

Robust Integrated Tactical Planning in Hybrid Multi-Echelon Manufacturing Systems

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Abstract

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The rise of Industry 4.0 has intensified demand for hybrid production systems delivering both standard and highly customized products. Yet, current production planning models largely overlook modular-structured products in multi-echelon job-shop environments, particularly under uncertainty. This study develops a robust tactical planning framework that captures defect risks, variable processing times, and capacity uncertainty for customized items. A mixed-integer programming model based on the cardinality-constrained robust optimization approach is formulated to jointly optimize order acceptance, machine activation, procurement, production, and inventory decisions, with the objective of maximizing profit across the manufacturing network.

To mitigate the effects of volatile manufacturing conditions, the framework allows the flexible activation of additional machine capacity modules within a budgeted uncertainty set. Computational experiments are conducted using literature-driven data to evaluate the model's behavior under a range of uncertainty realizations. Sensitivity analysis confirms that the selected uncertain parameters have the largest impact on profit. Complexity analysis shows that the model remains computationally efficient and scalable across different problem sizes.

An out-of-sample performance evaluation demonstrates that the robust approach's machine capacity module activation decisions outperform conventional deterministic planning by enhancing resilience and profitability under uncertainty, while maintaining operational efficiency with only marginal additional machine use. Most of the profit improvement is driven by reduced delays and cancellations, indicating a higher service level. These results highlight the model's ability to safeguard profitability and operational stability in complex, customization-driven production systems.

Keywords: Tactical Planning, Hybrid Manufacturing, Robust Optimization, Customization

Preface

This thesis has been prepared in “Manuscript-based” format under the supervision of Dr. Masoumeh Kazemi Zanjani from the Department of Mechanical, Industrial and Aerospace Engineering at Concordia University. This research was financially supported by the Canada Graduate Scholarships–Master’s (CGS M) program (awarded to Anthony Montanaro) and the Discovery Program (awarded to Masoumeh Kazemi Zanjani) by Natural Sciences and Engineering Research Council of Canada (NSERC). The article included in this thesis were coauthored and reviewed prior to submission for publication by Dr. Masoumeh Kazemi Zanjani. The author of this thesis served as the primary researcher, developing mathematical models, programming solution methods, analyzing and validating results, and composing the initial drafts of the articles. The thesis comprises one article, entitled “Robust Integrated Tactical Planning in Hybrid Multi-Echelon Manufacturing Systems”, co-authored with Dr. Kazemi Zanjani and submitted to “International Journal of Production Research” in July 2025.

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Chapter 1

Introduction

Recent advances in Industry 4.0 cyber-physical systems ([Dolgui, Ivanov, & Sokolov, 2020](#); [Ivanov, Dolgui, & Sokolov, 2019](#); [Ivanov, Tsipoulanidis, & Schönberger, 2021](#)) coupled with modular product design, along with flexible, additive, and digital manufacturing technologies have created a paradigm shift in many industries, where mass production has been substituted by mass customization and mass personalization ([Altendorfer & Minner, 2014](#); [Katoozian & Zanjani, 2022](#); [Wang, Ma, Yang, & Wang, 2017](#); [Xu, Xu, & Li, 2018](#)). One of the biggest challenges in this transition for the advanced technology sector is the high cost of producing custom-made products with intricate design features that are typically ordered in small volumes. To overcome this challenge, several industries, such as the semiconductor sector, adopt a hybrid manufacturing mode to balance the high investment costs against the scale offered by mass production, while still allowing the firm to capture the more profitable customized market ([Burkacky, de Jong, & Dragon, 2022](#); [Longauer, Vasvári, & Hauck, 2024](#)). A hybrid approach is often adopted when a firm seeks to differentiate itself or expand into additional market segments by offering greater product variety. ([Peeters & van Ooijen, 2020](#)).

Flexible and reconfigurable production lines are among the key enablers of hybrid manufacturing systems ([Bortolini, Galizia, & Mora, 2018](#); [Gkournelos et al., 2019](#); [Prasad & Jayswal, 2018](#)). For instance, advanced electronics (PCBs, smartphones, IoT devices) are built on surface mount assembly lines that include pick-and-place machines that are highly modular and use exchangeable

heads and feeder modules so one machine can be reconfigured for different boards (Fuji Corporation, 2024). While such technologies support rapid changeovers and flexible automation/control, hybrid manufacturing is highly prone to uncertainties arising in various facets related to the customization of products (Elyasi et al., 2024; Hsieh & Lathifah, 2022; Khakdaman, Wong, Zohoori, Tiwari, & Merkert, 2015).

Standard items are typically treated as commodity products that the company produces regularly in a push-based system, driven by sales forecasts. Demand fluctuations for these items are generally managed through safety stock. In contrast, forecasting demand for customized products is considerably more challenging due to their unique and often unpredictable nature. It is impossible to know in advance what design features and customization levels any given customer might order, as well as the exact order size. The uncertain design features of such items result in stochastic machine processing times and production cycle times. For instance, a PCB with irregular geometry (e.g., circular, L-shaped, etc.) would require more complex tooling, manual handling during assembly, and custom jigs and fixtures. The extended processing time, in turn, reduces overall production capacity and may lead to delays in order fulfillment. In addition, when manufacturing custom-made products with complex design features for the first time, it is common for a portion of the produced items to fail final quality tests. This is primarily due to suboptimal design and manufacturing processes, limitations of tooling and machinery, material behavior, and the learning curve associated with operators. These initial failures can lead to lost sales opportunities and a reduced service level.

The manufacturing delays and potential lost sales due to quality failures are generally difficult to offset through inventory buildup, given the unpredictable and customized nature of the products. Furthermore, since both standard and custom-made products are manufactured on the same production line, allocating machine time between standard items—with deterministic processing times—and customized items—with variable cycle times—presents a significant challenge in hybrid manufacturing environments. Outsourcing the manufacturing of complex product modules to external suppliers can be a viable option in certain industries; however, it often comes with a significant increase in production costs. Hence, manufacturers must strike a balance between improving customer service levels through flexibility and customization, and controlling production costs. Consequently, the batch size of custom orders is a key factor in offsetting the elevated production

costs inherent to customized manufacturing.

In this study, we consider a company producing modular-structured products in the advanced technology sector (e.g., semiconductors, optical/photonic devices, etc.) in both standard and custom-made categories in facilities equipped with reconfigurable production lines. While the standard products (e.g., optical sensors) are produced based on demand forecasts, the custom items (e.g., shape-sensing fiber optic sensor) are manufactured only upon receiving confirmed orders, allowing the company to evaluate each request based on design complexity and batch size before acceptance. Our objective is to develop an integrated capacity and production planning framework that explicitly accounts for the uncertainty associated with custom orders and the manufacturing challenges they entail.

A handful of studies in the literature investigate the production planning problem in hybrid make-to-stock and make-to-order (MTS/MTO) systems, including [Rafei, Rabbani, and Alimardani \(2013\)](#), [Khakdaman et al. \(2015\)](#), [Beemsterboer, Land, and Teunter \(2016\)](#), [Beemsterboer, Land, Teunter, and Bokhorst \(2017\)](#), [Pereira, Oliveira, and Carravilla \(2022\)](#). However, the majority of these models either focus on capacity planning or lot-sizing decisions in flow shop settings and a deterministic environment to maximize the fill rate of MTS products and minimize the production time of MTO items. In other words, existing approaches rarely account for modular-structured items with customizable modules in their bill of materials (BOM), and often overlook the inherent uncertainties in manufacturing such customized products—such as highly variable processing times and an elevated risk of quality defects. Moreover, by assuming a fixed set of alternative machine sequences for MTO items, they overlook the potential to activate additional machines or flexible capacity modules as a means to hedge against production uncertainty.

Building upon the classic MTS/MTO production planning frameworks in the literature, our main contribution lies in the development of a cardinality-constrained robust optimization model ([Bertsimas & Sim, 2004](#)) that optimizes procurement/outsourcing, machine capacity activation, production, and inventory levels in the above-mentioned hybrid manufacturing setting. The model is formulated based on a generic BOM corresponding to standard products. To incorporate custom orders, a subset of products' modules and components are assumed to be customizable at different degrees. Depending on the design complexity of such components, their processing time on the set

of required machines is modeled as an uncertain interval. In addition, the uncertain quality defect rate of customized items is accounted for by inflating the actual order size with a multiplier based on the expected final test failure rate. To control the degree of conservatism in the model, a limit (budget) is imposed to cap both the number of custom items subjected to worst-case processing times and the variability in order size due to quality-induced rejections. The objective of the model is to maximize the profit from selling standard items while simultaneously minimizing lost and delayed volumes of custom orders—thereby maximizing customer service level—under worst-case realizations of budgeted uncertainty.

We further develop a Monte-Carlo simulation framework to evaluate the effectiveness of machine activation decisions determined by the proposed robust model in terms of the expected profit and lost/delayed sales under uncertain conditions. We conduct extensive numerical experiments to demonstrate how the proposed robust model outperforms a deterministic approach by ensuring adequate machine activation to hedge against defect risks and extended processing times when producing complex customized items.

This paper is structured as follows. [Chapter 2](#) provides an overview of the current literature and research gaps. [Chapter 3](#) provides a detailed overview of robust optimization. [Chapter 4](#) summarizes the problem description and its formulation. The numerical experiments are provided in [Chapter 5](#). Conclusions and future research directions are highlighted in [Chapter 6](#).

Chapter 2

Literature Review

Research on hybrid production systems has gained momentum over the past decade. [Peeters and van Ooijen \(2020\)](#) provides a taxonomic review of the research in this area; classifying papers based on the approaches explored to investigate the topic. Most studies on this topic focus on capacity and production planning for the manufacturing environments that simultaneously produce both MTS and MTO products. However, these studies typically assume that MTO items are selected from a predefined product catalog, implying that the manufacturing sequence and resource requirements are already known during the production planning stage.

Among pioneering studies, we can refer to [Tsubone, Ishikawa, and Yamamoto \(2002\)](#) where a hierarchical framework was developed for designing a hybrid system in which MTS/MTO items are produced together. At the higher planning level, the buffer capacity is set as a design variable for determining production capacity, and the rule for allocating the production capacity to MTS/MTO products is adopted as a design variable at a lower level. The goal is to minimize the unfilled rate of MTS orders and manufacturing time of MTO items. The study mainly applies to flow-shop settings by considering a fixed production sequence for MTS products and a set of *a priori* manufacturing paths for MTO items.

[Aghezzaf and Van Landeghem \(2002\)](#) studies integrated inventory and production planning in a two-stage hybrid flow-shop production system. The first stage is a process production system that produces standard semi-finished products that are further processed in the second stage in a job-shop production system based on specific customer requirements. The proposed model aims

to optimize the production levels at both stages in addition to the inventory level of semi-finished products in the intermediate warehouse between the two stages. The objective is to strike a balance between the cost of carrying the inventory, the minimum batch size requirement in the first stage, and the required service level in the second stage. The proposed solution method involves solving the first stage model and using it as an input in the second stage model. Despite investigating a hybrid system, this study mainly focuses on adopting a postponement strategy in the context of MC. Hence, it fundamentally differs from our study where both standard and custom items are simultaneously produced on the same manufacturing lines in a multi-echelon manufacturing setting.

[Rafiei and Rabbani \(2012\)](#) combine a set of qualitative and quantitative methods for optimizing order acceptance/rejection policy, order due-date setting, lot-sizing, and capacity planning in systems producing in MTS/MTO modes. The proposed hierarchal production planning framework is composed of several stages. The first step determines high-priority MTO product families (for capacity allocation purposes) by using the analytic hierarchy process (AHP). The second step revolves around assigning production values to the MTS and MTS/MTO product families based on three qualitative criteria such as estimated contribution, reputation, and potential future sales. Lot sizes of MTS and MTS/MTO products and the capacity allocation are then optimized in the third stage with the aid of a mixed-integer-programming (MIP) model. To overcome the computational complexity of the proposed model, a backward lot-sizing heuristic is proposed.

[Rafiei et al. \(2013\)](#) combine the tactical planning framework proposed in ([Rafiei & Rabbani, 2012](#)) with operational-level production sequencing and preventive maintenance in a job shop environment producing both MTS and MTO items. The proposed bi-level planning approach focuses on order acceptance, lot sizing, capacity coordination, and potential due date negotiations at the first level to maximize profit. By considering the possibility of pre-emption, the goal of the second-level model is to minimize the earliness and tardiness of the jobs according to established priorities and due dates. The authors apply three meta-heuristics to solve the bi-level problem. The decision frameworks proposed in [Rafiei and Rabbani \(2012\)](#); [Rafiei et al. \(2013\)](#) fundamentally differ from the one presented in our model, which integrates order acceptance, capacity planning, outsourcing, lot-sizing, and inventory planning into a unified MIP model, while explicitly accounting for the uncertainty associated with customizable products in terms of design complexity and resource

requirements.

[Khakdaman et al. \(2015\)](#) tackle tactical planning in a hybrid MTS/MTO environment under demand and cost uncertainties. A bi-objective, scenario-based robust optimization model ([Mulvey, Vanderbei, & Zenios, 1995](#)) is proposed to optimize production, procurement, subcontracting, transportation, workforce, inventory, and backorder quantities in a flow shop environment. The objective is to minimize the total cost and maximize machine utilization. Although the study integrates several tactical decisions within a hybrid production environment, it overlooks the unique challenges associated with manufacturing custom-made products that, while sharing a common product structure with standard items, involve longer processing times and are more susceptible to quality defects.

