Effect of Customer Heterogeneity on Retailers' Decisions in a Duopoly

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Abstract

Effect of Customer Heterogeneity on Retailers' Decisions in a Duopoly

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This thesis investigates how customer heterogeneity influences strategic decisions in location-pricing problems within a duopoly setting. The market is modeled as a linear Hotelling line, where two competing retailers must determine their location, pricing, and service strategies. Customers are divided into two segments—premium and economy—each with distinct preferences and reservation prices, capturing market heterogeneity.

Two market entry structures are considered: simultaneous and sequential. In the simultaneous case, interactions are modeled using a Bertrand game; in the sequential case, a Stackelberg game framework is employed. Multiple models are developed to explore how different customer distributions and entry timings affect equilibrium outcomes.

The analysis reveals that customer heterogeneity significantly alters decisions. Retailers facing a heterogeneous market adopt different pricing, location, and service strategies compared to homogeneous settings. Furthermore, the spatial distribution of customers plays a critical role. When customers are uniformly distributed, firms tend to differentiate in location while converging on pricing and service levels. In contrast, non-uniform distributions lead to convergence across all strategic dimensions.

The study also finds that in sequential entry scenarios, the follower may achieve higher profits by offering superior service, especially in uniform markets. However, this advantage diminishes when customer distribution is non-uniform. The results underscore the importance of incorporating customer heterogeneity and spatial variation in analytical models, offering theoretical insights and managerial implications for retail competition in diverse markets.

"Au milieu de l'hiver, j'apprenais enfin qu'il y avait en moi un été invincible."

— Albert Camus

Dedication

To my parents, for always encouraging me to strive for excellence, especially in education and personal growth.

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Chapter 1

Introduction

1.1 Background and Motivation

Consumers who choose to purchase from a specific retailer may vary significantly in their demographics, shopping behaviors, brand preferences, and sensitivity to prices. On the other hand, retail formats differ in terms of marketing strategies, pricing models, store layouts, variety and quality of products, and—importantly—location Bonfrer, Chintagunta, and Dhar (2022); Hansen and Singh (2009).

Recognizing these differences, many firms differentiate their operations to serve distinct customer segments across heterogeneous markets. For example, Walgreens varies its pricing across store locations depending on neighborhood demographics Hansen and Singh (2009). Walmart operates multiple retail formats in Mexico, including supermarkets, hypermarkets, discount stores (Bodega Aurrera), and warehouse clubs. Similarly, Tesco differentiates by offering several store types such as Tesco Express, Tesco Superstores, Tesco Extra, and Tesco Fuel stations Bonfrer et al. (2022).

In fashion retailing, premium brands like Nordstrom and Saks Fifth Avenue have introduced lower-cost alternatives (Nordstrom Rack and Saks OFF 5th), which offer products of lower quality sourced differently. In the food sector, Kroger Inc. runs Fresh Fare, Food 4Less, and Kroger, each tailored to different customer groups Sedghi and Shavandi (2021).

CVS Health, for instance, strategically locates its stores and adjusts services to meet the needs of diverse communities. The company emphasizes community engagement through wellness programs and clinics, aiming to strengthen its presence in various neighborhoods. Additionally, CVS has been proactive in addressing cultural demographic shifts, preparing its business to better serve a changing customer base Llopis (2015).

Target Corporation has integrated Starbucks cafés into over 1,700 of its stores, enhancing the shopping experience by offering customers the convenience of enjoying a beverage during their visit. This collaboration not only increases customer dwell time but also adds value to the in-store experience. Furthermore, Target has introduced the ”Drive Up with Starbucks” service, allowing customers to add Starbucks items to their curbside pickup orders, thereby blending convenience with premium service Dooley (2023).

Costco Wholesale operates on a membership-based model, offering various tiers such as Gold Star and Executive memberships. This structure fosters customer loyalty and provides exclusive benefits, including discounted services and products. Costco's approach emphasizes value through bulk purchasing and member-only services, appealing to a broad customer base seeking quality and affordability Costco Wholesale (n.d.).

Retail competition is intense, especially when multiple retailers operate in the same region Salim, Prayoga, and Al Ikhsan (2023). As a result, retailers often differentiate their format based on the characteristics of the local market or to maintain a competitive edge Sedghi and Shavandi (2021). Since retail format decisions—such as location or core services—are made early and are costly to reverse, they must be supported by careful strategic planning Bonfrer et al. (2022).

Considering the inherent heterogeneity among customers, recent trends in customer differentiation underscore the importance of studying markets composed of individuals with varying preferences, utilities, and reservation prices. Accounting for such diversity is essential for developing realistic models that capture the complex decision-making processes and behavioral patterns observed in modern consumer markets.

1.2 Problem Context and Importance

In a retail duopoly, location decisions significantly shape the nature of competition. A retailer may choose to locate near a rival to benefit from market research spillovers, shared infrastructure, or logistical advantages, such as better road access and delivery systems Picone, Ridley, and Zandbergen (2009). However, proximity can also lead to price wars or reduced margins due to intensified competition, especially in homogeneous markets.

Retailers must also consider the spatial distribution of potential customers, demographic characteristics, and income heterogeneity before entering or expanding within a market Marianov, Eiselt, and Lüer-Villagra (2018). These factors influence not only demand levels but also the relative attractiveness of different store formats. For instance, wealthier neighborhoods may respond positively to premium offerings and additional services, whereas lower-income areas might favor basic formats with low prices.

Another critical factor in retail competition is service quality, which encompasses the retailer's environment, staff responsiveness, and supporting amenities. Studies show that higher perceived service quality leads to greater customer loyalty, repeat visits, and word-of-mouth, which can offset the disadvantage of higher prices or less convenient locations Hasibuan, Siregar, and Harahap (2022); Kumoro and Krisprimandoyo (2023); Pranoto, Haryono, and Assa (2022). The integration of services—such as cafés, pharmacies, or play areas—has emerged as a key strategy for retailers seeking to differentiate themselves.

Pricing strategy remains central to competitive positioning. Retailers aim to find equilibrium prices that balance customer acquisition with profitability Esmaeilzadeh and Taleizadeh (2016); Lu, Jiang, and Chai (2022). In markets where price-sensitive consumers dominate, even small changes in price can lead to significant shifts in market share Wang, Zhao, and Wei (2014). This is particularly important when service levels or distances are similar across competitors.

Moreover, coverage—defined as the spatial region from which a retailer can attract customers—is a fundamental metric in spatial competition models. In many models, such as Hotelling's line Hotelling (1990) or maximal covering location problems Pinto and Parreira (2014), it is assumed that customers select the closest or most beneficial option. Therefore, the joint optimization of location and pricing decisions is crucial to ensure maximum market coverage and profitability Drezner, Drezner, and Kalczynski (2015); Shan et al. (2019).

Understanding the interplay between location and pricing decisions in heterogeneous markets is crucial for firms seeking to optimize performance in competitive environments. In such contexts, ignoring customer diversity can lead to suboptimal strategies that fail to capture market potential or account for differentiated demand sensitivities. This thesis addresses this gap by exploring how firms can strategically position themselves and set prices when faced with spatially distributed customers who differ in preferences, willingness to pay, and sensitivity to non-price attributes such as service variety. By incorporating these real-world complexities into the modeling framework, this research aims to contribute to a deeper and more actionable understanding of competitive dynamics in modern retail and service markets.

1.3 Research Objectives and Scope

This research investigates how market heterogeneity affects retailers' decisions regarding price, location, and quality of service in a duopoly setting. We consider a Hotelling-line market composed of two customer groups: premium and economy. Two competing retailers, offering the same product, must decide simultaneously or sequentially on their location, pricing, and service strategy. The study models how these decisions are influenced by factors such as customer distribution, retailer entry order, and the extent of market coverage.

Building upon the motivation discussed in earlier sections, this thesis aims to model and determine the equilibrium pricing, location, quality, and number of services offered by the retailers under different competitive scenarios. Customer heterogeneity is explicitly captured by dividing the market into two segments—premium and economy—each with distinct preferences and reservation prices. To analyze strategic behavior under different market structures, we consider two entry modes. In the first, both retailers enter the market simultaneously, and their interactions are modeled as a Bertrand game. In the second, retailers enter sequentially, giving rise to a leader–follower dynamic, which is modeled using Stackelberg game theory. This framework allows us to examine how timing and asymmetry in decision-making power influence equilibrium outcomes in heterogeneous markets.

1.4 Research Questions

The research seeks to address the following questions:

(1) Does customer heterogeneity affect retailers' decisions regarding location, quality of services, and the number of services? If so, to what extent? Can customer heterogeneity be reasonably neglected or simplified in modeling without significantly affecting outcomes?

(2) Does the distribution of customers along the market line influence the pricing strategy and location decisions? How do decisions differ when customers are uniformly distributed versus when their distribution is non-linear?

(3) How does the retailers' mode of market entry—simultaneous versus sequential—affect decisions on price, location, and service level? Do equilibrium decisions change when retailers enter the market at the same time compared to when they enter sequentially? Under what conditions might the follower gain an advantage?

(4) Under various market conditions—including differences in customer distribution, market coverage, and entry mode—what is the best strategy for each retailer? Should a retailer focus solely on premium customers, economy customers, or attempt to serve both segments?

Several analytical models are developed and examined throughout this research to address these questions and provide insights into best retailer strategies under varying market conditions.

1.5 Main Findings Overview

The findings of this research highlight the significant impact of customer heterogeneity on retailers' decisions. In heterogeneous markets, the equilibrium outcomes for pricing, location, and service provision differ substantially from those observed in homogeneous settings. This underscores that customer heterogeneity should not be oversimplified or ignored in analytical models, as doing so can lead to misleading conclusions about best strategies.

Customer distribution also plays a critical role in shaping strategic decisions. When customers are uniformly distributed along the market line, competing retailers tend to differentiate in terms of location while converging on identical pricing strategies. In scenarios involving simultaneous market entry, both firms not only select symmetric prices but also adopt the same quality of service. However, under sequential entry, the second entrant typically offers a higher service level to gain a competitive edge, despite entering later.

In contrast, when the market features a non-uniform distribution of customers, both firms tend to converge on identical strategies across all dimensions—pricing, location, and service level—regardless of the timing of market entry. This convergence indicates a strategic flattening effect introduced by non-uniform demand, leading to symmetric equilibrium outcomes even under asymmetric entry conditions.

In the context of optimizing price, location, and the number of services, results reveal that in a uniformly distributed market, the follower sets a higher price and offers more services, ultimately achieving greater profitability than the leader. However, this advantage disappears in the case of non-uniform customer distribution, where both firms adopt identical pricing and service strategies.

1.6 Thesis Structure

The remainder of this thesis is organized as follows. Chapter 2 presents a comprehensive review of the literature on customer heterogeneity, choice models, Hotelling competition, location-pricing games, and service differentiation. Chapter 3 introduces the location-pricing problem in heterogeneous markets and outlines the key modeling assumptions. Chapter 4 develops game-theoretical models to analyze the impact of market heterogeneity on retailers' competition in terms of location, pricing, and service quality. Chapter 5 investigates how customer heterogeneity influences retailers' strategic decisions on location, pricing, and the number of services in the presence of an incumbent competitor. Finally, Chapter 6 concludes the thesis by summarizing the main findings and discussing their theoretical and managerial implications, along with potential avenues for future research

Chapter 2

Literature Review

The aim of this chapter is to provide a comprehensive overview of the existing literature related to location-pricing competition in retail markets, with a focus on game-theoretical modeling. The chapter begins by exploring foundational concepts in customer heterogeneity and differentiation followed by the classical and extended Hotelling models of spatial competition, which form the theoretical basis for many location-pricing frameworks. We then review key factors influencing customers' choice of retailers, such as price, service quality, and perceived value. The chapter continues with a detailed review of location-pricing games, including both simultaneous and sequential decision models, and concludes with recent advancements that incorporate customer heterogeneity and service differentiation. Finally, we review various customer choice models used to capture consumer behavior in competitive environments. This review lays the foundation for the development of the game-theoretical models proposed in this thesis.

2.1 Customer Differentiation and Heterogeneity

A central consideration in modeling market interactions is the diversity of consumer preferences. In the context of location-pricing games, this diversity can be modeled in two distinct but related ways: customer heterogeneity and customer differentiation. While often used interchangeably, these concepts have subtle differences with significant implications for modeling and strategic decisions.

2.1.1 Conceptual Framework

Customer heterogeneity refers to the natural variation among consumers in terms of preferences, sensitivities, and socio-economic attributes. This heterogeneity is typically treated as exogenous and is captured via continuous distributions or discrete segments (e.g., income levels, travel sensitivity, brand loyalty). In contrast, customer differentiation arises endogenously from firm strategies such as pricing, location, and service design, where firms deliberately segment the market to target distinct groups Moorthy (1984); Yang and Allenby (2003).

Modeling these variations is essential for understanding how firms can exploit asymmetries in demand to optimize profit. Both horizontal (taste-based) and vertical (quality- or price-based) differentiation are commonly embedded in spatial and pricing games Tirole (1988). In spatial models, heterogeneity often manifests as variation in customers' ideal points or willingness to travel, while in pricing games, it may appear as differing elasticities or reservation prices.

2.1.2 Classical Models of Heterogeneity and Differentiation

The heterogeneity of customer preferences has been a central theme in market analysis, particularly in pricing, product design, and location models. Early works such as Hotelling (1929) introduced spatial heterogeneity through consumer location, while Mussa and Rosen (1978) and Moorthy (1984) developed seminal models of vertical differentiation based on willingness to pay. Extensions by d'Aspremont, Gabszewicz, and Thisse (1979) addressed deficiencies in Hotelling's original specification by introducing quadratic transportation costs to ensure interior equilibria and more stable competition dynamics.

Varian (1980) introduced the idea of modeling the sales by considering consumer heterogeneity in the context of informational asymmetries. His model showed how firms might randomize prices or product offerings to serve distinct consumer types (e.g., informed vs. uninformed), leading to endogenous price dispersion even under symmetric competition.

Lacourbe (2012) emphasized that customers vary in their reservation utilities, highlighting the necessity of incorporating such differences into product line design. Anderson, de Palma, and Thisse (1992) provided a comprehensive framework for discrete choice and product differentiation under heterogeneous preferences, laying the groundwork for much of the subsequent literature.

More recent studies have explored markets segmented into discrete customer classes. Zhong, Pan, Xie, Cheng, and Lin (2020) and Cao, Wang, and Xie (2019) modeled two-segment markets, examining how firms adjust pricing, service levels, and product features in response to heterogeneity. DeSarbo, Ebbes, Fong, and Snow (2010) emphasized profitability-based segmentation and its implications for strategic targeting.

Theoretical contributions by Gabszewicz and Thisse (1979), Gabszewicz and Thisse (1980) and Tirole (1988) formalized customer heterogeneity in duopoly and oligopoly models. Additionally, empirical and game-theoretical studies such as those by Seim (2006), Draganska and Jain (2006), and Dubé, Hitsch, and Chintagunta (2010) further demonstrate how differentiation and segmentation shape competitive dynamics and firm profitability. Together, this body of research establishes customer heterogeneity—whether in price sensitivity, location, or quality perception—as a foundational consideration in location-pricing models and competitive strategy.

2.1.3 Strategic Firm Responses to Heterogeneous Markets

In the presence of heterogeneous demand, firms often adopt differentiation strategies to segment the market and extract greater consumer surplus. These strategies can involve spatial separation, differentiated pricing schemes, or varying levels of service quality. In game-theoretical models, firms anticipate the diverse preferences of customer groups and adjust their decisions accordingly, often in the form of two-stage or multi-stage games where pricing and location are chosen sequentially.

Vertical differentiation has been extensively used to model firm competition where customers differ in their willingness to pay for quality. Moorthy (1984) analyzed quality-based competition under duopoly and showed that when consumers are heterogeneous, firms have an incentive to position themselves at different quality levels to avoid price wars. This insight has direct implications for service-level decisions in location-pricing models, where service offerings can act as a proxy for quality.

In horizontally differentiated markets, spatial positioning plays a central role in firm strategy. d'Aspremont et al. (1979) demonstrated that when consumers are sensitive to location, firms choose to differentiate spatially to soften price competition.

Some models combine vertical and horizontal differentiation by allowing firms to vary both location and service quality or price simultaneously. These integrated frameworks show that customer heterogeneity increases equilibrium asymmetry: firms may locate closer to the more profitable segment and tailor their offerings to match specific sensitivities in travel cost, price elasticity, or quality perception Tabuchi (1994).

2.1.4 Summary

The literature on customer differentiation and heterogeneity reveals the central role of demand-side diversity in shaping firms' strategic decisions in pricing and spatial competition. Classical models established the foundational distinction between vertical and horizontal differentiation, while more recent approaches have expanded the analytical toolkit to capture increasingly complex market structures and consumer preferences. As firms face markets composed of distinct customer groups—whether defined by spatial location, price sensitivity, or service expectations—they must design strategies that align with heterogeneous demand patterns. These insights are particularly relevant for location-pricing games, where firm decisions on where to locate and how much to charge are deeply influenced by the distribution and composition of the customer base. Building on these contributions, the following section reviews the theoretical foundations and extensions of spatial competition, with particular attention to Hotelling-type models.

2.2 Hotelling-Type Spatial Competition

Ever since the competition for locating on a linear space was introduced by Hotelling (1929), many researchers studied variants of the problem. In the famous Hotelling problem, demand points are uniformly distributed on a line with finite length. Two retailers who sell the same product at a similar price compete on their location and try to optimize their position in a way that each can maximize her own market share. Each customer will travel to the nearest retailer to satisfy her demand. The travelling cost of the customers is linearly proportional to the traveling distance.

While Hotelling's original analysis laid the groundwork, later studies refined its assumptions and resolved technical issues. d'Aspremont et al. (1979) addressed discontinuities in demand by considering quadratic transportation costs, which led to existence and uniqueness of equilibrium and changed the strategic outcome from minimum to maximum differentiation. The circular city model of Salop (1979) eliminated boundary effects and enabled symmetric equilibrial in multi-firm settings. Additional contributions by Gabszewicz and Thisse (1980) and Eaton and Lipsey (1975) investigated the dynamics of entry and competition in differentiated industries.

Pinto and Parreira (2015) study a variant of the Hotelling problem in which the location of the two firms is fixed, and they only play a sub-game to decide on prices, with incomplete information about the production costs in each firm. Hernandez (2011) studies a Hotelling model in which products are either high quality or low quality. Their study shows that the degree of competition affects the ratio of the price of the two product types. Peng, Lu, Wu, Zhao, and Xiao (2020) analyze a new variant of the Hotelling problem in which the market is in the shape of a triangle, and three retailers try to optimize their locations in the existence of competition between them. Bar-Gill (2020) further considers the fact that customers are prone to making mistakes when choosing retailers. They quantify the effect of such mistakes in the Hotelling model. Kharbach and Chfadi (2022) consider the logistic costs of the retailers (such as transportation cost from the original firm to the retailer) in the original Hotelling model, and show that these costs can affect the equilibrium location and price strategies of the retailers. Variants of the Hotelling problem are further studied by Balestrieri and Izmalkov (2016) and Levanova and Gnusarev (2020), among others.

Together, these studies show that the Hotelling framework and its many extensions are powerful tools to study location decisions, price competition, product quality, customer behavior, and logistical considerations in spatially differentiated markets.

2.3 Factors Influencing Customer Choice of Retailers

Several factors contribute to customers' willingness to buy from a retailer. Among the most influential are price, product quality, level of service, promotional offers, and the selling channel. Unsurprisingly, the price offered by a retailer directly affects the number of potential customers and, consequently, the retailer's profit. Demand for a product is typically inversely related to its price: the lower the price, the higher the demand. For this reason, retailers aim to optimize their pricing strategies to maximize both demand coverage and market power Esmaeilzadeh and Taleizadeh (2016). In broader interpretations, price also encompasses time spent shopping and travel costs incurred by customers Panjaitan, Simanjorang, and Syahputra (2022); Yulistiawan, Dewi, Hananto, et al. (2023).

Price and service quality are often interdependent. A retailer offering higher prices may be able to justify them by providing superior services Panjaitan et al. (2022). Service quality has been empirically shown to be a critical factor influencing customer loyalty and, thus, retailer market share Li and Qi (2021).

Another significant determinant is coverage—defined as the maximum distance or radius a retailer can effectively serve. This concept is particularly relevant in spatial competition models, where customer locations influence market share Drezner et al. (2015). In competitive environments, customers tend to patronize the nearest retailer, as illustrated by classic Hotelling-type models Hotelling (1929); Pinto and Parreira (2014). Therefore, equilibrium location decisions are essential for maximizing market share Shan et al. (2019).

Beyond price and location, consumers consider the overall value they expect to derive from a purchase, which includes convenience, perceived fairness, and product availability Zeithaml (1988). In modern retail environments, selling channel—whether physical store, online platform, or hybrid—is a growing consideration. Research shows that customers often choose a retailer based not just on price but on the flexibility and reliability of the channel Verhoef, Kannan, and Thomas (2015). Omnichannel retail strategies that blend in-store and online experiences have been found to increase customer satisfaction and loyalty Juaneda-Ayensa, Abad, and Murillo (2016).

Empirical studies reinforce the impact of price, product quality, and service level on customer purchasing behavior. For example, Chaerudin and Syafarudin (2021) demonstrate through statistical analysis that retailer provided services significantly influence customer choice. Venkatesan, Mehta, and Bapna (2006) show that online retail markets exhibit price differentiation, where variations in service level and customer awareness enable retailers to pursue distinct pricing strategies.

Overall, customer utility depends on the price offered and the distance to the retailer Qi, Jiang, and Shen (2022); Fernández and Hendrix (2013). Customers act independently to maximize their own utility, considering service level, accessibility, brand reliability, and personal preferences Lin and Tian (2021); Levanova and Gnusarev (2020).

2.4 Location-Pricing Games

Location and pricing decisions are fundamental strategic choices for any retailer. As noted by Aboolian, Berman, and Krass (2008), location decisions are typically long-term and difficult to reverse, while pricing strategies are short-term and more flexible. Despite the fact that several studies have attempted to optimize location and pricing decisions separately, Hanjoul, Hansen, Peeters, and Thisse (1990) argue that treating these decisions independently often leads to suboptimal results. Since location decisions are based on the distribution of demand, and demand itself is sensitive to price, joint optimization is essential when retailers compete over market share.

The foundational work of Hotelling (1929) introduced spatial competition through the linear city model, in which firms compete by choosing locations. However, this model assumed fixed prices and homogeneous customer preferences. d'Aspremont et al. (1979) extended this framework by allowing simultaneous competition in both price and location, and demonstrated that equilibrium exists under quadratic transportation costs. Their work is considered one of the earliest models of location-pricing games.

A number of recent studies have analyzed joint location and pricing decisions in more complex environments. For instance, Esmaeilzadeh and Taleizadeh (2016) examine optimal pricing in a two-echelon supply chain under asymmetric market power. Sadeghi Dastaki, Setak, and Karimi (2021) investigate simultaneous decisions on location, pricing, and routing, proposing heuristic algorithms to solve the problem. Similarly, Zhang (2015), Diakova and Kochetov (2012), He, Cheng, Dong, and Wang (2013), Čvokić and Stanimirović (2020), and Thiel (2020) have developed models that jointly optimize pricing and location decisions, using a variety of analytical and computational methods. A recent review of competitive facility location models is provided in Mishra, Singh, and Gupta (2022).

In general, location-pricing games can be modeled as either simultaneous or sequential decision problems. In simultaneous-move games, firms decide on prices and locations at the same time, which can lead to analytical complexity and multiple equilibria Zhang (2015); Diakova and Kochetov (2012). Sequential models, often structured as Stackelberg games, assume that firms choose locations first and adjust prices later, allowing analysis of leader-follower dynamics He et al. (2013); Čvokić and Stanimirović (2020). These structures are particularly relevant in competitive retail markets where physical positioning is a long-term commitment, but price changes can occur dynamically.

Moreover, recent literature has introduced customer heterogeneity and differentiated services into location-pricing models. Esmaeilzadeh and Taleizadeh (2016) allow for partial market coverage and varying customer sensitivity to price, while Sadeghi Dastaki et al. (2021) include routing and service aspects in their optimization. This is in line with growing attention to behavioral and service-related factors in strategic decision-making, especially when competing retailers aim to capture distinct customer segments.

Given these developments, this thesis builds on the existing literature by formulating a game-theoretic model in which retailers compete on location, pricing, and service levels within a heterogeneous market. In contrast to models that assume a uniform customer base or identical service offerings, the proposed model accounts for two distinct customer groups with different sensitivities to price and service quality. Such integration contributes to the literature by addressing the strategic implications of customer differentiation in spatial competition.

2.4.1 Customer Heterogeneity and Service Differentiation in Location-Pricing Games

In competitive retail markets, customer heterogeneity and service differentiation have become crucial factors in shaping location-pricing strategies. Traditional models often assume homogeneous consumer preferences, but recent research has highlighted the limitations of such assumptions and extended the classical Hotelling framework to capture richer customer behavior and firm differentiation.

Vogel (2011) analyze a four-stage location-pricing game in which heterogeneous firms engage in spatial price discrimination. The study reveals that more productive firms tend to locate farther from competitors, thereby serving a wider range of customers at varying price points. This result underscores the strategic advantage of geographic isolation when customer preferences are heterogeneous.

Liang, Wang, and Zhang (2011) consider a duopoly setting in which firms simultaneously choose their locations and implement discriminatory pricing. They find that product and spatial differentiation becomes more pronounced as customer heterogeneity increases, allowing firms to mitigate price competition and capture distinct market segments. Their model demonstrates the non-trivial interaction between spatial location and differentiated service offerings in equilibrium outcomes.

Building on the role of uncertainty in consumer preferences, Yao, Zhao, Chen, Xu, and Long (2017) develop a modified Hotelling model where firms face uncertain demand from a heterogeneous population. They show that disadvantaged firms—such as late entrants or those with inferior service capabilities—can still survive by adopting location and pricing strategies that target niche segments of the market. This work highlights how heterogeneity in preference or service valuation can create competitive room for multiple firms.

