it underestimates the risk of shortages, potentially causing production delays. Similarly, if multiple suppliers raise prices but the model only anticipates supply chain uncertainties are often interconnected, mid-point values don't adequately capture cascading risks, ultimately reducing decision accuracy.

From a computational perspective, mid-point values add complexity without significant benefits. Since LINGO applies fixed worst-case deviations, it has to evaluate multiple scenarios separately, making the process inefficient. By restricting T to 0 (nominal conditions) or 3 (fully robust conditions), the model ensures a clearer strategy, either prioritizing profit when risks are low or fully preparing for potential disruptions. This approach enhances decision-making, efficiency, and overall supply chain resilience. Future research could explore dynamic robust optimization for more flexible risk management.

6.5. Keeping other robustness parameters at best-case and worst-case values

The decision to group other robustness parameters such as those related to transportation, processing, energy, packaging, and storage costs into unified best-case or worst-case scenarios reflects their inherent interdependencies. In real-world operations, these costs are often correlated.

For instance, if transportation costs increase due to external disruptions, it is likely that energy, processing, and other related costs will also rise. This interdependence is consistent with operational insights, such as those highlighted by Mulvey et al. (1995), where such parameters rarely vary in isolation.

Aligning these robustness parameters at unified extremes ensures the model accurately represents practical scenarios. Keeping them at best-case values reflects ideal operating conditions, optimizing for profitability. Conversely, grouping them at worst-case values accounts for potential cost surges, prioritizing resilience and feasibility under adverse conditions. This approach ensures the model provides meaningful insights for strategic decision-making while avoiding unrealistic assumptions about the independence of these costs.

Furthermore, maintaining unified best-case or worst-case scenarios supports computational efficiency and interpretability. As noted by Goldfarb and Iyengar (2003), introducing unnecessary granularity in correlated parameters can overly complicate the model without yielding proportionate improvements in decision quality. By grouping these parameters, the model simplifies the optimization process while still capturing the critical impacts of uncertainty on operational costs. This structured approach not only mirrors real-world cost behaviors but also allows the model to focus on actionable insights, enabling robust and practical decision-making in olive oil production.

6.6. Exclusion of statistical analysis

Statistical methods for modeling uncertainty typically rely on probability distributions based on historical data to predict potential outcomes. While these approaches can be effective, they require extensive, high-quality data that accurately reflects variations in key parameters.

However, in olive oil production, uncertainties such as supplier costs, yield fluctuations, and market dynamics are influenced by unpredictable external factors like weather conditions, global market volatility, and supply chain disruptions. Because these factors often lack sufficient historical data, a purely statistical approach may not be reliable.

To address this challenge, this study adopts a robust optimization framework that does not depend on probabilistic assumptions. Instead, it manages uncertainty by defining parameter ranges, making it particularly useful in real-world scenarios where data may be incomplete, unreliable, or affected by external sources. As Bertsimas and Sim (2002) demonstrated, robust optimization offers a flexible and computationally efficient way to balance robustness and optimality. Their framework allows decision-makers to account for the most significant uncertainties without being overly conservative, ensuring that solutions remain practical and feasible.

Earlier models, like those introduced by Soyster (1973), adopted a highly conservative approach, assuming that all parameters could reach their worst-case values simultaneously.

In contrast, Ben-Tal and Nemirovski (1998) introduced more refined models that focus on protecting against selected deviations while maintaining operational feasibility. This balance between robustness and performance is particularly important in dynamic systems like olive oil production, where multiple interconnected uncertainties can arise.

Mulvey et al. (1995) further emphasized the value of robust optimization in supply chain management, highlighting its ability to generate reliable solutions without requiring precise probabilistic data. Likewise, Bertsimas and Pachamanova (2008) demonstrated how robustness parameters can be adapted to address evolving uncertainties over time, making robust optimization a practical tool for managing dynamic and unpredictable environments. Additionally, Bienstock (1996) pointed out that statistical methods often struggle with assumptions about data distributions, which can lead to inaccuracies in decision-making when those assumptions do not hold.

By leveraging a robust optimization framework, this study ensures that decision-making remains resilient to worst-case scenarios while minimizing reliance on statistical models. This approach provides a practical and effective method for optimizing olive oil production, addressing the limitations of probabilistic techniques while maintaining operational feasibility and near-optimal performance under uncertainty.

Chapter 7: Results

This chapter presents the results of applying the Bertsimas & Sim (2002) robust optimization framework to APIA's olive oil supply chain. By analyzing the relationship between procurement, yield, operational costs, and revenue under varying scenarios, this section highlights the findings in terms of decision variables, auxiliary variables, and robustness parameters. The findings are contextualized within the proposed mathematical model, illustrating its capability to manage uncertainties while enhancing profitability. A complete set of results for all scenarios is presented in the appendix.

7.1. Objective value

The experiments are performed by varying two uncertain parameters: procurement cost (TS) and yield (TO). Both parameters have two levels:

- TS index: 0 (best) and 3 (worst)
- TO index: 0 (best) and 3 (worst)

This results in four combined scenarios represented as (TS, TO): (0,0), (0,3), (3,0), and (3,3).

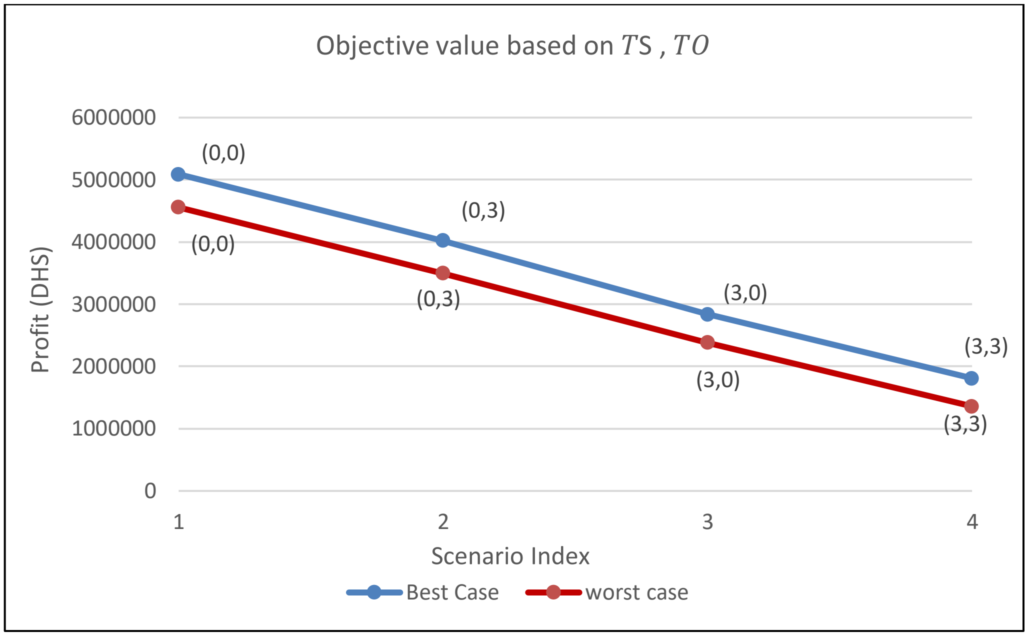
Each scenario is evaluated under two operational cost assumptions: best-case and worst-case operational costs. The objective is to measure the impact of these uncertainties on profit maximization.

Figure 1 shows how procurement costs (TS) and yield variability (TO) shape profitability, with both factors playing a significant role in financial performance.