[Beemsterboer et al. \(2016\)](#) explore a discrete-time Markov Decision Process to obtain optimal production policies for a hybrid MTO/MTS system with a positive lead time for MTO production. By considering one MTS and one MTO product, the proposed model decides which product should be manufactured in each period based on the state of inventory level for each product type. This study is further extended in [Beemsterboer et al. \(2017\)](#) by integrating MTS items in the control of an MTO job shop focusing on meeting the due dates of the MTO product while mitigating lost MTS sales. Four methods are proposed for this purpose that rely on creating orders of MTS items with fictitious due dates that are then considered in production planning alongside the rest of the orders. The proposed approaches are evaluated using discrete-event simulation. These studies are substantially different from our work as their main focus is deriving production control policies for 2 types of products.

[Pereira et al. \(2022\)](#) develop a multi-objective sales & operations planning (S&OP) model in a flow shop manufacturing setting producing MTS/MTO products. Focusing on the case of cable manufacturing, the company produces standard products to stock and customized products based on received orders in a deterministic environment. The model determines optimal order acceptance policy, purchasing, production, packing, and inventory levels while abiding by safety stock satisfaction levels for MTS items. The objectives are to maximize revenue from standard and custom products and to minimize operational and order rejection/delay costs for custom orders. Assuming that MTO items are derived from a predefined set of features, the proposed model is built upon a set of predetermined production sequences with deterministic processing times. Consequently, it

does not address the uncertainty involved in manufacturing customizable modular-structured products that may require significantly more resources than their standard counterparts. By adopting a systematic literature review approach, [Bhalla, Alfnes, Hvolby, and Oluyisola \(2023\)](#) develops a qualitative S&OP framework for the tactical planning process design to support delivery date setting in Engineer-to-order (ETO) contexts. The study identifies the main tactical planning activities managers in ETO companies should consider for the design of S&OP process and the required information inputs for coordinating the planning activities.

Several studies in the literature incorporate uncertainty into production and inventory planning problems (see, e.g., [Kazemi Zanjani, Nourelfath, and Ait-Kadi \(2010\)](#), [Chhaochhria and Graves \(2013\)](#), [Sanei Bajgiran, Kazemi Zanjani, and Nourelfath \(2017\)](#), [Cheng \(2024\)](#), [Elyasi et al. \(2024\)](#)) by exploring stochastic and robust optimization approaches. Nonetheless, they either focus on the manufacturing of standard products or do not address the challenging features of customized manufacturing in the category of MTO items. Conversely, a separate body of literature, such as [Kumar \(2007\)](#), [Wang et al. \(2017\)](#), [Yao and Xu \(2018\)](#), [Katoozian and Zanjani \(2022\)](#), concentrates solely on supply and production planning challenges associated with mass customization and mass personalization. These studies do not tackle the particularities of hybrid environments, as they do not consider producing both standard and custom items in the same production setting.

The above literature survey highlights the shortcomings of existing production planning frameworks proposed in the context of hybrid MTS/MTO manufacturing. First, the existing models rarely incorporate the product BOM structure in a multi-echelon job shop environment with reconfigurable and modular production lines. Most notably, the majority of these studies focus on a flow shop environment (with a set of alternative, pre-determined manufacturing sequences) without necessarily emphasizing the customized nature of MTO products and their corresponding challenges. In other words, the uncertain processing time and quality defect rates of customized items, associated with their uncertain design features, are typically overlooked. This study aims to fill this gap by explicitly incorporating the uncertain nature of customized manufacturing in an integrated tactical planning framework relying on products, featured with a generic structure, that can be manufactured simultaneously at standard and customized configurations in a modular, multi-echelon job shop setting.

Chapter 3

Robust optimization

One of the assumptions of linear optimization (LO) models is typically that quantities are known and will not change over the course of the planning horizon. In many cases, this is indeed true; however, uncertainty can be present in models in a number of ways. In cases where the probability distribution of the uncertain parameters is known, stochastic optimization will propose the best plan, based on the expected realization of the uncertainty present. However, if uncertain parameters realize in such a way that the feasible region is changed, solutions from different types of models, including stochastic optimization models, can become infeasible due to them no longer being included in the feasible region. Robust optimization is well-suited to combat this problem. Instead of proposing the best plan based on the expected realization of the uncertainty, robust optimization approaches provide a more conservative plan that will remain feasible even in worst-case scenarios (Ben-Tal, El Ghaoui, & Nemirovskiĭ, 2009).

To elaborate, an uncertain LO model (LO_U) is a collection

$$\{\min_x \{c^T x : Ax \leq b\}, (c, A, b) \in U(LO_U)\}$$

of problems, called instances, with the values of parameters varying in a given uncertainty set $U \subset R^{m \times n}$. An uncertainty set is parametrized, in an affine fashion, by an uncertainty vector ζ varying

within the perturbation set Z :

$$U = \left\{ \begin{array}{l} c^T = c_0^T + \sum_{l=1}^L c_l^T \zeta_l \\ a = a_0 + \sum_{l=1}^L a_l^T \zeta_l \\ b = b_0 + \sum_{l=1}^L b_l^T \zeta_l \end{array} \quad \zeta \in Z \subset R^L \right\}$$

where L denotes the number of uncertain parameters in the model. However, a family of optimization problems in the form of LO_U is not necessarily associated with feasibility or optimality on its own. The decision environment is assumed to follow three characteristics: all decision variables in LO_U represent "here and now" decisions; the decision maker is fully responsible for consequences of the decisions only when the realized uncertain is within the uncertainty set (U); and the constraints in LO_U are hard. Together, these assumptions dictate that feasible and meaningful solutions to LO_U must be fixed, and that those fixed solutions should satisfy all constraints for all realizations of the data in the uncertainty set (that is, the solutions are robust feasible). From these assumptions, three definitions can be described. Definition 1: a vector $x \in R^n$ is a robust feasible solution to LO_U if it satisfies all realizations of the constraints from the uncertainty set:

$$Ax \leq b \quad \forall (c, A, b) \in U$$

Definition 2: For a candidate solution x , the robust value $\hat{c}(x)$ of the objective in LO_U at x is the largest value of the true objective $c^T x$ over all realizations of the data from the uncertainty set:

$$\hat{c}(x) = \sup_{(c, A, b) \in U} [c^T x]$$

due to the above worst-case-oriented assumptions. Based on definitions 1 and 2, meaningful candidate solutions are outlined and their quality can be quantified. Thus, we seek the best robust value of the objective among all robust feasible solutions to the problem, which becomes the robust counterpart (RC) of LO_U . This synthesizes definition 3: the RC of LO_U is the optimization problem

of minimizing the robust value of the objective over all robust feasible solutions to the uncertain problem:

$$\min_x \left\{ \hat{c}(x) = \sup_{(c,A,b) \in U} (c^T x) : Ax \leq b, \forall (c, A, b) \in U \right\}$$

An optimal solution to the RC is called a robust optimal solution to LO_U , and is the best uncertainty-immunized solution associated with our problem.

Numerous approaches to robust optimization have been proposed over the decades. An early approach was proposed in [Soyster \(1973\)](#). Considering a particular row of the matrix \mathbf{A} , J_i is the set of coefficients in that row, i , that experiences uncertainty. Each entry a_{ij} , $j \in J$, is modelled as a symmetric and bounded random variable \tilde{a}_{ij} , $j \in J$ that takes values in the interval $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$. The robust formulation according to [Soyster \(1973\)](#) is:

$$\begin{aligned} & \max \mathbf{c}'\mathbf{x} \\ & \text{subject to } \sum_j a_{ij}x_j + \sum_{j \in J_i} \hat{a}_{ij}y_j \leq b_i & \forall i \\ & -y_j \leq x_j \leq y_j & \forall j \\ & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \\ & \mathbf{y} \geq \mathbf{0} \end{aligned}$$

With x^* as the optimal solution, at optimality, $y_j = |x_j^*|$ and thus:

$$\sum_j a_{ij}x_j^* + \sum_{j \in J_i} \hat{a}_{ij}|x_j^*| \leq b_i \quad \forall i$$

The random variable η_{ij} is then defined as $\eta_{ij} = (\tilde{a}_{ij} - a_{ij})/\hat{a}_{ij}$, which is an unknown but symmetric distribution and takes values in $[-1, 1]$. For any realization of \tilde{a}_{ij} , the solution can then be demonstrated to remain feasible:

$$\begin{aligned} \sum_j \tilde{a}_{ij}x_j^* &= \sum_j a_{ij}x_j^* + \sum_{j \in J_i} \eta_{ij}\hat{a}_{ij}x_j^* \\ &\leq \sum_j a_{ij}x_j^* + \sum_{j \in J_i} \hat{a}_{ij}|x_j^*| \end{aligned}$$

and thus we have a robust solution to our uncertain problem.

The robust optimization approach adopted in this paper is that of the price of robustness, proposed in [Bertsimas and Sim \(2004\)](#). The goal of this method is to withstand parameter uncertainty without excessively affecting the objective function. Another distinct feature of this approach is that it extends to discrete optimization problems well.

Considering a problem of the form $\mathbf{a}'_i \mathbf{x} \leq b_i$ with J_i as the set of coefficients $a_{ij}, j \in J_i$ that experience uncertainty. The uncertain parameters follow any symmetric distribution in an interval of $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$ with a mean of the nominal value a_{ij} . For every i , a parameter Γ_i , the budget of uncertainty, takes any real value in the interval $[0, |J_i|]$ and adjusts the conservatism of its constraint. It is unrealistic that every uncertain parameter $a_{ij}, j \in J_i$ will differ from their nominal value; the budget of uncertainty ensures that the model protects against cases where up to $\lfloor \Gamma_i \rfloor$ change and one coefficient a_{it} changes by $(\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it}$. This allows for an adjustable approach where the degree of conservatism is decided per uncertain constraint.

The implementation of this method occurs in a few steps. First, a protection function is implemented into the linear constraint, giving the nonlinear problem:

$$\max \mathbf{c}' \mathbf{x}$$

Subject to

$$\sum_j a_{ij} x_j + \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} y_{t_i} \right\} \leq b_i \quad \forall i$$

$$-y_j \leq x_j \leq y_j \quad \forall j$$

$$\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$$

$$\mathbf{y} \geq \mathbf{0}.$$

Two observations can be made here. When $\Gamma_i = 0$, the protection function $\beta(\mathbf{x}, \Gamma_i) = 0$, and we have the nominal version of the problem. When $\Gamma_i = |J_i|$, the formulation is equivalent to Soyster's, which is as conservative as possible. Thus, by varying $\Gamma_i \in [0, |J_i|]$, we can flexibly adjust the robustness of the model to match our desired level of conservatism.

y_j is equal to $|x_j^*|$ at optimality in the above version of the formulation. For a more compact

formulation, it can be written as

$$\begin{aligned}
\beta_i(\mathbf{x}^*, \Gamma_i) & \\
&\equiv \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} \hat{a}_{ij} |x_j^*| + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} |x_{t_i}^*| \right\} \\
&\equiv \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} y_{t_i} \right\}
\end{aligned}$$

which equals the objective function of the linear optimization problem:

$$\begin{aligned}
\beta_i(\mathbf{x}^*, \Gamma_i) &= \max \sum_{j \in J_i} \hat{a}_{ij} |x_j^*| z_{ij} \\
&\text{subject to} \\
&\sum_{j \in J_i} z_{ij} \leq \Gamma_i \\
&0 \leq z_{ij} \leq 1 \quad \forall j \in J_i.
\end{aligned}$$

The above linear optimization problem uses the auxiliary decision variable z_{ij} which indicates where along the distribution of the uncertain parameter a_{ij} our model is considering. To illustrate, if z_{12} is 0.5, that means the uncertain parameter will take a value of $a_{12} + 0.5\hat{a}_{12}$. By using maximization, it ensures that as many of these auxiliary variables as possible will take on their full value of 1, while targeting our parameters with the highest proportion of drift first. In other words, the problem's optimal solution will have the $\lfloor \Gamma_i \rfloor$ parameters with the largest uncertainty taking on their maximum drift, and one runner-up that will take on $(\Gamma_i - \lfloor \Gamma_i \rfloor)$ of its drift.

The protection function cannot be put into the model as-is because it would make the model non-linear. By taking the dual of the protection function, with z_i as the dual variable of the first

constraint and p_{ij} as the dual variable of the second constraint, we have:

$$\begin{aligned}
& \min \sum_{j \in J_i} p_{ij} + \Gamma_i z_i \\
& \text{subject to} \\
& z_i + p_{ij} \geq \hat{a}_{ij} |x_j^*| \quad \forall i, j \in J_i \\
& p_{ij} \geq 0 \quad \forall j \in J_i \\
& z_i \geq 0 \quad \forall i.
\end{aligned}$$

By strong duality, since the protection function is feasible and bounded, the dual problem is also feasible and bounded and their objective values coincide. Substituting the dual problem in the place of the protection function then gives the model:

$$\begin{aligned}
& \max \mathbf{c}'\mathbf{x} \\
& \text{subject to} \\
& \sum_j a_{ij} x_j + z_i \Gamma_i + \sum_j \in J_i p_{ij} \leq b_i \quad \forall i \\
& z_i + p_{ij} \geq \hat{a}_{ij} y_j \quad \forall i, j \in J_i \\
& -y_i \leq x_j \leq y_j \quad \forall j \\
& l_j \leq x_j \leq u_j \quad \forall j \\
& p_{ij} \geq 0 \quad \forall i, j \in J_i \\
& y_j \geq 0 \quad \forall j \\
& z_i \geq 0 \quad \forall i
\end{aligned}$$

which accomplishes our goal of linear robust optimization with a choosable amount of conservatism.