In a Bertrand competition framework, Cohen, Jacquillat, and Song (2023) introduce capacity constraints and inventory allocation across customer types, incorporating both vertical differentiation and price discrimination. Their results suggest that firms can strategically allocate limited resources to high-value customer segments and extract surplus through differentiated pricing policies. This has implications for firms operating in service-based industries where capacity is constrained and demand is non-uniform.

These models collectively demonstrate that accounting for customer heterogeneity and service differentiation significantly alters strategic outcomes in location-pricing games. Firms not only position themselves to maximize market coverage but also calibrate their prices and service levels to match the preferences and sensitivities of different customer groups. In this thesis, such insights are embedded into a model that features two distinct consumer classes and allows retailers to compete through a combination of price, location, and service differentiation.

2.5 Customer Choice Models

Understanding how customers make choices is foundational to modeling market dynamics, especially in location-pricing games where firms compete on spatial positioning, pricing, and service differentiation. Customer choice models serve as the analytical bridge between firm decisions and customer behavior, allowing researchers to predict market share and optimize strategic variables accordingly.

2.5.1 Classical Discrete Choice Models

The most widely used approach in modeling customer preferences is the discrete choice framework, particularly the multinomial logit (MNL) model. In its standard form, the MNL assumes that each customer selects the option that provides the highest utility, with utility decomposed into systematic (observable) and random (unobservable) components. The probability that customer i chooses product j is given by:

$$P_{ij} = \frac{\exp(V_{ij})}{\sum_{k=1}^{J} \exp(V_{ik})} \tag{1}$$

where V_{ij} is the deterministic utility component for alternative j, often modeled as a function of price, distance, and product characteristics McFadden (1974).

Despite its elegance and computational simplicity, the MNL model is criticized for its independence of irrelevant alternatives (IIA) property, which implies unrealistic substitution patterns. To address this, researchers have extended the model to nested logit Ben-Akiva and Lerman (1985), allowing for correlated alternatives within pre-defined nests, and mixed logit Train (2009), which incorporates customer-level heterogeneity through random coefficients.

2.5.2 Extensions for Customer Heterogeneity

To capture more realistic consumer behavior, many researchers have introduced heterogeneity in preferences through random utility models. The mixed logit model (also known as the random parameters logit) allows utility coefficients to vary across individuals, thus capturing variations in sensitivity to price, distance, and service features Train (2009). Another extension is the latent class model, which assumes that the population consists of several unobserved segments, each with distinct preference structures Greene and Hensher (2003). These models are particularly useful in capturing discontinuities in customer behavior, which are often observed in multi-product or multi-service markets.

2.5.3 Behavioral Choice and Perception Effects

Classical models generally assume rational, utility-maximizing agents. However, empirical evidence suggests that real customers often deviate from this ideal due to bounded rationality, heuristics, and emotional biases Simon (1955). Behavioral choice models incorporate these deviations, offering more accurate demand predictions under certain market conditions.

An important behavioral dimension is the perception of quality. Customers often make choices not only based on observable product attributes but also on perceived value, brand reputation, and service experience. In location-pricing games, this means that even spatially disadvantaged retailers can capture market share if their perceived quality compensates for other shortcomings.

Furthermore, studies have shown that reference price effects and loss aversion play critical roles in shaping customer preferences Winer (1986). These effects can distort traditional demand functions and lead to nonlinear, asymmetric price responses, which significantly affect equilibrium pricing and positioning strategies.

2.5.4 Applications in Location-Pricing Games

In location-pricing games, customer choice models are instrumental in linking firm strategies to market outcomes. Retailers choose locations and prices to maximize profit, while customers evaluate options based on utility derived from price, distance, and other factors—such as perceived quality. The MNL model is commonly embedded in these spatial models due to its tractability, enabling closed-form expressions for demand and profit functions Dukes and Gal-Or (2003).

In more complex settings, mixed logit and nested logit models allow for asymmetric responses and multiple competing firms.

Recent contributions emphasize the importance of modeling perceived service quality alongside price and distance. Firms may be able to offset locational disadvantages by investing in quality enhancements or branding strategies that shift perceived utility. In such models, equilibrium pricing and positioning depend not only on cost structures but also on how customers interpret quality signals Nevo (2001); Erdem and Swait (1998).

2.5.5 Critical Review and Research Opportunities

While the existing literature provides a robust framework for modeling customer choice, several limitations remain. First, many models assume static environments where customer preferences and market structure are fixed. In reality, customer preferences evolve over time due to external factors (e.g., social influence, online reviews), which are rarely accounted for in location-pricing models.

Second, although advanced models like mixed logit can incorporate heterogeneity, they often require intensive data and computation, which can limit their practical application in real-time decision-making contexts. Simplified approximations, while computationally appealing, may fail to capture the nuanced behaviors of segmented or highly differentiated markets. Third, perceived quality —though acknowledged in some models—remains underexplored in spatial competition. Most formulations treat quality as exogenous or perfectly observed, whereas empirical studies suggest that customer perception is shaped by complex, context-dependent cues. Modeling how these cues interact with location and pricing decisions presents a promising avenue for future research.

Lastly, incorporating behavioral insights—such as fairness perceptions, inertia, or reference dependence—can enrich the predictive power of existing models. Doing so may help explain market anomalies where traditional utility-based frameworks fall short, such as why some customers remain loyal despite inferior offers in price or proximity.

2.5.6 Summary

In summary, customer choice models provide a crucial analytical foundation for studying firm behavior in location-pricing games. From classical MNL formulations to more sophisticated extensions that account for heterogeneity, perception, and behavioral deviations, these models enable researchers to capture the diversity and complexity of real-world consumer decision-making. As competition in modern markets increasingly hinges on differentiated services and subjective customer experiences, incorporating elements such as perceived quality and bounded rationality becomes essential. Future research can build on these foundations by integrating dynamic choice behavior, contextual perception, and data-driven personalization into strategic firm-level models.

2.6 Use of Hotteling models over customer choice models

In this thesis, we examine the effect of the spatial distribution of customers on retailers' equilibrium decisions, along with the effect of customer heterogeneity. While customer choice models are effective in capturing heterogeneity in consumer preferences and allowing for flexible representations of differentiation, they typically abstract away from the explicit role of geography. As a result, they may not fully capture how location influences competitive interactions among retailers.

In contrast, the Hotelling model provides a natural and tractable framework for incorporating both the spatial distribution of customers and the strategic positioning of retailers. By embedding consumers along a geographical (or characteristic) space, this class of models allows retailers to make simultaneous decisions regarding location, pricing, and level of differentiation. Moreover, it captures the trade-off between attracting a larger market share by moving closer to competitors (leading to partial or full dedifferentiation) and maintaining distinctiveness to soften price competition.

Therefore, Hotelling-type models are particularly well suited for analyzing how spatial considerations interact with competitive strategies, enabling a richer understanding of equilibrium outcomes in markets where both location and differentiation play a central role.

2.7 Conclusion

This chapter has reviewed the relevant literature on location-pricing competition and strategic retail decision-making. It began with a discussion on the role of customer heterogeneity and differentiation in shaping competitive strategies. We then explored classical and extended Hotelling models, which form the theoretical foundation for spatial competition. Followed by an overview of key factors that influence customer choices, including price, quality, and service attributes. Subsequently, we reviewed game-theoretical models that integrate location and pricing decisions, highlighting both simultaneous and sequential approaches. Finally, various customer choice models were examined to understand how consumers select among competing retailers based on utility-based frameworks. The insights gathered from the literature point to a research gap in modeling competitive strategies that simultaneously account for location, pricing, service levels, and heterogeneous customer preferences. The next chapter introduces a formal game-theoretical model that addresses these dimensions in a unified framework.

Chapter 3

Competition for Price and Location

3.1 Problem definition

In this chapter, we model the joint location and pricing decisions of two retailers in a heterogeneous market comprising two types of customers. Both retailers offer the same product. Premium customers exhibit a higher willingness to pay, indicating a greater reservation price for the product. Conversely, economy customers possess a lower willingness to pay and may be unwilling or unable to bear high costs for the product. The market is conceptualized as a Hotelling line.

The product may be every day low value products like groceries, or one time high value purchases like cellphones. The model and the results are valid in both situations.

We explore two distinct cases. Initially, we examine the problem assuming that both customer groups are uniformly distributed across the market.

Following this, we explore cases where each customer group is concentrated at one end of the demand line.

Two primary situations are studied within each case. First, both retailers simultaneously enter the market, modeled using Bertrand game theory. Second, one retailer (the leader) is already present in the market, and the second retailer (follower) decides on market entry, location, and pricing strategy based on the leader's prior decisions. To address the latter, Stackelberg game theory is employed.

3.1.1 Notations

To model the problem, some parameters and variables must be defined:

Parameters:

R_P: Reservation price of premium customers.

R_E: Reservation price of economy customers.

λ :Length of the demand line.

t: Customers' transportation cost per unit distance.

γ: Fraction of market consisted of premium customers.

Π_i : Profit function of retailer i. $i \in \{1, 2\}$.

D_{ij}: Demand of customer type j satisfied by retailer i where $i \in \{1, 2\}$, $j \in \{P, E\}$.

I_{ij}: The interval of the demand of customer type j, being covered by retailer i. $i \in \{1, 2\}$, $j \in \{P, E\}$.

Decision variables:

P_i: price set by retailer i, $i \in \{1, 2\}$.

X_i: location of retailer i along the X axis,$i \in \{1, 2\}$.

We consider the market to be a linear space, from 0 to λ. Also, γ percent of the customers are premium, and $(1 - \lambda)$ percent are economy customers.

Without loss of generality, let's assume that retailer 1 will locate on the left side, and retailer two will locate on the right side of the Hotelling line. Let's denote the location of retailer one as X_1 and the location of retailer two as X_2. Figure 5.1 shows the relative location of the retailers in the market.

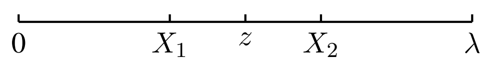

Figure 3.1: Depiction of the decision space

Consider an arbitrary customer located at point y on the line. The utility for this customer to buy from retailer one is the reservation price of the customer for the product minus the traveling cost to retailer one and the price that retailer one sets:

$$U = R - p_1 - t|(y - X_1)| \quad (2)$$

There are customers both to the right and to the left of retailer one who wish to buy from them. Some customers are indifferent between buying a product from retailer one or not buying a product at all, meaning their utility to buy from retailer one is zero. This condition sets U equal to zero:

$$|y - X_1| = \frac{R_j - p_1}{t}, j \in \{P, E\} \tag{3}$$

According to (193), the farthest premium customer that can be served by retailer one is located $\frac{R_P - P_1}{t}$ units away in either direction (right side or left side of retailer one). Similarly, the farthest economy customer who may choose to buy from retailer one is situated $\frac{R_E - P_1}{t}$ units away in either direction.

Figure 5.2 illustrates this. The orange line represents the range of premium customers who might choose retailer one, while the blue line indicates the range of economy customers willing to purchase from retailer one.

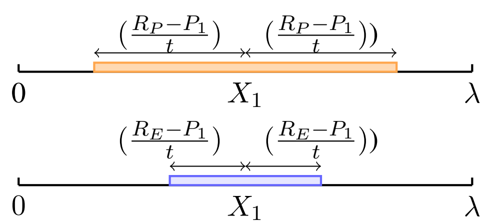

Figure 3.2: Maximum distance a retailer can serve

Similarly, the farthest premium and economy customers that can be served by retailer two are located at $\frac{R_P - P_2}{t}$ and $\frac{R_E - P_2}{t}$ unit distances away from it, respectively.

We assume that a customer will make a purchase from a retailer only when their utility for buying products from that retailer is positive. It is important to highlight that if the utility for purchasing from both retailers is positive, the customer will choose the retailer offering the higher utility for their purchase.

Additionally, let's contemplate a point z situated between the two points X_1 and X_2, where a customer is indifferent regarding the choice between purchasing from retailer one and retailer two.That is:

$R_P - P_1 - t(z - X_1) = R_P - P_2 - t(X_2 - z)$

$$z = \frac{X_1 + X_2}{2} + \frac{P_2 - P_1}{2t} \tag{4}$$

Considering (4), the intuitive interpretation is that if P_1 and P_2 are equal, the customer positioned precisely at the midpoint between the two retailers remains indifferent about buying from either retailer, signifying an equal market share for both retailers. If $P_2 > P_1$, the indifferent customer leans toward the right side of the line, indicating a greater market share for retailer one. Conversely, if $P_2 < P_1$, the indifferent customer leans toward the left side of the line, highlighting a larger market share for retailer two.

The interval that each retailer can serve for each customer group is as follows:

$$
\begin{aligned}
I_{1_P} &= [max\{0, X_1 - \frac{R_P - P_1}{t}\}, min\{X_1 + \frac{R_P - P_1}{t}, \frac{X_1 + X_2}{2} + \frac{P_2 - P_1}{2t}\}] \\
I_{1_E} &= [max\{0, X_1 - \frac{R_E - P_1}{t}\}, min\{X_1 + \frac{R_E - P_1}{t}, \frac{X_1 + X_2}{2} + \frac{P_2 - P_1}{2t}\}] \\
I_{2_P} &= [max\{\frac{X_1 + X_2}{2} + \frac{P_2 - P_1}{2t}, X_2 - \frac{R_P - P_2}{t}\}, min\{\lambda, X_2 + \frac{R_P - P_2}{t}\}] \\
I_{2_E} &= [max\{\frac{X_1 + X_2}{2} + \frac{P_2 - P_1}{2t}, X_2 - \frac{R_E - P_2}{t}\}, min\{\lambda, X_2 + \frac{R_E - P_2}{t}\}]
\end{aligned} \tag{5}
$$

To simplify (5), let's assume that retailer one is initially located at point 0. While she is willing to move to the right to attract more customers, even drawing some from retailer two, she avoids moving too far to the right to prevent losing customers on the left side. Therefore, the leftmost premium customer covered by retailer one is located at zero, resulting in X_1 being at $\frac{R_P - P_1}{t}$. Given that the maximum covering distance for economy customers is less than that for premium customers, the leftmost economy customer covered by retailer one is at $X_1 - \frac{R_E - P_1}{t}$. Similarly, the rightmost premium customer covered by retailer two is at point λ, making X_2 at $\lambda - \frac{R_P - P_2}{t}$. Additionally, the rightmost economy customer covered by retailer two is at $X_2 + \frac{R_E - P_2}{t}$. Figure 5.3 demonstrates these relations.

$$
X_1 = \frac{R_P - P_1}{t} \tag{6}
$$

$$X_2 = \lambda - \frac{R_P - P_2}{t} \tag{7}$$

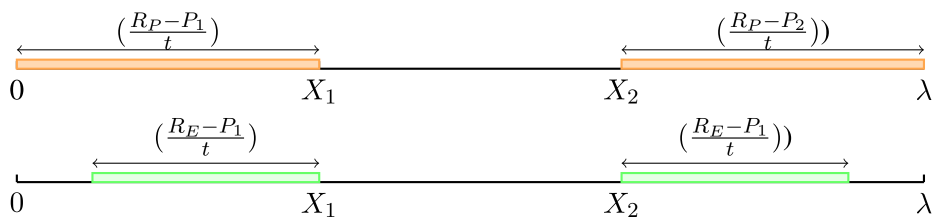

Figure 3.3: The equilibrium location of the two retailers based on the problem parameters

Having determined the location of retailers, X_1 and X_2, we can simply find point z, where a customer, (regardless of their group), is indifferent between choosing either retailer.

$$z = \frac{X_1 + X_2}{2} + \frac{P_2 - P_1}{2t} = \frac{\lambda}{2} + \frac{P_2 - P_1}{2t} \tag{8}$$

3.2 Customers are uniformly distributed along the demand line and retailers enter the market simultaneously

In this situation, no retailer previously existed in the market, and both retailers will enter the market simultaneously. Hence, Bertrand game theory will be used to model and solve the problem. Each retailer must set a price. They must set a lower price if they want to attract both customer groups and can set a higher price if they want to focus only on premium customers.

Three different cases may occur by comparing $(X_1 + \frac{R_P - P_1}{t})$, $(X_2 - \frac{R_P - P_2}{t})$, and z. The problem should be solved separately in each of these three cases:

3.2.1 Both premium and economy customers are partially served

Here, the coverage range for each retailer is relatively short, resulting in non-overlapping potential markets for retailer one and retailer two. This situation is illustrated in figure 3.4, where the potential market segment of retailer one is shown in red, and the potential market segment for retailer two is depicted in blue. Such a situation occurs when travel costs are high and reservation prices for both customer groups are low. Because there is no overlap between the two markets, no competition arises, allowing each retailer to maximize their profit independently of the other.

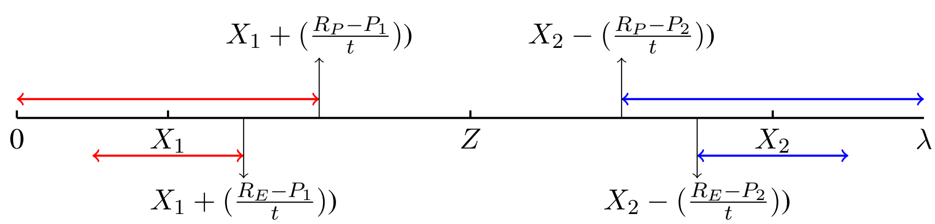

Figure 3.4: Partial coverage of both customer groups

It can be inferred from figure 3.4 that:

$$
\begin{aligned}
X_1 + \frac{R_P - P_1}{t} < Z \\
X_2 - \frac{R_P - P_2}{t} > Z \\
\Rightarrow P_1 + P_2 > 2R_P - \frac{\lambda t}{2}
\end{aligned}
\tag{9}
$$

After solving the problem, we must check to see if the equilibrium prices satisfy the conditions in (9) .

The profit function of each retailer i is the sum of the premium and economy demand the retailer can cover, multiplied by the price the retailer sets. It should be noted that γ percent of customers are premium an $(1-\gamma)$ percent are economy customers. Therefore, the number of premium customers retailer i can serve is $2\gamma D_{i_P}$ and the number of economy customers she can serve is $2(1-\gamma)D_{i_P}$.

$$
\pi_i = 2(\gamma D_{i_P} + (1-\gamma)D_{i_E})P_i \tag{10}
$$

To normalize the profit function, we divide the sum of the length of the demand line being covered, by λ.

$$
\pi_i = P_i \times \frac{2}{\lambda} \times (\gamma \frac{R_P - P_i}{t} + (1-\gamma)\frac{R_E - P_i}{t}) \tag{11}
$$

The first-order condition results:

$$
P_i^* = \frac{\gamma R_P + (1-\gamma)R_E}{2} \tag{12}
$$

$$\pi_i^* = \frac{(\gamma R_P + (1-\gamma)R_E)^2}{2t\lambda} \tag{13}$$

Checking the initial conditions, P_1^* and P_2^* are equilibrium prices when $(1-\gamma)R_E + \frac{\lambda t}{2} < (2-\gamma)R_P$.

Looking through the equilibrium results, the two retailers set the same price, but choose different locations, and earn the same profit. Moreover, the equilibrium price is a weighted average of the reservation price of the two customer groups.Increase in the reservation price of customers will increase the equilibrium price the retailers sets, as well as the profit they earn.

3.2.2 All premium customers and part of economy customers can be served

This case arises when each retailer has the capacity to potentially serve more than half of the premium customers but less than half of the economy customers. In this situation, competition ensues between retailer one and retailer two for dominance in the premium customers' market. Retailer one caters to premium customers situated between points 0 and Z, while retailer two serves premium customers located from point Z to point λ.

For economy customers, there is a clear lack of overlap in potential markets. Specifically, the rightmost economy customer that retailer one can accommodate is positioned to the left of point Z, and the leftmost economy customer that retailer two can serve is situated to the right of point Z. Figure 5.7 provides a visual depiction of this situation.

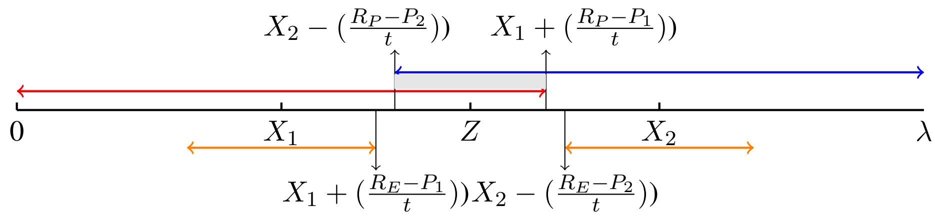

Figure 3.5: Overlap only in premium customers' market

In Figure 5.7, the red line represent the potential market of premium customers for retailer one, while the blue line illustrates the potential market of premium customers for retailer two. Customers within the gray shaded segment of the demand line experience positive utility for purchasing from either retailer. As assumed earlier, customers in this segment will choose to make a purchase from the retailer that offers them greater utility. Therefore, premium customers located to the left of point Z choose retailer one, while those to the right of point Z choose retailer two.

The potential markets of economy customers for retailer one and two are shown in orange. It is evident that the potential markets for economy customers do not overlap.

Considering the potential markets of each retailer for premium and economy customers in this case:

$$\begin{aligned} X_1 + \frac{R_P - P_1}{t} > Z \\ X_1 + \frac{R_E - P_1}{t} < Z \\ X_2 - \frac{R_P - P_2}{t} < Z \\ X_2 - \frac{R_E - P_2}{t} > Z \\ \Rightarrow R_P + R_E - \frac{\lambda t}{2} < P_1 + P_2 < 2R_P - \frac{\lambda t}{2} \end{aligned} \tag{14}$$

The demand that each retailer covers for each of the two customer groups can be determined:

$I_{1,P} = [0, Z] \Rightarrow D_{1,P} = \frac{1}{\lambda} \times \gamma(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t})$

$I_{2,P} = [0, Z] \Rightarrow D_{2,P} = \frac{1}{\lambda} \times \gamma(\frac{\lambda}{2} + \frac{P_1 - P_2}{2t})$

$D_{i_E} = \frac{1}{\lambda} \times 2(1 - \gamma)(\frac{R_E - P_i}{2t})$

Having determined the amount of demand for each retailer, we can form the profit functions:

$$\pi_1 = \frac{1}{\lambda} \times P_1[\gamma(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t}) + 2(1 - \gamma)\frac{R_E - P_1}{t}] \tag{15}$$

$$\pi_2 = \frac{1}{\lambda} \times P_2[\gamma(\frac{\lambda}{2} + \frac{P_1 - P_2}{2t}) + 2(1 - \gamma)\frac{R_E - P_2}{t}] \tag{16}$$

The first-order condition results:

$$P_1^* = P_2^* = \frac{R_E}{2} + \frac{\gamma \lambda t}{8(1 - \gamma)} \tag{17}$$

$$\pi_1^* = \pi_2^* = R_E(\frac{\gamma}{4} + \frac{(1-\gamma)}{\lambda t}) + \frac{\gamma^2 \lambda t}{16(1-\gamma)} - \frac{\gamma}{4} \tag{18}$$

An increased ratio of premium customers leads to a higher equilibrium price. Additionally, the equilibrium price increases linearly with an increase in the reservation price of economy customers.

As γ increases, the equilibrium profit also increases for the same value of R_E. Additionally, the figure demonstrates the linear relationship between equilibrium profit and R_E for fixed values of γ.

Finally, for the equilibrium decisions to constitute a Nash equilibrium, condition (14) must be satisfied:

$$R_P + R_E - \frac{\lambda t}{2} < R_E + \frac{\gamma \lambda t}{4(1-\gamma)} < 2R_P - \frac{\lambda t}{2} \tag{19}$$

Note: The equilibrium price and profit in this case do not depend on the reservation price of premium customers (i.e. they are independent of the parameter R_P). This is because all premium customers can be served.

3.2.3 Both premium and economy customers can be fully served

Here, the reservation price for both premium and economy customers is high enough to cause the potential markets of the two retailers to overlap for both groups of customers.

Figure (5.10) is a depiction of this situation. The red arrows indicate the

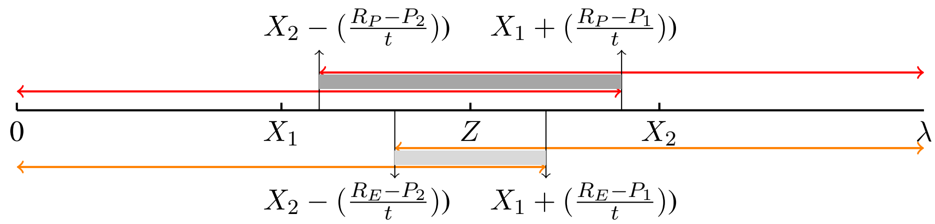

Figure 3.6: Overlap in premium and economy customers' market

potential market for premium customers for each retailer. Demand points within the dark gray shaded area have positive utility to buy from either retailer. However, customers will choose the retailer providing higher utility. Thus, premium customers in the interval $[0, Z]$ will opt to buy from retailer one, while those in $[Z, \lambda]$ will choose retailer two.

Similarly, the orange arrows indicate the potential market for economy customers for each retailer. Economy customers within the light gray shaded area of the demand line have positive utility to buy from both retailers. However, they will choose the retailer that provides higher utility. Economy customers in the interval [0,Z] will opt to buy from retailer one, while those in [Z,λ] will choose retailer two.

The rightmost premium and economy customers that retailer one can serve are located to the right of point Z. Similarly, the leftmost premium and economy customers that retailer two can serve are located to the left of point Z. That is:

$$
\begin{aligned}
X_1 + \frac{R_P - P_1}{t} > Z \\
X_1 + \frac{R_E - P_1}{t} > Z \\
X_2 - \frac{R_P - P_2}{t} < Z \\
X_2 - \frac{R_E - P_2}{t} < Z \\
P_1 + P_2 < R_P + R_E - \frac{\lambda t}{2}
\end{aligned}
\tag{20}
$$

Retailer one serves customers located in the interval [0,Z], and retailer two serves customers in the interval [Z, λ].