- In the best-case operational scenario, where all conditions are optimal, profit starts at 5,089,770 DHS ($TS = best, TO = best$) but declines as uncertainty increases. A drop in yield alone ($TS = best, TO = worst$) reduces profit to 4,022,252 DHS, while a rise in procurement costs ($TS = worst, TO = best$) leads to a steeper decline to 2,835,832 DHS, suggesting that procurement costs put more financial pressure on profitability than yield fluctuations. The worst-case combination of both factors ($TS = worst, TO = worst$) results in 1,810,861 DHS, highlighting the compounding effect of rising costs and declining yields.
- Under worst-case operational costs, where inefficiencies further constrain profitability, the trend is similar but starts at a lower level (4,556,720 DHS, ($TS = best, TO = best$). A worsening TO alone ($TS = best, TO = worst$) reduces profit to 3,496,598 DHS, while a rise in procurement costs ($TS = worst, TO = best$) drops it to 2,382,911 DHS, reinforcing that procurement costs have a stronger impact than yield variability. The lowest profit, 1,360,074 DHS ($TS = worst, TO = worst$), confirms the severe financial strain caused when both factors deteriorate together.

These findings highlight that while both procurement costs and yield variability reduce profitability, rising TS is the bigger risk. Cost control is essential for financial stability, but mitigating both procurement and yield uncertainties together is key to long-term resilience.

Figure 1 : Impact of olive cost and yield variability on profit fluctuations



7.2. Supplier selection and procurement quantities ($Xs(i)$)

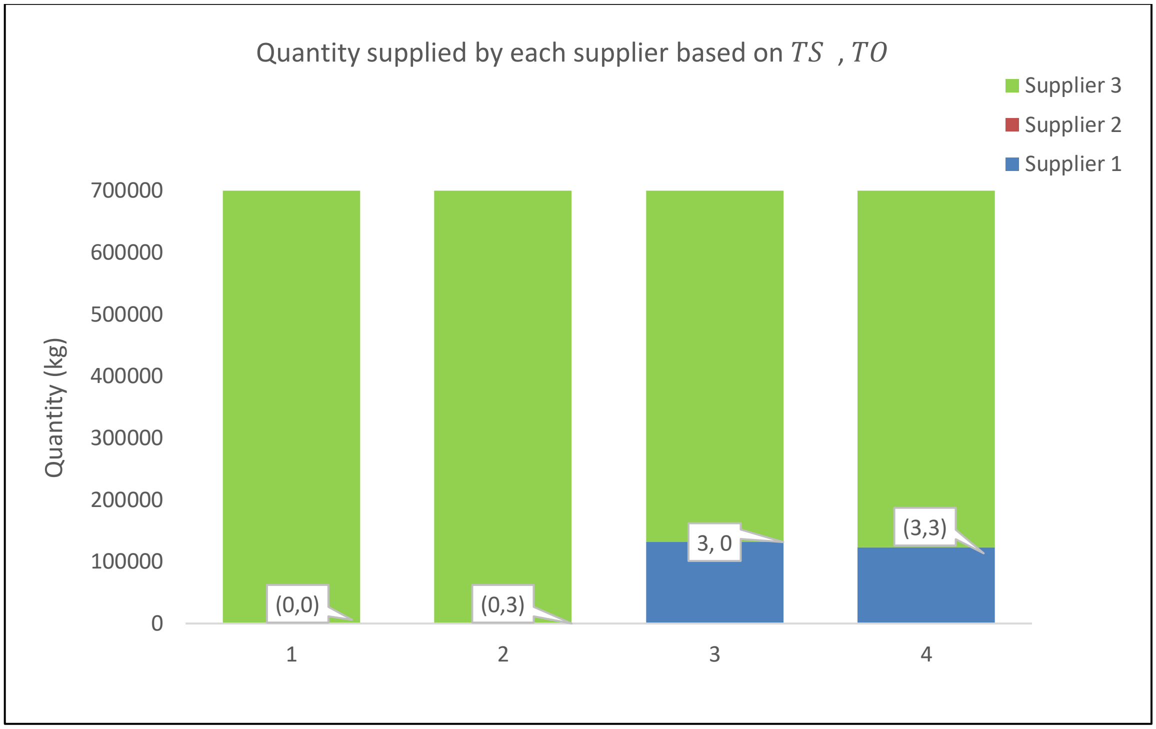
Figure 2 shows that procurement costs (TS) have the biggest influence on supplier selection, while yield variability (TO) affects how much can be procured rather than which suppliers are chosen.

- When procurement costs are low and yields are stable ($TS = best, TO = best$), the model relies entirely on Supplier 3, sourcing 700,000 kg to maximize cost efficiency. Even when $,TO$ worsens ($TS = best, TO = worst$), supplier selection remains unchanged, meaning that fluctuations in yield alone do not trigger supplier shifts as long as costs stay low.

- However, when TS increases ($TS = worst, TO = best$), the model begins diversifying, adding Supplier 1 (131,683.2 kg) while reducing reliance on Supplier 3 (568,316.8 kg). In the worst-case scenario ($TS = worst, TO = worst$), Supplier 1's share decreases slightly to 123,148.1 kg, while Supplier 3's share rises to 576,851.9 kg. This suggests that higher procurement costs push the model to diversify, while worsening yields primarily affect procurement feasibility rather than supplier choice.

Interestingly, Supplier 2 is never selected, likely due to higher costs or less favorable terms. These results highlight the importance of a flexible procurement strategy that responds to cost changes while ensuring supply stability, balancing cost efficiency with risk mitigation.

Figure 2 : Impact of olive cost and yield variability on optimal supply quantity



7.3. Production quantity (Xo)

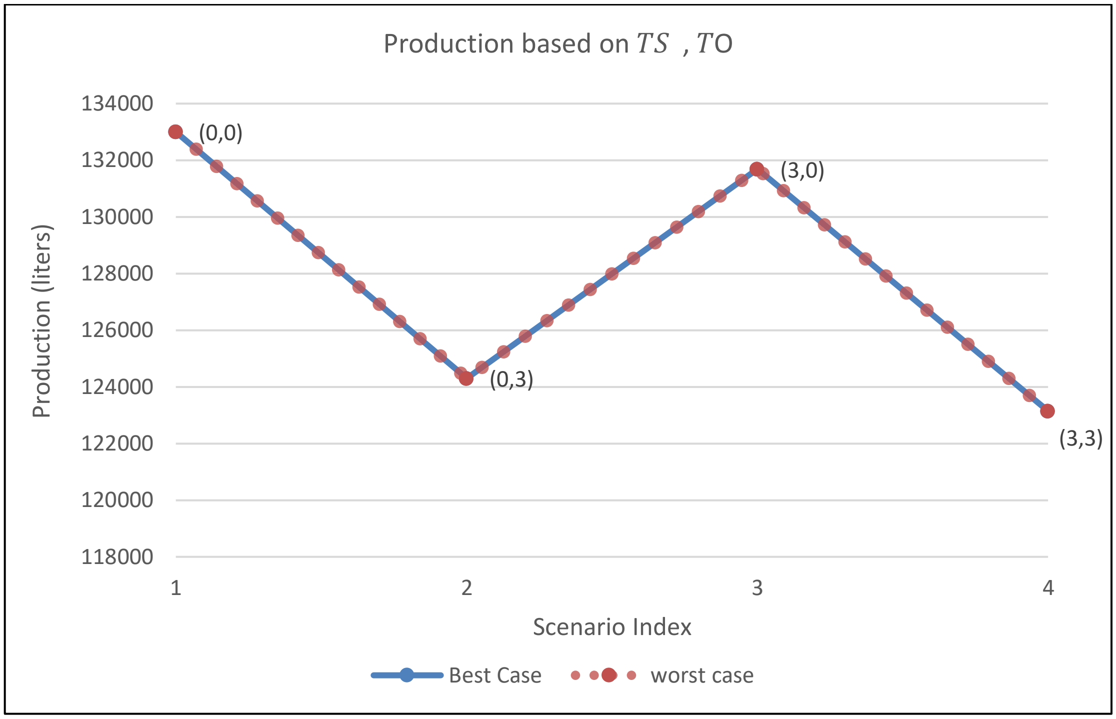
Figure 3 shows that yield variability (TO) has a bigger impact on production than procurement costs (TS).