Chapter 4

Problem description and formulation

4.1 Problem description

Inspired by the manufacturing of advanced optical devices, such as high-power laser modules and other optical assemblies, this study focuses on a job-shop setting that produces modular-structured items that can be either ordered in a customized configuration (e.g., components of medical scanners, avionic systems, etc.) or produced in a standard configuration (e.g., laser cutters). Custom-made final items are manufactured based on received orders, while standard final items are fulfilled according to sales forecasts. We further consider a generic bill-of-material (BOM) structure, in terms of the number of levels and component/sub-assembly composition. Within this framework, standard final products are composed of standard modules and components; whereas, customized final products feature customizable semi-finished products, sub-assemblies, and components, an example of which is provided in [Figure 4.1](#).

We further assume a job shop setting, comprising of general-purpose and reconfigurable/programmable machines (e.g., lathe machines, 3D printers, CNC mills, robots, etc.) that are grouped into several stations. Every item in the shop is assigned a set of machines on which it must be processed. Each item must meet its processing requirements before it can be used in a superordinate item or sold. [Figure 4.1](#) shows this flow from the lower to upper echelons in the manufacturing network under investigation.

Some BOM items other than the final products (e.g., complex customized modules/components)

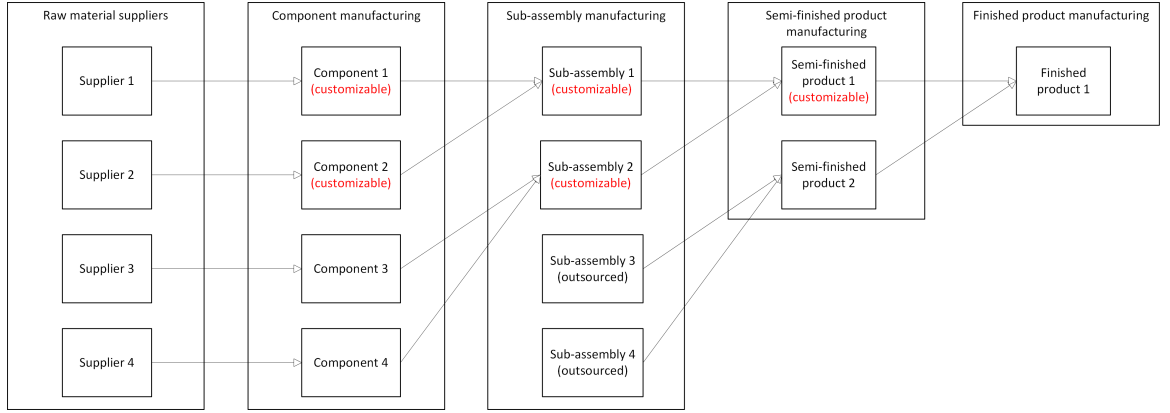


Figure 4.1: Multi-echelon structure of the production system.

can also be procured from a third-party supplier in its completed state, skipping these production steps. We further consider a modular-capacity job shop, similar to the surface-mount lines with pick-and-place machines, where more machines (or capacity modules) can be added to each station to compensate for production capacity loss and order size fluctuations due to potential quality defects.

The standard or custom status of an item affects the processing time and cost of the associated components when making a finished product. More precisely, customizable components in a BOM (e.g., lenses, or sensors) typically require a more complex design, manufacturing steps, and manpower skills; hence, would spend more time at some stations. Therefore, while nominal values are considered for processing times of standard finished products, and BOM items at a standard configuration, the processing times are increased when custom-made finished products or subordinate items are being produced. Furthermore, manufacturing custom-made items for the first time is often susceptible to quality and testing failures (due to design, manufacturing, and tooling limitations as well as operators' learning curve), which can jeopardize the fulfillment of these orders within their promised due dates.

We develop a decision model for integrated tactical planning in the above-mentioned hybrid manufacturing system, producing a set of standard and custom-designed modular-structured products. The key decisions include whether to accept custom orders, how many additional production modules to activate, and the determination of procurement, production, and inventory holding quantities across the planning horizon. It is noteworthy that inventories of both standard and custom-made items are carried over the planning horizon to balance the production capacity with,

respectively, forecast and quality defect rates. The goal is to maximize revenue at minimum production, inventory, and rejected/delayed orders' costs under production capacity limitations and BOM configurations.

To incorporate uncertain processing time and quality defects of customizable products, we adopt a cardinality-constrained robust optimization approach (Bertsimas & Sim, 2004). This method relies on optimizing the production, inventory, and fulfillment strategies under the worst-case outcomes of uncertainty; hence protecting the firm from exceeding capacity restrictions. However, planning only for the worst-case outcomes will severely limit the achievable profit of any given plan. Alternatively, a budget of uncertainty is considered that limits the extent to which the manufacturer must be prepared for worst-case outcomes. The framework models processing time uncertainty for custom items through random intervals around nominal values and accounts for quality-related defects by adjusting order sizes with multipliers derived from average defect rates. Specifically, the standard deviation of each custom order is derived as the product of the corresponding item's average defect rate and its order size. Higher design complexity is typically associated with a higher defect rate. Subsequently, an uncertainty budget is introduced to adjust the number of standard deviations added to the actual order size for each custom item. This multiplier regulates the level of production readiness required to compensate for quality defects and ensures that the production volume aligns with the actual order size during custom item manufacturing.

4.2 Problem Formulation

In this section, we first present mathematical notations followed by the uncertain mixed-integer-programming (MIP) model formulated for integrated tactical planning in the hybrid manufacturing environment described above. We further elaborate on the uncertainty sets adopted to model the random order size and processing time of customizable products and formulate the robust counterpart of the uncertain MIP model accordingly.

4.2.1 Notations

Indexes and sets

| | |
|--------------------|---|
| C | Set of items' customization levels, indexed by c . $c = 1$ corresponds to standard and $c = 2$ represents customized. |
| P_c | Set of items at customization level c , indexed by p . |
| $P_c^C \subset P$ | Subset of components at customization level c . |
| $P_c^F \subset P$ | Subset of finished items at customization level c . |
| $P_{pc} \subset P$ | Set of direct successors of item p in the BOM at customization level c . |
| M | Set of machines, indexed by m . |
| P_m^c | Set of items at customization level c processed on machine m , indexed by p . |
| T | Set of time periods, indexed by t . |
| O_{p2} | Set of custom orders for finished item p at customization level 2, indexed by o_{p2} . |

Parameters

Procurement

| | |
|-----------|---|
| k_{pc} | Procurement cost of item p at customization level c . |
| q_{pct} | Maximum quantity of item p at customization level c available to purchase at time t . |

Production

| | |
|----------------------|--|
| λ_{spc} | Quantity of item p used in super-ordinate item s at customization level c . |
| g_{mt} | Amount of time available at machine m in period t . |
| $\tilde{\tau}_{p2m}$ | Random production time of item p at customization level 2 on machine m . |
| $\bar{\tau}_{pcm}$ | Nominal production time of item p at customization level c on machine m . |
| $\Delta\tau_{p2m}$ | Drift on production time of product p at customization level 2 on machine m . |
| v_{pc} | Unit production cost of item p at customization level c . |
| a_m | Unit module activation cost for machine m . |
| y | Limit on the number of extra machines (or extra capacity modules) that can be activated per station. |
| Γ_m^M | Budget of uncertainty per machine m . |

Inventory

| | |
|-----------|--|
| h_{pct} | Unit holding cost of item p at customization level c at time t . |
| i_{pc} | Initial inventory of item p at customization level c . |

Sales

| | |
|-----------------------|--|
| s_{p1} | Unit selling price of finished item p at customization level 1. |
| $\phi_{o_{p2}}$ | Unit selling price of finished item p at customization level 2 in order o_{p2} . |
| $r_{o_{p2}}$ | Penalty factor for rejecting order o_{p2} . |
| $\bar{\eta}_{o_{p2}}$ | Nominal order size of finished item p at customization level 2 in order o_{p2} . |
| $\sigma_{o_{p2}}$ | Standard deviation of the order size for custom item p in order o_{p2} , reflecting item's potential defect rate, represented as a percentage of nominal order size. |
| $d_{o_{p2}}$ | Due date of order o_{p2} , indicated as a time period $t \in T$. |
| f_{p1t} | Forecasted demand for finished item $p \in P_1^F$ in time period t . |
| $\rho_{o_{p2}}$ | Price depreciation for each time period of delay for order o_{p2} . |
| $\Gamma_{o_{p2}}^O$ | Budget of uncertainty for the order size of custom item p in order o_{p2} , defined as the number of standard deviation drifts from the nominal value $\bar{\eta}_{o_{p2}}$. This budget aims to limit order size inflation due to quality defects. |

Decision variables

Procurement

| | |
|-----------|--|
| Q_{pct} | Quantity of item $p \in P_c^C$ at customization level c purchased in time period t . |
|-----------|--|

Production

| | |
|---------------------|---|
| X_{pct} | Quantity of item p at customization level c produced in time period t . |
| Y_{mt} | Number of extra machines of type m to activate at time t . All stations start with one machine and their capacity can be augmented by activating more machines (or extra capacity modules). |
| δ_{p2m} | Binary variable. Takes a value of 1 if product p at customization level 2 takes maximum time drift on machine m . 0 otherwise. |
| μ_m, ψ_{p2m} | Dual variables corresponding to the protection function defined for uncertain machine capacity constraints. |

Inventory

| | |
|-----------|--|
| I_{pct} | Inventory of item p at customization level c at time t . |
|-----------|--|

Sales

| | |
|------------|--|
| A_{op2t} | Order dispatching binary variable. 1 if order o_{p2} is dispatched in period t . 0 otherwise. |
| R_{op2} | Order refusal binary variable. 1 if order o_{p2} is not accepted during the planning horizon. 0 otherwise. |
| D_{op2} | Number of days of delay of order o_{p2} relative to the due date. |
| F_{p1t} | Quantity of standard finished item $p \in P_1^F$ sold in period t . |

4.2.2 Uncertain Mathematical Programming Model

$$\max Z = Rev_{Custom} + Rev_{Standard} - Cost \quad (1)$$

Subject to

$$Rev_{Custom} = \sum_{p \in P_2^F} \sum_{o_{p2} \in O_{p2}} \eta_{o_{p2}} \phi_{o_{p2}} (1 - r_{o_{p2}} R_{o_{p2}} - \rho_{o_{p2}} D_{o_{p2}}) \quad (2)$$

$$Rev_{Standard} = \sum_{p \in P_1^F} \sum_{t \in T} s_p F_{p1t} \quad (3)$$

$$Cost = \sum_{t \in T} \sum_{p \in P_c} \sum_{c \in C} k_{pc} Q_{pct} + \sum_{t \in T} \sum_{p \in P_c} \sum_{c \in C} h_{pct} I_{pct} + \sum_{p \in P_c} \sum_{c \in C} \sum_{t \in T} \sum_{m \in M} v_{pc} \tau_{pcm} X_{pct} + \sum_{m \in M} \sum_{t \in T} a_m Y_{mt} \quad (4)$$

Procurement limits

$$Q_{pct} \leq q_{pct} \quad \forall p \in P_c^C, c \in C, t \in T \quad (5)$$

Production capacities

$$\sum_{p \in P_m^1} \bar{\tau}_{p1m} X_{p1t} + \sum_{p \in P_m^2} \tilde{\tau}_{p2m} X_{p2t} \leq g_{mt} (1 + Y_{mt}) \quad \forall m \in M, t \in T \quad (6)$$

$$Y_{mt} \leq y \quad \forall m \in M, t \in T \quad (7)$$

Inventory balance

$$I_{pct} = I_{p,c,t-1} + Q_{pct} + X_{pct} - \sum_{s \in P_{pc}} \lambda_{spc} X_{sct} \quad \forall t = \{2, \dots, T\}, p \in P_c^C, c \in C \quad (8)$$

$$I_{pc1} = i_{pc} + X_{pc1} + Q_{pc1} - \sum_{s \in P} \lambda_{spc} X_{sc1} \quad \forall p \in P_c^C, c \in C \quad (9)$$

$$I_{p1t} = I_{p,1,t-1} + X_{p1t} - F_{p1t} \quad \forall t = \{2, \dots, T\}, p \in P_1^F \quad (10)$$

$$I_{p11} = i_{p1} + X_{p11} - F_{p11} \quad \forall p \in P_1^F \quad (11)$$

$$I_{p,2,t-1} + X_{p2t} - I_{p2t} = \sum_{o_{p2} \in O_{p2}} \left(\bar{\eta}_{o_{p2}} + \Gamma_{o_{p2}}^O \sigma_{o_{p2}} \right) A_{o_{p2}t} \quad \forall t = \{2, \dots, T\}, p \in P_2^F \quad (12)$$

$$i_{p2} + X_{p21} - I_{p21} = \sum_{o_{p2} \in O_{p2}} \left(\bar{\eta}_{o_{p2}} + \Gamma_{o_{p2}}^O \sigma_{o_{p2}} \right) A_{o_{p2}1} \quad \forall p \in P_2^F \quad (13)$$