The objective functions of the two retailers are:

$$
\pi_1 = P_1 D_1 = \frac{1}{\lambda} \times P_1(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t}) \tag{21}
$$

$$
\pi_2 = P_2 D_2 = \frac{1}{\lambda} \times P_2(\frac{\lambda}{2} + \frac{P_1 - P_2}{2t}) \tag{22}
$$

The first-order condition results:

$$
P_1^* = P_2^* = R_E - \frac{\lambda t}{4} \tag{23}
$$

$$
\pi_i^* = \frac{1}{2}(R_E - \frac{\lambda t}{4}) \tag{24}
$$

Finally, to satisfy conditions in (20):

$$R_E > \frac{\lambda t}{4} \tag{25}$$

Note: The equilibrium price and profit in this case do not depend on the fraction of premium and economy customers (i.e. they are independent of the parameter γ). This is because all customers, whether premium or economy, can be served.

Note: In 3.2.1, the equilibrium price and profit depend on the reservation prices of both economy and premium customers. However, in 3.2.2 and 3.2.3, they depend only on the reservation price of economy customers. This difference arises because in the first case, neither economy nor premium customers can be served completely. Conversely, in the latter two cases, all premium customers can be served, making economy customers the primary focus in decision-making.

Tables 3.1 summarize these three cases that may occur. Figure 3.7 depicts the changes in the equilibrium price value with variations in the reservation price of economy customers, considering specific values of γ and transportation costs. It is evident that the price function exhibits three segments, each corresponding to one of the three cases studied.

Table 3.1: Summary of 3.2

condition	equilibrium price (P^*)	equilibrium profit (π^*)
$(1-\gamma)R_E + \frac{\lambda t}{2} < (2-\gamma)R_P$	$\frac{\gamma R_P + (1-\gamma)R_E}{2}$	$\frac{(\gamma R_P + (1-\gamma)R_E)^2}{2\lambda t}$
$R_P + R_E - \frac{\lambda t}{2} < R_E + \frac{\gamma \lambda t}{4(1-\gamma)} < 2R_P - \frac{\lambda t}{2}$	$\frac{R_E}{2} - \frac{\gamma \lambda t}{8(1-\gamma)}$	$R_E(\frac{\gamma}{4} + \frac{(1-\gamma)}{\lambda t}) - \frac{\gamma^2 \lambda t}{16(1-\gamma)} + \frac{\gamma}{t}$
$R_E > \frac{\lambda t}{4}$	$R_E - \frac{\lambda t}{4}$	$\frac{1}{2}(R_E - \frac{\lambda t}{4})$

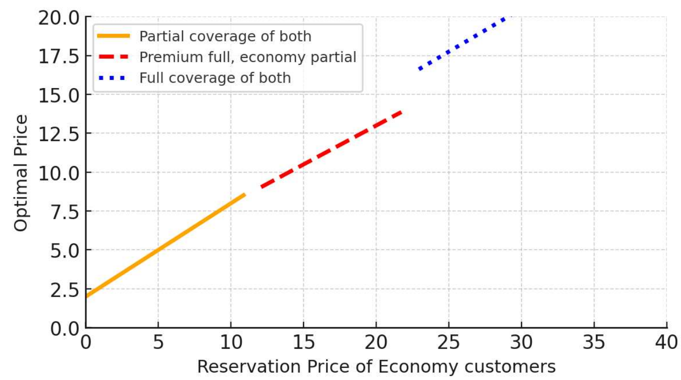

Figure 3.7: Comparison of equilibrium price values in the three cases

Proposition 1: When the two classes of customers are uniformly distributed and both retailers make simultaneous decisions, they end up offering the same price while choosing different locations.

Explanation of proposition 1: This is mathematically proven earlier. Intuitively, the market share between the two retailers depends on the price difference. When prices are unequal, the retailer with lower profit will always have an incentive to adjust their price to increase profitability.

Proposition 2: If $\frac{R_E - P_i}{t} > \frac{\lambda}{2}$ for both retailers, the equilibrium decision for both would be to co-locate at point $X_1^* = X_2^* = \frac{\lambda}{2}$ (the midpoint of the demand line) and set equal prices, $P_1 = P_2 = R_E - \frac{\lambda t}{2}$.

Explanation of proposition 2: When $\frac{R_E - P_i}{t} > \frac{\lambda}{2}$ for both retailers, they can cover all customers of either type. The problem then resembles the classic Hotelling model, where both retailers end up co-locating at the center of the demand line with the same price.

Proposition 3: The difference between R_E and R_P does not influence the equilibrium price or the equilibrium profit. In other words, the magnitude of $\frac{R_P}{R_E}$ does not impact the equilibrium price and profit.

Explanation of proposition 3: In Section 3.2.1, the equilibrium profit and price depend on both R_E and R_P, specifically on their magnitudes rather than their difference. In other words, higher values of both R_E and R_P lead to greater equilibrium prices and profits. However, in Sections 3.2.2 and 3.2.3, both the equilibrium price and profit depend solely on R_E.

3.3 Customers are uniformly distributed along the demand line and retailers enter the market sequentially

In this situation, one retailer is already present in the market, and their location and pricing decisions are known. The second retailer is considering entering the market and must decide on their location and pricing. Without loss of generality, let's assume the first retailer is the incumbent, and the second retailer is contemplating entry. The Stackelberg game theory is employed to determine their equilibrium decisions.

In the following sections, a comprehensive analysis of various possible cases is provided:

3.3.1 Both the leader and the follower focus on premium customers

Here again, without loss of generality, let's assume that retailer one (the leader) will locate to the left side of retailer two. Obviously, the leader does not want to lose customers on the left side of the line. Hence, the leader will locate at $X_1 = \frac{R_P - P_1}{t}$. The follower, aiming to maximize her share of the market, locates at $X_2 = \lambda - \frac{R_P - P_2}{t}$.

Case one: Partial coverage

This occurs when the reservation price of premium customers is less than the transportation cost, and the potential market of the leader and the follower is small enough that they do not overlap. Consequently, each retailer will have a separate market.

$I_{1,P} = [0, 2\frac{R_P - P_1}{t}]$

$I_{2,P} = [\lambda - 2\frac{R_P - P_2}{t}, \lambda]$

$D_{i_P} = 2\gamma\frac{R_P - P_i}{t}$, $i \in \{1, 2\}$

$$\pi_i = \frac{p_1\gamma}{\lambda}(2\frac{R_P - P_i}{t}) \tag{26}$$

$$P_i^* = \frac{R_P}{2} \tag{27}$$

$$\pi_i^* = \gamma \frac{R_P^2}{2\lambda t} \tag{28}$$

Here, the equilibrium price and equilibrium profit of both the leader and the follower are equal.

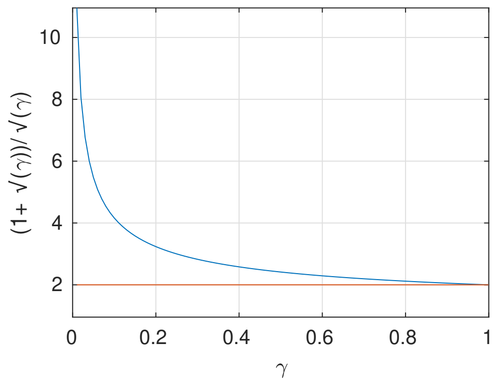

Figure 3.8: Values of $\frac{1+\sqrt(\gamma)}{\sqrt(\gamma)}$

Note: Comparing (27) and (12), it becomes evident that the equilibrium price is strictly higher when the two retailers focus solely on premium customers rather than serving both customer groups. This difference arises because the reservation price of premium customers is greater than that of economy customers.

Note: Comparing (28) and (13), shows that if $\frac{R_P}{R_E} > \frac{1+\sqrt{\gamma}}{\sqrt{\gamma}}$, it is more profitable to serve only premium customers rather than serving both customers. To facilitate the analysis, $\frac{1+\sqrt{\gamma}}{\sqrt{\gamma}}$ is plotted in figure 3.8. Since R_E is strictly less than R_P, the fraction $\frac{R_p}{R_E}$ is always greater than one. It can be seen that only when the majority of the market consists of premium customers (γ is equal or very close to 1), and the difference between R_L and R_E is high, it is more profitable to cater only to premium customers. However, when γ is not extremely big, it is more profitable to serve both groups of customers.

Case two: Full coverage

In such a case, where the reservation price of premium customers compared to the travel cost is significant enough to cause an overlap in the potential market of the leader and the follower, the leader, positioned to the left of the line, will choose a location to cover the maximum possible number of customers on her left side. Thus, she will locate at $X_1^* = \frac{R_P - P_1}{t}$.

The follower, who also wants to maximize her share of the market, will try to locate at a point on the right side of the line to cover the customers on the right side. Hence, she will locate at $X_2^* = \lambda - \frac{R_P - P_2}{t}$.

$I_{1,P} = [0, Z]$

$I_{2,P} = [Z, \lambda]$

Hence, the demand covered by each retailer would be:

$D_{1,P} = \gamma(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t})$

$D_{2,P} = \gamma(\frac{\lambda}{2} - \frac{P_2 - P_1}{2t})$

Solving the Stackelberg problem:

$$\pi_2 = \frac{P_2}{\lambda}\gamma(\frac{\lambda}{2} + \frac{P_1 - P_2}{2t}) \tag{29}$$

$$\pi_1 = \frac{P_1}{\lambda}\gamma(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t}) \tag{30}$$

$$P_1^* = \frac{3\lambda t}{2} \tag{31}$$

$$\pi_1^* = \gamma\frac{9(\lambda)^2 t}{16} \tag{32}$$

$$P_2^* = \frac{5\lambda t}{4} \tag{33}$$

$$\pi_2^* = \gamma\frac{25\lambda^2 t}{32} \tag{34}$$

Comparing the two equilibrium prices, it is evident that $P_1^* > P_2^*$. Additionally, the equilibrium profit of the follower, π_2^*, is strictly greater than the equilibrium profit of the leader,π_1^*. This disparity is attributed to the follower's advantage of deciding after the leader, particularly in symmetric games where every situation and condition are the same for both retailers.

Note: Comparing (23) with (31) and (33) indicates that, when the entire market can be served, the equilibrium price is higher when retailers exclusively cater to premium customers.

Note: Comparing (24) with (32) reveals that, in the case of full coverage

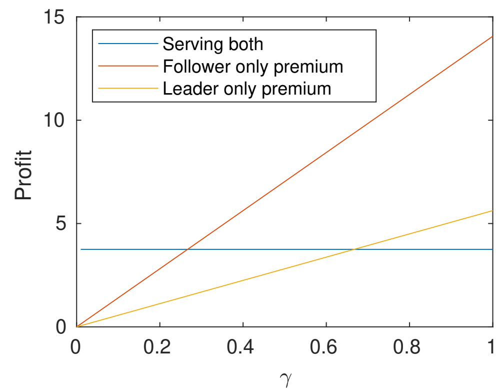

Figure 3.9: Comparison of different strategies in full coverage of demand

when $\frac{2t}{9} < \gamma$, it is to the best of the retailers to server only premium customers. Otherwise, it is more profitable to consider both customer groups. This is illustrated in figure 3.9.

3.3.2 The leader focuses on premium customers while the follower focuses on economy customers

In this case, since each retailer focuses on a different customer type, there is no overlap between the two target markets, and the retailers can even co-locate.

Case one: Partial coverage

The first retailer cannot cover all premium customers. She should locate anywhere in the $[\frac{R_P-P_1}{t}, \lambda - \frac{R_P-P_1}{t}]$ interval to maximize her market share. Her equilibrium price and profit would be same as (27),(28), respectively.

Similar analysis can be done for the second retailer.The second retailer can locate at any point in the $[\frac{R_L-P_2}{t}, \lambda - \frac{R_E-P_2}{t}]$ interval.

$$P_2^* = \frac{R_E}{2} \tag{35}$$

$$\pi_2^* = (1-\gamma)\frac{R_E^2}{2\lambda t} \tag{36}$$

Case two: Full coverage

The first retailer can serve the entire premium market and locates at the center of the demand line, point $\frac{\lambda}{2}$.

$$P_1^* = R_P - \frac{\lambda t}{2} \tag{37}$$

$$\pi_1^* = \gamma(R_P - \frac{\lambda t}{2}) \tag{38}$$

The second retailer can cover the entire economy market, and locates at the center of the demand line, point $\frac{\lambda}{2}$.

$$P_2^* = R_E - \frac{\lambda t}{2} \tag{39}$$

$$\pi_2^* = (1-\gamma)\,(R_E - \frac{\lambda t}{2}) \tag{40}$$

Results are summarized in table 3.2 and table 3.3:

Table 3.2: equilibrium decisions serving only premium customers

X_1^*	P_1^*	π_1^*	Condition
$[\frac{R_P}{2t}, \lambda - \frac{R_P}{2t}]$	$\frac{R_P}{2}$	$\gamma \frac{R_P^2}{2t}$	$R_P < \frac{\lambda t}{2}$
$[0, \lambda]$	$R_P - \frac{\lambda t}{2}$	$\gamma(R_P - \frac{\lambda t}{2})$	$R_P > \frac{\lambda t}{2}$

Table 3.3: equilibrium decisions serving only economy customers

X_2^*	P_2^*	π_2^*	Condition
$[\frac{R_R}{2t}, \lambda - \frac{R_E}{2t}]$	$\frac{R_E}{2}$	$(1-\gamma)\frac{R_E^2}{2\lambda t}$	$R_E < \frac{\lambda t}{2}$
$[0, \lambda]$	$R_E - \frac{\lambda t}{2}$	$(1-\gamma)(R_E - \frac{\lambda t}{2})$	$R_E > \frac{\lambda t}{2}$

Note: When a retailer targets only one group of customers, it is evident that the retailer focusing on premium customers ends up setting a higher price in each case (partial or full coverage of the market).

Note: In terms of profit, when the market can only be partially covered, focusing exclusively on premium customers is more profitable if $(\frac{R_P}{R_E}) > \sqrt{\frac{1-\gamma}{\gamma}}$. Otherwise, it is more profitable to cater exclusively to economy customers. Figure 3.10 depicts this relation.When the proportion of premium customers is small, it is in the retailers's best interest to cater exclusively to economy customers. However, when the proportion of premium customers is moderate or large, it is more profitable for the retailer to exclusively serve premium customers.

Note: Comparing profits when full market coverage is possible reveals that, for small values of γ, it is in the retailer's best interest to cater exclusively to economy customers. However, for greater values of γ (indicating a higher concentration of premium customers), a retailer would be better off

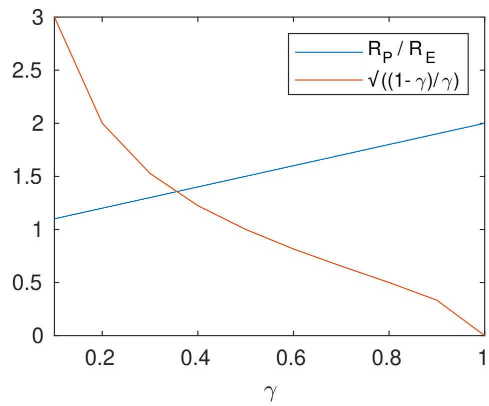

Figure 3.10: The relationship between the fraction of reservation prices in partial coverage

serving exclusively premium customers if $(\gamma - \frac{1}{2})\lambda t < \gamma R_P - (1-\gamma)R_E$. Figure 3.11 shows this relation.

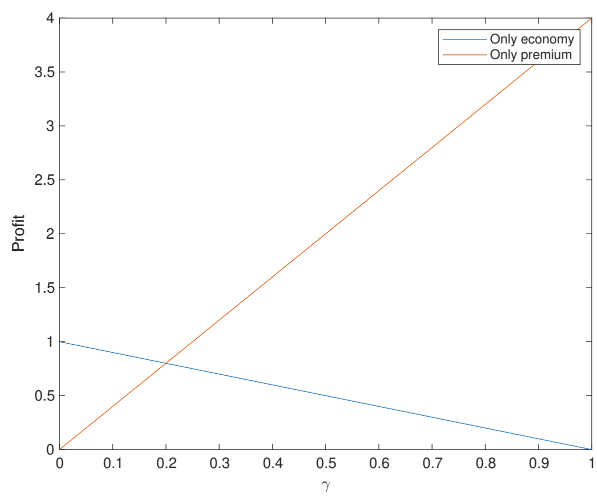

Figure 3.11: Profit when catering exclusively to either group in full market coverage

3.3.3 The leader focuses on premium customers while the follower focuses on both customers

Since only one follower targets economy customers, there is no competition in that segment. Consequently, retailers will compete exclusively for the premium customer market. The equilibrium locations will be same as in table 3.2.

Case one: Partial coverage

The objective functions of the retailers are:

$$\pi_1 = \frac{P_1}{\lambda}(2\gamma\frac{R_P - P_1}{t}) \tag{41}$$

$$\pi_2 = \frac{2P_2}{\lambda}(\gamma(\frac{R_P - P_2}{t}) + (1-\gamma)\frac{R_E - P_2}{t}) \tag{42}$$

The equilibrium price and profit for the first retailer are same as (27) and (28), respectively. The equilibrium price and profit for the second retailer are same as (12) and (13), respectively.

Case two:Full coverage

Here, there is an overlap in the potential market of the leader and the follower for the premium customers

$I_{1_H} = [0, Z]$

$I_{2_H} = [Z, \lambda]$

$I_{2_L} = [x_2 - \frac{R_E - P_2}{t}, x_2 + \frac{R_E - P_2}{t}]$

Hence, the demand covered by each retailer would be:

$D_{1_P} = \gamma(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t})$

$D_{2_P} = \gamma(\frac{\lambda}{2} - \frac{P_2 - P_1}{2t})$

$D_{2_E} = 2(1-\gamma)(\frac{R_E - P_2}{t})$

$$\pi_1 = \frac{P_1}{\lambda}\gamma(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t}) \tag{43}$$

$$\pi_2 = \frac{P_2}{\lambda}(\gamma(\frac{\lambda}{2} - \frac{P_2 - P_1}{2t}) + 2(1-\gamma)\frac{R_E - P_2}{t}) \tag{44}$$

Solving the Stackelberg game:

$$P_1^* = \lambda t + \frac{2(1-\gamma)tR_E}{\gamma} \tag{45}$$

$$P_2^* = \lambda t + \frac{(1-\gamma)R_E t}{\gamma} + \frac{2(1-\gamma)R_E}{\gamma} \tag{46}$$

Proposition 4:

i) In 3.3.3 in partial coverage, the price offered by the first retailer is strictly greater than the price offered by the second retailer ($P_1^* > P_2^*$).

ii) In the same problem, $\pi_2^* > \pi_1^*$ if and only if $\frac{R_P}{R_E} < \frac{\sqrt{\gamma}+1}{\sqrt{\gamma}}$.

iii) In 3.3.3. in full coverage , the price set by the follower is strictly greater than the price set by the leader ($P_2^* > P_1^*$). In the same problem, the equilibrium profit of the follower is also strictly greater than the equilibrium profit of the leader ($\pi_2^* > \pi_1^*$).

Explanation of proposition 4:

i) Comparing (13) and (27), given that $R_E < R_P$, it is evident that any linear combination of R_P and R_E will also be less than R_P. Intuitively, as the leader exclusively targets premium customers and the follower caters to both premium and economy customers, the follower must set a lower price.

ii) Intuitively, as the leader exclusively serves premium customers at a higher price compared to the follower, who caters to both customer groups, the leader stands to gain more profit if the percentage of premium customers significantly outweighs the percentage of economy customers.

iii) It can easily be inferred from (45) and (46) that ($P_2^* > P_1^*$). As the follower targets both customer groups, she has a larger customer base. Offering a higher price, her profit is consequently greater than that of the leader.

3.3.4 The leader focuses on economy customers, while the follower focuses on premium customers

This is same as 3.3.2, except that the positions of the leader and the follower are changed.

3.3.5 Both the leader and the follower focus on economy customers

Partial Coverage When the market can be partially served, the equilibrium price and profit of both retailers is similar to (35) and (36), respectively.

Full coverage

In such a case, where the reservation price of economy customers compared to the travel cost is significant enough to cause an overlap in the potential market of the leader and the follower, the leader, positioned to the left of the line, will choose a location to cover the maximum possible number of customers on her left side. Thus, she will locate at $X_1^* = \frac{R_E - P_1}{t}$.

The follower, who also wants to maximize her share of the market, will try to locate at a point on the right side of the line to cover the customers on the right side. Hence, she will locate at $X_2^* = \lambda - \frac{R_E - P_2}{t}$.

$I_{1_E} = [0, Z]$

$I_{2_E} = [Z, \lambda]$

Hence, the demand covered by each retailer would be:

$D_{1_P} = (1-\gamma)(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t})$

$D_{2_P} = (1-\gamma)(\frac{\lambda}{2} - \frac{P_2 - P_1}{2t})$

Solving the Stackelberg problem:

$$\pi_2 = \frac{P_2}{\lambda}(1-\gamma)(\frac{\lambda}{2} + \frac{P_1 - P_2}{2t}) \tag{47}$$

$$\pi_1 = \frac{P_1}{\lambda}(1-\gamma)(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t}) \tag{48}$$

$$P_1^* = \frac{3\lambda t}{2} \tag{49}$$

$$\pi_1^* = (1-\gamma)\frac{9(\lambda)^2 t}{16} \tag{50}$$

$$P_2^* = \frac{5\lambda t}{4} \tag{51}$$

$$\pi_2^* = (1-\gamma)\frac{25\lambda^2 t}{32} \tag{52}$$

Comparing the two equilibrium prices, it is evident that $P_1^* > P_2^*$. Additionally, the equilibrium profit of the follower, π_2^*, is strictly greater than the equilibrium profit of the leader,π_1^*. This disparity is attributed to the follower's advantage of deciding after the leader, particularly in symmetric games where every situation and condition are the same for both retailers.

3.3.6 The leader focuses on economy customers, while the follower focuses on both customers

Since only the follower targets premium customers, there is no competition for them. The retailers will compete only for the economy market.
The leader will locate at point $X_1 = \frac{R_E - P_1}{t}$, and the follower will locate at point $X_2^* = \lambda - \frac{R_P - P_2}{t}$.

Case one: Partial coverage

For the leader, the equilibrium price and equilibrium profit is same as (35) and (36). For the follower, the equilibrium price and equilibrium profit would be same as (12) and (13) respectively.

Note: From the above analysis, it is evident that the price set by the follower is strictly greater than that of the leader. This is due to the fact that, the leader only targets economy customers, which have low reservation price, but the follower targets both premium and economy customers and the reservation price of the premium customers is greater than that of the economy ones.

Moreover, the profit of the follower surpasses that of the leader. This is primarily attributed to the fact that the follower caters to both types of customers, whereas the leader exclusively targets economy customers.

Case two: Full coverage

Here,the leader serves the economy demand in the interval $[0, Z]$. Hence, $D_{1,E} = (1-\gamma)(\frac{\lambda}{2} + \frac{P_2 - P1}{2t})$. Forming the objective function of the leader results in:

$$\pi_1 = \frac{P_1}{\lambda}(1-\gamma)(\frac{\lambda}{2} + \frac{P_2 - P1}{2t}) \tag{53}$$

Moreover, the follower covers the demand in the interval $[Z, \lambda]$, so the demand of economy customers that the follower covers is: $D_{2,E} = (1-\gamma)(\frac{\lambda}{2} + \frac{P_1 - P2}{2t})$. The follower can cover all the premium demand. Therefore, $D_{2,H} = \gamma\lambda$. The objective function of the follower is:

$$\pi_2 = \frac{P_2}{\lambda}(\gamma\lambda + (1-\gamma)(\frac{\lambda}{2} + \frac{P_1 - P2}{2t})) \tag{54}$$

Solving the Stackelberg game:

$$P_1^* = \frac{\lambda t(3-\gamma)}{2(1-\gamma)} \tag{55}$$

$$P_2^* = \frac{\lambda t(\gamma+5)}{4(1-\gamma)} \tag{56}$$

$$\pi_1^* = \frac{\lambda t(\gamma^2 - 6\gamma + 9)}{16(1-\gamma)} \tag{57}$$

$$\pi_2^* = \frac{\lambda t(\gamma^2 + 10\gamma + 25)}{32(1-\gamma)} \tag{58}$$

To compare (55) and (56) they are both plotted in figure 3.12. The plot shows that the price set by the leader is strictly less than the price set by the follower ($P_1^* < P_2^*$).

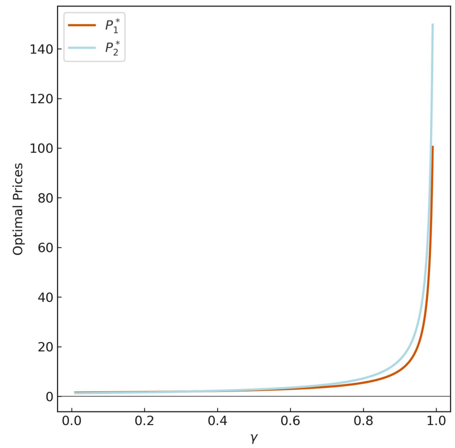

Figure 3.12: Comparison of prices in (55) and (56)

Note: Since the price set by the leader is less that that of the follower, the leader serves a greater portion of the economy market. However, the follower serves all premium customers as well. Therefore, the follower earns more profit.

3.4 Customers are non-uniformly distributed along the demand line and retailers enter the market simultaneously

In this case, the market is once again presumed to be a linear space, ranging from 0 to λ. However, the distribution of customers is no longer uniform. Premium customers are more abundant on the left side of the line, decreasing linearly as one moves to the right. Conversely, the number of economy customers is minimal on the left side but rises linearly toward the right side of the line. Figure (non-uniform-depiction) provides a straightforward illustration of this non-linear distribution scenario.