- When costs are low and yields are stable ($TS = best, TO = best$), production reaches its peak at 133,000 liters. But when yields drop ($TS = best, TO = worst$), production falls to 124,299.1 liters, showing how directly yield availability affects output.

- On the other hand, when procurement costs increase but yields remain stable ($TS = worst, TO = best$), production only decreases slightly to 131,683.2 liters, meaning higher costs have less impact on production feasibility. In the worst-case scenario ($TS = worst, TO = worst$), production drops further to 123,148.1 liters, confirming that yield variability plays the biggest role in determining output.

These results highlight that while controlling procurement costs is important, managing yield fluctuations is even more critical to maintaining stable production levels.

Figure 3 : Impact of olive cost and yield variability on optimal production volume



7.4. Uncertainty protection mechanism

This section presents the protection variables calculated by the robust optimization model to help APIA manage uncertainty in its supply chain. These values represent the additional quantities or capacities needed to protect the system against worst-case scenarios, where procurement costs, yields, transportation, warehousing, and processing conditions are at their most unfavorable.

The data was generated by solving the primal robust optimization model in LINGO, using the worst-case scenario uncertainty set defined in Chapter 6. The model follows the Bertsimas and Sim (2002) approach, which introduces protection variables as part of the solution. These variables indicate the extra measures the system needs to take whether in procurement, transportation, or operations to maintain stability when facing uncertainty.

As shown in Figure 4, Supplier 3 requires the largest protection. Its procurement adjustment reaches 1,442,130 kg, which is significantly higher than Supplier 1 (283,240.7 kg) and Supplier 2 (246,296.3 kg). This means the model anticipates the need for up to 1.4 million additional kilograms from Supplier 3 to hedge against the worst-case risks in procurement costs and yields.

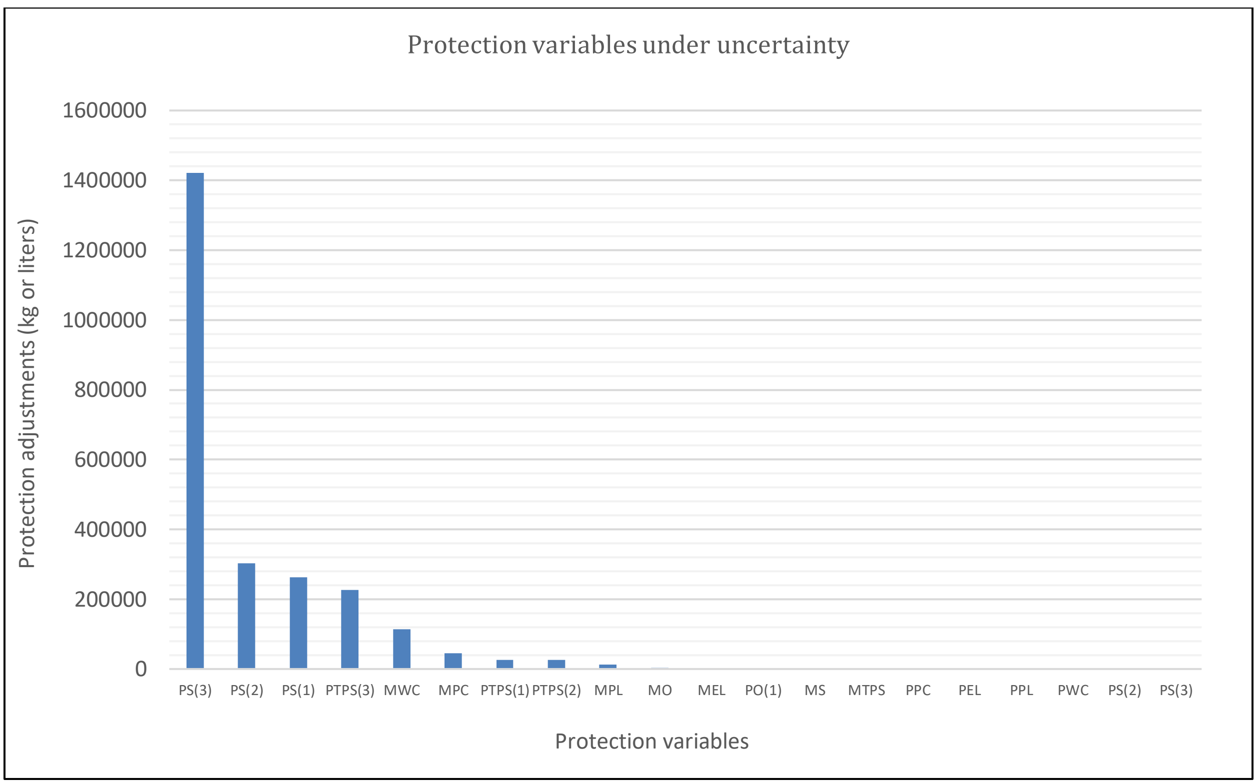
The reason Supplier 3 is the most vulnerable is due to its central role in the procurement strategy. As explained in Section 7.2 and shown in Figure 2, the model relies heavily on Supplier 3, especially when procurement costs are favorable. This heavy reliance increases risk exposure when uncertainties occur, requiring larger protective measures.

The same trend appears in transportation. Supplier 3 needs an additional 230,740.7 kg of transportation capacity to manage potential disruptions, compared to 82,120.3 kg for Supplier 1 and 91,560.4 kg for Supplier 2. This highlights the logistical risks tied to Supplier 3's dominant role in the supply chain.

The model also recommends extra capacity for warehousing and processing to absorb fluctuations in supply and production. It identifies the need for an additional 115,370 kg of warehousing space and 43,101 liters of processing capacity under worst-case conditions.

These findings confirm that Supplier 3 presents the highest risk, not just in procurement but across the supply chain. Its high protection values underline the importance of reducing dependency on a single supplier. To improve resilience, APIA should consider diversifying its suppliers and strengthening transportation, storage, and processing capabilities.

Figure 4 : Protection variables under maximum uncertainty



7.5. Comparative analysis: Evaluating Model 1 (Robust Optimization) against Model 2 (Ellipsoid Approach)

The previous section focused on evaluating the performance of Model 1, based on the framework of Bertsimas and Sim (2002). This section extends the analysis by comparing its effectiveness against an alternative Model 2, assessing the trade-offs between strict robustness and probabilistic feasibility in decision-making under uncertainty.

Parameters for comparative study

- Bertsimas & Sim model ($T = 3$): This approach represents a fully robust scenario, where up to three uncertain coefficients can simultaneously reach their worst-case deviations. By incorporating uncertainty directly into constraints, the model ensures strict feasibility, leading to conservative but highly reliable decisions under extreme conditions.