Sales limits

$$R_{o_{p2}} + \sum_{t \in T: t \geq d_{o_{p2}}} A_{o_{p2}t} = 1 \quad \forall o_{p2} \in O_{p2}, p \in P_2^F \quad (14)$$

$$\sum_{t \in T: t < d_{o_{p2}}} A_{o_{p2}t} = 0 \quad \forall o_{p2} \in O_{p2}, p \in P_2^F \quad (15)$$

$$D_{o_{p2}} = \sum_{t \in T: t \geq d_{o_{p2}}} t A_{o_{p2}t} - d_{o_{p2}}(1 - R_{o_{p2}}) \quad \forall o_{p2} \in O_{p2}, p \in P_2^F \quad (16)$$

$$F_{p1t} \leq f_{p1t} \quad \forall p \in P_1^F, t \in T \quad (17)$$

Domain constraints

$$Q_{pct}, X_{pct}, Y_{mt}, I_{pct}, D_{o_{p2}}, F_{p1t} \geq 0 \quad \forall p \in P, c \in C, t \in T, m \in M, o_{p2} \in O_{p2} \quad (18)$$

$$Y_{mt} \in \mathbb{Z}_0^+ \quad \forall m \in M, t \in T \quad (19)$$

$$A_{o_{p2}t}, R_{o_{p2}} \in \{0, 1\} \quad \forall p \in P, t \in T, o_{p2} \in O_{p2} \quad (20)$$

The objective function in this problem is to maximize profit, which is calculated as the sum of the revenue from standard and custom finished product sales minus the associated purchasing, inventory holding, and production costs. Equation (2), calculating the sales revenue of custom orders, corder rejection penalty cost for refusing orders, tunable to the firm's wants, in addition to the penalty cost of delayed dispatches. The goal is to create an incentive to accept custom orders and minimize delivery delays.

Constraints (5) ensure the quantity of each item procured in each period is limited by the quantity of that item available in that period to be purchased from external suppliers. Constraints (6) limit the quantity produced according to machine capacity, and activate additional machines and/or capacity modules as needed to meet demand. Considering that each machine can potentially produce both standard and customizable items, the total capacity consumption by all items is separated for the two categories of items. While the standard items require a deterministic (nominal) processing time

on each machine, the custom-made items have a random processing time $\tilde{\tau}_{p2m}$ that will be modeled in the following sub-section. As such, constraints (6) are not well-defined. Their robust counterpart will be formulated in the next subsection. Constraints (7) limit the number of additional machines to activate in each station.

The next set of five constraints (8)-(13) maintain the balance of inventory for semi-finished and finished items with the production quantities and demand while respecting the BOM structure. These constraints are paired per category (semi-finished, final standard, and final customized items), with the first set providing the inventory balance across all periods other than the first and the second set providing the inventory balance for the first period. Constraints (8) and (9), in particular, detail the inventory balancing of non-finished items across all periods. More specifically, they update the inventory level of such items as the difference between the quantities purchased from external suppliers along with the volumes produced in-house and the quantities required in superordinate items, quantified through the BOM coefficients (λ_{spc}). It is noteworthy that the inclusion of customization index c in λ_{spc} is necessary to ensure the stock level of standard items can be updated in case they are required for manufacturing both standard and custom-made super-ordinate BOM items. On the other hand, if a superordinate item is standard, all its subordinate BOM items must be standard; hence, λ_{spc} for $c=2$ has to be set to zero. Finished products, whose inventory balance is found in constraints (10)-(13), are only sourced from in-house production and inventory from previous periods. However, finished products are consumed in the fulfillment of forecasted sales or received custom orders. Constraints (10) and (11) balance quantities for standard finished items (at $c = 1$) against forecasted sales across all periods. Constraints (12) and (13) balance quantities for custom-made finished items (at $c = 2$), against accepted custom orders across all periods. These constraints are protected against random defect rates of custom-made items and their impact on the production volume. The goal is to protect the production plan against lost and delayed orders in this category. This is achieved by adding multipliers of order size standard deviation (σ_{op2}) to the nominal order size. As mentioned earlier, the standard deviation of the order size is calculated by multiplying the average defect rate for the corresponding custom item by its actual (nominal) order size. Furthermore, the multipliers Γ_{op2}^O , representing the budget of uncertainty for inflating the order size, can be adjusted to create a robust order fulfillment strategy that is protected against random

quality defects.

Constraints (14) and (15) detail how custom orders are handled in the system. Constraints (14) make it so any single custom order is either accepted and fulfilled on or after its due date, or altogether rejected. Constraints (15) ensure a custom order is never accepted and fulfilled before its due date. Constraints (16) calculate the number of periods a custom order is delayed if it was accepted at all. Finally, constraints (17) ensure that standard items sold do not surpass the quantity forecasted.

4.3 Modelling Uncertain Production Time for Custom Orders

We model the uncertain production time of customizable items on the machines, $\tilde{\tau}_{p2m}$, as a box uncertainty set, represented within the interval $[\bar{\tau}_{p2m}, \bar{\tau}_{p2m} + \Delta\tau_{p2m}]$. If the production times of all items processed on each machine take their maximum drift from the nominal value, the worst-case outcome will be reflected in the uncertain capacity constraints (6). This could potentially lead to a significant drop in profit as the result of rejecting custom orders or long delays in their delivery due to capacity shortages. To control the level of conservatism of the model, a budget of uncertainty (Γ_m) is introduced per machine. It would represent the maximum number of items that are allowed to take their worst-case production time on that machine. This budget is within the following limit, where the budget of zero corresponds to the deterministic model.

$$0 \leq \Gamma_m \leq |P_m^2|$$

4.4 Robust Counterpart of Capacity Constraints

Using the uncertainty set defined in section 4.3, the goal of the robust optimization approach in (Bertsimas & Sim, 2004) is to protect model (1)-(20) against the violation of production capacity constraints as long as the outcomes of uncertain production times do not exceed the uncertainty budget. To guarantee the feasibility of this constraint under worst-case processing times while respecting the budget of uncertainty, a protection function, $\beta(X_{p2t}, \Gamma_m)$, is introduced in the left-side of capacity constraints as follows:

$$\sum_{p \in P_m^2} X_{p1t} \bar{\tau}_{p1m} + \beta(X_{p2t}, \Gamma_m) \leq g_{mt}(1 + Y_{mt}) \quad \forall m \in M, t \in T.$$

The expanded version of the protection function is formulated as follows:

$$\beta(X_{p2t}, \Gamma_m) \equiv \max \sum_{p \in P_m^2} \Delta \tau_{p2m} \delta_{p2m} X_{p2t}$$

Subject to

$$\sum_{p \in P_m^2} \delta_{p2m} \leq \Gamma_m$$

$$\delta_{p2m} \in (0, 1) \quad \forall p \in P_m^2.$$

where the binary decision variable δ_{p2m} takes 1 if a product at customization level $c = 2$ takes its maximum drift on its production time at machine m , and 0 otherwise. To integrate the nonlinear protection function into the capacity constraint in a linear fashion, the above maximization problem is dualized as follows:

$$\min \mu_m \Gamma_m + \sum_{p \in P_m^2} \psi_{p2m}$$

Subject to

$$\mu_m + \psi_{p2m} \geq \Delta \tau_{p2m} X_{p2t} \quad \forall p \in P_m^2, m \in M$$

$$\mu_m \geq 0 \quad \forall m \in M$$

$$\psi_{p2m} \geq 0 \quad \forall p \in P_m^2, m \in M.$$

where μ_m and ψ_{p2m} represent the dual variables corresponding to the constraints of the protection function. By strong duality theorem, the objective function of the dual model is equal to the objective function of primal. Hence, the primal protection function can be substituted by its dual and the robust counterpart of uncertain capacity constraints (6) can be formulated as follows:

$$\sum_{p \in P_m^1} X_{p1t} \bar{\tau}_{p1m} + \sum_{p \in P_m^2} X_{p2t} \bar{\tau}_{p2m} + \mu_m \Gamma_m^M + \sum_{p \in P_m^2} \psi_{p2m} \leq g_{mt}(1 + Y_{mt}) \quad \forall m \in M, t \in T \quad (21)$$

$$\mu_m + \psi_{p2m} \geq \Delta \tau_{p2m} X_{p2t} \quad \forall m \in M, p \in P_m^2, t \in T \quad (22)$$

$$\mu_m, \psi_{p2m} \geq 0 \quad \forall p \in P_m^2, m \in M \quad (23)$$

Considering that constraints (21) must be satisfied for all feasible μ_m and ψ_{p2m} , their feasibility would hold for the dual variables that minimize the (dual of) protection function. Therefore, the “*min*” term has been removed from the left side of these constraints.

Chapter 5

Numerical experiments

In this section, we first describe the experimental settings used to validate the proposed robust tactical planning model. Next, a sensitivity analysis is conducted on the deterministic version of the model to justify the choice of uncertain parameters in the robust model. In addition, the performance of the robust model is analyzed under different uncertainty budgets. We also briefly analyze the computational complexity of this model by increasing the size of problem instances. Finally, Monte-Carlo simulation experiments are conducted to further analyze the out-of-sample performance of this model and also estimate the value of the robust solution via comparing its performance with a deterministic model.

All models were run in Python using Docplex on an Intel Core i7 3.80GHz CPU with 32 GB of RAM running Windows 11. The optimality gap was changed from the default value to 0.1%.

5.0.1 Experimental Design

This section will provide context for the hypothetical case study inspired by the literature used to validate the proposed robust tactical planning model. We consider two levels of customization (standard and custom-made). However, including several levels of customization, distinguished by design complexity level, in the proposed model is quite straightforward. We further consider a BOM where each parent item has two successors. As such, each finished product has one standard and one custom-made semifinished item (module). Standard semifinished items use only standard components, and custom semifinished products use only custom components in our base case. However,

the proposed robust model can be easily generalized to products with various BOM structures. An example of this can be seen in [Figure 5.1](#). This configuration was used in a hypothetical system with 2 final products in the experiments run, for a total of 30 unique items to be processed in the system.

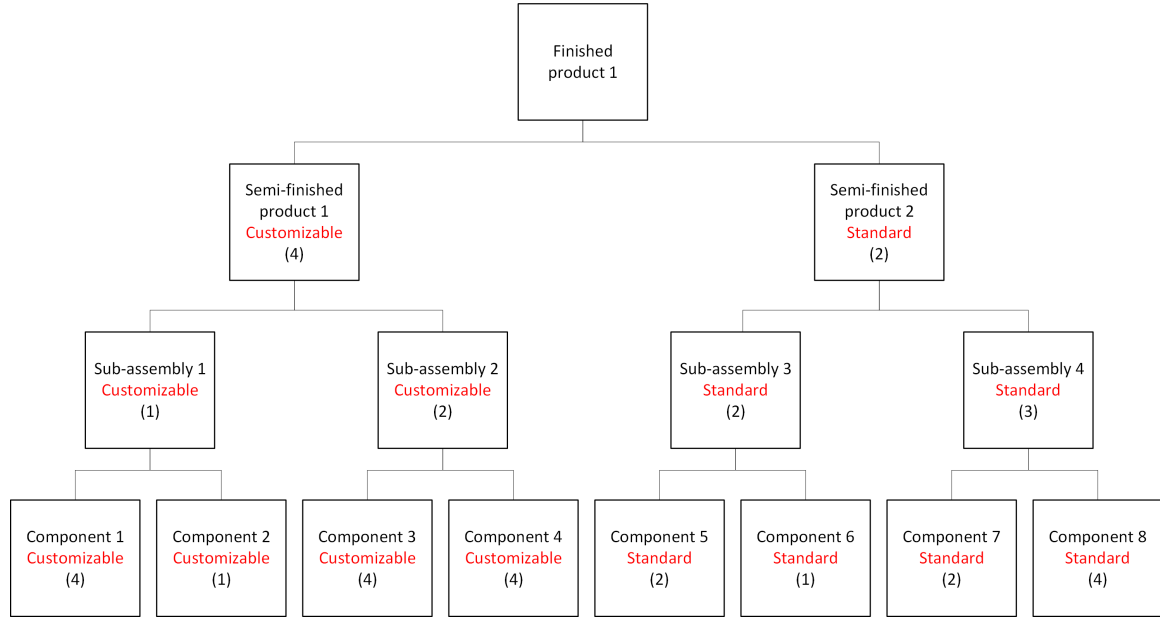


Figure 5.1: An example of a customizable bill-of-materials.

Each item had a set of machines to be processed on. This set was randomly selected with a size between 2 and 3 machines per item. The total number of machines used in the system is 50. This number was decided on through experimentation to find a balance between assigning multiple items to the machines while ensuring not every machine has multiple items assigned to maintain the realism of modeling a job shop.

[Table 5.1](#) presents the values used for all parameters, that are mainly inspired from the literature, with some presented as ranges within which values were generated. This table only presents the nominal values for these parameters; as described in [section 4.1](#), customizable items experience increased costs and processing time, scaling with their customization level. This scaling in our experiments follows a rule of increasing by $1 + \frac{c}{2}$ for each level of customization if the customizable component is being used in a custom final product. The rows affected by this scaling are: procurement cost, processing time, and production cost. The due dates of custom orders are randomly spread over the planning horizon.