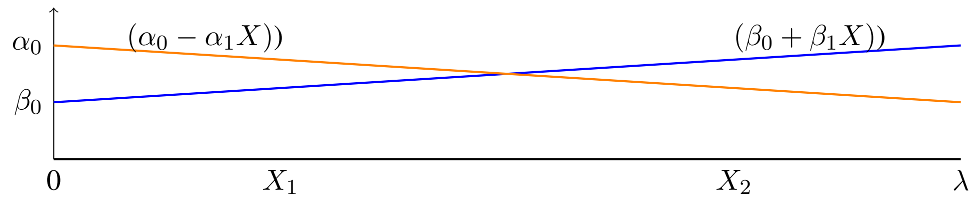

Figure 3.13: Depiction of non-uniform situation

In Figure 3.13, the orange line depicts the distribution of premium customers, peaking on the left side of the demand line and gradually decreasing towards the right. Meanwhile, the blue line illustrates the distribution of economy customers, starting low on the left side and increasing towards the right.

This situation is common in many cities, where wealthier individuals are concentrated in certain neighborhoods, while less affluent residents reside in others (Sedghi, Shavandi, & Abouee-Mehrizi, 2017).

A few new parameters need to be defined here:

α_0: the number of premium customers at point 0.

α_1: the rate at which the number of premium customers decreases along the demand line.

β_0: the number of economy customers at point 0.

β_1: the rate at which the number of economy customers increases along the demand line.

Hence, the number of premium and economy customers at an arbitrary point

X, when $0 < X < \lambda$, would be $(\alpha_0 - \alpha_1 X)$ and $(\beta_0 + \beta_1 X)$ respectively .

3.4.1 Both customer groups are partially served

Retailer one will cover as many demand points as possible. Thus, retailer one will cover the interval $[X_1 - \frac{R_P - P_1}{t}, X_1 + \frac{R_P - P_1}{t}]$ for premium customers, and the interval $[X_1 - \frac{R_E - P_1}{t}, X_1 + \frac{R_E - P_1}{t}]$ for economy customers. Retailer two will also aim to cover as many demand points as possible, covering the interval $[X_2 - \frac{R_P - P_2}{t}, X_2 + \frac{R_P - P_2}{t}]$ for premium customers, and the interval $[X_2 - \frac{R_E - P_2}{t}, X_2 + \frac{R_E - P_2}{t}]$ for economy customers.

Figure 3.14 illustrates this situation, where the orange shaded region corresponds to the premium customers served by retailer one, and the blue shaded region corresponds to the economy customers served by retailer one.

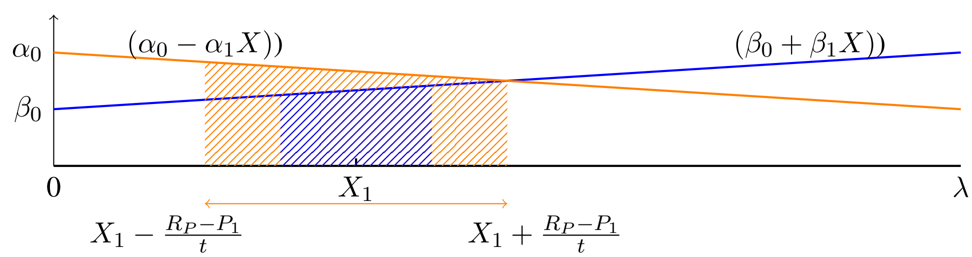

Figure 3.14: Partial coverage for both customer groups

The maximization problems of retailers are:

$$\pi_i = P_i\left(\int_{X_i - \frac{R_P - P_i}{t}}^{X_i + \frac{R_P - P_i}{t}} (\alpha_0 - \alpha_1 X)\, dX + \int_{X_i - \frac{R_E - P_i}{t}}^{X_i + \frac{R_E - P_i}{t}} (\beta_0 + \beta_1 X)\, dX\right) \quad (59)$$

Subject to:

$$\frac{R_P - P_i}{t} < \frac{\lambda}{2} \quad \text{for } i \in \{1,2\} \tag{60}$$

$$\frac{R_E - P_i}{t} < \frac{\lambda}{2} \quad \text{for } i \in \{1,2\} \tag{61}$$

The objective function (59) maximizes the total revenue of the retailer from selling to customers; the sum of the number of the premium and economy customers being served, multiplied by the price of the product.

Constraints (60) and (61) ensure that the premium and economy customers can partially be served.

The equilibrium solutions are:

$$X_1^* = X_2^* = \frac{(\alpha_1 + \beta_1)(\alpha_0 R_P + \beta_0 R_E) - 2(\alpha_0 + \beta_0)(\alpha_1 R_P + \beta_1 R_E)}{2(\alpha_0 R_P + \beta_0 R_E)(\alpha_1 + \beta_1) + (\alpha_1 R_P + \beta_1 R_E)(\alpha_1 + \beta_1 - 2\alpha_0 - 2\beta_0)} \tag{62}$$

$$P_1^* = P_2^* = \frac{\alpha_1 R_P + \beta_1 R_E}{\alpha_1 + \beta_1} \tag{63}$$

Equation (62) provides a closed-form expression for the equilibrium co-located position of the two retailers. It shows that, as the reservation price of premium customers (R_P) increases, retailers have stronger incentives to move toward the left side of the market, where premium customers are initially more concentrated. Since the density of premium customers decreases from left to right, while that of economy customers increases, firms locate closer to segments with higher willingness to pay.

It can be readily deduced from (63) that the equilibrium price is a convex combination of R_P and R_E. The weight assigned to each term in this combination depends on the slope at which the number of premium customers decreases along the demand line and the slope at which the number of economy customers increases, respectively.

Proposition 5: In the non-uniform case, the retailers will ultimately converge to co-locating and setting the same price.

Explanation of proposition 5: When the distribution of customers is non-uniform, there exists a unique point where customer concentration is maximized. This point becomes a focal competition point for both retailers. Intuitively, both retailers will inevitably co-locate at this point. If either retailer chooses a different location, there will always be an incentive to relocate to attract more customers. As the retailers co-locate, they must set the same price; otherwise, customers will prefer the retailer with the lower price, compelling the higher-priced retailer to lower their price.

3.4.2 All premium and a subset of economy customers are served

In this case, the reservation price for premium customers is sufficiently high to accommodate all premium customers ($R_P > \frac{\lambda t}{2}$), while the reservation price for economy customers is relatively lower, allowing for partial service ($R_E < \frac{\lambda t}{2}$). As both retailers co-locate (as per proposition 5), they will share the same market for both premium and economy customers. Each potential customer will randomly choose either retailer. Specifically, each potential customer has a 0.5 probability of choosing retailer one and an equally likely 0.5 probability of choosing retailer two.

Given that the density of economy customers is higher on the right side of the line compared to the left side, retailers will aim to position themselves to the left of the line. This strategic placement allows them to capture the entire demand on the left side of the line and maximize their coverage of the demand on the right side of the line as much as possible.

Figure 3.15 illustrates this situation, with retailer one positioned near the right end of the demand line—where economy customers are concentrated—in order to maximize the number of economy customers served

$$X_1^* = X_2^* = \lambda - \frac{R_E - P_i^*}{t} \tag{64}$$

The maximization problem for the retailers follows:

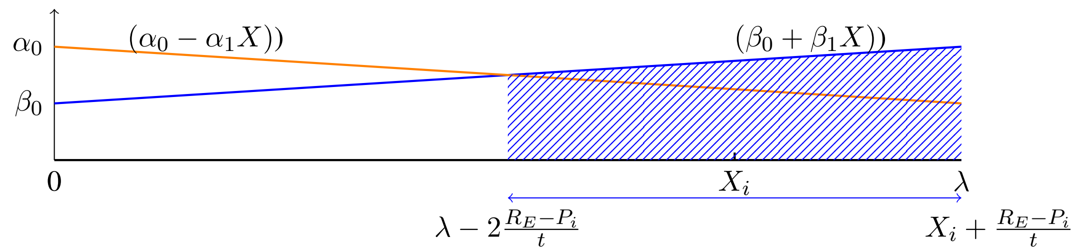

Figure 3.15: Partial coverage for economy and full coverage of premium customers

$$\pi_i = \frac{P_i}{2}\left(\int_{\lambda - 2\frac{R_E - P_i}{t}}^{\lambda} (\beta_0 + \beta_1 x)\, dx + \int_0^{\lambda} (\alpha_0 - \alpha_1 x)\, dx\right) \tag{65}$$

Subject to:

$$\frac{R_P - P_i}{t} > \frac{\lambda}{2} \quad \text{for } i \in \{1, 2\} \tag{66}$$

$$\frac{R_E - P_i}{t} < \frac{\lambda}{2} \quad \text{for } i \in \{1, 2\} \tag{67}$$

The objective function (65) represents the total revenue of the retailers. It is defined as the price multiplied by the sum of all premium customers and the portion of economy customers who can be served. As established in the previous subsection, the retailers are expected to co-locate. Consequently, each customer is assumed to randomly select between the two retailers, resulting in an equal split—half of the potential customers choose Retailer one and the other half choose Retailer two.

Constraint (66) ensures that all premium customers can be served. Specifically, it requires that the maximum distance a retailer can cover for premium customers is greater than half of the market length. Since a retailer serves customers on both its left and right, this condition implies that the retailer has the capacity to cover the entire market line for premium customers.

Constraint (67) ensures that the economy customers can partially be served.

Solving the optimization problem, the equilibrium price is:

$$P_i^* = \frac{2R_E\beta_1 - \beta_0 t - \beta_1\lambda t + \sqrt{(R_E\beta_1)^2 - R_E\beta_1 t(\beta_0 + \beta_1\lambda) + (\beta_0 + \beta_1\lambda)^2 t^2}}{3\beta_1} \tag{68}$$

It is evident from (68) that the price depends solely on the distribution of economy customers along the demand line, not the premium ones. Intuitively, since all premium customers are served in this context, the distribution of premium customers does not impact the price. However, the manner in which economy customers are distributed becomes crucial for maximizing retailers' profit.

Notably, when $\beta_0 = 0$, the equilibrium price simplifies to $P_i^* = \frac{2R_E - \lambda t + \sqrt{R_E^2 - R_E\lambda t + \lambda^2 t^2}}{3}$ That is, when the initial number of economy customers is zero, the equilibrium price does not depend on β_0.

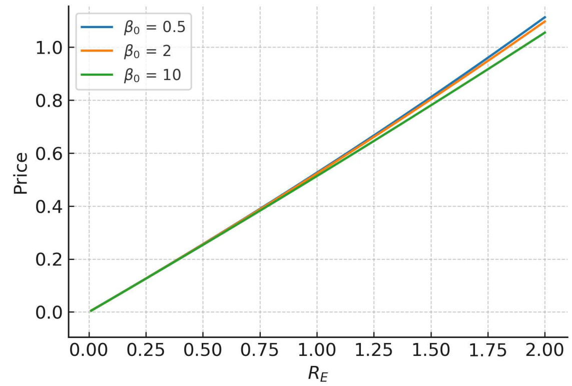

Figure 3.16: Effect of β_0 on equilibrium price

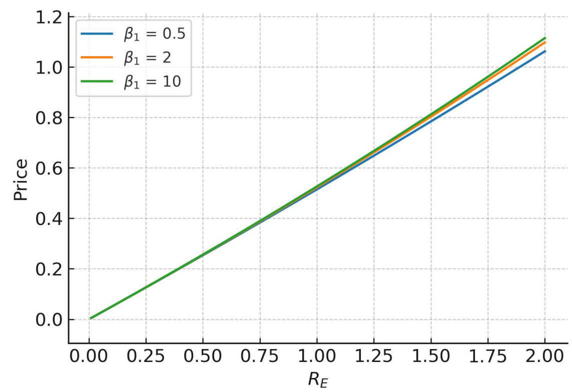

Figure 3.17: Effect of β_1 on equilibrium price

Figures 3.16 and 3.17 investigate the effect of β_0, β_1 and R_E on equilibrium price.It is clear that the equilibrium price increases as the reservation price of economy customers increases.

Furthermore, an increase in β_0 (keeping β_1 and R_E constant) results in an decrease in the equilibrium price. This is because, with other parameters fixed, an increase in β_0 implies an increase in the number of economy customers near the retailers' location. Consequently, retailers can decrease the price they set and still serve the same number of customers.

Finally, an increase in β_1 (holding β_0 and R_E constant) results in an increase in the equilibrium price. This is attributed to the heightened slope, causing an increase in the number of economy customers around the retailers' location. Consequently, the retailer can increase the price, as she can cover more customers close to her location.

3.4.3 Both customer groups are fully served

In this situation, the reservation prices for both premium and economy customers exceed transportation costs, ensuring that all customers can be accommodated ($R_P > \frac{\lambda t}{2}$ and $R_E > \frac{\lambda t}{2}$). The profit functions are:

$$\pi_i = \frac{P_i}{2}(\int_0^\lambda (\beta_0 + \beta_1 x)\,dx + \int_0^\lambda (\alpha_0 - \alpha_1 x)\,dx) \tag{69}$$

Subject to:

$$\frac{R_P - P_i}{t} > \frac{\lambda}{2} \quad \text{for } i \in \{1, 2\} \tag{70}$$

$$\frac{R_E - P_i}{t} > \frac{\lambda}{2} \quad \text{for } i \in \{1, 2\} \tag{71}$$

The objective function (69) represents the total revenue of the retailers. It is defined as the price multiplied by the sum of all premium and economy customers. As established in proposition 5, the retailers are expected to co-locate. Consequently, each customer is assumed to randomly select between the two retailers, resulting in an equal split—half of the potential customers choose Retailer one and the other half choose Retailer two.

Constraint (70) ensures that all premium customers can be served. Specifically, it requires that the maximum distance a retailer can cover for premium customers is greater than half of the market length. Since a retailer serves customers on both its left and right, this condition implies that the retailer has the capacity to cover the entire market line for premium customers.

Similarly, constraint (71)ensures that all economy customers can be served.

The equilibrium results follow:

$$X_1^* = X_2^* = \frac{\lambda}{2} \tag{72}$$

$$P_i^* = R_E - \frac{\lambda t}{2} \tag{73}$$

$$\pi_i = \frac{1}{2}\left(R_E - \frac{\lambda t}{2}\right)\left[(\beta_0 + \alpha_0)\lambda + \frac{(\beta_1 - \alpha_1)\lambda^2}{2}\right] \tag{74}$$

Comparing the equilibrium prices in (73) and (23), under full market coverage the equilibrium price is higher when customers are uniformly distributed than when they are non-uniformly distributed along the demand line.

Proposition 6: When all premium and economy customers can be served, the equilibrium price is higher when customers are uniformly distributed along the demand line, compared to the situation in which customers are non-uniformly distributed.

Explanation of proposition 6: When the whole market can be served and customers are uniformly distributed, the retailers do not co-locate, and each serves a separate half of the market. However, when customers are non-uniformly distributed along the demand line, the two retailers will co-locate, and each will cover the entire market.

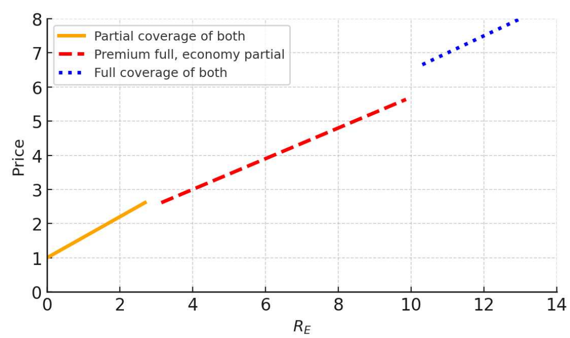

Figure 3.18: Changes in the equilibrium price with the increase in reservation price

Figure 3.18 shows the changes in the equilibrium price in relation to the reservation price. It can be seen that the equilibrium prices generally increase as the customers' reservation prices rise. The three sections on the plot, each colored differently, correspond to one of the three cases studied.

3.5 Customers are non-uniformly distributed, and retailers take turn entering the market

In this situation, one retailer already exists in the market, and the second retailer decides entering the market while knowing the decisions the first retailer has made. In this section we analyze the result of concentration on different customer groups.

3.5.1 The leader and the follower both focus on premium customers

The equilibrium decisions in this case depends on the length of the demand line. The equilibrium varies whether all demand points can be served or not.

Case (I): Partial coverage

Here, both retailers will locate at point $X_1 = X_2 = \frac{R_P - P_i}{t}$, to maximize their share of the premium customers' market. Figure 3.19 depicts this situation.

$$X_1^* = X_2^* = \frac{R_P - P_i^*}{t} \tag{75}$$

Since both retailers will co-locate, the potential customers will randomly choose any of the retailers with probability of 0.5. The retailers also have to set the same prices. The profit functions are:

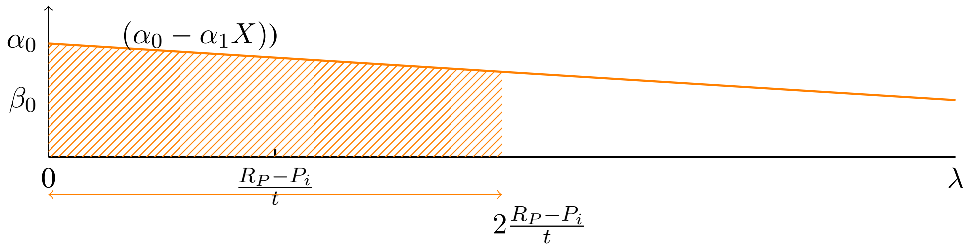

Figure 3.19: Partial coverage for premium customers

$$\pi_i = \frac{1}{2} \times \int_0^{2\frac{R_P - P_i}{t}} P_i(\alpha_0 - \alpha_1 x)\, dx \tag{76}$$

Subject to:

$$\frac{R_P - P_i}{t} < \frac{\lambda}{2} \quad \text{for } i \in \{1, 2\} \tag{77}$$

The equilibrium prices are:

$$P_i^* = \frac{2R_P\alpha_1 - \alpha_0 t\sqrt{(R_P\alpha_1)^2 - (R_P\alpha_1)(\alpha_0 t) + (\alpha_0 t)^2}}{3\alpha_1} \tag{78}$$

Considering the equilibrium price in (78), if $\alpha_0 = 0$, the equilibrium price is simplified to $P_i^* = \frac{2R_P}{3}$.

Case (II): Full coverage

All demand points can be covered here. Therefore, the leader and the follower will locate at the center of the line ($X_1 = X_2 = \frac{\lambda}{2}$). The profit functions are:

$$\pi_i = \frac{1}{2} P_i \int_0^{\lambda} (\alpha_0 - \alpha_1 x)\, dx \tag{79}$$

Subject to:

$$\frac{R_P - P_i}{t} > \frac{\lambda}{2} \quad \text{for } i \in \{1, 2\} \tag{80}$$

The equilibrium results are:

$$P_i^* = R_P - \frac{\lambda t}{2} \tag{81}$$

$$\pi_i = \frac{1}{2} \left(R_P - \frac{\lambda t}{2} \right) \left(\alpha_0 \lambda - \frac{\alpha_1 \lambda^2}{2} \right) \tag{82}$$

3.5.2 The leader focuses on premium customers, while the follower focuses on economy customers

In this case, the leader and the follower will not have any competition, as they serve totally different customers. Therefore, each of the retailers makes their decisions regardless of the decisions made by the other.

Considering the leader, if $R_P < \frac{\lambda t}{2}$, she will locate at point $X_1^* = \frac{R_P - P_1}{t}$, and set her price same as (78). However, if $R_P > \frac{\lambda t}{2}$, she will locate at $X_1^* = \frac{\lambda t}{2}$, and set her price same as (81).

Considering the follower, two cases may happen:

Case (I): Partial coverage

Only part of the economy customers can be served. the follower will locate at point $X_2 = \lambda - \frac{R_E - P_2}{t}$, and will cover demand points in the interval $[\lambda - 2\frac{R_E - P_2}{t}, \lambda]$. Her profit function would be:

$$\pi_2 = \int_{\lambda - 2\frac{R_E - P_2}{t}}^{\lambda} P_2(\beta_0 + \beta_1 x)\, dx \tag{83}$$

Subject to:

$$\frac{R_P - P_i}{t} < \frac{\lambda}{2} \quad \text{for } i \in \{1, 2\} \tag{84}$$

The equilibrium price would be same as (68) .

Case (II): Full coverage

the follower would be able to serve the whole demand , and will locate at point $X_2^* = \frac{\lambda}{2}$.

$$\pi_2 = \int_{0}^{\lambda} P_2(\beta_0 + \beta_1 x)\, dx \tag{85}$$

The follower sets her price same as in (73).

$$\pi_2 = \left(R_E - \frac{\lambda t}{2}\right)\left(\beta_0 \lambda + \frac{\beta_1 \lambda^2}{2}\right) \tag{86}$$

3.5.3 The leader focuses on premium customers, while the follower focuses on both customer groups

In this case, the leader focuses on premium customers, while the follower serves both customer groups. The retailers compete for the premium market, but there is no competition for economy customers. Premium customers are prioritized due to higher profitability.

Case (I): Partial coverage

Both retailers will locate at point $X_1 = X_2 = \frac{R_P - P}{t}$. They will both set their prices same as in (78).

Case (II): Full coverage

Both retailers will locate at the center of the demand line, and set their price equal to (81).

It goes beyond saying that,since the leader serves both customer groups, her profit would be more than that of the follower in either case.

3.5.4 The leader focuses on economy customers, while the follower focuses on premium customers

This is the same as 3.5.2, but the leader and follower have switched places.

3.5.5 The leader and the follower both focus on economy customers

In this case, there will be a competition between the leader and the follower, as they both focus only on economy customers. No matter what conditions apply, they would end up co-locating with the same price.

Case (I): Partial coverage

Since the reservation price is low compared to traveling costs, only part of the economy customers can be served. Both retailers will co-locate at point $X_1 = X_2 = \lambda - \frac{R_E - p_1}{t}$, to be able to serve economy customers in the interval $[\lambda - 2\frac{R_E - P_1}{t}, \lambda]$, where economy customers are most concentrated. They will set their equilibrium price same as in (68).

Case (II): Full coverage

Since the reservation price is high, compared to transportation costs, all economy customers can be served. Both retailers will co-locate at the middle of the demand line, setting their price equal to (73).

3.5.6 The leader focuses on economy customers, while the follower focuses on customhouse groups

The retailers compete for the economy market, but there is no competition for premium customers.

Case (I): Partial coverage

The leader will locate at point $X_1 = \lambda - \frac{R_E - P_1}{t}$, so that she can cover premium demands in the interval $[\lambda - 2\frac{R_E - P - 1}{t}, \lambda]$. She would set her offering price same as in (68).

The follower will co-locate at the same point and set the same price. She will also cover premium customers in the interval $[\lambda - \frac{R_E + R_P}{t}, \lambda]$.

Case (II): Full coverage

Both retailers will co-locate at the center of the demand line, setting their price as in (73).

3.6 Analysis of results

This section discusses and consolidates the results from the previous section.

3.6.1 Customers are uniformly distributed along the demand line and both retailers enter the market simultaneously

When the two retailers simultaneously decide on their location and price, their equilibrium strategy is to avoid co-location and instead select different locations. However, the equilibrium pricing strategy is to always set the same price. This differs from the traditional Hotelling problem, where both retailers co-locate. The key difference is that in the problem analyzed here, customers' reservation costs are also considered, and some customers may not be served.

Table 3.4 summarizes how equilibrium decisions depend on the parameters of the problem when retailers decide simultaneously. As shown in this table, the equilibrium price and profit consistently depend on the length of the demand line (interpreted as the number of customers), the travel cost per unit distance, and the distribution of the two customer types along the demand line (representing the percentage of customers of each type).

Moreover, when there is no overlap in the potential markets of the two retailers for both premium and economy customers, the equilibrium price and profit depend on both R_P and R_E. However, when there is no overlap in potential markets for economy customers but an overlap in the market for premium customers, the equilibrium price (and consequently the equilibrium profit) depends solely on R_E.

This is because, when all premium customers have relatively high reservation prices, they can all be served. Therefore, maximizing each retailer's profit involves serving as many economy customers as possible, given that both already serve half of the premium customers. As a result, the equilibrium price set by each retailer in this case depends only on R_E and not on R_P.

Finally, when there is overlap in the potential market for both economy and premium customers, the equilibrium price (and consequently the equilibrium profit) depends only on R_E and not on R_P. In this scenario, the reservation prices for both customer groups are sufficiently high relative to transportation costs, allowing all customers to be served. By definition, the reservation price of economy customers is strictly lower than that of premium customers ($R_E < R_P$). Therefore, the minimum of the two reservation prices is R_E. Since the equilibrium price cannot exceed the customers' reservation price, R_E serves as an upper bound for the equilibrium price in this case.

Table 3.4: equilibrium price in uniform scenario simultaneous entering

Situation	equilibrium price
Both customers can be served partially	Function of R_E,R_P,λ,t and γ.
All premium and part of economy customers can be served	Function of R_E,λ,t and γ.
All customers can be served	Function of R_E,λ,t and γ.

When comparing the equilibrium prices across the three cases, the highest price occurs in Case (I), followed by Case (II), with the lowest price in Case (III). Concurrently, the number of potential customers is smallest in Case (I), increases in Case (II), and is largest in Case (III). This observation suggests that the fewer the potential customers, the higher the price.

For equilibrium profit, the larger the potential market, the higher the equilibrium profit. Thus, the equilibrium profit is highest in Case (III), followed by Case (II), and lowest in Case (I).

Finally, it is worth noting that the difference or ratio between the reservation prices of economy and premium customers does not affect the equilibrium price or profit.

3.6.2 Customers are uniformly distributed along the demand line, and retailers take turns entering the market

When one retailer is already established in the market and a second retailer is considering entry, the equilibrium strategy for both is to avoid co-location. The findings on equilibrium pricing strategies are summarized in Table 3.5.

Table 3.5: Results of the Stackelberg Game in uniform case

	Follower focuses on premium	Follower focuses on economy	Follower focuses on Both
Leader focuses on Premium	When there is no overlap in markets, both set the same price and earn the same profit. However, when there is an overlap in markets, the follower sets a lower price but receives more profit.	The leader's set price is higher than that of the follower. Their profits depend on the proportion of each customer group.	In partial coverage, the follower always gains more profit. In full coverage, the follower gains more profit if the percentage of premium customers is not very high.
Leader focuses on Economy	The leader's set price is less than that of the follower. Their profits depend on the proportion of each customer group.	When there is no overlap in markets, both offer the same price and earn the same profit. However, when there is an overlap in markets, the follower offers a lower price but receives more profit.	The follower always gains more profit.