- Ellipsoid model ($\Sigma = 0.14, \Sigma = 0.45$) : The Ellipsoid Model provides probabilistic feasibility, meaning that constraints hold with a probability dependent on Σ. The probability of feasibility is computed as:

$$P_{\text{feasibility}} = e^{-\frac{\Sigma^2}{2}}$$

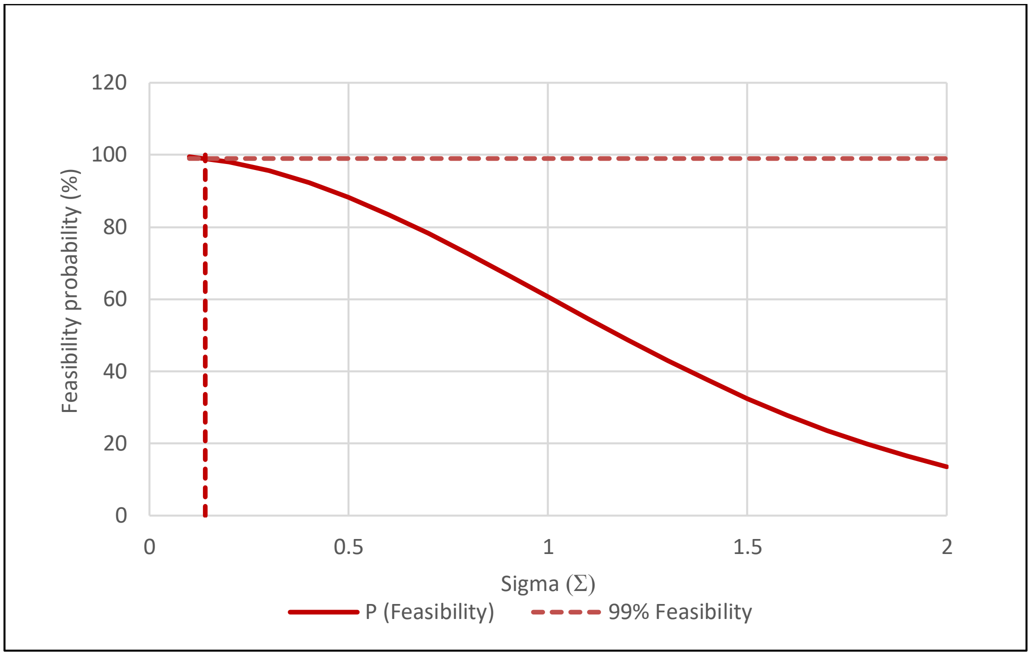
  The impact of Σ on feasibility and financial performance varies:

- For $\Sigma = 0.14$, feasibility is approximately 99%, offering a moderate balance between robustness and flexibility.

- For $\Sigma = 0.45$, feasibility decreases significantly, but profitability increases, reflecting a riskier decision-making approach. This setting prioritizes expected performance over strict feasibility, making it more suitable for less risk-averse environments.

To further illustrate the effect of Σ on feasibility, Figure 5 provides a visualization of how increasing Σ leads to a gradual decline in feasibility probability. This figure was developed using data generated from the Ellipsoid model simulations conducted in this study.

Figure 5 : Effect of sigma on feasibility in the Ellipsoid model



Performance comparison

Table 7 presents a comparative evaluation of both models based on key performance metrics, including profitability, feasibility guarantees, and robustness control mechanisms.

Table 7 : Comparative analysis of the Bertsimas & Sim model and the Ellipsoid model

Model	Profit (DHS)	Feasibility Guarantee	Robustness Control
Bertsimas & Sim ($T = 3$)	1,360,074	Guaranteed (Hard constraints)	Up to 3 worst-case deviations
Ellipsoid Model ($\Sigma = 0.14$)	1,019,595	99% (Probabilistic feasibility)	Handles correlated uncertainties
Ellipsoid Model ($\Sigma = 0.45$)	2,082,928	90% Probabilistic feasibility	Handles correlated uncertainties

This comparison highlights a key trade-off between strict feasibility and probabilistic feasibility. The Bertsimas & Sim model ($T = 3$) guarantees feasibility even in the worst-case scenario, making it a highly reliable choice for high-risk environments where constraint violations are unacceptable. However, this strict robustness can limit financial performance by prioritizing stability over flexibility.

On the other hand, the Ellipsoid Model, particularly at higher Σ values, introduces greater profitability potential by leveraging correlated uncertainties, but this comes at the expense of reduced feasibility guarantees. As Σ increases, the risk of constraint violations rises, making it a more adaptable but riskier approach.

In our case, where ensuring feasibility is a top priority, the Bertsimas & Sim model is the preferred choice, as it provides the necessary robustness to withstand extreme uncertainty while maintaining operational stability. While the Ellipsoid Model may offer higher expected returns, its probabilistic feasibility introduces a level of risk that is not acceptable in this context. By prioritizing reliability over flexibility, the Bertsimas & Sim model ensures that decisions remain stable under worst-case conditions, making it the best fit for APIA's strategic needs.

Chapter 8: Conclusion, recommendation and research perspectives

8.1. Conclusion

This study demonstrates that the Bertsimas & Sim (2002) robust optimization framework is well-suited for APIA's olive oil supply chain, offering strong feasibility guarantees under uncertainty. While the Ellipsoid Model provides a probabilistic alternative, the primary results emphasize the effectiveness of the primal model in managing procurement decisions, yield fluctuations, and operational constraints with greater control and reliability.

This thesis has presented a robust optimization framework customized for the intrinsic uncertainties within the olive oil production supply chain. The model integrates procurement, production, operational costs, and revenue considerations into one model to yield feasible and robust decisions. With the use of extreme T values for the robustness parameters, this framework allows decision-makers to study the critical trade-offs between risk mitigation and profit maximization. This offers a structured approach to navigate the unpredictable nature of supply chains in the industry.

The decision to adopt robust optimization, rather than statistical methods, reflects the practical constraints faced by the industry, such as limited and unreliable historical data. This approach prioritizes bounded uncertainty, ensuring feasibility across worst-case scenarios without requiring detailed probabilistic assumptions. This focus on practical applicability aligns with the operational realities of olive oil production, where seasonal variability, supply chain disruptions, and market dynamics necessitate adaptive planning.

8.2. Recommendations for APIA

To enhance APIA's supply chain resilience and long-term profitability, a strategic and data-driven approach is necessary to mitigate key risks related to procurement costs, yield variability, and transportation uncertainties. The findings highlight that procurement costs have the most significant impact on profitability, with supplier 3 being the most cost-effective yet highly vulnerable to fluctuations. To reduce risk exposure, APIA should diversify its procurement strategy by securing flexible contracts, exploring alternative suppliers, and negotiating price-lock agreements during low-cost periods to stabilize procurement expenses.

Beyond procurement, yield variability presents a direct threat to production stability, making supply continuity a critical priority. APIA can mitigate this risk by strengthening supplier relationships, investing in predictive yield analytics, and maintaining strategic inventory buffers to prevent supply disruptions during low-yield seasons. Collaboration with farmers on sustainable farming practices and technological improvements can further enhance supply consistency.

Transportation risks also pose significant challenges, particularly for supplier 3, where fluctuating costs add uncertainty to procurement operations. To minimize disruptions, APIA should assess alternative transport providers, negotiate more flexible logistics agreements, and implement real-time tracking systems to improve supply chain visibility. These measures will help anticipate transportation risks and adjust procurement schedules accordingly.

Additionally, fluctuations in warehousing and processing costs require efficiency improvements to maintain stable operations. Implementing a just-in-time (JIT) inventory strategy, refining processing workflows, and conducting regular cost-benefit analyses will help APIA balance cost efficiency with supply chain flexibility.

To build a robust and optimized supply chain, APIA must integrate scenario-based risk management strategies, data-driven decision-making, and proactive supplier and logistics coordination. Strengthening collaboration with suppliers through long-term agreements, risk-sharing mechanisms, and sustainability-focused partnerships will improve resilience against external market shifts. By adopting these targeted strategies, APIA can reduce financial risk, enhance operational efficiency, and establish a more sustainable and competitive position in the olive oil industry.

While uncertainty is a constant in the olive oil industry, APIA can transform it into a competitive advantage through cost-sensitive procurement, strategic supplier diversification, adaptive yield management, and proactive risk mitigation. A dynamic, data-driven approach will enhance APIA's profitability, supply chain stability, and long-term resilience in a rapidly changing market.