Table 5.1: Ranges used for generating parameters data

| Name (units) | Values |
|---|---|
| Number of time periods | 5 |
| Unit procurement cost (\$) | [0.02, 0.5] |
| Processing time (seconds) | [2, 10] |
| Unit production cost (\$) | [0.01, 0.02] |
| Holding cost, components/finished products | 10% of procurement cost/1% of selling price |
| Machine hours available per period | [32, 60] |
| Limit on additional machine activations | 3 |
| Selling price of standard items (\$) | [3, 7] |
| Forecasted demand of a standard item per period | [1000, 11000] |
| Number of custom orders due per period | [1, 3] |
| Selling price of items in custom order (\$) | $2 \times$ standard selling price |
| Number of items in custom order | [10000, 15000] |
| Penalty per period of delay | 7% of selling price |
| Custom order rejection penalty cost | $2 \times$ order revenue |

5.0.2 Sensitivity Analysis on the Deterministic Model

Sensitivity analysis on the deterministic model was used to justify the choice of random parameters to incorporate into the hybrid robust tactical planning model. The goal was to determine the parameters that have the highest impact on the optimal profit of the deterministic model.

Experimental Factors

Four factors (parameters) were hypothesized to have a significant impact on the optimal objective value of the (deterministic) hybrid tactical planning model through preliminary screening testing. The four factors include: the production time per item, τ_{pcm} ; the forecasted demand for standard items, f_{p1t} ; the number of items in a custom order, η_{op2} ; and the limit on the number of additional machines available for activation, y . These four factors were tested in a full factorial design at high and low values of double and half of nominal values. Thus, the test conducted was a 2^4 factorial experiment. 20 different seeds were used to create 20 replications for each experiment. The response variable is the profit returned by the deterministic model.

Experimental Results

The results were obtained using the statsmodels library in Python. The detailed numerical results are presented in Table 5.2. The interaction and main effect plots, along with residual plots can be found in Figure 5.2 to Figure 5.10. A Pareto plot (Figure 5.11) was generated to aid in visualization of the effects of these factors on the model, with an additional line indicating the threshold for statistical significance level, α , of 0.05. Using this plot, we conclude that the two most significant factors are τ_{pcm} and η_{op2} , followed by their interaction.

Table 5.2: Statistical test results for the designed experiments.

| Factor(s) | Sum of squares | Degrees of freedom | F | PR(>F) |
|-----------------------------|--------------------------|--------------------|-----------------------|------------------------|
| τ_{pcm} | 6.57×10^{13} | 1 | 1.79×10^2 | 2.34×10^{-32} |
| f | 7.47×10^{12} | 1 | 2.03×10^1 | 9.46×10^{-6} |
| $\tau_{pcm}:f$ | 3.58×10^{11} | 1 | 9.72×10^{-1} | 3.25×10^{-1} |
| η_{op2} | 3.00×10^{14} | 1 | 8.14×10^2 | 5.78×10^{-88} |
| $\tau_{pcm}:\eta_{op2}$ | 3.72×10^{13} | 1 | 1.01×10^2 | 1.06×10^{-20} |
| $f:\eta_{op2}$ | 4.44×10^{10} | 1 | 1.20×10^{-1} | 7.29×10^{-1} |
| $\tau_{pcm}:f:\eta_{op2}$ | 1.99×10^{10} | 1 | 5.40×10^{-2} | 8.16×10^{-1} |
| y | 3.66×10^{13} | 1 | 9.95×10^1 | 1.84×10^{-20} |
| $\tau_{pcm}:y$ | 1.44×10^{13} | 1 | 3.90×10^1 | 1.42×10^{-9} |
| $f:y$ | 1.59×10^{11} | 1 | 4.31×10^{-1} | 5.12×10^{-1} |
| $\tau_{pcm}:f:y$ | 1.22×10^{11} | 1 | 3.31×10^{-1} | 5.66×10^{-1} |
| $\eta_{op2}:y$ | 1.74×10^{13} | 1 | 4.72×10^1 | 3.66×10^{-11} |
| $\tau_{pcm}:\eta_{op2}:y$ | 5.22×10^{12} | 1 | 1.42×10^1 | 1.99×10^{-4} |
| $f:\eta_{op2}:y$ | 1.66×10^9 | 1 | 4.52×10^{-3} | 9.46×10^{-1} |
| $\tau_{pcm}:f:\eta_{op2}:y$ | 2.73×10^9 | 1 | 7.42×10^{-3} | 9.31×10^{-1} |
| Residual | 1.11913×10^{14} | 304 | | |

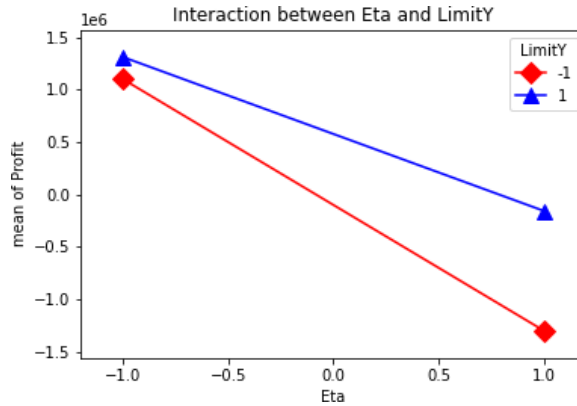


Figure 5.2: Interactions between η_{op2} and y .

Increasing either of these parameters results in a decrease in profit. The decrease in profit occurs for τ_{pcm} because an increase in production time per item is an effective reduction in machine

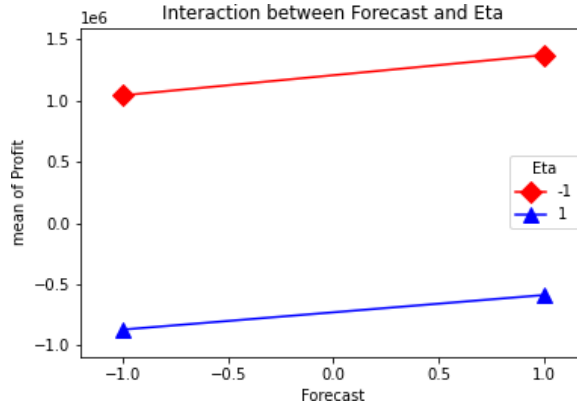


Figure 5.3: Interactions between f_{p1t} and η_{op2} .

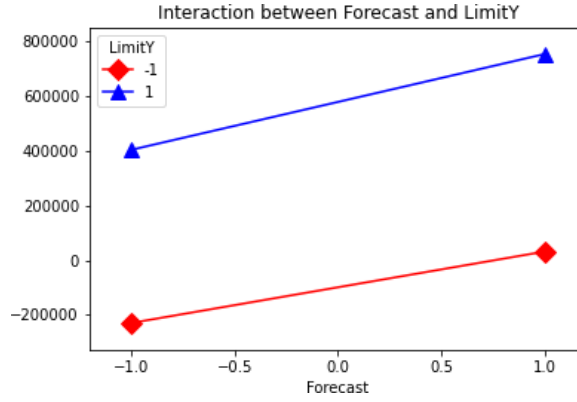


Figure 5.4: Interactions between f_{p1t} and y .

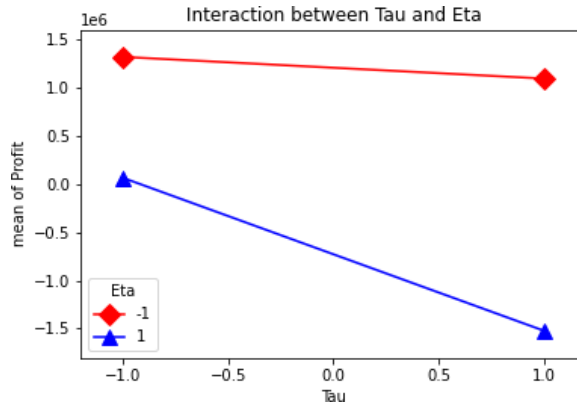


Figure 5.5: Interactions between τ_{pcm} and η_{op2} .

capacity; this results in fewer units being produced and sold, and thus a decrease in profit. For η_{op2} , the decrease in profit occurs because increasing order sizes without increasing capacity results in the firm being unable to meet the demand from its custom orders. When the firm cannot meet

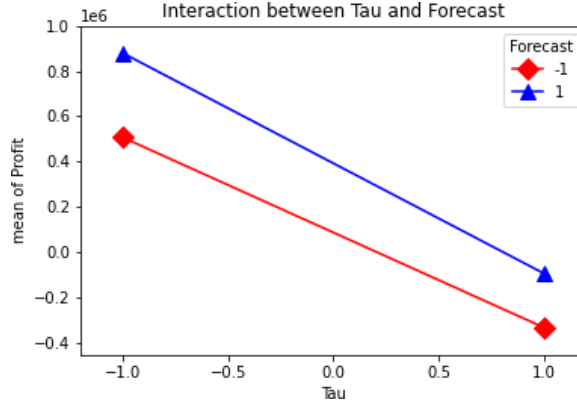


Figure 5.6: Interactions between τ_{pcm} and f_{plt} .

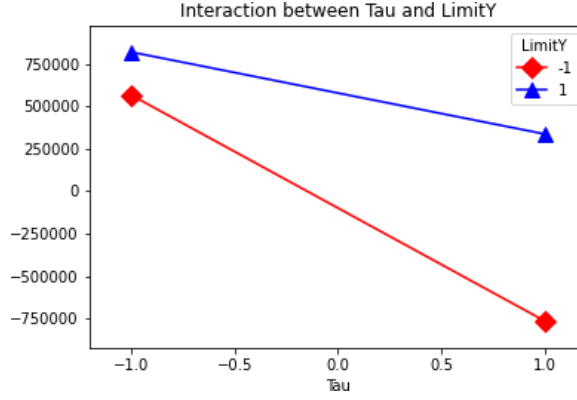


Figure 5.7: Interactions between τ_{pcm} and y .

this demand, it must reject the order, incurring the rejection penalty cost, representing a decrease in service level. This further justifies the choice of uncertain parameters in the robust model, which affected constraints (6), (12), and (13).

5.0.3 Performance Analysis of the Robust Hybrid Tactical Planning Model

The goals of these experiments are to ascertain the performance of the robust model under different uncertainty budgets representing the combinations of custom order design features and size. Design feature uncertainty, in particular, affects the machine processing time factor. Our goal is to compare the profit and the number of extra machines activated under different budgets.

The robust model was run for three budgets of uncertainty per uncertain factor, with 20 seeds used to create 20 replications. For production time uncertainty per machine, 50%, 70%, and 100%

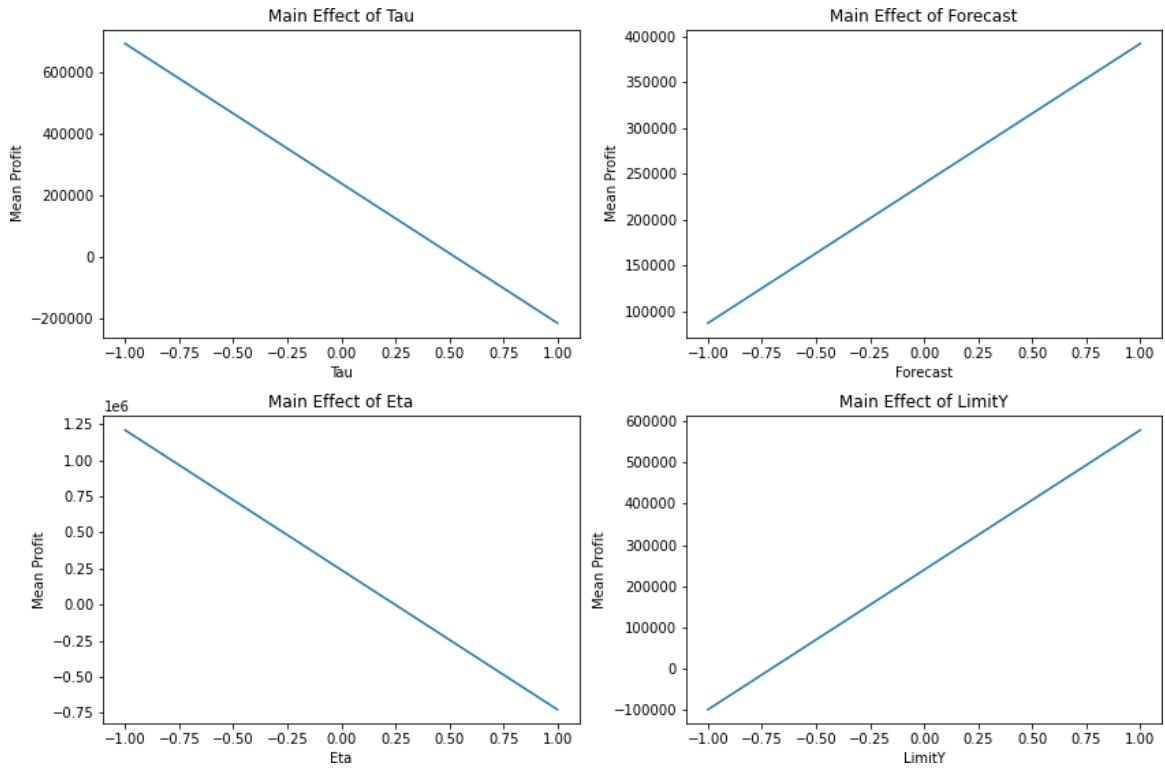


Figure 5.8: Main effects of each experimental factor on profit.