It can be easily inferred that focusing solely on economy customers is not advisable for the first mover (the leader). In all scenarios, regardless of the follower's decisions, the follower achieves a higher profit than the leader.

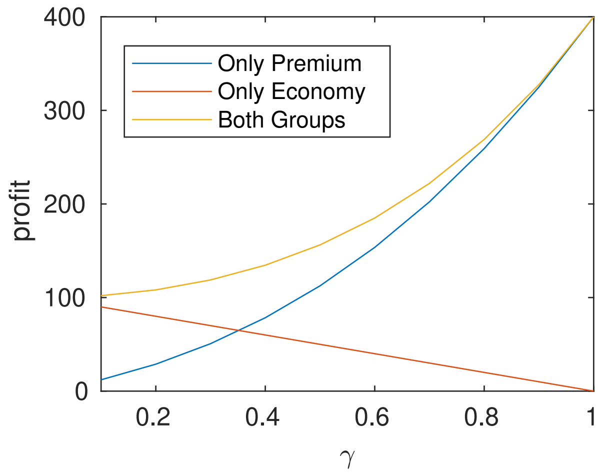

Figure 3.20: Comparison of Profits Across Different Strategies in Partial Demand Coverage

Moreover, it is in the leader's best interest to serve both customer groups when the market can only be partially served.

Figure 3.20 illustrates this result. It is evident that the profit is strictly higher when a retailer serves both customer groups, as opposed to focusing on only one.

Figure 3.21 compares a retailer's profit under three strategies: targeting both customer groups, focusing exclusively on premium customers, and focusing solely on economy customers, in situations where all demand can be fulfilled. When the retailer serves the entire market, focusing only on premium customers results in a higher equilibrium price than when both groups are served. However, serving both groups expands the customer base. Thus, when the proportion of premium customers (γ) is low, targeting both groups

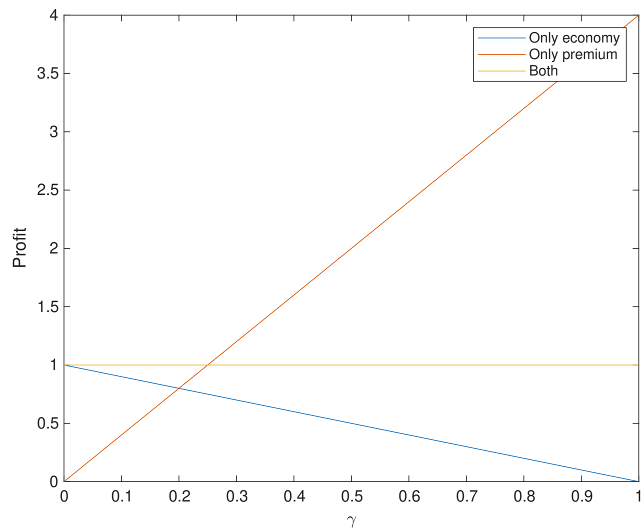

Figure 3.21: Comparison of Profits Across Different Strategies in Full Demand Coverage

yields greater profit, despite requiring a lower price. Conversely, when premium customers predominate, focusing on them alone and setting a higher price is more profitable.

Table 3.6: Follower's best response on leader's decisions in uniform scenario

Leader's focus	Follower's Best Response
Premium customers	When the market can only be partially served, focus on both customer groups. If the market can be fully served and the proportion of premium customers is high ($\frac{2t}{9} < \gamma$), exclusively cater to premium customers. Otherwise, serve both customer groups.
Economy customers	When the market can only be partially served, always focus on both customer groups. When the entire market can be served, if the proportion of premium customers is significant, exclusively focus on premium customers. Otherwise, focus on both customer groups.
Both customer groups	When the market can only be partially served, always focus on both customer groups. When the entire market can be served, if the proportion of premium customers is significant, exclusively focus on premium customers. Otherwise, focus on both customer groups.

Table 3.6 illustrates the follower's best response to the various decisions the leader can make. When the market can be fully served, if the proportion

of premium customers is high, the leader would be better off focusing exclusively on premium customers. Otherwise, the leader's best strategy is to target both customer groups.

3.6.3 Customers are non-uniformly distributed along the demand line and both retailers enter the market simultaneously

this section examines a non-uniform distribution of the two customer groups, premium and economy, along the demand line. Premium customers are concentrated on the left side of the line, decreasing linearly toward the right, while economy customers are sparse on the left and increase linearly toward the opposite end.

In simultaneous decision-making, both retailers will choose to co-locate. However, this shared location is not necessarily at the center of the demand line, contrary to the typical outcome in the Hotelling problem.

When neither all premium customers nor all economy customers can be fully served, the shared location of the two retailers depends on the reservation prices of both customer groups (R_P, R_E) and the distribution of these groups along the demand line ($\alpha_0, \alpha_1, \beta_0, \beta_1$). There should be a point around which the cumulative number of customers is maximized, considering the maximum distance a retailer can serve premium or economy customers. The location of this point depends on the distribution and reservation prices of both customer groups. In this case, both retailers set the same price, which is a convex combination of the two customer groups' reservation prices.

When all premium customers can be served but only a limited number of economy customers can be served, the equilibrium strategy for both retailers is to co-locate. However, despite the Hotelling problem, they do not co-locate at the center. Instead, their common location is more aligned with the right side of the demand line when the number of economy customers is higher. Since both retailers can serve all premium customers but only a limited number of economy customers, it is logical for them to choose a location where they can serve the maximum number of economy customers (closer to the right side of the line), while also serving all premium customers.

Table 3.7: equilibrium strategies for simultaneous entering in non-uniform distribution

Coverage of customers	location	price (function of)
Partial coverage of both groups	Co-location (not center)	R_E,R_P,α_1 , β_1.
All premium and part of economy	Co-location (aligned to right)	R_E,β_0 ,β_1, λ, t.
Full coverage of both groups	Co-location (center)	R_E,λ,t

When all premium and economy customers can be served (i.e. the reservation prices of both customer groups are sufficiently high relative to the travel costs), both retailers will co-locate at the center of the demand line. The equilibrium pricing strategy involves setting the same price, which depends solely on the reservation price of the economy customers, the length of the demand line, and the travel cost. When all customers can be served, the reservation price of the economy customers sets an upper bound for the equilibrium price. Since the farthest customer from the retailers is at a distance of $\frac{\lambda}{2}$, the maximum price that allows coverage of all customers is $R_E - \frac{\lambda t}{2}$.

The discussion presented here is summarized in table 3.7.

3.6.4 Customers are non-uniformly distributed along the demand line, and retailers take turns entering the market

In the non-uniform case, both retailers co-locate. This outcome holds for both sequential and simultaneous entering mode when customers are non-uniformly distributed along the demand line.

When the market can be fully served, both retailers will co-locate at the center of the demand line to reach all customers. However, when only partial market coverage is possible, the retailers will co-locate near either end of the demand line, depending on the customer type they target.

The equilibrium prices and profits for different strategies under full coverage are presented in Table 3.8. It is evident that the retailers set the same price when serving only economy customers or targeting both customer groups. However, the price is higher when they focus exclusively on premium customers.

Comparing the equilibrium profits reveals that focusing exclusively on economy customers is a dominated strategy. A retailer always earns strictly higher profits when serving both customer groups than when targeting only economy customersIf the number of premium customers is significantly greater than that of economy customers, it may be more beneficial to focus exclusively on premium customers. In all other cases, serving both customer groups yields higher profits.

Table 3.8: equilibrium decisions in non-uniform scenario under full coverage

Target customers	Price	Profit
Premium	$R_P - \frac{\lambda t}{2}$	$\left(R_P - \frac{\lambda t}{2}\right)\left(\alpha_0\lambda - \frac{\alpha_1\lambda^2}{2}\right)$
Economy	$R_E - \frac{\lambda t}{2}$	$\left(R_E - \frac{\lambda t}{2}\right)\left(\beta_0\lambda + \frac{\beta_1\lambda^2}{2}\right)$
Both	$R_E - \frac{\lambda t}{2}$	$\left(R_E - \frac{\lambda t}{2}\right)\left[(\beta_0 + \alpha_0)\lambda + \frac{(\beta_1-\alpha_1)\lambda^2}{2}\right]$

The results for this scenario are summarized in Table 3.9. When both the leader and the follower target the same customer group, they set identical prices and achieve equal profits. The strategy of targeting only economy customers is dominated by the strategy of serving both customer groups. Focusing solely on premium customers is more profitable than serving both customer groups only when the number of premium customers is substantially higher than that of economy customers. In all other cases, it is in the retailer's best interest to target both customer groups.

Table 3.10 mentions the best strategies for the follower, based on the leader's action. It can be seen that,when the leader focuses on economy customers, the follower would be better off focusing on premium customers or focusing on both customer types, to gain a higher profit.

Table 3.9: Results of the Stackelberg Game in non-uniform scenario

	Follower focuses on premium	Follower focuses on economy	Follower focuses on both
Leader focuses on Premium	Both retailers set the same price, and earn the same profit	The leader's set price is higher than that of the follower. Their profits depend on the number of customers in each group.	The leader sets a higher price; The profit of the follower is greater, unless when the number of premium customers is considerably greater than that of the economy customers.
Leader focuses on Economy	The leader's set price is less than that of the follower. Their profits depend on the number of customer in each group.	Both retailers set the same price and earn the same profit.	The follower sets the same price and always gains more profit.

Table 3.10: Follower's best response based on leader's action in non-uniform scenario

Leader's Focus	Follower's best response
Premium Customers	Focus on both customer groups. Only if the number of premium customers is extremely greater than that of economy customers, focus only on premium customers.
Economy Customers	Focus on both customer groups. Only if the number of premium customers is extremely greater than that of economy customers, focus only on premium customers.
Both Customers	Focus on both customer groups. Only if the number of premium customers is extremely greater than that of economy customers, focus only on premium customers.

It goes beyond saying that, for the leader, choosing to focus only on economy customers is not suggested. When the leader focuses only on premium customers, the follower may obtain at least as much as her (by also focusing only on economy customers), or gain even more profit (by focusing on premium customers or on both customer groups).

3.6.5 Comparison of results from uniform and non-uniform cases

Comparing the uniform and non-uniform scenarios reveals that customer distribution along the demand line significantly influences retailers' equilibrium decisions and profits.

When customers are uniformly distributed along the demand line, retailers will never co-locate, whereas in the non-uniform scenario, they will always co-locate.

In the uniform scenario, the results of a sequential game differ from those of a simultaneous game. In the simultaneous game, both retailers set the same price and earn equal profits. However, in the sequential game, the follower sets a lower price and earns more profit by attracting more customers.

In the non-uniform scenario, the retailers will always set the same price and earn the same profit, regardless of whether they enter the market simultaneously or sequentially.

3.7 Conclusion

This paper studies the competition between two retailers for the location, and price of the same good, under the condition that the demand market is made up of two types of customers: premium customers, whose reservation price for the product is high, and economy customer, whose reservation price for the product is low. We analyze the problem in two different situations. In the first situation, the two groups of customers are uniformly distributed along the market. In the second situation however, the number of customers of either group is high on one side of the demand market and decreases linearly as one moves toward the other side of the demand market. The demand market is considered to be a traditional Hotelling line.

In each of the two situations, we consider two different scenarios: when the two retailers decide simultaneously on their location and price (model where Bertrand game theory is used), and when the two retailers take turns to enter the market and decide about their location and price (model where Stackelberg game theory is used).

The results of our study show that the heterogeneity of customers has a considerable impact on the retailers' decision of location and price, and that the heterogeneity of customers should not be neglected by simplifying the model and considering only one type of customer.

We demonstrate that the equilibrium strategy for a retailer—whether to target premium customers, economy customers, or both—depends on the distribution of customers along the demand line (uniform versus non-uniform) and the retailers' mode of entry (simultaneous versus sequential). We outline the best strategy for retailers under each of these settings

When the two customer groups are uniformly distributed along the demand line, the two retailers choose different locations but set the same price in the Bertrand game. In the Stackelberg game, they also choose different locations; however, the follower charges a lower price and earns higher profits by capturing a larger market share.

Nevertheless, when the distribution of the two groups of customers is non-uniform along the demand line, they will end up co-locating and offering the same price. However, this same location is not necessarily in the middle of the demand line, despite the traditional Hotelling problem.

Future research could extend this work by incorporating three customer groups (economy, premium, and premium plus) to further analyze market heterogeneity. Another promising direction would be to investigate how collaboration between the two retailers influences their profits in non-homogeneous markets.

Moreover, in this paper, for the non-uniform scenario, we examined the case in which the number of customers in each group changes linearly along the demand line. Future research could explore different and more general forms of non-uniform customer distributions along the demand line.

Finally, considering variations in product quality presents another interesting avenue for future research. In this paper, we assumed the existence of a single product that both customer groups are interested in. Future studies could instead examine scenarios where products with different quality levels are available, and each customer group exhibits a distinct willingness to pay for higher quality.

Chapter 4

Competition for Quality of Services, Price and Location

4.1 Introduction

In modern retail markets, firms increasingly differentiate themselves not only through price and location, but also through the quality of services they provide. In this thesis, service quality refers to a bundle of value-enhancing attributes that improve customer experience beyond the core product. These attributes include after sales support, warranties and extended warranties,hassle free product return, personalized assistance and recommendations, comfortable waiting areas, entertainment and refreshing in the waiting areas, as well as store atmosphere, layout and decoration, and other non-price actions that increase perceived customer value.

Recent empirical research highlights the importance of these service elements.

Environmental aspects of the retail space also play a crucial role. A 2024 study by MOUKRIM, GABER, DIOUCH, and SALAMI (2024) finds that store design, ambient cues, and spatial layout significantly determine customer satisfaction and perceived value. Likewise, Faria, Carvalho, and Vale (2022) provide evidence that service quality and store design jointly serve as strategic differentiation variables in retail competition.

From a strategic perspective, service quality can be interpreted as a retailer investment that enhances consumer utility beyond product and price characteristics. Loyalty programs and membership tiers operate as mechanism-design tools that raise switching costs and segment customers by usage intensity.

In this thesis, service quality is represented by the decision variable Q_i, capturing the intensity of service features offered by retailer i. These features include, but are not limited to, higher-tier membership plans, enhanced in-store atmosphere and decoration, improved customer support services, and more generous discount or reward structures. Increasing Q_i raises customer utility across both premium and economy segments, consistent with recent findings that service investments meaningfully influence consumer preferences, satisfaction, and long-term loyalty. Q_i is considered to be continuous variable, with minim Q_m.

Incorporating service quality into the competitive model allows for a more realistic representation of retail dynamics, where firms routinely invest in service bundles to differentiate themselves, enhance customer experience, and strengthen competitive positioning in the market.

4.2 Problem Definition

In this problem, we consider two retailers evaluating entry into a heterogeneous market. The market is represented as a bounded continuum of length λ, consistent with the classical Hotelling framework and its many extensions in the literature. The demand side consists of two distinct customer segments: premium customers, who place a high value on service quality, and economy customers, who are less sensitive to service enhancements and more price-oriented. The two retailers decide on their market locations, pricing strategies, and—unlike in the previous chapter—the level of service quality they intend to offer.

The analysis builds directly on the previous chapter, where competition was restricted to location and price. Here, we extend the model by incorporating quality of services as a third strategic variable. This addition enables a richer characterization of retailer competition and allows us to study how non-price service attributes influence market coverage and consumer choice.

We investigate two alternative market structures. In the first structure, both customer groups are uniformly distributed along the demand line. In the second structure, the distribution is non-uniform: the number of premium customers is highest at one end of the line and decreases linearly toward the opposite end, whereas the number of economy customers exhibits the opposite pattern—starting lower at the premium-dominated side and increasing linearly toward the other end of the market. This structure reflects markets where customer types are spatially concentrated and gradually transition from one segment to the other.

Under each of these two market structures, we examine two distinct modes of entry for the retailers. In the first mode, both retailers enter the market simultaneously and make their decisions regarding location, pricing, and quality of services at the same time. In the second mode, one retailer is already established in the market and has chosen its location, price, and quality of services in advance. A second retailer then contemplates entering the market and makes its own decisions while taking the incumbent's strategies as given. This setting represents a sequential entry case, where the entrant behaves as a follower and the incumbent as a leader.

4.2.1 Notations

Problem parameters and decision variables are:

Parameters

λ: Length of the demand line.

N: Total number of customers in the market.

t_P: Transportation cost per unit of distance for premium customers.

t_E: Transportation cost per unit of distance for economy customers.

For simplicity and for the sake of comparison, we assume that both customer types incur the same transportation cost, that is,$t_P = t_E = t$. This assumption allows us to focus on differences in preferences for service quality rather than on differences in travel behavior. Moreover, in many Hotelling-type models, a common transportation parameter is standard and facilitates analytical comparison across customer types.

γ: Fraction of the market composed of premium customers.

C: Cost incurred for improving the quality of services by one unit.

d_{ij}: Maximum distance retailer i can cover to serve customer type j, where $i \in \{1, 2\}$ and $j \in \{\text{premium}, \text{economy}\}$.

Π_i: Profit function of retailer i, where $i \in \{1, 2\}$.

Q_m: Minimum level of service quality expected by customers.

θ_j: A constant marginal utility parameter for customer type j, where $j \in \{\text{premium}, \text{economy}\}$.

The parameter θ_j measures how much the utility of a customer of type j changes for each additional unit of service quality offered by a retailer. We interpret θ_j as the incremental sensitivity to small improvements in service quality, relative to a given baseline level.

Note that, while premium customers have a higher overall valuation for high service quality, their marginal sensitivity parameter satisfies $\theta_{\text{premium}} <$

θ_{economy}, indicating that small improvements in service quality generate relatively larger utility gains for economy customers.

Decision variables

P_i: price of the products set by retailer i, $i \in \{1, 2\}$.

X$_i$: location of retailer i along the demand line, $i\in \{1, 2\}$.

Q$_i$: level of quality of services provided by retailer i, $i \in \{1, 2\}$.

It should be noted that providing a service quality level of Q incurs a cost of CQ for each retailer.

Suppose a customer of type j is located at an arbitrary point y_j in the interval $[0, \lambda]$, and that retailer i is located at the point X_i offering the service quality level Q_i and setting the price P_i (figure 4.1). The customer's utility depends upon the quality of services offered by the retailer, and the time spent to access the store:

$$U(j) = \theta_j Q_i - P_i - t|X_i - y_j| \tag{87}$$

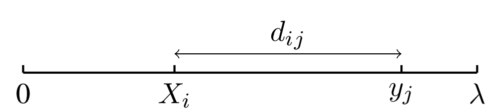

Figure 4.1: Maximum distance that retailer i can cover

The customer will choose to buy from retailer i only if their utility to buy from the retailer is greater than or equal to zero. Putting $U(j) = 0$, we have:

$|X_i - y_j| = \frac{\theta_j Q_i - P_i}{t}$. That is, the far-most customer of type j that retailer i can serve is at distance:

$$d_{ij} = \frac{\theta_j Q_i - P_i}{t} \tag{88}$$

It it is assumed that, when the utility of a customer to buy from both retailers is positive, the customer will choose to buy from the retailer that provides higher utility for the customer.

Figure 5.2 illustrates the farthest economy and premium customers that a retailer located at point X can serve. The segment of demand from economy customers that the retailer can cover is shown in orange, while the segment from premium customers is shown in blue.

Specifically, a retailer located at an arbitrary point X, setting price P and offering service quality level Q, can potentially serve premium customers located within the interval $[X - \frac{\theta_P Q - P}{t},\ X + \frac{\theta_P Q - P}{t}]$. Similarly, the retailer can serve economy customers located within the interval $[X - \frac{\theta_E Q - P}{t},\ X + \frac{\theta_E Q - P}{t}]$.

Because economy customers have a higher marginal utility for service quality ($\theta_{\mathrm{P}} < \theta_{\mathrm{E}}$), the retailer is able to cover a wider portion of the demand line for economy customers.

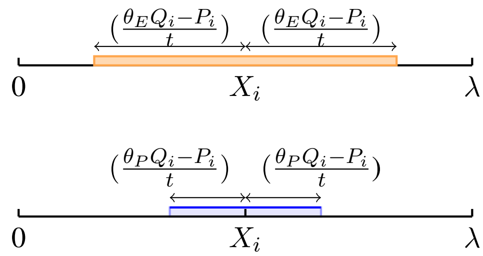

Figure 4.2: Maximum market share of retailer i

In order to analyze the retailers strategy we first make assumption regarding customer distribution. We consider two cases associated with the distribution of both customer groups along the demand line. In the first case, customers are uniformly distributed along the demand line, while in the second case, the customers are non-uniformly distributed. Furthermore, we will consider two sub-cases within each case, regarding retailers' entrance mode to the market. In the first sub-case, both retailers enter the market simultaneously, while in the second, they enter the market sequentially.

4.3 Customers are uniformly distributed along the demand line and both retailers enter the market simultaneously

In the first case, both premium and economy customers are distributed uniformly on the demand line. Two retailers try to maximize their own profit, by optimizing their location, price and quality of services. Here, both retailers make simultaneous decisions regarding their location, pricing, and quality of services, lacking knowledge of each other's intentions. The interplay between transportation costs and quality-related utility leads to variations in the equilibrium choices. Bertrand game theory is used to study this situation.

As evident from (88), a retailer's coverage area depends upon t, θ and Q.

4.3.1 Only a subset of both economy and premium customers can be served by the retailers

When the traveling costs and retail price are relatively high compared to the monetary value of quality of services for both premium and economy customers ($\theta_P Q - p < \frac{\lambda t}{4}$, $\theta_E Q - p < \frac{\lambda t}{4}$), each retailer can serve only part of customers, and some customers are not served at all by either retailer.

Without loss of generality, let's assume that retailer one locates at the left side of retailer two. Retailer one locates at point $X_1 = d_{1E}$ to serve the maximum number of economy customers on her left, and retailer two locates at point $X_2 = \lambda - d_{2E}$ to serve maximum number of economy customers on her right. Figure (5.3) depicts this situation. The orange segments show the intervals of the demand line that each retailer cover economy customers, while the green segments depict the intervals where premium customers are served by each retailer.

$$X_1 = \frac{\theta_E Q_1 - P_1}{t} \tag{89}$$

$$X_2 = \lambda - \frac{\theta_E Q_2 - P_2}{t} \tag{90}$$

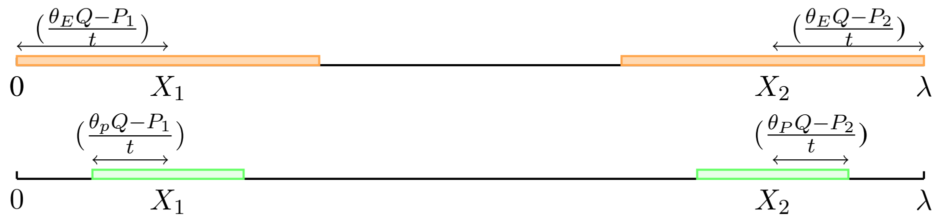

Figure 4.3: Retailers' potential market in partial coverage of economy and premium customers

Therefore, retailer one can cover the economy demand in the interval $[0, 2\frac{\theta_E Q_1 - P_1}{t}]$ and premium demand in the interval $[\frac{(\theta_E - \theta_P)Q_1}{t}, \frac{(\theta_E + \theta_P)Q_1 - 2P_1}{t}]$. Retailer two can cover economy demand in the interval $[\lambda - 2\frac{\theta_E Q_2 - P_2}{t}, \lambda]$ and premium demand in the interval $[\lambda + \frac{P_2 - (2\theta_E - \theta_P)Q_2}{t}, \lambda - 2\frac{(\theta_E - \theta_P)Q_2}{t}]$.

Hence, retailer i can serve a segment of the demand line of length $2\frac{\theta_P Q_i - P_i}{t}$ for premium customers, and a segment of length $2\frac{\theta_E Q_i - P_i}{t}$ for economy customers.

Each retailer seeks to maximize their own profit. The profit is defined as the revenue generated from product sales minus the cost of providing a specific level of service quality. The revenue depends on the number of products sold, which is proportionate to the market demand they are able to cover. Hence, profit of retailer i is:

$$\pi_i = \sum_{j=E,P} \frac{2d_{ij}}{\lambda} P_i N - CQ_i \tag{91}$$

The profit of each retailer is calculated as the fraction of the demand they can cover, multiplied by the total number of customers and the price charged, minus the cost associated with providing the chosen level of service quality.

The response function of retailer i is as follows:

$$\pi_i = \frac{2NP_i}{\lambda}[\gamma(\frac{\theta_P Q_i - P_i}{t}) + (1-\gamma)(\frac{\theta_E Q_i - P_i}{t})] - CQ_i \tag{92}$$

Satisfying the first order conditions results :

$$P_i^* = \frac{C\lambda t}{2N(\gamma\theta_P + (1-\gamma)\theta_E)} \tag{93}$$

$$Q_i^* = \frac{C\lambda t}{N(\gamma\theta_P + (1-\gamma)\theta_E)^2} \tag{94}$$

$$\pi_i^* = \frac{C^2\lambda t}{(\gamma\theta_P + (1-\gamma)\theta_E)^2 N}(\frac{N}{2} - 1) \tag{95}$$

It can be observed from (93) that the equilibrium price increases as the traveling cost(t) and cost of providing quality of services (C) increase. On the other hand, the equilibrium price decreases as the perceived utility of customers increases. When customers get a high utility from quality of services, they would be satisfied with a lower level of services, to provide which the retailer needs less expenses on service quality.

Moreover, (94) shows that the equilibrium quality level is inversely proportional with the square of the utility of the services. This phenomenon occurs because when the perceived service utility increases, customers can attain the same satisfaction level even with a lower quality of services.