8.3. Research perspectives

The robust optimization framework developed in this thesis lays a strong foundation for tackling the uncertainties and complexities of the olive oil production supply chain. However, there are still many opportunities for future research to refine its accuracy, expand its scope, and make it more applicable to other industries facing similar challenges.

One major limitation of the current model is its reliance on historical data, which may be incomplete or unreliable. Future studies could address this by incorporating more advanced data sources, such as real-time data from Internet of Things (IoT) devices and sensors. These technologies could continuously monitor key variables, allowing for a more precise and adaptable optimization process. Additionally, integrating machine learning and artificial intelligence could help the model predict uncertainties related to supplier behavior, market fluctuations, and yield variations. By adjusting its parameters in response to real-time inputs, the model could become more responsive to the dynamic nature of the olive oil supply chain.

While robust optimization is effective in managing uncertainty without requiring detailed probabilistic assumptions, a hybrid approach that combines it with probabilistic models could provide even greater flexibility. Future research could explore integrating robust optimization with statistical methods like Monte Carlo simulations, which would allow the model to account for different levels of uncertainty. This would enable decision-makers to maintain stability under worst-case scenarios while also leveraging probabilistic forecasts when reliable data is available, striking a balance between risk management and strategic planning.

Currently, the model primarily focuses on balancing risk management with profit maximization. However, decision-makers in the olive oil industry and other sectors often must consider multiple, sometimes conflicting priorities, such as environmental sustainability, social responsibility, and long-term supply chain resilience. Future research could explore multiobjective optimization, which would allow the model to optimize financial performance while also incorporating sustainability goals and broader strategic considerations. Expanding the framework in this way would provide businesses with more comprehensive decision-making tools that integrate economic, environmental, and social factors.

Although this thesis focuses on procurement and production, robust optimization techniques could also be applied to other key areas of the supply chain, such as inventory management, demand forecasting, and logistics. A more holistic approach to supply chain optimization would help improve coordination across these interconnected areas. For example, incorporating demand forecasting could enhance responsiveness to market fluctuations, while improved inventory management strategies could prevent shortages or overstocking. Expanding the model in this way would create a more complete and practical solution for managing uncertainty across the supply chain.

Collaboration among supply chain stakeholders is another promising area for future research. Developing models that incorporate risk-sharing mechanisms and information-sharing strategies between suppliers, distributors, and retailers could significantly improve supply chain resilience. Technologies like blockchain could enable secure and transparent data exchanges, fostering greater trust and coordination among stakeholders. Exploring collaborative strategies for mitigating risk could provide valuable insights into strengthening supply chain stability in unpredictable environments.

A crucial next step in advancing this research is testing the model in real-world settings. While the theoretical framework outlined in this thesis provides a strong foundation, its practical effectiveness needs to be validated through pilot projects in actual olive oil production or similar industries. Implementing the model in an operational environment would provide valuable insights into its strengths and areas for improvement, helping refine it based on real-world feedback. Additionally, developing user-friendly decision-support tools could make advanced optimization techniques more accessible to industry professionals, facilitating their practical adoption.

In conclusion, while the robust optimization framework presented in this thesis offers a promising approach to managing uncertainty in the olive oil supply chain, there is still plenty of room for further development. By incorporating real-time data sources, extending optimization to other areas of the supply chain, and conducting real-world testing, future research can enhance the model's accuracy, adaptability, and practical value. The continued evolution of this framework holds great potential, not just for the olive oil industry, but also for other sectors dealing with similar challenges related to variability, risk, and complexity.

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Appendix

Table A1: The LINGO code (Bertsimas & Sim Model)

```
SETS:
Suppliers: Xs, As, Bs, Ys, Ps, Ao, Bo, Yo, Po, Atps, Btps, Ptps;
ENDSETS
DATA:
Suppliers= 1 2 3;
As= 13.00 14.50 12.29;
Bs=2.00 2.30 2.50;
Ao=0.18 0.17 0.19;
Bo=0.03 0.02 0.02;
Ar=130.05;
Br=30.00;
Atps = 1.79 1.70 1.70;
Btps = 0.20 0.20 0.40;
Apc = 3.73;
Bpc = 0.35;
Ael = 2.29;
Bel = 0.01;
Apl = 1.34;
Bpl = 0.09;
Awc = 2.05;
Bwc = 0.20;
WarehouseProdCapacity = 700000;
MaxOilProcessingCapacity = 300000;
Ts = 3;
To = 3;
Tr = 0;
Ttps = 3;
Tpc = 1;
Tel = 1;
Tpl = 1;
Twc = 1;
ENDDATA
! Objective function ;
MAX = Rev - C;
C = C1+C2+C3+C4+C5+C6;
! Purchasing constraint;
@SUM(Suppliers(i): As(i)*Xs(i) + Ps(i)) + Ms*Ts - C1 < 0;
@FOR(Suppliers(i): Ms + Ps(i) - Bs(i)*Ys(i) > 0);
@FOR(Suppliers(i): Xs(i) - Ys(i) < 0);
! Yield constraint;
@SUM(Suppliers(i): Ao(i)*Xs(i) - Po(i)) - Mo*To - Xo > 0;
@FOR(Suppliers(i): Mo + Po(i) - Bo(i)*Yo(i) > 0);
! other costs ;
@SUM(Suppliers(i): Atps(i) * Xs(i) + Ptps(i)) + Mtps * Ttps - C2 < 0;
@FOR(Suppliers(i): Mtps + Ptps(i) - Btps(i) * Ys(i) > 0);
Apc * Xo + Ppc + Mpc * Tpc - C3 < 0;
@FOR(Suppliers(i): Mpc + Ppc - Bpc * Yo(i) > 0);
Ael * Xo + Pel + Mel * Tel - C4 < 0;
@FOR(Suppliers(i): Mel + Pel - Bel * Yo(i) > 0);
Apl * Xo + Ppl + Mpl * Tpl - C5 < 0;
@FOR(Suppliers(i): Mpl + Ppl - Bpl * Yo(i) > 0);
@SUM(Suppliers(i): Awc * Xs(i) + Pwc) + Mwc * Twc - C6 < 0;
@FOR(Suppliers(i): Mwc + Pwc - Bwc * Ys(i) > 0);
@FOR(Suppliers(i): Xo - Ys(i) < 0);
Ar * Xo - Pr - Mr * Tr - Rev > 0;
@FOR(Suppliers(i): Mr + Pr - Br * Yo(i) > 0);
@FOR(Suppliers(i): Xo - Yo(i) < 0);
! Capacity;
@sum(Suppliers(i): Xs(i)) <= WarehouseProdCapacity;
Xo <= MaxOilProcessingCapacity;
```