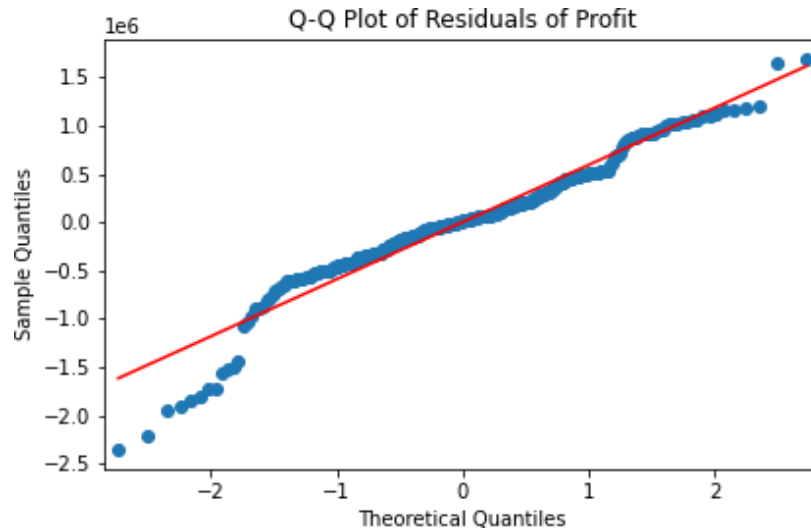


Figure 5.9: Q-Q plot of residuals of profit vs line of best fit of a standard normal distribution.

uncertainty budgets were considered. Through experiments, a drift of 150% of nominal processing time was selected that matched with the order of magnitude of nominal processing times for standard items. For order size, the budget of uncertainty was set as 1, 2, and 3 standard deviations above the

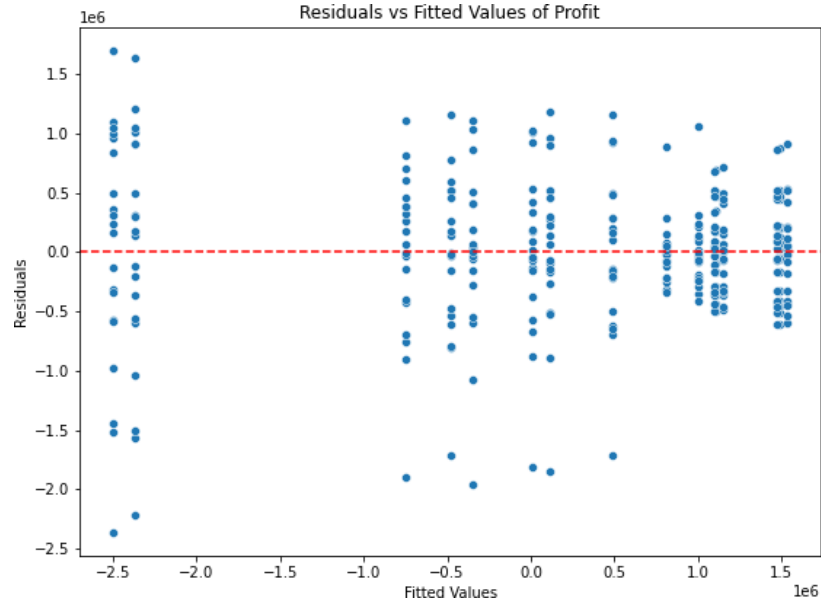


Figure 5.10: Residuals vs fitted values of profit.

nominal value. The drift (standard deviation) on order size of custom orders was set to 5% of the nominal value, which is justified based on the average defect rate of such items.

The results of running the robust model for each combination of budgets of uncertainty across all 20 replications in terms of Average Profit and Average Activated Machine Hours are presented in [Table 5.3](#). The plots for both KPIs are also provided in [Figure 5.12](#) and [Figure 5.13](#). A budget of uncertainty of 0 for both production time and order size represents the deterministic model, which is included in the results for reference.

The results in [Figure 5.12](#) are aligned with the findings in [subsection 5.0.2](#), demonstrating that rising levels of these two factors lead to a decline in profit. This occurs because inflating the order size in anticipation of quality defects, combined with extended processing times, tightens the production capacity. therefore, this leads to increased order refusals and delays, incurring additional costs. According to these results, the impact of the processing time on profit is less significant as compared with the order size (quality defect) fluctuations.

According to [Figure 5.13](#), when the nominal order size is considered for custom orders (zero order size budget, representing zero quality defects), increasing the machine processing time uncertainty budget does not show consistent trends in terms of scheduled machine hours. More specifically, when 70% of custom items take their maximum production drift on the machines, more

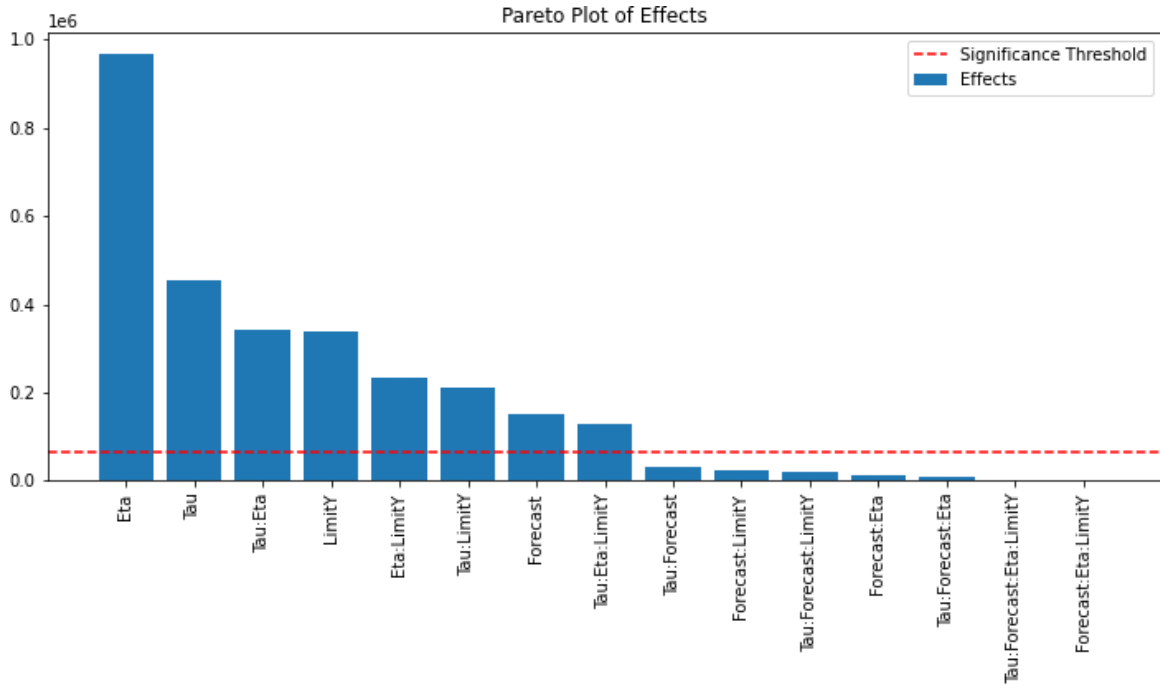


Figure 5.11: A Pareto plot of the effects of the experimental factors on profit (95% confidence level).

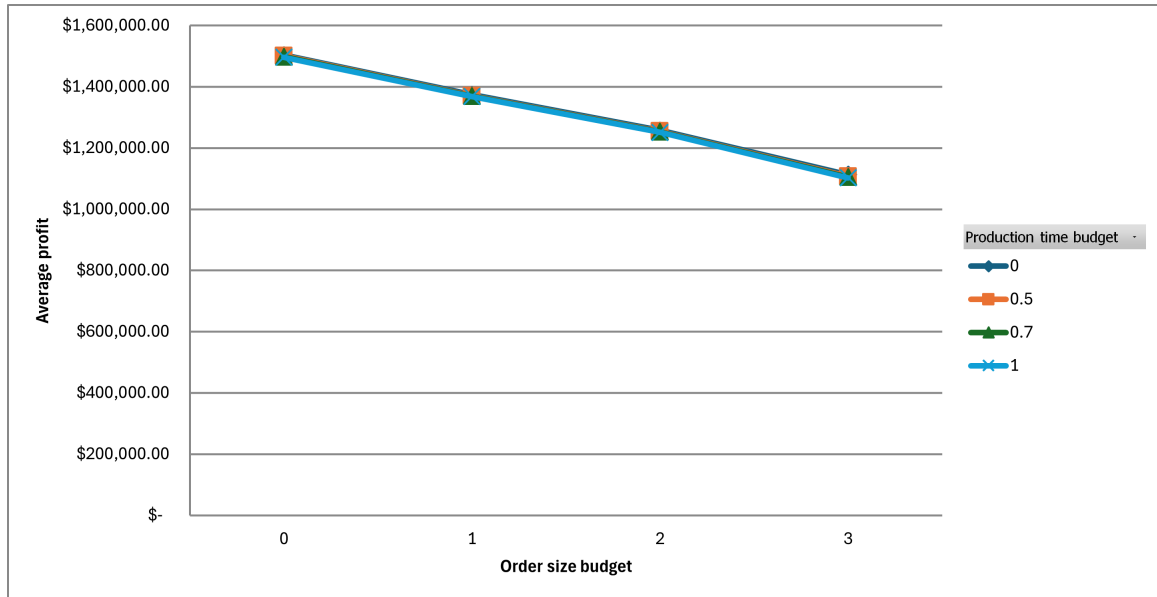


Figure 5.12: Average Profit of the Robust Model by Paired Uncertainty Budgets

machines are activated as compared with the deterministic model. For 50% and 100% budgets, on the contrary, the robust model changes the machine activation policy; hence the total scheduled machine hours is decreased. When the order size budget is increased only by one σ of the nominal

Table 5.3: Impact of Budget of Uncertainty on Machine Activation and Profit

| Budget of Uncertainty (Production Time (%)) | Budget of Uncertainty (Order Size) | Average Profit (\$) | Average Activated Machine Hours |
|---|--|---------------------------|---------------------------------------|
| 0 | 0 | \$1,503,926 | 19206 |
| 0 | 1 | \$1,374,099 | 19157 |
| 0 | 2 | \$1,257,723 | 19309 |
| 0 | 3 | \$1,113,195 | 19335 |
| 50 | 0 | \$1,501,320 | 19152 |
| 50 | 1 | \$1,370,451 | 19182 |
| 50 | 2 | \$1,254,308 | 19333 |
| 50 | 3 | \$1,106,876 | 19401 |
| 70 | 0 | \$1,498,505 | 19310 |
| 70 | 1 | \$1,369,552 | 19270 |
| 70 | 2 | \$1,253,266 | 19315 |
| 70 | 3 | \$1,105,376 | 19440 |
| 100 | 0 | \$1,496,256 | 19216 |
| 100 | 1 | \$1,368,225 | 19163 |
| 100 | 2 | \$1,251,350 | 19291 |
| 100 | 3 | \$1,102,777 | 19440 |

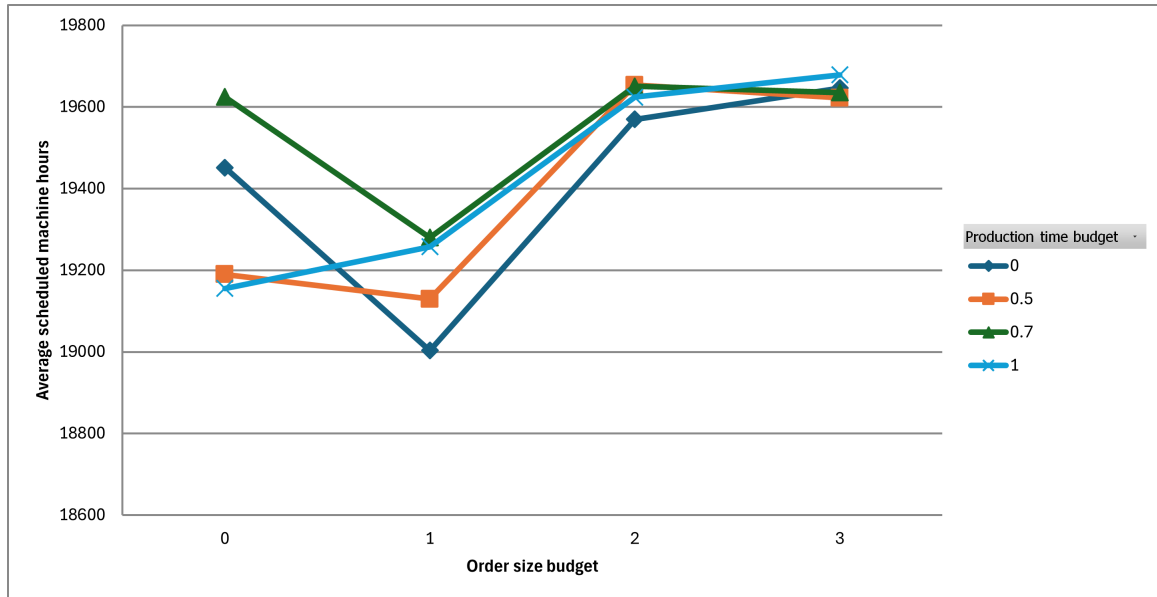


Figure 5.13: Average Scheduled Machine Hours in the robust model by Paired Uncertainty Budgets

value, the scheduled machine hours are decreased as a result of the change in machine activations determined by the robust model to minimize the lost and delayed orders. For higher order size budgets, representing protection against higher ratios of quality defects, the results indicate an increase

in terms of the number of machine hours scheduled (through activating more capacity modules) and this trend is increasing by tightening the budget of uncertainty. This occurs for two reasons. First, when the production time budget is increased, it is equivalent to expecting the majority of custom-made products to take more time to produce. When production time per item increases, more machines need to be scheduled during the planning horizon to match this increase. Second, when the order size budget is increased, this increases the number of products expected to be produced (to offset the impact of quality defects on the actual order size); hence, more machines must be scheduled to match the increase in demand.