4.3.2 All economy customers and only a subset of premium customers can be served by the retailers

This case arises when the monetary value of the utility that economy customers derive from service quality is sufficiently high relative to travel costs and retail price, i.e.,($\theta_E Q - p > \frac{\lambda t}{4}$). Under this condition, all economy customers prefer to purchase, and therefore the entire segment of economy customers is covered by the two retailers. In contrast, premium customers face higher travel costs and retail price compared to the quality-related utility they receive, i.e., ($\theta_P Q - p < \frac{\lambda t}{4}$). As a result, only a subset of premium customers are willing to purchase, and the coverage of premium customers remains partial.

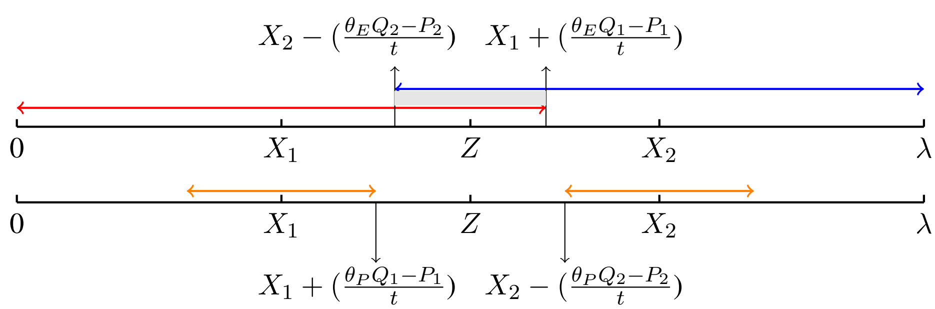

Figure 4.4: Overlap only in economy customers' market.

Figure 4.4 depicts this situation. The interval marked in red represents the potential market of economy customers for retailer one, while the blue interval represents the potential market of economy customers for retailer two. The gray shaded segment indicates economy customers that can be served by either retailer one or retailer two. The orange intervals indicate the potential premium markets for retailers one and two. As depicted in the figure, there is no overlap among premium customers markets in this case. The two retailers will locate at points X_1 and X_2, same as in (89) and (90) respectively.

We need to define the point where a customer is indifferent in choosing either retailers. Calling this point Z, the customers located at point Z have the same utility for buying products from either retailer.

$$Z = \frac{X_1 + X_2}{2} + \frac{\theta_E}{2t}(Q_1 - Q_2) - \frac{P_1 - P_2}{2t} = \frac{\lambda}{2} + \frac{\theta_E}{t}(Q_1 - Q_2) - \frac{P_1 - P_2}{t} \tag{96}$$

Looking at the mathematical representation of point Z in (96), the location of point Z depends on the difference between the prices the two retailers offer, as well as the difference between the level of the services that two retailers provide. If both retailers offer the same price and provide the same level of service, point Z would be at the center of the demand line. This formula also shows that the retailer who offers lower price, or provides higher level of services will have more share of the demand market.

The response functions of the two retailers are as follows:

$$\pi_1 = \frac{P_1 N}{\lambda}\left((1-\gamma)Z + 2\gamma(\frac{\theta_P Q_1 - P_1}{t})\right) - CQ_1 \tag{97}$$

$$\pi_2 = \frac{P_2 N}{\lambda}\left((1-\gamma)(\lambda - Z) + 2\gamma(\frac{\theta_P Q_2 - P_2}{t})\right) - CQ_2 \tag{98}$$

solving to first order conditions:

$$Z = \frac{\lambda}{2} + \frac{\theta_E}{t}(Q_1 - Q_2) - \frac{P_1 - P_2}{t} \tag{99}$$

The partial derivatives of Z with respect to each decision variable are

$$\frac{\partial Z}{\partial P_1} = -\frac{1}{t}, \qquad \frac{\partial Z}{\partial Q_1} = \frac{\theta_E}{t}, \tag{100}$$

$$\frac{\partial Z}{\partial P_2} = \frac{1}{t}, \qquad \frac{\partial Z}{\partial Q_2} = -\frac{\theta_E}{t} \tag{101}$$

Since Z depends on prices and qualities, the chain rule must be applied.

Retailer 1. The first-order condition with respect to price is

$$\frac{\partial \pi_1}{\partial P_1} = \frac{N}{\lambda}\left[\left((1-\gamma)Z + \gamma\frac{2(\theta_P Q_1 - P_1)}{t}\right) + P_1\left((1-\gamma)\frac{\partial Z}{\partial P_1} - \frac{2\gamma}{t}\right)\right] = 0. \tag{102}$$

Substituting $\frac{\partial Z}{\partial P_1} = -\frac{1}{t}$ yields

$$\frac{\partial \pi_1}{\partial P_1} = \frac{N}{\lambda}\left[(1-\gamma)Z + \gamma\frac{2(\theta_P Q_1 - P_1)}{t} - \frac{(1-\gamma+2\gamma)P_1}{t}\right] = 0. \tag{103}$$

The first-order condition with respect to quality is

$$\frac{\partial \pi_1}{\partial Q_1} = \frac{N}{\lambda}\left[P_1\left((1-\gamma)\frac{\partial Z}{\partial Q_1} + \frac{2\gamma\theta_P}{t}\right)\right] - C = 0. \tag{104}$$

Using $\frac{\partial Z}{\partial Q_1} = \frac{\theta_E}{t}$, this becomes

$$\frac{N}{\lambda}P_1\left(\frac{(1-\gamma)\theta_E + 2\gamma\theta_P}{t}\right) = C. \tag{105}$$

Retailer 2. Similarly, the first-order condition with respect to price is

$$\frac{\partial \pi_2}{\partial P_2} = \frac{N}{\lambda}\left[(1-\gamma)(\lambda - z) + \gamma\frac{2(\theta_P Q_2 - P_2)}{t} - \frac{(1-\gamma+2\gamma)P_2}{t}\right] = 0, \tag{106}$$

and the first-order condition with respect to quality is

$$\frac{N}{\lambda}P_2\left(\frac{(1-\gamma)\theta_E + 2\gamma\theta_P}{t}\right) = C. \tag{107}$$

From (105) and (107), the equilibrium prices satisfy

$$P_i^* = \frac{C\lambda t}{N\big((1-\gamma)\theta_E + 2\gamma\theta_P\big)}. \tag{108}$$

Substituting $P_1 = P_2 = P^*$ into the price first-order conditions implies

$$Q_i^* = \frac{(3\gamma+1)P^* - \frac{(1-\gamma)\lambda t}{2}}{2\gamma\theta_P} \tag{109}$$

$$Q^* = \frac{\lambda t}{2\gamma\theta_P}\left(\frac{(3\gamma+1)C}{N\big((1-\gamma)\theta_E + 2\gamma\theta_P\big)} - \frac{1-\gamma}{2}\right). \tag{110}$$

$$\pi_i^* = \frac{C\,\lambda\,t}{4N\gamma\theta_P\,A^2}\left[4C\gamma\theta_P(\gamma+1) - 2C(3\gamma+1)A - N(\gamma-1)A^2\right] \tag{111}$$

Where

$$A = (1-\gamma)\theta_E + 2\gamma\theta_P. \tag{112}$$

The equilibrium price and service quality expressions reveal that premium customers exert a disproportionately stronger influence on retailers' decisions compared to economy customers. Although the equilibrium price depends on a weighted average of the utilities of both customer groups, the utility of premium customers is multiplied by a factor of two.

A similar pattern emerges in the equilibrium service quality expression, where only the premium customers' valuation of service quality, θ_P, appears in the denominator.

This result arises because all economy customers are fully covered in equilibrium, while only a subset of premium customers is served. Hence, the mass of economy customers is fixed, whereas the number of premium customers captured by each retailer is endogenously determined by pricing and service quality decisions. This endogeneity explains why the valuation of service quality by premium customers has a stronger influence on the equilibrium decisions.

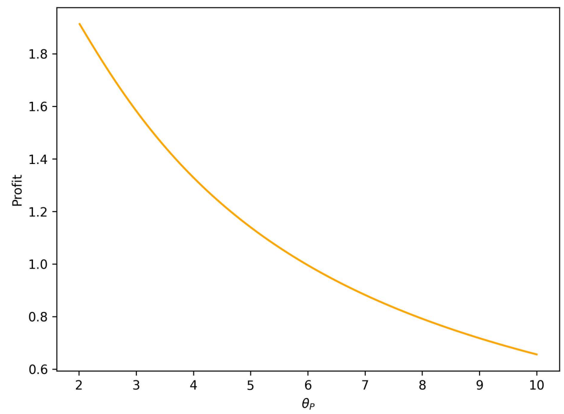

Figure 4.5: Effect of θ_P on profit

Figure 4.5 illustrates the impact of θ_P on the equilibrium profit. As the utility drawn from service quality for premium customers increases, the equilibrium profit decreases. This occurs because an increase in θ_P lowers the equilibrium price more significantly than it affects the equilibrium service quality level. Specifically, the equilibrium price is inversely proportional to θ_P, whereas the equilibrium service quality level is inversely proportional to the square root of θ_P. As a result, the overall profit declines with increasing θ_P.

As the proportion of premium customers (γ) increases, the profit of each retailer declines. This is because the number of economy customers - who can be fully served - decreases, while the number of premium customers — of whom only a subset can be served — increases. Consequently, the total number of customers effectively served diminishes, leading to a reduction in overall profit.

Figure 4.6 depicts the impact of the percentage of premium customers and their utility on the equilibrium level of service quality. The equilibrium quality level decreases with an increase in the percentage of premium customers. This is because the utility of premium customers for quality of servicess is consistently lower than that of economy customers. Consequently, a higher percentage of premium customers results in fewer customers prioritizing service quality, leading to a decrease in the overall quality of services. Furthermore, the service quality diminishes as the utility of service quality increases for premium customers. A heightened utility level indicates that customers are more content with a minor change in the quality level. Consequently, elevated utility values for quality of servicess imply that premium customers find satisfaction even with lower levels of service quality.

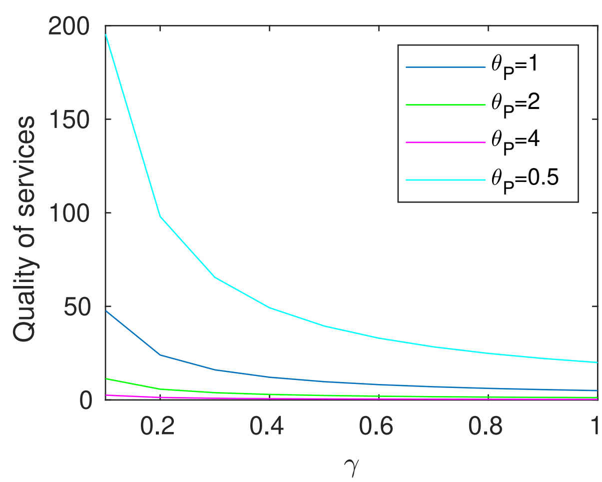

Figure 4.6: Effect of γ and θ_P on quality of services

4.3.3 The retailers are able to serve all customers, both premium and economy

When the traveling cost and retail price of both customer groups are relatively low compared the monetary value of the quality of service ($\theta_P Q - p > \frac{\lambda t}{4}$, $\theta_E Q - p > \frac{\lambda t}{4}$), all demand points for both customer groups can be covered. Some customers can be served by either retailer. Figure 5.10 depicts this situation. The two retailers will locate at points X_1 and X_2, same as in (89) and (90) respectively.

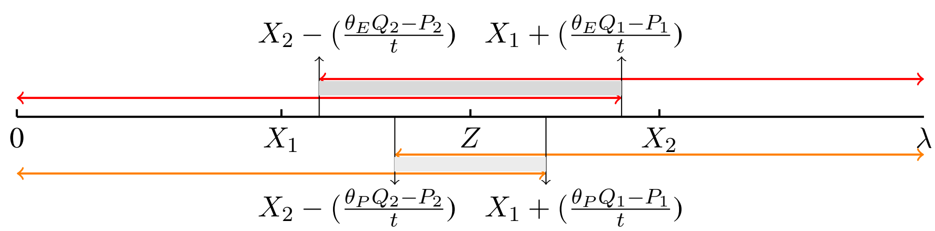

Figure 4.7: Overlap in both customers' market.

The response functions of the retailer are:

$$\pi_1 = \frac{NP_1}{\lambda}(Z) - CQ_1 \tag{113}$$

$$\pi_2 = \frac{NP_2}{\lambda}(\lambda - Z) - CQ_2 \tag{114}$$

Solving the first order conditions:

Since $\frac{\partial \pi_i}{\partial Q_i} = -C$, a constant negative value independent of either variable,

Q_i should take the minimum possible value, which is Q_m.

$$Q_i^* = Q_m \tag{115}$$

Also, since π_i is increasing in P_i, the equilibrium price should be the maximum possible price, that allows coverage of the entire market.

$$P_i^* = \theta_P Q_m - \frac{\lambda t}{4} \tag{116}$$

$$\pi_i = \frac{N}{2}(\theta_P Q_m - \frac{\lambda t}{4}) - CQ_m \tag{117}$$

The equilibrium results reveal that the equilibrium decisions only depend on the utility of the premium customers (θ_P), and not on the percentage of them(γ), or the characteristics of the economy customers. This can be explained by the fact that all customers of both groups are covered completely in this case. Therefore, the percentage of either group is not important.

Summing up the results in section 4.3:

Proposition 4.1. *When customers are uniformly distributed along the demand line and the two retailers enter the market simultaneously, the equilibrium price P_i^* and equilibrium service quality Q_i^* depend on whether the premium and economy segments can be fully or partially served. For each retailer $i \in \{1, 2\}$:*

(i) ***Partial coverage of both segments:***

If $(\frac{\theta_P Q_i - P_i}{t} < \frac{\lambda}{4})$ *and* $(\frac{\theta_E Q_i - P_i}{t} < \frac{\lambda}{4})$*, then* $(P_i^*, Q_i^*) \in \mathcal{F}(C)$*.*

(ii) ***Full coverage of economy, partial coverage of premium:***

If $(\frac{\theta_P Q_i - P_i}{t} < \frac{\lambda}{4})$ *and* $(\frac{\theta_E Q_i - P_i}{t} > \frac{\lambda}{4})$*, then* $(P_i^*, Q_i^*) \in \mathcal{F}(C)$*.*

(iii) ***Full coverage of both segments:***

If $(\frac{\theta_P Q_i - P_i}{t} > \frac{\lambda}{4})$ *and* $(\frac{\theta_E Q_i - P_i}{t} > \frac{\lambda}{4})$*, then* $(P_i^*, Q_i^*) \notin \mathcal{F}(C)$*.*

Here, $\mathcal{F}(C)$ denotes the set of equilibrium solutions in which the equilibrium service quality depends on the quality cost parameter C.

Proof. When $\frac{\theta_P Q_i - P_i}{t} < \frac{\lambda}{4}$ and $\frac{\theta_E Q_i - P_i}{t} < \frac{\lambda}{4}$, for $i \in \{1, 2\}$ (see Section 4.3.1), or when $\frac{\theta_P Q_i - P_i}{t} < \frac{\lambda}{4}$ and $\frac{\theta_E Q_i - P_i}{t} > \frac{\lambda}{4}$, for $i \in \{1, 2\}$ (see Section 4.3.2), the extent of market coverage depends on the utility margin $\theta_j Q_i - P_i$. Since this market size is directly affected by the chosen service quality Q_i, the size of the covered region depends on how much customers value service quality. Therefore, both the equilibrium price and the equilibrium level of service quality become functions of the cost parameter C.

However, when $\frac{\theta_P Q_i - P_i}{t} > \frac{\lambda}{4}$ and $\frac{\theta_E Q_i - P_i}{t} > \frac{\lambda}{4}$, for $i \in \{1, 2\}$ (see Section 4.3.3), full coverage of demand is achievable because transportation costs are sufficiently low relative to the utility margin. In this case, each retailer serves the entire market, and market share no longer depends on small changes in service quality. Consequently, the equilibrium price and equilibrium service quality become independent of the quality cost parameter C. □

Proposition 4.2 (Symmetric Pricing and Quality of Services with Distinct Locations under Uniform Demand)**.** *When demand points are uniformly distributed along the Hotelling line and both retailers choose their strategies simultaneously, the equilibrium features identical prices and identical levels of quality of services for both retailers. Nevertheless, the retailers locate at distinct positions along the demand line in order to maximize their respective profits.*

Explanation. Under a uniform distribution of demand, symmetric pricing and service quality choices arise because customers on either side of the market evaluate both retailers identically when prices, quality of servicess, and transportation costs are the same. Consequently, any deviation in price or quality of service by one retailer would reduce its profit, leading both firms to adopt identical strategies in these dimensions.

However, despite symmetry in price and service quality, the retailers equilibriumly choose distinct locations. If both firms were to co-locate, they would have to split the market equally, which generally results in lower profits compared to differentiating their spatial positions. This effect is especially significant when full market coverage is not guaranteed; choosing different locations allows each retailer to secure a larger portion of the demand that lies closer to its position, thereby increasing profitability.

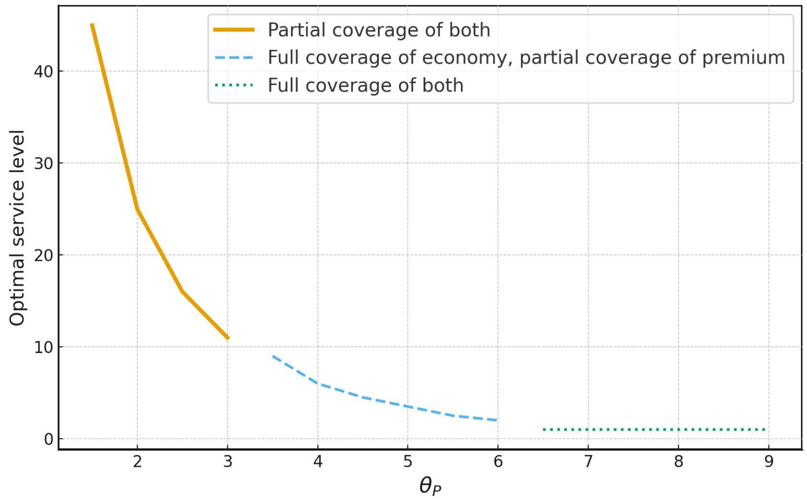

Figure 4.8: Effect of utility of quality of services on quality of services

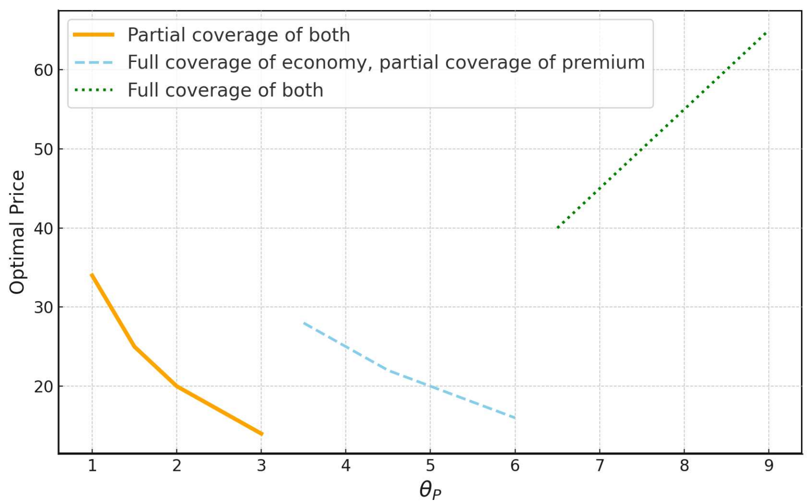

Figure 4.9: Effect of utility of quality of services on equilibrium price

Figure 4.8 demonstrates the effect of the utility of services on the equilibrium level of quality of services in the three cases discussed in 4.3. It is clear that, as the utility for services increases, the quality of services decreases until it reaches a constant value—the minimum quality of services—and does not change further. As the utility for the quality of services increases, customers will be satisfied with a lower service quality. Therefore, retailers can reduce the level of service quality when the utility for service quality increases. When the utility is high enough, retailers can set the quality of services to the minimum possible.

Figure 4.9 depicts the effect of utility from the quality of services on the pricing policy in the three cases discussed in 4.3. It is clear from the plot that, in cases 4.3.1 and 4.3.2, where the demand can be served partially, the price decreases as the utility for service quality increases. This is due to the fact that, as discussed earlier, with the increase in utility for service quality, the level of quality decreases, resulting in lower costs for providing services at a lower quality level. Therefore, as utility increases, quality of services and price will decrease. However, in 4.3.3, where all demand can be served, the quality of services will stay constant with an increase in utility for the quality of services. Therefore, the cost of providing services will also remain unchanged. With the increase in the utility level, customers will be more satisfied with the minimum quality of services and will be willing to pay more for the product. Hence, the price increases with an increase in utility for the quality of services in 4.3.3.

4.4 Customers are uniformly distributed along the demand line and retailers enter the market sequentially

In this case, a pre-existing retailer (referred to as the leader) is already established within the market, having established factors such as location, pricing, and quality standards. Another retailer (referred to as the follower) contemplates market entry. The follower seeks to make informed choices, taking into account the established decisions of the leader. Different cases are examined based on the customer group that the leader and the follower target.

4.4.1 Both leader and follower target premium customers

Case one: Partial coverage

The two retailers opt to locate same as in (89) and (90), respectively.As there is no intersection between the potential markets, each retailer independently maximizes their profit function. In the profit function, only premium customers are considered, and both the price and the quality services are decided upon with respect to this group. However, the additional profit obtained from serving economy customers at the same equilibrium price and quality of services is also calculated.

$$\pi_i = \frac{2\gamma p_i}{\lambda}(\frac{\theta_P Q_i - P_i}{t})N - CQ_i \tag{118}$$

The first-order condition results:

$$P_i^* = \frac{\theta_P Q_i^*}{2} \tag{119}$$

$$P_i^* = \frac{\lambda t C}{2N\gamma\theta_P} \tag{120}$$

$$Q_i^* = \frac{\lambda t C}{N\gamma\theta_P^2} \tag{121}$$

$$\pi_i^* = \frac{\lambda t C^2}{\gamma\theta_P^2 N}(\frac{N}{2} - 1) \tag{122}$$

$$\pi_i^{additional} = \frac{(1-\gamma)C^2\lambda t}{N\gamma^2\theta_p^2}(\frac{\theta_E}{\theta_P} - \frac{1}{2}) \tag{123}$$

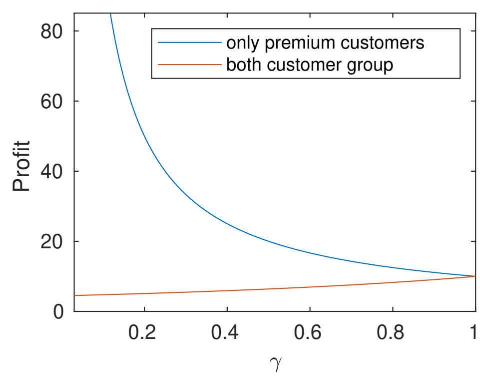

Figure 4.10: Serving premium vs both premium and economy customers

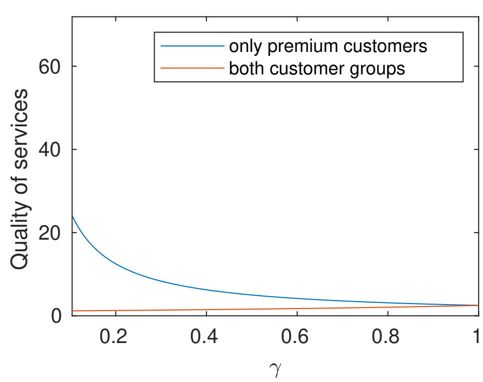

Figure 4.11: Comparison of quality of services serving premium or both groups

When comparing the equilibrium pricing and quality levels between this case and 4.3.1, it becomes evident that the outcomes are highly identical. The sole distinction lies in the fact that in this situation, only one type of customer is being focused.

Figure 4.10 compares the profit of serving premium customers versus serving both customer groups when demand can be served partially. It is always in the best interest of a retailer to focus only on premium customers rather than serving both economy and premium customers.

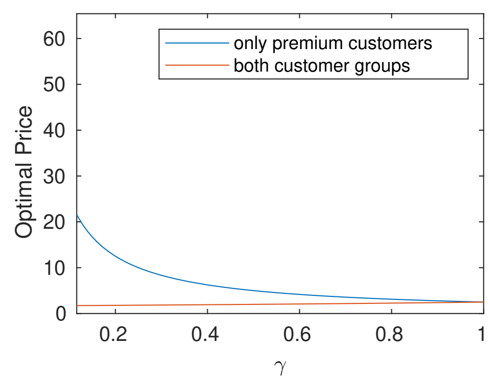

Figure 4.12: Pricing for premium vs both customer groups

Moreover, figures 4.11 and 4.12 compare the equilibrium quality of services and the equilibrium pricing strategy, respectively, for serving exclusively premium customers and both customer groups when demand can be fulfilled partially. It is evident that both the quality of services and the offered price are strictly greater when only premium customers are served. Since the utility that premium customers derive from services is lower than that of economy customers, serving only premium customers requires a higher quality service, which is more costly compared to the case where both customer groups are served.

It is worth noting that the graphs intersect at the point $\gamma = 1$, which indicates that all customers are premium customers. At this point, serving only premium customers is equivalent to serving both customer groups.