Table A2: Formulation (Bertsimas & Sim Model)

```
MODEL:
[_1] MAX= REV - C;
[_2] C - C1 - C2 - C3 - C4 - C5 - C6 = 0;
[_3] - C1 + 3 * MS + 13 * XS_1 + PS_1 + 14.5 * XS_2 + PS_2 + 12.29 * XS_3 + PS_3 <=0;
[_4] MS - 2 * YS_1 + PS_1 >= 0;
[_5] MS - 2.3 * YS_2 + PS_2 >= 0;
[_6] MS - 2.5 * YS_3 + PS_3 >= 0;
[_7] XS_1 - YS_1 <= 0;
[_8] XS_2 - YS_2 <= 0;
[_9] XS_3 - YS_3 <= 0;
[_10] - 3 * MO - XO + 0.18 * XS_1 - PO_1 + 0.17 * XS_2 - PO_2 + 0.19 * XS_3 - PO_3>= 0;
[_11] MO - 0.03 * YO_1 + PO_1 >= 0;
[_12] MO - 0.02 * YO_2 + PO_2 >= 0;
[_13] MO - 0.02 * YO_3 + PO_3 >= 0;
[_14] - C2 + 3 * MTPS + 1.79 * XS_1 + PTPS_1 + 1.7 * XS_2 + PTPS_2 + 1.7 * XS_3 +PTPS_3 <= 0;
[_15] MTPS - 0.2 * YS_1 + PTPS_1 >= 0;
[_16] MTPS - 0.2 * YS_2 + PTPS_2 >= 0;
[_17] MTPS - 0.4 * YS_3 + PTPS_3 >= 0;
[_18] - C3 + 3.73 * XO + PPC + MPC <= 0;
[_19] PPC + MPC - 0.35 * YO_1 >= 0;
[_20] PPC + MPC - 0.35 * YO_2 >= 0;
[_21] PPC + MPC - 0.35 * YO_3 >= 0;
[_22] - C4 + 2.29 * XO + PEL + MEL <= 0;
[_23] PEL + MEL - 0.01 * YO_1 >= 0;
[_24] PEL + MEL - 0.01 * YO_2 >= 0;
[_25] PEL + MEL - 0.01 * YO_3 >= 0;
[_26] - C5 + 1.34 * XO + PPL + MPL <= 0;
[_27] PPL + MPL - 0.09 * YO_1 >= 0;
[_28] PPL + MPL - 0.09 * YO_2 >= 0;
[_29] PPL + MPL - 0.09 * YO_3 >= 0;
[_30] - C6 + 3 * PWC + MWC + 2.05 * XS_1 + 2.05 * XS_2 + 2.05 * XS_3 <= 0;
[_31] PWC + MWC - 0.2 * YS_1 >= 0;
[_32] PWC + MWC - 0.2 * YS_2 >= 0;
[_33] PWC + MWC - 0.2 * YS_3 >= 0;
[_34] XO - YS_1 <= 0;
[_35] XO - YS_2 <= 0;
[_36] XO - YS_3 <= 0;
[_37] - REV + 130.05 * XO - PR >= 0;
[_38] PR + MR - 30 * YO_1 >= 0;
[_39] PR + MR - 30 * YO_2 >= 0;
[_40] PR + MR - 30 * YO_3 >= 0;
[_41] XO - YO_1 <= 0;
[_42] XO - YO_2 <= 0;
[_43] XO - YO_3 <= 0;
[_44] XS_1 + XS_2 + XS_3 <= 700000;
[_45] XO <= 300000;
END
```

Table A3: Best-case operational costs (Bertsimas & Sim Model)

Variable	TS=best TO=best	TS=best TO=worst	TS=worst TO=best	TS=worst TO=worst
Objective value	5089770	4022252	2835832	1810861
AR	130.05	130.05	130.05	130.05
BR	30.00	30.00	30.00	30.00
APC	3.73	3.73	3.73	3.73
BPC	0.35	0.35	0.35	0.35
AEL	2.29	2.29	2.29	2.29
BEL	0.01	0.01	0.01	0.01
APL	1.34	1.34	1.34	1.34
BPL	0.09	0.09	0.09	0.09
AWC	2.05	2.05	2.05	2.05
BWC	0.20	0.20	0.20	0.20
WAREHOUSEPRODCAPACITY	700000.00	700000.00	700000.00	700000.00
MAXOILPROCESSINGCAPACITY	300000.00	300000.00	300000.00	300000.00
TS	0.00	0.00	3.00	3.00
TO	0.00	3.00	0.00	3.00
TR	0.00	0.00	0.00	0.00
TTPS	0.00	0.00	0.00	0.00
TPC	0.00	0.00	0.00	0.00
TEL	0.00	0.00	0.00	0.00
TPL	0.00	0.00	0.00	0.00
TWC	0.00	0.00	0.00	0.00
REV	17296650.00	16165090.00	17125400.00	16015420.00
C	12206880.00	12142840.00	14289560.00	14204560.00
C1	8603000.00	8603000.00	10683520.00	10662100.00
C2	1190000.00	1190000.00	1201851.00	1201083.00
C3	496090.00	463635.50	491178.20	459342.60
C4	304570.00	284644.90	301554.50	282009.30
C5	178220.00	166560.70	176455.40	165018.50
C6	1435000.00	1435000.00	1435000.00	1435000.00
MS	1750000.00	1750000.00	0.00	0.00
MO	3990.00	2485.98	3950.50	2462.96
XO	133000.00	124299.10	131683.20	123148.10
MTPS	280000.00	280000.00	227326.70	230740.70
PPC	0.00	0.00	0.00	0.00
MPC	46550.00	43504.67	46089.11	43101.85
PEL	0.00	0.00	0.00	0.00
MEL	1330.00	1242.99	1316.83	1231.48
PPL	0.00	0.00	0.00	0.00
MPL	11970.00	11186.92	11851.49	11083.33
PWC	0.00	0.00	0.00	0.00
MWC	140000.00	140000.00	113663.40	115370.40
PR	0.00	0.00	0.00	0.00
MR	3990000.00	3728972.00	3950495.00	3694444.00
XS(1)	0.00	0.00	131683.20	123148.10
XS(2)	0.00	0.00	0.00	0.00
XS(3)	700000.00	700000.00	568316.80	576851.90
AS(1)	13.00	13.00	13.00	13.00
AS(2)	14.50	14.50	14.50	14.50
AS(3)	12.29	12.29	12.29	12.29
BS(1)	2.00	2.00	2.00	2.00
BS(2)	2.30	2.30	2.30	2.30
BS(3)	2.50	2.50	2.50	2.50
YS(1)	133000.00	124299.10	131683.20	123148.10

Variable	TS=best TO=best	TS=best TO=worst	TS=worst TO=best	TS=worst TO=worst
YS(2)	133000.00	124299.10	131683.20	123148.10
YS(3)	700000.00	700000.00	568316.80	576851.90
PS(1)	0.00	0.00	263366.30	246296.30
PS(2)	0.00	0.00	302871.30	283240.70
PS(3)	0.00	0.00	1420792.00	1442130.00
AO(1)	0.18	0.18	0.18	0.18
AO(2)	0.17	0.17	0.17	0.17
AO(3)	0.19	0.19	0.19	0.19
BO(1)	0.03	0.03	0.03	0.03
BO(2)	0.02	0.02	0.02	0.02
BO(3)	0.02	0.02	0.02	0.02
YO(1)	133000.00	124299.10	131683.20	123148.10
YO(2)	133000.00	124299.10	131683.20	123148.10
YO(3)	133000.00	124299.10	131683.20	123148.10
PO(1)	0.00	1242.99	0.00	1231.48
PO(2)	0.00	0.00	0.00	0.00
PO(3)	0.00	0.00	0.00	0.00
ATPS(1)	1.79	1.79	1.79	1.79
ATPS(2)	1.70	1.70	1.70	1.70
ATPS(3)	1.70	1.70	1.70	1.70
BTPS(1)	0.20	0.20	0.20	0.20
BTPS(2)	0.20	0.20	0.20	0.20
BTPS(3)	0.40	0.40	0.40	0.40
PTPS(1)	0.00	0.00	0.00	0.00
PTPS(2)	0.00	0.00	0.00	0.00
PTPS(3)	0.00	0.00	0.00	0.00

Table A4: Worst-case operational costs (Bertsimas & Sim Model)