It is noteworthy that, the increases in available machine hours in all experiments are less than 1.2%, indicating that the robust model addresses uncertainty by optimizing machine activation strategies rather than merely expanding capacity by increasing the number of activated machines. As a result of a more efficient machine activation plan, the manufacturing of custom orders can be evenly distributed across the planning horizon while still meeting order due dates.

Complexity Analysis of the Robust Model

Table 5.4 shows the effects of increasing the number of final products and complexity of the bill of materials and its effect on the CPU time required to solve the robust model. These results are illustrated in a graph in Figure 5.14. As expected, the time to solve the model increases as more complexity is added to the BOM. However, even at the highest level of complexity tested, the time to solve the robust model does not exceed 48 seconds. These results demonstrate the scalability of the proposed robust optimization model for large-scale problem instances.

5.0.4 Out-of-Sample Performance Analysis of the Robust Model

In this section, the goal is to compare the performance of the deterministic and robust tactical planning models in terms of profit under uncertain order feature and size (out-of-sample) scenarios with the aid of Monte-Carlo simulations experiments. We further estimate the value of the robust solution (VRS), as the difference between the expected profit of robust and deterministic models determined over the generated random machine time and order size scenarios within the simulation experiments. We will first detail the steps used to conduct the simulations, and then present the

Table 5.4: Average CPU time (seconds) of the robust model

| Number of final products | BOM Levels | | |
|--------------------------|------------|-------|-------|
| | 4 | 5 | 6 |
| 2 | 0.19 | 0.23 | 0.36 |
| 3 | 0.25 | 0.55 | 0.67 |
| 4 | 0.31 | 0.67 | 1.09 |
| 5 | 0.84 | 1.39 | 2.62 |
| 6 | 1.20 | 1.23 | 4.52 |
| 7 | 0.85 | 1.18 | 8.59 |
| 8 | 1.02 | 2.35 | 6.41 |
| 9 | 1.28 | 2.91 | 9.25 |
| 10 | 1.06 | 3.14 | 14.74 |
| 11 | 4.18 | 18.99 | 12.58 |
| 12 | 5.87 | 8.46 | 13.80 |
| 13 | 2.89 | 12.44 | 36.41 |
| 14 | 6.74 | 11.02 | 19.88 |
| 15 | 6.60 | 19.11 | 48.18 |

results.

Monte-Carlo Simulation Process

The simulation process, outlined in [Figure 5.15](#), is run for both deterministic and robust models under different uncertainty budgets. Once the models are solved, the machine activation decisions, Y_{mt} , are recorded. Afterward, production time and order size drifts are randomly generated from the corresponding uncertainty sets and fed into the deterministic model. This step is followed by solving the deterministic model after fixing the machine activations decisions to the optimal values obtained from the deterministic and robust models. This procedure is repeated for several iterations (1000 in our experiments) and the average and standard deviation of profit and lost/delayed sale costs are calculated over all simulation runs. The drift on production time of custom items was generated using a uniform distribution with an interval of $[\bar{\tau}_{p2m}, \bar{\tau}_{p2m} + \Delta\tau_{p2m}]$. To create a more realistic simulation, only 50% of custom items experienced a drift in production times. The drift on order size was generated using a truncated Normal distribution that only includes the portion greater than or equal to the mean. The Normal distribution has a mean of $\bar{\eta}_{op2}$ and a standard deviation of σ_{op2} .

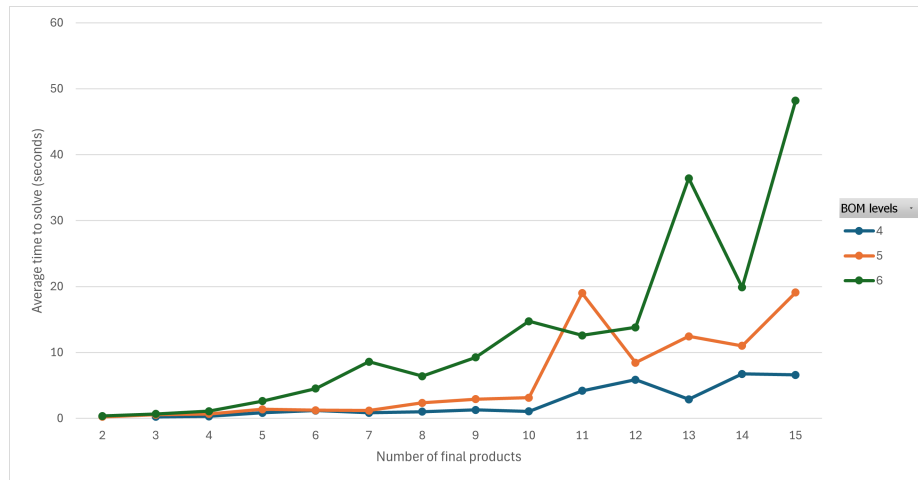


Figure 5.14: Effect of number of products and BOM levels on CPU time of the robust model

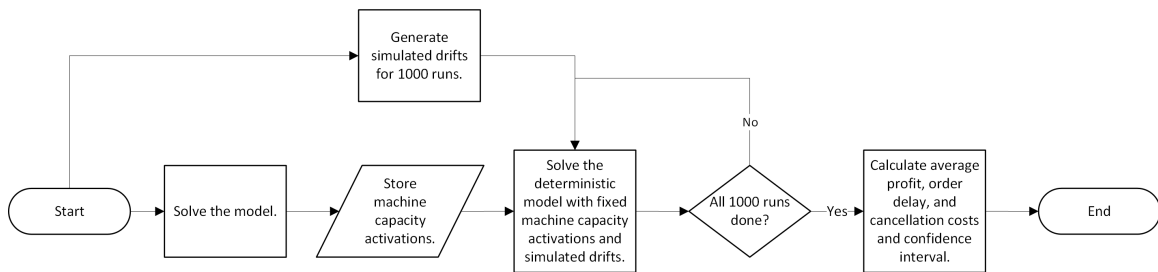


Figure 5.15: The Monte-Carlo simulation process

Value of the Robust Solution

The detailed results of the Monte-Carlo simulation runs can be found in table [Table 5.5](#). This table includes two columns summarizing the statistical confidence intervals, constructed at a 95% confidence level for profit and the VRS, using the 1000 simulation runs for each pair of budgets of uncertainty. To construct the VRS interval, the profit difference of robust and the deterministic models are recorded for each simulation run. This table also entails the average relative VRS (%), calculated by dividing the average VRS (over 1000 runs) over the average profit reported by the robust model. The service cost reduction interval column contains a 95% confidence interval on the expected reduction in costs of order delays and rejections in the robust model as compared with the deterministic one.

These results show a great deal about the value of adopting a robust optimization approach versus a deterministic method. A graph of the mean VRS is found in [Figure 5.16](#). From this

Table 5.5: Results of Monte-Carlo simulation runs

| Budget of uncertainty of production time (%) | Budget of uncertainty of order size | Profit (\$) | VRS (\$) | Average VRS (%) | Service cost reduction (\$) |
|--|-------------------------------------|------------------|----------------|-----------------|-----------------------------|
| 0 | 0 | [369115, 474315] | [0, 0] | 0.0 | [0, 0] |
| 0 | 1 | [406896, 512889] | [34860, 41495] | 8.3 | [29118, 41303] |
| 0 | 2 | [418022, 524442] | [45313, 53721] | 10.5 | [43106, 55463] |
| 0 | 3 | [405116, 510964] | [32655, 39994] | 7.9 | [28020, 40619] |
| 20 | 0 | [363080, 467712] | [-8034, -4604] | -1.5 | [-6653, -3997] |
| 20 | 1 | [399221, 505028] | [26314, 34505] | 6.7 | [20558, 32748] |
| 20 | 2 | [421072, 527701] | [48097, 57245] | 11.1 | [45491, 57504] |
| 20 | 3 | [404994, 510831] | [32526, 39868] | 7.9 | [26061, 42610] |
| 50 | 0 | [380775, 486005] | [7937, 15413] | 2.7 | [9567, 20002] |
| 50 | 1 | [397447, 503078] | [24383, 32711] | 6.3 | [18894, 30970] |
| 50 | 2 | [416950, 523315] | [44155, 52680] | 10.3 | [42194, 54188] |
| 50 | 3 | [405089, 510935] | [32619, 39974] | 7.9 | [26172, 42489] |
| 70 | 0 | [380773, 486002] | [7934, 15410] | 2.7 | [13483, 16087] |
| 70 | 1 | [398720, 504495] | [25801, 33984] | 6.6 | [19641, 32996] |
| 70 | 2 | [419904, 526479] | [46922, 56032] | 10.9 | [42370, 58508] |
| 70 | 3 | [405763, 511605] | [33275, 40662] | 8.1 | [26366, 42683] |

graph, the upward trend of the VRS can be observed as the uncertainty budget increases. This trend indicates the value of adopting a robust optimization approach is more pronounced when the level of conservatism (budget) is increased. Reductions in service costs indicate an increase in service level, as these costs are only incurred when an order is delayed or rejected. A graph of the mean reduction in cost is presented in [Figure 5.17](#). The robust model successfully reduces service costs, increasing the service level overall. Increases in service level follow an upward trend, indicating that the robust model results in fewer orders being delayed or canceled, and only improves upon this metric as budgets of uncertainty are increased.

A peak is observed in both the service level gains and the VRS at 2 standard deviations of the order size budget. This peak is present across all production time budgets tested. This peak indicates that over-protection against quality defects will decrease the benefits of the robust model. However, even in situations of an excess of order size budget, the robust model still performs better than the deterministic model across all production time budgets.

The only outlier in the results, where service costs are increased in the robust model and the VRS is negative, occurs when the firm accounts for a 20% drift in production time and no drift in

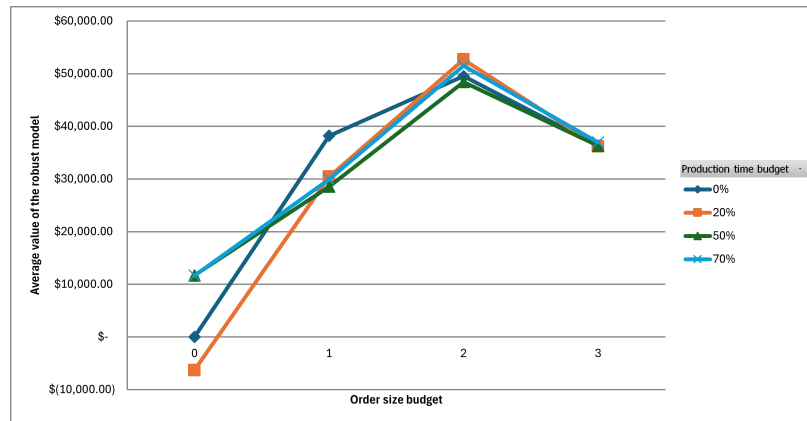


Figure 5.16: Mean VRS by paired budgets of uncertainty

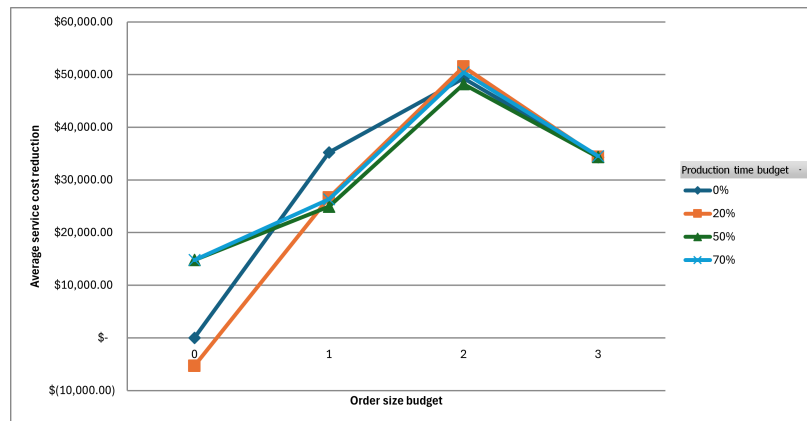


Figure 5.17: Mean reduction in service costs of the robust model by paired budgets of uncertainty

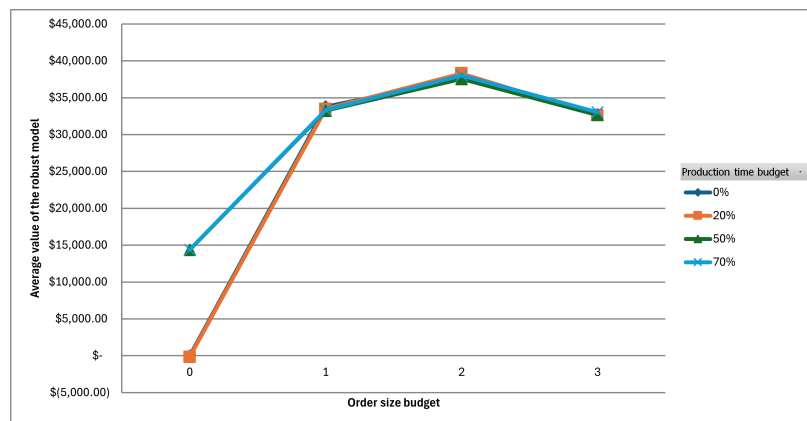


Figure 5.18: Mean VRS under the pessimistic simulation runs by paired budgets of uncertainty

order size. This outlier occurs because the budget of uncertainty for production time alone is not sufficient to protect against the drift experienced on both production time and inflated order size

in the simulation runs. These results are not observed when only the production time budget is increased under production time budgets of 50 and 70% . Indeed, those budgets are sufficient to shield against some drift experienced both in terms of order size and production time.