Case two: Full coverage

In this case, both retailers collectively have the capacity to fulfill the entire demand line. Their respective locations would coincide with (89) and (90). The response functions for each would be :

$$\pi_1 = P_1\gamma(\frac{1}{2} + \frac{\theta_P}{\lambda t}(Q_1 - Q_2) - (\frac{P_1 - P_2}{\lambda t}))N - CQ_1 \tag{124}$$

$$\pi_2 = P_2\gamma(\frac{1}{2} + \frac{\theta_P}{\lambda t}(Q_2 - Q_1) + (\frac{P_1 - P_2}{\lambda t}))N - CQ_2 \tag{125}$$

solving to equilibriumly results:

$$P_i^* = \frac{\lambda t C}{N\gamma\theta_P} \tag{126}$$

$$Q_2^* = Q_1^* + \frac{\lambda t C}{\gamma\theta_P^2} - \frac{\lambda t}{2\theta_P} \tag{127}$$

$$Q_1^* = \frac{\lambda t}{\gamma\theta_P - C} \tag{128}$$

$$Q_2^* = \frac{\lambda t}{\gamma\theta_P - C} + \frac{\lambda t C}{\gamma\theta_P^2} - \frac{\lambda t}{2\theta_P} \tag{129}$$

$$\pi_1 * = \frac{\lambda t C}{\theta_P}(1 - \frac{C}{\gamma\theta_P} - \frac{1}{\gamma\theta_P - C})$$

$$\pi_1^{additional} = (1-\gamma)(\frac{\lambda t C}{N\gamma\theta_P})(\frac{\lambda}{2} + \frac{\lambda t\theta_E}{t\theta_P}(\frac{1}{2} - \frac{C}{\gamma\theta_P}))(\frac{N}{\lambda}) \tag{130}$$

$$\pi_2^* = \lambda t C[\frac{\gamma+1}{\theta_P} - \frac{1}{\gamma\theta_P - C} - \frac{C}{\gamma\theta_P^2}] \tag{131}$$

$$\pi_2^{additional} = (1-\gamma)(\frac{\lambda t C}{N\gamma\theta_P})(\frac{\lambda}{2} - \frac{\lambda t\theta_E}{t\theta_P}(\frac{1}{2} - \frac{C}{\gamma\theta_P}))(\frac{N}{\lambda}) \tag{132}$$

When assessing the equilibrium decisions of the two retailers in this case, it becomes evident that both retailers present identical pricing. Nonetheless, the second retailer provides a superior service quality (assuming that $C > \frac{\gamma\theta_P}{2}$). As a result, the follower caters to a larger segment of the premium customer market demand and consequently earns a higher profit.This outcome corresponds with the principle that in a symmetric game with identical information, the follower's profit is either greater than or equal to that of the leader.

4.4.2 The Leader focuses on premium customers while the follower focuses on economy customers

As each retailer concentrates on a distinct customer group, each one will make their decisions independently, regardless of the choices made by the other retailer.

Case one: Partial coverage

The leader can locate at any point in the interval $[\frac{\theta_P Q_1 - P_1}{t}, \lambda - \frac{\theta_P Q_1 - P_1}{t}]$. The leader's response function, as well as their equilibrium pricing, quality level, and resulting profit, resemble those of (119),(120),(121) and (122) respectively.

Case two: Full coverage

The leader will locate at the center of the line, at point $\frac{\lambda}{2}$. The response function of the leader would be as follows:

$$\pi_1 = \frac{1}{\lambda} p_1 \gamma \lambda N - CQ_1 \tag{133}$$

Solving to first order conditions results:

$$P_1^* = \theta_P Q_m - \frac{\lambda t}{2} \tag{134}$$

$$Q_1^* = Q_m \tag{135}$$

$$\pi_1^* = \gamma N(\theta_P Q_m - \frac{\lambda t}{2}) - CQ_m \tag{136}$$

$$\pi_1^{additional} = (1-\gamma)N(\theta_P Q_m - \frac{\lambda t}{2}) \tag{137}$$

This result shares similarities with the outcome in section 4.3.3, while differing in the aspect that, in this scenario, the equilibrium price consistently surpasses that of case 4.3.3. This can be attributed to the fact that only premium customers are being served here, and they are inclined to pay higher prices for enhanced quality. Moreover, the quality level is elevated when exclusively catering to premium customers, as opposed to serving both customer groups. Lastly, the overall profit in this case is greater when $(\theta_P - \theta_E < \frac{2C}{1-\gamma})$.

The same reasoning can be applied to the follower. We choose the price and level of services by considering only economy customers. The extra profit form serving premium customers with the same price and quality of services is also presented.

Case one: Partial coverage

The follower can locate at any point in the interval $[\frac{\theta_E Q_2 - P_2}{t}, \lambda - \frac{\theta_E Q_2 - P_2}{t}]$. The response function of the follower is:

$$\pi_2 = \frac{2P_2}{\lambda}(1-\gamma)(\frac{\theta_E Q_2 - P_2}{t})N - CQ_2 \tag{138}$$

The equilibrium decisions of the follower would be:

$$P_2^* = \frac{\lambda t C}{2N(1-\gamma)\theta_E} \tag{139}$$

$$Q_2^* = \frac{\lambda t C}{N(1-\gamma)\theta_E^2} \tag{140}$$

$$\pi_2^* = \frac{\lambda t C^2}{N(1-\gamma)\theta_E^2}(\frac{N}{2} - 1) \tag{141}$$

$$\pi_2^{additional} = \frac{\gamma C^2 \lambda t}{N(1-\gamma)^2\theta_E^2}(\frac{\theta_P}{\theta_E} - \frac{1}{2}) \tag{142}$$

Case two: Full coverage

The follower will locate at the center of the line, at point $\frac{\lambda}{2}$. The response function of the follower would be:

$$\pi_2 = \frac{1}{\lambda}p_2(1-\gamma)\lambda N - CQ_2 \tag{143}$$

$$P_2^* = \theta_E Q_m - \frac{\lambda t}{2} \tag{144}$$

$$Q_2^* = Q_m \tag{145}$$

$$\pi_2^* = (1-\gamma)N(\theta_E Q_m - \frac{\lambda t}{2}) - CQ_m \tag{146}$$

$$\pi_2^{additional} = \gamma N(\theta_E Q_m - \frac{\lambda t}{2}) \tag{147}$$

Contrasting this scenario with 4.3.3, the equilibrium quality of services remains the same in both cases. However, the equilibrium price is higher in the present case, where the retailer focuses exclusively on economy customers, compared to 4.3.3, where the retailer serves both customer groups.

This outcome can be explained by the fact that economy customers derive greater utility from service quality. Consequently, at the same quality of services, economy customers are willing to pay more than premium customers, and a mix of premium and economy customers.

Figure 4.13 depicts a comparison of profits in full demand coverage, for three different scenarios: serving only premium customers, serving only economy customers, or serving both customer groups. The profit functions are presented in (136), (146) and (117) respectively.

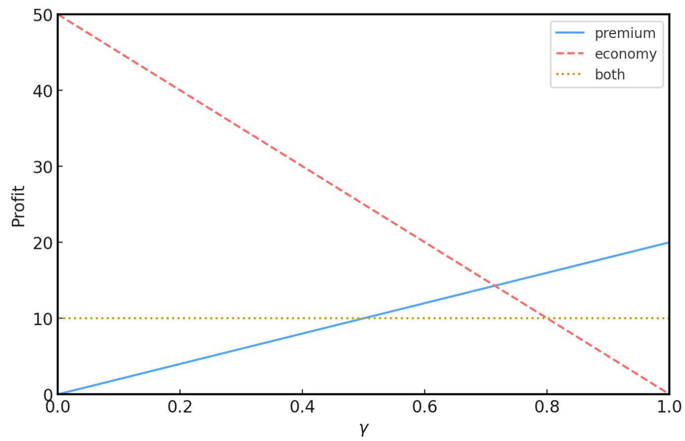

Figure 4.13: Comparison of profits in full demand coverage, exclusively serving one group

It can be observed that serving both customer groups is never the equilibrium strategy for retailers when they have the option to focus exclusively on a single group. When the proportion of premium customers is low or moderate, it is more profitable for the retailer to focus solely on economy customers. Conversely, when the proportion of premium customers is high, the retailer achieves greater profit by serving only premium customers.

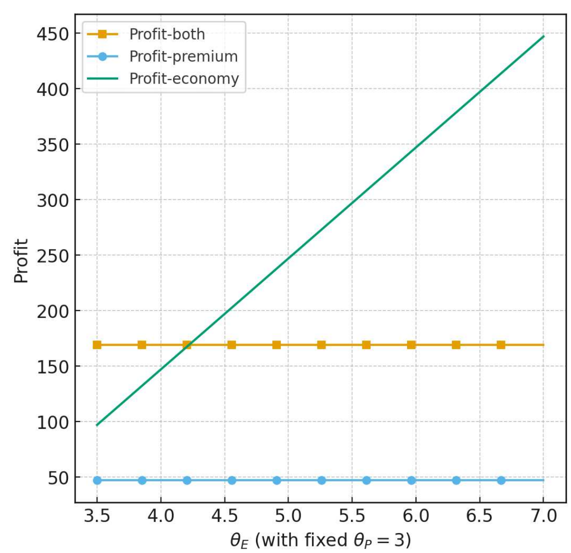

Figure 4.14: Comparison of profits in full coverage, serving both groups

Figure 4.14 depicts a comparison of profits for three different strategies: setting equilibrium price and quality of services based on premium customers (and serving the economy customers with same price and quality of services), based on economy customers (and swerving premium customers with the same price and quality of services), or based on both customer groups, when the demand can be completely covered.

When the difference in service-quality utility between premium and economy customers is small, it is equilibrium for retailers to set price and service quality by considering both customer groups. However, when this utility difference is moderate or large, retailers are better off making their decisions solely with respect to economy customers.

4.4.3 The leader focuses on economy customers and the follower focuses on the premium customers

This scenario is identical to 4.4.2, with the sole distinction being the reversal of positions between the leader and the follower.

4.4.4 Both leader and follower focus on economy customers

Here, since both the leader and the follower focus only on economy customers, there is no competition on premium customers.

Case one: Partial coverage

The response function, equilibrium price, equilibrium quality and equilibrium price would be same as (119), (120), (122) and (122), respectively.

Case two: Full coverage

Here, both retailers together can serve the whole demand line. Their location would be same as (89) and (90) respectively. The response functions would be:

$\pi_1 = P_1(1-\gamma)(\frac{1}{2} + \frac{\theta_E}{\lambda t}(Q_1 - Q_2) - (\frac{P_2 - P_1}{\lambda t}))N - CQ_1$

$\pi_2 = P_2(1-\gamma)(\frac{1}{2} + \frac{\theta_E}{\lambda t}(Q_2 - Q_1) + (\frac{P_1 - P_2}{\lambda t}))N - CQ_2$

$$P_i^* = \frac{\lambda t C}{N(1-\gamma)\theta_E} \tag{148}$$

$$Q_2^* = Q_1^* + \frac{\lambda t C}{(1-\gamma)\theta_E^2} - \frac{\lambda t}{2\theta_E} \tag{149}$$

$$Q_1^* = \frac{\lambda t}{(1-\gamma)\theta_E - C} \tag{150}$$

$$Q_2^* = \frac{\lambda t}{(1-\gamma)\theta_E - C} + \frac{\lambda t C}{(1-\gamma)\theta_E^2} - \frac{\lambda t}{2\theta_E} \tag{151}$$

$$\pi_1 * = \frac{\lambda t C}{\theta_E}(1 - \frac{C}{(1-\gamma)(\theta_E)} - \frac{1}{(1-\gamma)\theta_E - C})$$

$$\pi_2^* = \lambda t C[\frac{\gamma+1}{\theta_E} - \frac{1}{(1-\gamma)\theta_E - C} - \frac{C}{(1-\gamma)\theta_E^2}] \quad (152)$$

When assessing the equilibrium decisions of the two retailers in this case, it becomes evident that both retailers present identical pricing. Nonetheless, the second retailer provides a superior service quality (assuming that $C > \frac{(1-\gamma)\theta_E}{2}$). As a result, the follower caters to a larger segment of the economy customer market demand and consequently earns a higher profit.This outcome corresponds with the principle that in a symmetric game with identical information, the follower's profit is either greater than or equal to that of the leader.

4.5 Customers are non-uniformly distributed along the demand line and retailers enter the market simultaneously

The second situation explored in this paper involves the non-uniform distribution of two customer groups: premium customers and economy customers, along the demand line. This situation is considered more realistic. In this context, it is assumed that the number of premium customers is relatively low on the left side of the demand line and increases gradually as we move towards the right side. Conversely, the number of economy customers is higher on the left side of the demand line and gradually decreases as we progress towards the right side. This scenario is often encountered in densely populated metropolitan areas, where wealthier individuals tend to reside in specific neighborhoods, while less affluent residents live in other neighborhoods. Figure 5.14 illustrates this scenario.

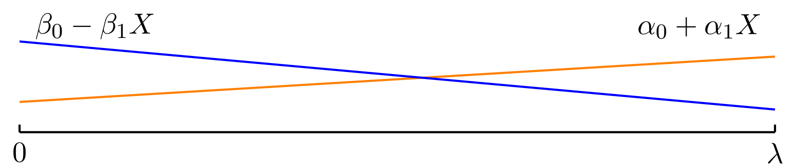

Figure 4.15: Depiction of the non-uniform Scenario

Some parameters need to be defined here:

α_0: the number of premium customers at the left side of the demand line (point 0).

α_1: the rate with which the number of premium customers increase when moving toward the right side of the demand line.

β_0: the number of economy customers at the left side of the demand line (point 0).

β_1: the rate with which the number of economy customers decrease along the demand line.

At any given point x on the demand line, where ($0 < x < \lambda$), the premium demand is ($f_P(x) = \alpha_0 + \alpha_1 x$) and the economy demand is ($f_E(x) = \beta_0 - \beta_1 x$).

N.B. Throughout section 4.5, we assume $\alpha_1 \neq \beta_1$. The number of premium and economy customers at any given point are $f_P(x) = \alpha_0 + \alpha_1 x$, and $f_E(x) = \beta_0 - \beta_1 x$, respectively. Hence, the total number of customers along the market is $f_T(x) = f_P(x) + f_E(x) = \alpha_0 + \beta_0 + (\alpha_1 - \beta_1)\, x$. When $\alpha_1 = \beta_1$, the total density becomes uniform (constant in x). Consequently, $\alpha_1 = \beta_1$ falls outside the scope of our non-uniform analysis and is excluded. All results below are stated under the maintained assumption $\alpha_1 \neq \beta_1$.

N.B. It was previously proved in chapter 3 that the two retailers will always co-locate when the distribution of customers is non-uniform. This result is used in the following analysis.

4.5.1 Only a subset of both economy and premium customers can be served by retailers

In this setting, neither the premium nor the economy customer groups can be fully served. Consequently, each retailer serves only a portion of the demand line from both customer types. It is clear that the potential market size for economy customers is strictly larger than that of the premium customers (since $\theta_E > \theta_P$). Therefore, each retailer must strategically choose her location, price, and service quality to maximize profit. The corresponding profit functions of the retailers are presented below.

$$\pi_i = \frac{P_i}{2}(\int_{X_i-\frac{\theta_P Q_i - P_i}{t}}^{X_i+\frac{\theta_P Q_i - P_i}{t}} (\alpha_0 + \alpha_1 X)\,dx + \int_{X_i-\frac{\theta_E Q_i - P_i}{t}}^{X_i+\frac{\theta_E Q_i - P_i}{t}} (\beta_0 - \beta_1 X)\,dx) - CQ_i \tag{153}$$

Subject to:

$$\frac{\theta_{pQi} - p_i}{t} > 0 \tag{154}$$

$$\frac{\theta_E Q_i - p_i}{t} > 0 \tag{155}$$

$$\frac{\theta_p Q_i - p_i}{t} < \frac{\lambda}{2} \tag{156}$$

$$\frac{\theta_E Q_i - p_i}{t} < \frac{\lambda}{2} \tag{157}$$

Equation (153) presents the profit function of each retailer. It is defined as the sum of the revenues from the economy and premium customer segments—each calculated as the number of customers served multiplied by the corresponding price—minus the cost of providing the service.

Assuming that each retailer i chooses a location X_i, which is to be determined, the retailer can serve premium customers within the interval $[X_i - \frac{\theta_P Q - P}{t}, X_i + \frac{\theta_P Q - P}{t}]$, and economy customers within the interval $[X_i - \frac{\theta_E Q - P}{t}, X_i + \frac{\theta_E Q - P}{t}]$ respectively. Accordingly, in the profit function, these intervals are used as the bounds of integration over the spatial distributions of premium and economy customers, respectively, to capture the total number of customers the retailer is able to serve.

Constraints (154) and (155) ensure that the maximum distance a retailer can cover is positive for premium and economy customers, respectively.

Constraints (156) and (157) ensure that both groups of customers can only be partially served. In other words, the maximum distance a retailer can cover for each group of customers is strictly less than half of the length of the demand line. Since each retailer covers a symmetric interval around her location, the total serviceable interval is twice the maximum distance.

The equilibrium solutions are as follows:

$$X_1^* = X_2^* = \frac{2C + \theta_P \alpha_0 + \theta_E \beta_0}{\theta_P \alpha_1 + \theta_E \beta_1} - \frac{2C(\theta_E + \theta_P)(\beta_1 - \alpha_1)}{(\beta_1 \theta_E)^2 - (\alpha_1 \theta_P)^2} \tag{158}$$

$$P_1^* = P_2^* = \frac{\lambda t(\beta_1 \theta_E - \alpha_1 \theta_P)}{\beta_1(\theta_P - \theta_E)} \tag{159}$$

$$Q_1^* = Q_2^* = \frac{\lambda t(\beta_1 - \alpha_1)}{\theta_P - \theta_E} \tag{160}$$

Analyzing the equilibrium location expression in (158), the solution comprises two distinct components. The first term (first fraction) represents a weighted center of demand based on customer density and cost structure, while the second term (second fraction) accounts for customer heterogeneity by adjusting for differences in the spatial distribution of premium and economy customers.

If ($\alpha_1 > \beta_1$), the number of premium customers increases with a higher rate. The second fraction becomes negative, which shifts the equilibrium location towards the right side of the line, where the premium customers are more densely concentrated.

If ($\alpha_1 < \beta_1$), the number of economy customers increases with a higher rate. The second fraction becomes positive, which shifts the equilibrium location towards the left side of the line, where the economy customers are more densely concentrated.

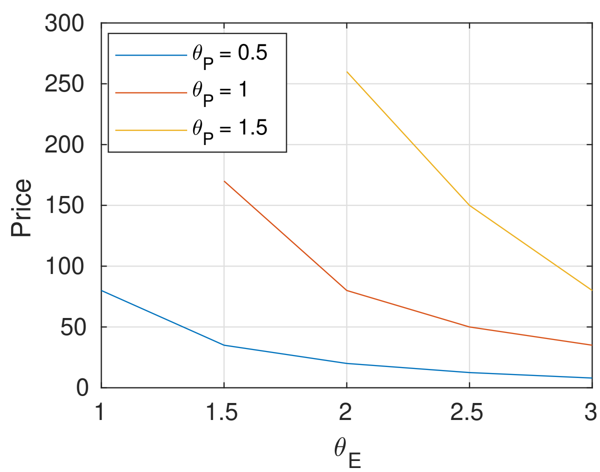

Figure 4.16: Effect of customers' utility on price

Figure 4.16 Depicts the effect of customers' utility of the equilibrium price in 3.2.1.(I). It can be seen that as the difference between the utility of economy and premium customers increases, the equilibrium price decreases. Moreover, when values of the utility of economy and premium customers change, but their ratio stays constant, the equilibrium price does not change. That is, the ratio $\frac{\theta_P}{\theta_E}$ is important in determining the equilibrium price, not the values of θ_P and θ_E.

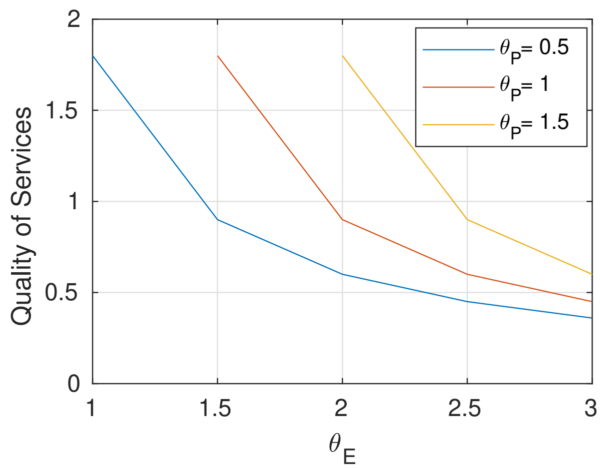

Figure 4.17: Effect of customers' utility on quality

Figure4.17 demonstrated the effect of customers' utility on the equilibrium service quality of services. It is clear that the equilibrium quality of services decreases as the difference between the utility of economy and premium customers increases. More over, regardless of the values of utility of economy and premium customers, if their difference stays unchanged, the quality of services will remains unchanged. That is, $(\theta_E - \theta_P)$ is important in determining the equilibrium quality of services.

It is important to note that in Figures 4.16 and 4.17, some lines do not begin at the vertical axis. This is due to the condition $(\theta_E > \theta_P)$, which must be satisfied for the model to be valid. As a result, the plots are only shown for combinations of parameter values that meet this condition, and not for the full domain of θ_E and θ_P.

4.5.2 All economy customers and only a subset of premium customers can be served by the retailers

Here, the perceived utility of economy customers from quality of services is high, compared to the utility of transportation costs. However, the perceived utility of premium customers is low compared to the utility of transportation costs. Therefore, each retailer can potentially cover the whole demand for economy customers. Nevertheless, each retailer can only partially serve the premium customers.

Wherever the retailers locate, they can potentially serve all economy customers. Therefore, it is the premium customers who have a key roll in finding the location of the retailers. Each retailer wants to maximize the number of premium customers she serves. Therefore, she has to locate where the concentration of premium customers is higher, namely toward the right side of the demand line. Hence, both retailers will co-locate at point $X_i = \lambda - \frac{\theta_P Q_i - P_i}{t}$, to serve the maximum number of economy customers.

$$X_i^* = \lambda - \frac{\theta_P Q_i - P_i}{t} \tag{161}$$

The response function of the retailers is:

$$\pi_i = \frac{P_i}{2}(\int_{\lambda-2(\frac{\theta_P Q_i - P_i}{t})}^{\lambda} (\alpha_0 + \alpha_1 X)\, dx + \int_0^{\lambda} (\beta_0 - \beta_1 X)\, dx) - CQ_i \quad (162)$$

Subject to:

$$\frac{\theta_p Q_i - p_i}{t} > 0 \quad (163)$$

$$\frac{\theta_p Q_i - p_i}{t} < \frac{\lambda}{2} \quad (164)$$

The equilibrium solutions are:

$$p_i^* = \frac{C(\alpha_0 t + \alpha_1 \lambda t)}{D} + \frac{CtR}{D} \quad (165)$$

$$Q_i^* = \frac{1}{2\alpha_1 \theta_P} (\alpha_0 t + \alpha_1 \lambda t + tR) \quad (166)$$

Where:

$$D = \theta_P \alpha_0^2 + 2\theta_P \alpha_0 \alpha_1 \lambda + \theta_P \alpha_1^2 \lambda^2 - \beta_1 \theta_P \alpha_1 \lambda^2 + 2\beta_0 \theta_P \alpha_1 \lambda - 6C\alpha_1 \quad (167)$$

$$R = \sqrt{\frac{\alpha_0^2 \theta_P^2 + 2\alpha_0 \alpha_1 \lambda \theta_P^2 + \alpha_1^2 \lambda^2 \theta_P^2 + 2\alpha_1 \beta_0 \lambda \theta_P^2 - \alpha_1 \beta_1 \lambda^2 \theta_P^2 - 6C\alpha_1 \theta_P}{\theta_P^3 D}} \quad (168)$$

Table 4.1 demonstrates a few examples of how the equilibrium price, quality of services and profit change, with changes in the utility of premium customers for quality of services.

Table 4.1: equilibrium price, quality of services, and profit for selected values of θ_P

θ_P	Price	Quality of services	Profit
1.0	95.26	100.00	4456.56
1.5	99.95	69.99	4790.01
2.0	99.97	52.50	4842.51
2.5	100.00	41.97	4874.01

As observed in Table 4.1, increasing θ_P leads to a moderate rise in price, a sharp decline in the quality of services, and an overall increase in total profit. Notably, the profit curve exhibits a concave shape, indicating diminishing marginal gains.

4.5.3 All premium and economy customers can be served

In this situation, all premium and economy customers can potentially be served by either retailer. Therefore, both retailers locate at the center of the demand line:

$$X_1^* = X_2^* = \frac{\lambda}{2} \tag{169}$$

The profit function of each of the retailers is:

$$\pi_i = \frac{P_i}{2}(\int_0^{\lambda} (\alpha_0 + \alpha_1 X)\, dx + \int_0^{\lambda} (\beta_0 - \beta_1 X)\, dx) - CQ_i \tag{170}$$

Subject to:

$$\frac{\theta_p Q_i - p_i}{t} > \frac{\lambda}{2} \tag{171}$$

$$Q_i \geq Q_m \tag{172}$$

Solving to first-order conditions:

$$P_i^* = \theta_p Q_m - \frac{\lambda t}{2} \tag{173}$$

$$Q_i^* = Q_m \tag{174}$$

$$\pi_i = \frac{1}{2}(\alpha_1 + \beta_1)\lambda(\theta_p Q_m - \frac{\lambda t}{2}) - CQ_m \tag{175}$$

Since the utility for quality of services is high for both premium and economy customers, they remain satisfied even with a low quality of service. Therefore, the retailers choose to offer the minimum quality of service quality, Q_m, to decrease costs, and increase profitability.

Looking at the structure of the formulas for equilibrium price and profit, both depend solely on the utility of premium customers. This is because all customers are being served, and since premium customers value service quality less than economy customers, only θ_P (the utility parameter of premium customers) influences the outcome.

Moreover, the length of the demand line, denoted by λ, negatively affects both the equilibrium price and the profit. This is because, as λ increases, the average distance between customers and retailers also increases. Consequently, retailers must lower their prices to remain attractive to customers located farther away.