Variable	**TS=best TO=best**	**TS=best TO=worst**	**TS=worst TO=best**	**TS=worst TO=worst**
Objective value	4556720	3496598	2382911	1360074
AR	130.05	130.05	130.05	130.05
BR	30.00	30.00	30.00	30.00
APC	3.73	3.73	3.73	3.73
BPC	0.35	0.35	0.35	0.35
AEL	2.29	2.29	2.29	2.29
BEL	0.01	0.01	0.01	0.01
APL	1.34	1.34	1.34	1.34
BPL	0.09	0.09	0.09	0.09
AWC	2.05	2.05	2.05	2.05
BWC	0.20	0.20	0.20	0.20
WAREHOUSEPRODCAPACITY	700000.00	700000.00	700000.00	700000.00
MAXOILPROCESSINGCAPACITY	300000.00	300000.00	300000.00	300000.00
TS	0.00	0.00	3.00	3.00
TO	0.00	3.00	0.00	3.00
TR	0.00	0.00	0.00	0.00
TTPS	3.00	3.00	3.00	3.00
TPC	1.00	1.00	1.00	1.00
TEL	1.00	1.00	1.00	1.00
TPL	1.00	1.00	1.00	1.00
TWC	1.00	1.00	1.00	1.00
REV	17296650.00	16165090.00	17125400.00	16015420.00
C	12739930.00	12668500.00	14742490.00	14655340.00
C1	8603000.00	8603000.00	10683520.00	10662100.00
C2	1523200.00	1519720.00	1481851.00	1481083.00
C3	542640.00	507140.20	537267.30	502444.40
C4	305900.00	285887.90	302871.30	283240.70
C5	190190.00	177747.70	188306.90	176101.90
C6	1575000.00	1575000.00	1548663.00	1550370.00
MS	1750000.00	1750000.00	0.00	0.00
MO	3990.00	2485.98	3950.50	2462.96
XO	133000.00	124299.10	131683.20	123148.10
MTPS	0.00	0.00	0.00	0.00
PPC	0.00	0.00	0.00	0.00
MPC	46550.00	43504.67	46089.11	43101.85
PEL	0.00	0.00	0.00	0.00
MEL	1330.00	1242.99	1316.83	1231.48
PPL	0.00	0.00	0.00	0.00
MPL	11970.00	11186.92	11851.49	11083.33
PWC	0.00	0.00	0.00	0.00
MWC	140000.00	140000.00	113663.40	115370.40
PR	0.00	0.00	0.00	0.00
MR	3990000.00	3728972.00	3950495.00	3694444.00
XS(1)	0.00	0.00	131683.20	123148.10
XS(2)	0.00	0.00	0.00	0.00
XS(3)	700000.00	700000.00	568316.80	576851.90
AS(1)	13.00	13.00	13.00	13.00
AS(2)	14.50	14.50	14.50	14.50
AS(3)	12.29	12.29	12.29	12.29
BS(1)	2.00	2.00	2.00	2.00
BS(2)	2.30	2.30	2.30	2.30
BS(3)	2.50	2.50	2.50	2.50

Variable	TS=best TO=best	TS=best TO=worst	TS=worst TO=best	TS=worst TO=worst
YS(1)	133000.00	124299.10	131683.20	123148.10
YS(2)	133000.00	124299.10	131683.20	123148.10
YS(3)	700000.00	700000.00	568316.80	576851.90
PS(1)	0.00	0.00	263366.30	246296.30
PS(2)	0.00	0.00	302871.30	283240.70
PS(3)	0.00	0.00	1420792.00	1442130.00
AO(1)	0.18	0.18	0.18	0.18
AO(2)	0.17	0.17	0.17	0.17
AO(3)	0.19	0.19	0.19	0.19
BO(1)	0.03	0.03	0.03	0.03
BO(2)	0.02	0.02	0.02	0.02
BO(3)	0.02	0.02	0.02	0.02
YO(1)	133000.00	124299.10	131683.20	123148.10
YO(2)	133000.00	124299.10	131683.20	123148.10
YO(3)	133000.00	124299.10	131683.20	123148.10
PO(1)	0.00	1242.99	0.00	1231.48
PO(2)	0.00	0.00	0.00	0.00
PO(3)	0.00	0.00	0.00	0.00
ATPS(1)	1.79	1.79	1.79	1.79
ATPS(2)	1.70	1.70	1.70	1.70
ATPS(3)	1.70	1.70	1.70	1.70
BTPS(1)	0.20	0.20	0.20	0.20
BTPS(2)	0.20	0.20	0.20	0.20
BTPS(3)	0.40	0.40	0.40	0.40
PTPS(1)	26600.00	24859.81	26336.63	24629.63
PTPS(2)	26600.00	24859.81	26336.63	24629.63
PTPS(3)	280000.00	280000.00	227326.70	230740.70

Table A5: The LINGO code (Ellipsoid model)

```
SETS:
Suppliers: Xs, As, Bs, Ys, Ao, Bo, Yo, Atps, Btps, Zs, Zo, Ztps;
ENDSETS
DATA:
Suppliers= 1 2 3;
As= 13.00 14.50 12.29;
Bs=2.00 2.30 2.50;
Ao=0.18 0.17 0.19;
Bo=0.03 0.02 0.02;
Ar=130.05;
Br=30.00;
Atps = 1.79 1.70 1.70;
Btps = 0.20 0.20 0.40;
Apc = 3.73;
Bpc = 0.35;
Ael = 2.29;
Bel = 0.01;
Apl = 1.34;
Bpl = 0.09;
Awc = 2.05;
Bwc = 0.20;
Sigma=0.14;
WarehouseProdCapacity = 700000;
MaxOilProcessingCapacity = 300000;
ENDDATA
! Objective function ;
MAX = Rev - C;
C = C1+C2+C3+C4+C5+C6;
! Purchasing constraint;
@SUM(Suppliers(i): As(i)*Xs(i) + Bs(i)*Ys(i)) + Sigma * ELPs - C1 < 0;
ELPs * ELPs = @SUM(Suppliers(i): Bs(i) * Bs(i) * Zs(i) * Zs(i));
@FOR(Suppliers(i): Xs(i) - Zs(i) - Ys(i) < 0);
! Yield constraint;
@SUM(Suppliers(i): Ao(i)*Xs(i) - Bo(i)*Yo(i)) - Sigma * ELPo - Xo > 0;
ELPo * ELPo = @SUM(Suppliers(i): Bo(i) * Bo(i) * Zo(i) * Zo(i));
@FOR(Suppliers(i): Xo- Zo(i) - Yo(i) < 0);
! other costs ;
@SUM(Suppliers(i): Atps(i) * Xs(i)+ Btps(i)* Ys(i)) + Sigma * ELPtps - C2 < 0;
ELPtps * ELPtps = @SUM(Suppliers(i): Btps(i) * Btps(i) * Ztps(i)* Ztps(i));
@FOR(Suppliers(i): Xs(i) - Ztps(i) - Ytps < 0);
@SUM(Suppliers(i): Apc * Xo+ Bpc* Yo(i)) + Sigma * ELPpc - C3 < 0;
ELPpc * ELPpc = Bpc * Bpc * Zpc * Zpc;
@FOR(Suppliers(i): Xo - Zpc - Ypc < 0);
@SUM(Suppliers(i): Ael * Xo + Bel* Yo(i)) + Sigma * ELPel - C4 < 0;
ELPel * ELPel = Bel * Bel * Zel * Zel;
@FOR(Suppliers(i): Xo- Zel - Yel < 0);
@SUM(Suppliers(i): Apl * Xo+ Bpl* Yo(i)) + Sigma * ELPpl - C5 < 0;
ELPpl * ELPpl = Bpl * Bpl * Zpl * Zpl;
@FOR(Suppliers(i): Xo - Zpl - Ypl < 0);
@SUM(Suppliers(i): Awc * Xs(i)) + Sigma * ELPwc - C6 < 0;
ELPwc * ELPwc = Bwc * Bwc * Zwc * Zwc;
@FOR(Suppliers(i): Xs(i) - Zwc - Ywc < 0);
! Final production risk consideration;
Ar * Xo - Sigma * ELPr - Rev > 0;
ELPr * ELPr = Br * Br * Zr * Zr;
Xo - Zr - Yr < 0;
! Capacity;
@SUM(Suppliers(i): Xs(i)) <= WarehouseProdCapacity;
Xo<=MaxOilProcessingCapacity;
! Probability factor;
Probability = @EXP(-Sigma * Sigma / 2);
```