Further analysis reveals that the main share of the VRS comes from budget related to order size variations, a result that is expected from the previous conclusions drawn from the designed experiments. Combining these findings with the results presented in [subsection 5.0.3](#), the robust model proves to be quite effective in protecting the production plan against profit and service level decline due to uncertainty in design features. Given that the preserved decision is the machine activations, this means that the robust model achieves these savings through superior machine scheduling practices.

[Figure 5.18](#) presents the VRS under more pessimistic simulation settings, where 100% of custom orders experience production times drift. The robust model proves to be superior to the deterministic model under the most pessimistic scenarios and continues to show an upward trend in VRS as the uncertainty budget is increased.

5.0.5 Managerial Insights

The findings indicate that preemptively increasing order sizes for customizable final products, to offset anticipated defects during initial production in a hybrid manufacturing environment, adversely impacts the firm's overall profitability. This occurs because the firm must either reject or significantly delay orders for which it did not properly schedule the capacity to produce. Adopting a robust optimization approach by considering moderate uncertainty budgets can protect, to some extent, against service level losses to unexpected quality defects and inflated order size through more efficient machine activation. Nonetheless, when the quality defects are anticipated to be significantly high, the company might consider rejecting such intricate orders by focusing on the manufacturing of less complex products. Alternatively, firms could invest in advanced design and manufacturing technologies to reduce the risk of failures in producing such items, especially when the order sizes of custom products are large enough to justify the investment.

We also highlighted the importance of accounting for stochastic machine-hour requirements

in the tactical planning model, given the uncertainty associated with the design features of customized items. Intuition would dictate that scheduling more machine hours would be best to protect against this source of uncertainty. However, our results revealed that using a robust optimization approach could lead to fewer machines scheduled overall, with more targeted activations that protect against machine time uncertainty, resulting in better service level and profit. Considering the significant impact of this source of uncertainty on profitability and service level, the firm could invest in machine-learning (ML) and artificial intelligence (AI) algorithms to predict a more accurate estimation of machine time requirements based on the design features of customized items.

Chapter 6

Conclusion

In this study, we developed a robust integrated tactical planning model for hybrid multi-echelon manufacturing systems, simultaneously producing standard and custom-made modular-structured products. The proposed model explicitly incorporates the uncertain processing time and quality defects experienced when manufacturing custom items with complex design specifications. More specifically, the increased processing time was modeled by adding a positive drift to nominal processing times of standard counterparts, whereas, random quality defect rates were represented by inflating the actual order size. By budgeting the level of variations for the above uncertain factors across all custom orders, the model determines the optimal custom-order acceptance policy, capacity module activation, outsourcing, production, and inventory levels such that the overall profit under worst-case stochastic outcomes is maximized.

Our numerical experiments showcased the effectiveness of the robust optimization approach in reducing the cost of production and losses in service level under uncertain design features and manufacturing complexity of customized products. Prevention in losses of profit under uncertainty is primarily observed when assigning a relatively large budget of uncertainty to order size fluctuations. A budget assigned to random machine time due to design feature uncertainty further enhances this prevention. The protection, on the contrary, does not necessarily imply an increased number of activated machines and elevated production costs. The proposed robust tactical planning model rather achieves high protection against profit and service level losses by providing a more efficient machine activation plan and distributing the manufacturing of large orders across the planning horizon.

The current study can be extended by leveraging ML and AI algorithms to obtain a more accurate estimation of processing time and defect rates incurred for producing customized products with intricate design features and manufacturing requirements. In the same vein, embedding these predictive algorithms within the proposed decision framework would provide a predictive-prescriptive tactical planning tool that can dynamically adjust the capacity and production plans in response to the evolution of custom orders.

References

- Aghezzaf, E.-H., & Van Landeghem, H. (2002). An integrated model for inventory and production planning in a two-stage hybrid production system. *International Journal of Production Research*, 40(17), 4323–4339. doi: 10.1080/00207540210159617
- Altendorfer, K., & Minner, S. (2014). A comparison of make-to-stock and make-to-order in multi-product manufacturing systems with variable due dates. *IIE Transactions*, 46(3), 197–212. doi: 10.1080/0740817X.2013.803638
- Beemsterboer, B., Land, M., & Teunter, R. (2016). Hybrid MTO-MTS production planning: An explorative study. *European Journal of Operational Research*, 248(2), 453–461. doi: 10.1016/j.ejor.2015.07.037
- Beemsterboer, B., Land, M., Teunter, R., & Bokhorst, J. (2017). Integrating make-to-order and make-to-stock in job shop control. *International Journal of Production Economics*, 185, 1–10. doi: 10.1016/j.ijpe.2016.12.015
- Ben-Tal, A., El Ghaoui, L., & Nemirovskiĭ, A. S. (2009). *Robust optimization*. Princeton: Princeton University Press.
- Bertsimas, D., & Sim, M. (2004). The Price of Robustness. *Operations Research*, 52(1), 35–53.
- Bhalla, S., Alfnes, E., Hvolby, H.-H., & Oluyisola, O. (2023). Sales and operations planning for delivery date setting in engineer-to-order manufacturing: A research synthesis and framework. *International Journal of Production Research*, 61(21), 7302–7332. doi: 10.1080/00207543.2022.2148010
- Bortolini, M., Galizia, F. G., & Mora, C. (2018). Reconfigurable manufacturing systems: Literature review and research trend. *Journal of manufacturing systems*, 49, 93–106.

- Burkacky, O., de Jong, M., & Dragon, J. (2022). Strategies to lead in the semiconductor world. *MGI, April, 3*.
- Cheng, Z. (2024). Stochastic dynamic production planning in hybrid manufacturing and remanufacturing system with random usage durations. *International Journal of Production Research*, 62(7), 2331–2349. doi: 10.1080/00207543.2022.2117870
- Chhaochhria, P., & Graves, S. C. (2013). A forecast-driven tactical planning model for a serial manufacturing system. *International Journal of Production Research*, 51(23-24), 6860–6879. doi: 10.1080/00207543.2013.852266
- Dolgui, A., Ivanov, D., & Sokolov, B. (2020). Reconfigurable supply chain: the x-network. *International Journal of Production Research*, 58(13), 4138–4163. doi: 10.1080/00207543.2020.1774679
- Elyasi, M., Altan, B., Ekici, A., Özener, O. Ö., Yanıkoğlu, İ., & Dolgui, A. (2024). Production planning with flexible manufacturing systems under demand uncertainty. *International Journal of Production Research*, 62(1-2), 157–170. doi: 10.1080/00207543.2023.2288722
- Fuji Corporation. (2024). *Nxtr-s smart factory platform*. Retrieved from <https://smt.fuji.co.jp/en/product/nxtr-s>
- Gkournelos, C., Kousi, N., Bavelos, A. C., Aivaliotis, S., Giannoulis, C., Michalos, G., & Makris, S. (2019). Model based reconfiguration of flexible production systems. *Procedia CIRP*, 86, 80–85.
- Hsieh, C.-C., & Lathifah, A. (2022). Ordering and waste reuse decisions in a make-to-order system under demand uncertainty. *European Journal of Operational Research*, 303(3), 1290–1303. doi: 10.1016/j.ejor.2022.03.041
- Ivanov, D., Dolgui, A., & Sokolov, B. (2019). The impact of digital technology and industry 4.0 on the ripple effect and supply chain risk analytics. *International Journal of Production Research*, 57(3), 829–846. doi: 10.1080/00207543.2018.1488086
- Ivanov, D., Tsipoulanidis, A., & Schönberger, J. (2021). Digital supply chain, smart operations and industry 4.0. In D. Ivanov, A. Tsipoulanidis, & J. Schönberger (Eds.), *Global supply chain and operations management* (pp. 521–581). Cham: Springer International Publishing. doi: 10.1007/978-3-030-72331-6_16

- Katoozian, H., & Zanjani, M. K. (2022). Supply network design for mass personalization in Industry 4.0 era. *International Journal of Production Economics*, 244. doi: 10.1016/j.ijpe.2021.108349
- Kazemi Zanjani, M., Noureldath, M., & Ait-Kadi, D. (2010). A multi-stage stochastic programming approach for production planning with uncertainty in the quality of raw materials and demand. *International Journal of Production Research*, 48(16), 4701–4723.
- Khakdaman, M., Wong, K. Y., Zohoori, B., Tiwari, M. K., & Merkert, R. (2015). Tactical production planning in a hybrid Make-to-Stock-Make-to-Order environment under supply, process and demand uncertainties: A robust optimisation model. *International Journal of Production Research*, 53(5), 1358–1386. doi: 10.1080/00207543.2014.935828
- Kumar, A. (2007). From mass customization to mass personalization: A strategic transformation. *International Journal of Flexible Manufacturing Systems : Design, Analysis, and Operation of Manufacturing and Assembly Systems*, 19(4), 533–547. doi: 10.1007/s10696-008-9048-6
- Longauer, D., Vasvári, T., & Hauck, Z. (2024). Investigating make-or-buy decisions and the impact of learning-by-doing in the semiconductor industry. *International Journal of Production Research*, 62(11), 3835–3852. doi: 10.1080/00207543.2023.2250009
- Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust Optimization of Large-Scale Systems. *Operations Research*, 43(2), 264–281.
- Peeters, K., & van Ooijen, H. (2020). Hybrid make-to-stock and make-to-order systems: A taxonomic review. *International Journal of Production Research*, 58(15), 4659–4688. doi: 10.1080/00207543.2020.1778204
- Pereira, D. F., Oliveira, J. F., & Carravilla, M. A. (2022). Merging make-to-stock/make-to-order decisions into sales and operations planning: A multi-objective approach. *Omega*, 107. doi: 10.1016/j.omega.2021.102561
- Prasad, D., & Jayswal, S. (2018). A review on flexibility and reconfigurability in manufacturing system. *Innovation in Materials Science and Engineering: Proceedings of ICEMIT 2017, Volume 2*, 187–200.
- Rafiei, H., & Rabbani, M. (2012). Capacity coordination in hybrid make-to-stock/make-to-order production environments. *International Journal of Production Research*, 50(3), 773–789.

doi: 10.1080/00207543.2010.543174

- Rafiei, H., Rabbani, M., & Alimardani, M. (2013). Novel bi-level hierarchical production planning in hybrid MTS/MTO production contexts. *International Journal of Production Research*, 51(5), 1331–1346. doi: 10.1080/00207543.2012.661089
- Sanei Bajgiran, O., Kazemi Zanjani, M., & Nourelfath, M. (2017). Forest harvesting planning under uncertainty: A cardinality-constrained approach. *International Journal of Production Research*, 55(7), 1914–1929. doi: 10.1080/00207543.2016.1213915
- Soyster, A. L. (1973). Convex Programming with Set-Inclusive Constraints and Applications to Inexact Linear Programming. *Operations Research*, 21(5), 1154–1157.
- Tsubone, H., Ishikawa, Y., & Yamamoto, H. (2002). Production planning system for a combination of make-to-stock and make-to-order products. *International Journal of Production Research*, 40(18), 4835–4851. doi: 10.1080/00207540210158834
- Wang, Y., Ma, H.-S., Yang, J.-H., & Wang, K.-S. (2017). Industry 4.0: A way from mass customization to mass personalization production. *Advances in Manufacturing*, 5(4), 311–320. doi: 10.1007/s40436-017-0204-7
- Xu, L. D., Xu, E. L., & Li, L. (2018). Industry 4.0: State of the art and future trends. *International Journal of Production Research*, 56(8), 2941–2962. doi: 10.1080/00207543.2018.1444806
- Yao, Y., & Xu, Y. (2018). Dynamic decision making in mass customization. *Computers & Industrial Engineering*, 120, 129–136. doi: 10.1016/j.cie.2018.04.025