Table A6: Formulation (Ellipsoid model)

```
[_1] MAX= REV - C;
[_2] C - C1 - C2 - C3 - C4 - C5 - C6 = 0;
[_3] - C1 + 0.14 * ELPS + 13 * XS_1 + 2 * YS_1 + 14.5 * XS_2 + 2.3 * YS_2 + 12.29 *XS_3 + 2.5 * YS_3 <= 0;
[_4] ELPS * ELPS = ( 2 * 2 * ZS_1 * ZS_1 + 2.3 * 2.3 * ZS_2 * ZS_2 + 2.5 * 2.5 *ZS_3 * ZS_3);
[_5] XS_1 - YS_1 - ZS_1 <= 0;
[_6] XS_2 - YS_2 - ZS_2 <= 0;
[_7] XS_3 - YS_3 - ZS_3 <= 0;
[_8] - 0.14 * ELPO - XO + 0.18 * XS_1 - 0.03 * YO_1 + 0.17 * XS_2 - 0.02 * YO_2 +0.19 * XS_3 - 0.02 * YO_3 >= 0;
[_9] ELPO * ELPO = ( 0.03 * 0.03 * ZO_1 * ZO_1 + 0.02 * 0.02 * ZO_2 * ZO_2 + 0.02* 0.02 * ZO_3 * ZO_3);
[_10] XO - YO_1 - ZO_1 <= 0;
[_11] XO - YO_2 - ZO_2 <= 0;
[_12] XO - YO_3 - ZO_3 <= 0;
[_13] - C2 + 0.14 * ELPTPS + 1.79 * XS_1 + 0.2 * YS_1 + 1.7 * XS_2 + 0.2 * YS_2 +1.7 * XS_3 + 0.4 * YS_3 <= 0;
[_14] ELPTPS * ELPTPS = ( 0.2 * 0.2 * ZTPS_1 * ZTPS_1 + 0.2 * 0.2 * ZTPS_2 *ZTPS_2 + 0.4 * 0.4 * ZTPS_3 * ZTPS_3);
[_15] - YTPS + XS_1 - ZTPS_1 <= 0;
[_16] - YTPS + XS_2 - ZTPS_2 <= 0;
[_17] - YTPS + XS_3 - ZTPS_3 <= 0;
[_18] - C3 + 11.19 * XO + 0.14 * ELPPC + 0.35 * YO_1 + 0.35 * YO_2 + 0.35 * YO_3 <=0;
[_19] ELPPC * ELPPC = 0.35 * 0.35 * ZPC * ZPC;
[_20] XO - ZPC - YPC <= 0;
[_21] XO - ZPC - YPC <= 0;
[_22] XO - ZPC - YPC <= 0;
[_23] - C4 + 6.87 * XO + 0.14 * ELPEL + 0.01 * YO_1 + 0.01 * YO_2 + 0.01 * YO_3 <=0;
[_24] ELPEL * ELPEL = 0.01 * 0.01 * ZEL * ZEL;
[_25] XO - ZEL - YEL <= 0;
[_26] XO - ZEL - YEL <= 0;
[_27] XO - ZEL - YEL <= 0;
[_28] - C5 + 4.02 * XO + 0.14 * ELPPL + 0.09 * YO_1 + 0.09 * YO_2 + 0.09 * YO_3 <=0;
[_29] ELPPL * ELPPL = 0.09 * 0.09 * ZPL * ZPL;
[_30] XO - ZPL - YPL <= 0;
[_31] XO - ZPL - YPL <= 0;
[_32] XO - ZPL - YPL <= 0;
[_33] - C6 + 0.14 * ELPWC + 2.05 * XS_1 + 2.05 * XS_2 + 2.05 * XS_3 <= 0;
[_34] ELPWC * ELPWC = 0.2 * 0.2 * ZWC * ZWC;
[_35] - ZWC - YWC + XS_1 <= 0;
[_36] - ZWC - YWC + XS_2 <= 0;
[_37] - ZWC - YWC + XS_3 <= 0;
[_38] - REV + 130.05 * XO - 0.14 * ELPR >= 0;
[_39] ELPR * ELPR = 30 * 30 * ZR * ZR;
[_40] XO - ZR - YR <= 0;
[_41] XS_1 + XS_2 + XS_3 <= 700000;
[_42] XO <= 300000;
END
```

Table A7: Results (Ellipsoid model)

Variable	Value (Σ = 0.14)	Value (Σ = 0.45)
Objective value	1019595	2082928
AR	130.05	130.05
BR	30.00	30.00
APC	3.73	3.73
BPC	0.35	0.35
AEL	2.29	2.29
BEL	0.01	0.01
APL	1.34	1.34
BPL	0.09	0.09
AWC	2.05	2.05
BWC	0.20	0.20
SIGMA	0.14	0.45
WAREHOUSEPRODCAPACITY	700000.00	700000.00
MAXOILPROCESSINGCAPACITY	300000.00	300000.00
REV	17197380.00	16981570.00
C	16177790.00	14898650.00
C1	10353000.00	9390500.00
C2	1470000.00	1190000.00
C3	1479728.00	1461160.00
C4	908466.00	897065.90
C5	531591.50	524920.60
C6	1435000.00	1435000.00
ELPS	0.00	1750000.00
ELPO	5452.26	5383.84
XO	132236.70	130577.30
ELPTPS	0.00	0.00
YTPS	700002.00	700000.00
ELPPC	0.00	0.00
ZPC	0.00	0.00
YPC	132236.70	130577.30
ELPEL	0.00	0.00
ZEL	0.00	0.00
YEL	132236.70	130577.30
ELPPL	0.00	0.00
ZPL	0.00	0.00
YPL	132236.70	130577.30
ELPWC	0.00	0.00
ZWC	0.00	0.00
YWC	700000.00	700000.70
ELPR	0.00	0.00
ZR	0.00	0.00
YR	132236.70	130577.30
PROBABILITY	0.99	0.90
XS(1)	0.00	0.00
XS(2)	0.00	0.00
XS(3)	700000.00	700000.00
AS(1)	13.00	13.00
AS(2)	14.50	14.50
AS(3)	12.29	12.29
BS(1)	2.00	2.00
BS(2)	2.30	2.30
BS(3)	2.50	2.50
YS(1)	0.00	0.00
YS(2)	0.00	0.00
YS(3)	700000.00	0.00
AO(1)	0.18	0.18
AO(2)	0.17	0.17
AO(3)	0.19	0.19

Variable	Value (Σ = 0.14)	Value (Σ = 0.45)
BO(1)	0.03	0.03
BO(2)	0.02	0.02
BO(3)	0.02	0.02
YO(1)	0.00	0.00
YO(2)	0.00	0.00
YO(3)	0.00	0.00
ATPS(1)	1.79	1.79
ATPS(2)	1.70	1.70
ATPS(3)	1.70	1.70
BTPS(1)	0.20	0.20
BTPS(2)	0.20	0.20
BTPS(3)	0.40	0.40
ZS(1)	0.00	0.00
ZS(2)	0.00	0.00
ZS(3)	0.00	700000.00
ZO(1)	132236.70	130577.30
ZO(2)	132236.70	130577.30
ZO(3)	132236.70	130577.30
ZTPS(1)	0.00	0.00
ZTPS(2)	0.00	0.00
ZTPS(3)	0.00	0.00