Proposition 4.3. *When customers are non-uniformly distributed along the demand line:*

(i) If

$$\frac{\theta_p Q_i - P_i}{t} < \frac{\lambda}{2} \quad \textit{and} \quad \frac{\theta_E Q_i - P_i}{t} < \frac{\lambda}{2},$$

then

$$(P_i^*, Q_i^*, \pi_i^*) \in \mathcal{F}(\theta_P, \theta_E).$$

(ii) If

$$\frac{\theta_E Q_i - P_i}{t} > \frac{\lambda}{2} \quad \textit{and} \quad \frac{\theta_P Q_i - P_i}{t} < \frac{\lambda}{2},$$

then

$$(P_i^*, Q_i^*, \pi_i^*) \in \mathcal{F}(\theta_P).$$

(iii) If

$$\frac{\theta_E Q_i - P_i}{t} > \frac{\lambda}{2} \quad \textit{and} \quad \frac{\theta_P Q_i - P_i}{t} > \frac{\lambda}{2},$$

then

$$(P_i^*, \pi_i^*) \in \mathcal{F}(\theta_P) \quad \textit{and} \quad Q_i^* = Q_{min}.$$

Proof. The proof proceeds by examining each case separately:

(i) When $\left(\frac{\theta_P Q_i - P_i}{t} < \frac{\lambda}{2}\right)$ and $\left(\frac{\theta_E Q_i - P_i}{t} < \frac{\lambda}{2}\right)$, both premium and economy markets are only partially covered. Therefore, the retailer's market share depends R_P and R_E. Hence, the equilibrium decisions and profit are also functions of R_P and R_E (section 4.5.1).

(ii) When $\left(\frac{\theta_P Q_i - P_i}{t} < \frac{\lambda}{2}\right)$ and $\left(\frac{\theta_E Q_i - P_i}{t} > \frac{\lambda}{2}\right)$, all economy customers are served, while the economy group is only partially served. Consequently, the market share depends solely on R_E. The equilibrium decisions are functions of both R_E and R_P (section 4.5.2).

$\left(\frac{\theta_P Q_i - P_i}{t} > \frac{\lambda}{2}\right)$ and $\left(\frac{\theta_E Q_i - P_i}{t} > \frac{\lambda}{2}\right)$, both groups are fully served. Hence, the entire market is covered, and the market share is independent of R_P and R_E. Consequently, the price and profit are only functions of R_E and not R_P, while the equilibrium number of services is constant and not dependent to either reservation price (section 4.5.3).

□

Proposition 4.4. *When customers are non-uniformly distributed along the* *demand line the equilibrium of the game is symmetric. In particular,*

$$P_1^* = P_2^*, \qquad N_1^* = N_2^*, \qquad \pi_1^* = \pi_2^*.$$

4.6 Customers are non-uniformly distributed along the demand line and retailers take turn entering the market

In this situation, one retailer already exists in the market, and the second retailer decides entering the market while knowing the decisions the first retailer has made. In this section we analyze the result of concentration on different customer groups.

4.6.1 Both the leader and the follower focus on premium customers

Since both retailers only consider premium customers, there will be a competition between the two retailers to gain more market share of the premium customers.

Case 1: Partial coverage

Both retailers will co-locate at a place where they can serve the maximum number of premium customers.Since the premium customers are more concentrated toward the right side of the demand line, the two retailers will locate at the right side. Both retailers can cover the distance of $\frac{\theta_p Q - P}{t}$. Therefore, they co-locate at the point $(\lambda - \frac{\theta_P Q - P}{t})$ to cover premium customers in the interval $[\lambda - 2\frac{\theta_P Q - P}{t}, \lambda]$.

The response function of each retailer would be as follows:

$$\pi_i = \frac{P_i}{2}(\int_{\lambda - 2\frac{\theta_P Q - P}{t}}^{\lambda} (\alpha_0 + \alpha_1 X)\, dx) - CQ_i \tag{176}$$

Solving to first-order conditions results:

$$P_i^* = \frac{Ct(\frac{\theta_P^2(\alpha_0+\alpha_1\lambda)^2 - 3C\alpha_1\theta_p}{\sqrt{\theta_P^3(\theta_P(\alpha_0+\alpha_1\lambda)^2 - 2C\alpha_1)}}))}{2\theta_p(\alpha_0 + \alpha_1\lambda t)^2 - 6C\alpha_1} \tag{177}$$

$$Q_i^* = \frac{\alpha_0 t + \alpha_1\lambda t + \frac{t\theta_P(\theta_P(\alpha_0+\alpha_1\lambda)^2 - 3C\alpha_1)}{\sqrt{\theta_P^3(\theta_P(\alpha_0+\alpha_1\lambda)^2 - 2C\alpha_1)}}}{2\alpha_1\theta_p} \tag{178}$$

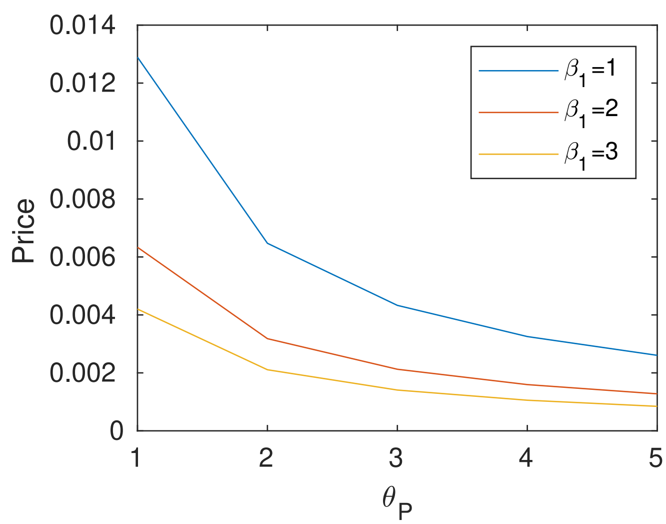

Figure 4.18: Effect of utility on price when both retailers focus on premium customers

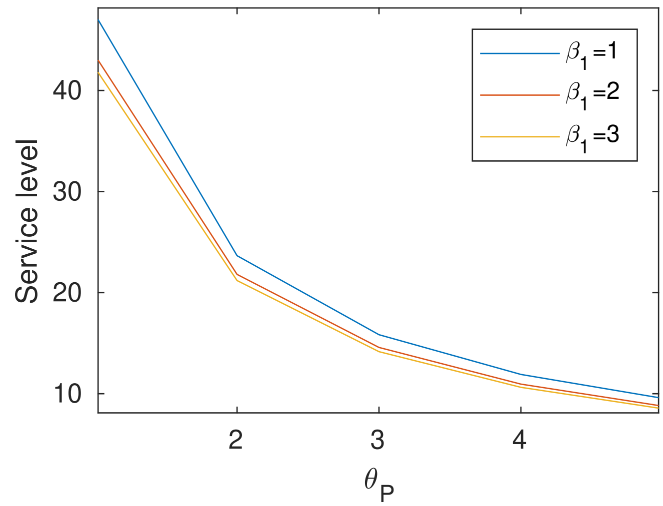

Figure 4.19: Effect of utility on quality of services when both retailers focus on premium customers

Figure 4.20 depicts the effect of utility of premium customers and their number on the equilibrium price.

Figure 4.19 depicts the effect of utility of premium customers and their number on the equilibrium quality of services.

Case 2: Full coverage

Here, the two retailers can cover the whole demand for premium customers. Therefore, they will co-locate at the center of the demand line $\frac{\lambda}{2}$. The reaction function of the retailers is:

$$\pi_i = \frac{P_i}{2}(\int_0^\lambda (\alpha_0 + \alpha_1 X)\, dx) - CQ_i \tag{179}$$

Subject to:

$$Q_i \geq Q_m \tag{180}$$

Solving to first-order conditions:

$$P_i^* = \theta_P Q_m - \frac{\lambda t}{2} \tag{181}$$

$$Q_i^* = Q_m \tag{182}$$

$$\pi_i = \frac{1}{2}\alpha_1 \lambda(\theta_p Q_m - \frac{\lambda t}{2}) - CQ_m \tag{183}$$

4.6.2 The leader focuses on premium and the follower focuses on economy customers

Here, since each retailer focus on a different group of customer, there is no competition among the retailers.

The leader will act same as in 4.6.1. For the follower same analysis counts:

Case one: Partial coverage

The follower cannot cover all economy customers and can only cover economy customers partially. Therefore,she locates towards the left side of the demand line where the concentration of the economy customers is the highest. Hence, she will locate at

$$\pi_2 = \frac{P_2}{2}\left(\int_0^{2\frac{\theta_E Q - P_2}{t}} (\beta_0 - \beta_1 X)\, dx\right) - CQ_2 \tag{184}$$

Solving to first order conditions results:

$$P_i^* = \frac{Ct\left(\frac{\beta_0^2\theta_E^2 - 3C\beta_1\theta_E - C\beta_0 t}{\sqrt{\theta_E^3(\theta_E\beta_0^2 - 2C\beta_1)}}\right)}{6C\beta_1 - 2\theta_E\beta_0^2} \tag{185}$$

$$Q_i^* = \frac{t\left(\beta_0 + \frac{\beta_0^2\theta_E^2 - 3C\beta_1\theta_E}{\sqrt{\theta_E^3(\theta_E\beta_0^2 - 2C\beta_1)}}\right)}{2\beta_1\theta_E} \tag{186}$$

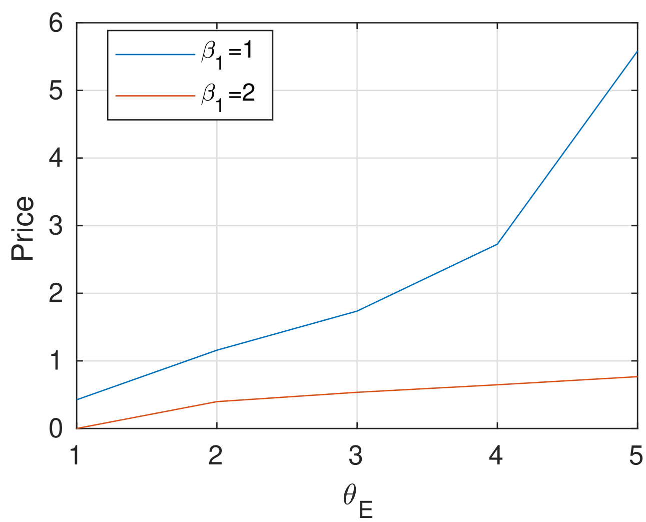

Figure 4.20: Effect of utility on price

Figure 4.20 depicts the effect of utility and the number of economy customers on the equilibrium price when the market can be partially served. It shows that the equilibrium price increases as the utility for quality of services increases. Moreover, as β_1 increases, or in other words, the number of customers decreases, the equilibrium prices increase so that the retailer can cover more customers.

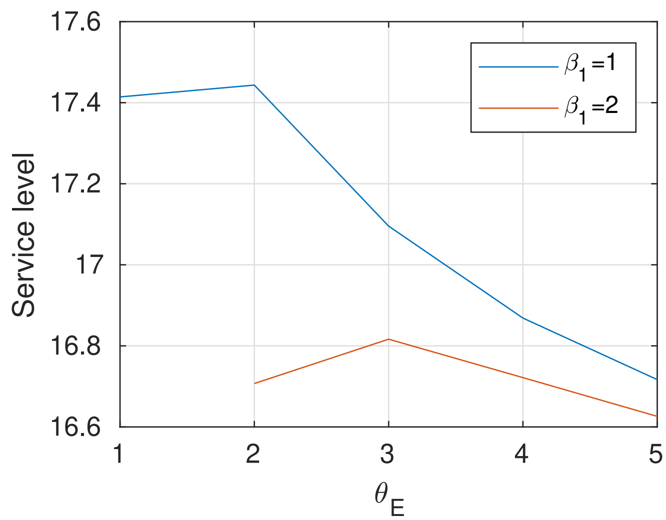

Figure 4.21: Effect of utility on quality of services

Figure 4.21 depicts the effect of the utility of economy customers and their number on the equilibrium quality of services offered by the retailer. In general, the equilibrium level of services decreases as the utility increases. This is because higher utility levels result in customers being satisfied with lower quality of services. Moreover, the figure shows that with an increase in β_1, or in other words, a decrease in the number of economy customers being served, the equilibrium service quality decreases. This is because as the number of economy customers decreases, retailers reduce the quality of services to minimize costs, offer lower prices, and attract more customers.

Case two: Full coverage

Here, the follower can cover all demand of economy customers. Therefore, she locates at point $X_2 = \frac{\lambda}{2}$. Her reaction function is:

$$\pi_2 = \frac{P_i}{2}(\int_0^{\lambda} (\beta_0 - \beta_1 X)\, dx) - CQ_2 \tag{187}$$

Subject to:

$$Q_i \geq Q_m \tag{188}$$

Solving to first order conditions results:

$$P_2^* = \theta_E Q_m - \frac{\lambda t}{2} \tag{189}$$

$$Q_2^* = Q_m \tag{190}$$

$$\pi_i = \frac{1}{2}\beta_1 \lambda(\theta_E Q_m - \frac{\lambda t}{2}) - CQ_m \tag{191}$$

4.6.3 The leader focuses on economy and the follower focuses on premium customers

This is same as section 4.6.2, just that the place of the leader and the follower is exchanged.

4.6.4 Both the leader and the follower focus on premium customers

Since both retailers only consider focus of economy customers, there will be a competition between the two retailers to gain more market share of the premium economy. The results are same as section 4.6.2.

4.7 Analysis of the results

The results are analyzed and compared in this section. The first subsection examines the outcomes of games in which customers are uniformly distributed along the demand line, while the second subsection focuses on games with a non-uniform customer distribution. The final subsection provides a comparison between these two customer distributions.

4.7.1 Customers are uniformly distributed along the demand line

When customers are uniformly distributed across the market, and the two retailers simultaneously decide on their locations, prices, and quality of servicess, they ultimately choose different locations but identical prices and service qualities. It is never equilibrium for both retailers to locate at the same point, since this would force them to share both the market and the resulting profits. However, they must converge on the same price and quality of services, as any deviation creates an incentive for undercutting. Specifically, if one retailer were to offer a lower price or higher service quality, they would capture a larger market share, which in turn compels the other retailer to respond by lowering their price or raising their service quality. This process leads to an equilibrium in which prices and quality of servicess are identical, while locations remain differentiated.

Depending on the extent of traveling costs and the utility of premium and economy customers for the quality of services, three different cases may occur. These cases are:

(i) when traveling costs are high, and utility of both customer groups for quality of services is low, both premium and economy customers can only be served partially. In such a case, the utility of both premium and economy customers plays a role in determining the equilibrium quality of services, equilibrium price, and, of course, the equilibrium profit of the two retailers.

Moreover, the ratio of the utility of premium and economy customers for the quality of services is not important. It is their individual values that make a difference in equilibrium decisions. In other words, it is not crucial how much the utility of economy customers is greater than that of premium customers. Only the individual values of these two utility levels have an impact on equilibrium decisions.

(ii) When traveling costs are low compared to the utility of economy customers for quality of services, but high compared to the utility of premium customers, all economy customers can be served. However, only part of the premium customers can be served. In this case, the equilibrium pricing and quality of services decisions depend only on the utility of premium customers and are independent of the utility of economy customers. This is because all economy customers can already be served, but the extent of the utility of premium customers determines how many premium customers can be served, thereby determining the equilibrium pricing and quality of services.

(iii) Finally, when the extent of traveling costs is low compared to the utility of both premium and economy customers for the quality of services, all customers of both groups can be completely served. In such a case, the quality of services is independent of the utility of either customer group and is set at the minimum level possible. However, the utility of premium customers affects the equilibrium price and, consequently, the equilibrium profit of retailers.

It is important to note that as the utility of customers for the quality of services increases (going from case (i), to case (ii) to case (iii)), the quality of services decreases, and the equilibrium price increases. Moreover, more customers can be served, so the market share increases. Therefore, with an increase in price and market size, and a decrease in the quality of services (which imposes fewer costs for providing services), the equilibrium profit will increase.

When the two retailers take turns entering the market and deciding on the equilibrium pricing and quality of services strategies, various cases may arise.

If one retailer focuses on premium customers and the other retailer focuses on economy customers, the two retailers would make their decisions separately, and their potential markets are independent of one another.

However, when both retailers target the same customer group—such as both retailers catering to premium customers or serving exclusively the economy customers—they have to compete for their market shares. In such a situation, when customers can be partially served, the two retailers choose different locations but offer the same price and quality of services. However, if the market can be served entirely, interesting results are obtained. The second retailer will offer the same price as the first retailer but provide a higher quality of services. Therefore, the second retailer will have a greater market share and obtain greater profit compared to the first retailer.

Table 4.2 mesmerizes the equilibrium price, quality of services, and profit for three different scenarios: serving only premium customers, serving only economy customers, or serving both customer groups when the market can be served partially.

Table 4.2: equilibrium decision in uniform distribution, when market is served partially

	Serving only Premium	Serving only Economy	Serving both
equilibrium price	$\frac{\lambda tC}{2N\gamma\theta_P}$	$\frac{\lambda tC}{2N(1-\gamma)\theta_E}$	$\frac{\lambda tC}{2N(\gamma\theta_P+(1-\gamma)\theta_E)}$
equilibrium quality	$\frac{\lambda tC}{N\gamma\theta_P^2}$	$\frac{\lambda tC}{N(1-\gamma)\theta_E^2}$	$\frac{\lambda tC}{(\gamma\theta_P+(1-\gamma)\theta_E)^2}$
equilibrium Profit	$\frac{\lambda tC^2}{\gamma\theta_P^2 N}(\frac{N}{2}-1)$	$\frac{\lambda tC^2}{(1-\gamma)\theta_E^2 N}(\frac{N}{2}-1)$	$\frac{\lambda tC^2}{(\gamma\theta_P+(1-\gamma)\theta_E)^2 N}(\frac{N}{2}-1)$

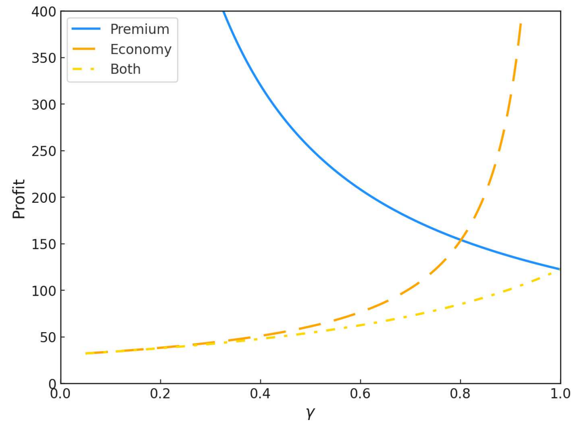

Figure 4.22: Comparison of strategies in partial coverage, serving both groups

Figure 4.22 illustrates the profit comparison across the three strategies when neither retailer can exclusively cater to a single customer group. For low to moderate values of γ, it is more profitable to determine the equilibrium price and service quality with respect to premium customers and apply these decisions to both groups. For high values of γ, however, it becomes equilibrium to base decisions on economy customers and extend the same price and service quality to premium customers. Notably, setting price and quality of servicess jointly for both groups is never the most profitable strategy.

Figure 4.23 compares the three strategies in the case where retailers can exclusively serve one customer group. The results show that, regardless of the proportion of premium customers, focusing solely on premium customers yields the highest profit when demand can only be partially covered—that is, when travel costs are high and the utility derived from service

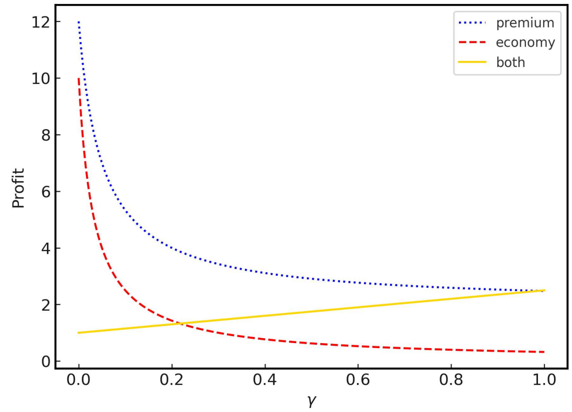

Figure 4.23: Comparison of strategies in partial coverage, exclusively serving one group

quality is low.

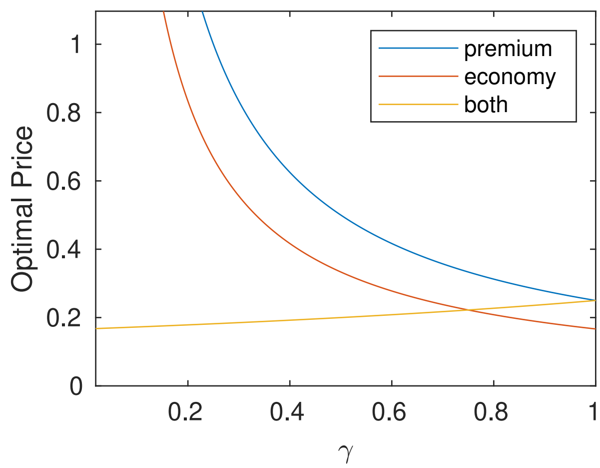

Figure 4.24: Comparison of equilibrium price for different strategies in partial coverage

Figure 4.24 demonstrates a comparison of equilibrium price for the three strategies. It can be seen that the price is greatest when focusing only on premium customers.

Table 4.3 mentions the equilibrium price, quality of services and profit for three different scenarios: serving only premium customers, serving only economy customers, and catering to both customer groups, when the market

Table 4.3: equilibrium decision in uniform distribution, when whole market can be served

	Serving only Premium	Serving only Economy	Serving both
equilibrium price	$\theta_P Q_m - \frac{\lambda t}{2}$	$\theta_E Q_m - \frac{\lambda t}{2}$	$\theta_P Q_m - \frac{\lambda t}{4}$
equilibrium quality	Q_m	Q_m	Q_m
equilibrium Profit	$\gamma N(\theta_P Q_m - \frac{\lambda t}{2}) - CQ_m$	$(1-\gamma)N(\theta_P Q_m - \frac{\lambda t}{2}) - CQ_m$	$\frac{N}{2}(\theta_P Q_m - \frac{\lambda t}{4}) - CQ_m$

can be fully served.

It can be seen that the equilibrium quality of services is identical for all three strategies. Moreover, the equilibrium price is greater when serving economy customers that premium customers. This can be explained by the fact that, economy customers have a higher utility for quality of services, and tend to pay more cost for the same level of quality of services, compared to premium customers.

If $Q_m(\theta_E - \theta_P) > \frac{\lambda t}{4}$, the price would be greatest when focusing of economy customers. Otherwise, the price would be greatest when focusing on both customer groups.

Profits of different strategies in full coverage are compared using graphs in section 4.4.2. For small and moderate values of γ it is more profitable to set the price based on economy customers. For high values of γ, considering both customer groups for deciding on price and quality of services is more profitable.

Table 4.4 mentions the best response of the follower to each strategy taken by the leader.

Table 4.4: Best response of the follower based on leader's decision

		Follower's best response
Leader's Decision	Premium	When the market can only be partially served, focus on both customer groups. If the market can be served entirely and the proportion of premium customers is high, exclusively cater to premium customers. Otherwise, cater to both customer groups.
	Economy	Serve both customer groups, regardless of the market's partial or full coverage.
	Both	When the market can only be partially served, always focus on both customer groups. When the entire market can be served, if the proportion of premium customers is significant, exclusively focus on premium customers. Otherwise, focus on both customer groups.

4.7.2 Customers are non-uniformly distributed along the demand line

In this sub-section we try to find the best strategies for retailers comparing the outputs in similar case. We aim to answer the following questions for the non-uniform situation:

1- When the entire market can be covered, what is the best strategy: focusing on both customer groups, exclusively serving premium customers, or catering solely to economy customers?

2- What is the best strategy when only a portion of the market can be served?

3- In the leader-follower game, when the leader focuses on premium customers, what strategy is more advantageous for the follower: concentrating on premium customers, targeting economy customers, or serving both?

4- In the leader-follower game, when the leader focuses on economy customers, what strategy is more advantageous for the follower: concentrating on premium customers, targeting economy customers, or serving both?

Table 4.5: equilibrium decision in non-uniform Situation, when market is served completely.

	Serving only Premium	Serving only Economy	Serving both
Price	$\theta_P Q_m - \frac{\lambda t}{2}$	$\theta_E Q_m - \frac{\lambda t}{2}$	$\theta_P Q_m - \frac{\lambda t}{2}$
Quality	Q_m	Q_m	Q_m
Profit	$\frac{1}{2}\alpha_1\lambda(\theta_p Q_m - \frac{\lambda t}{2}) - CQ_m$	$\frac{1}{2}\beta_1\lambda(\theta_E Q_m - \frac{\lambda t}{2}) - CQ_m$	$\frac{1}{2}(\alpha_1+\beta_1)\lambda(\theta_P Q_m - \frac{\lambda t}{2}) - CQ_m$

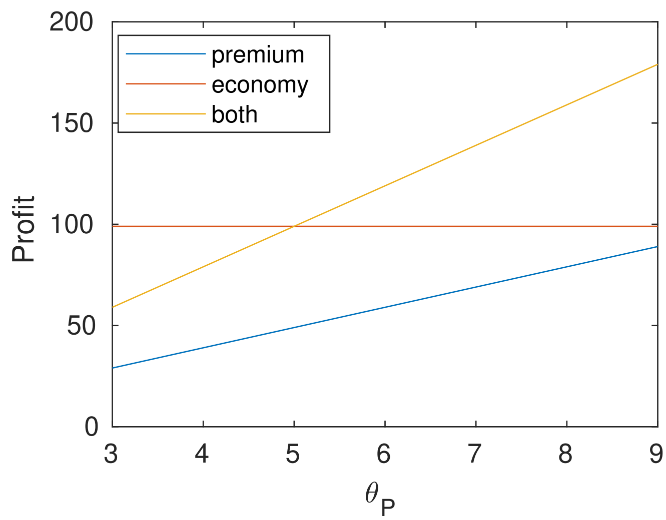

Figure 4.25: Profit of different strategies in 3.2.2

Figure 4.25 depicts the profit when serving only premium customers, serving only economy customers, and catering to both customer groups, for a fixed value of $\theta_E = 10$ and different values of θ_P, assuming full market penetration. It is evident that when the utility of premium customers is small compared to that of economy customers, it is more profitable to serve only economy customers. However, as the proportion of premium customers' utility to economy customers' utility increases, serving both customer groups brings more profit to the retailers. It is beyond doubt that

Table 4.6: Best responses of the follower in non-linear distribution

		Follower's best response
Leader's Decision	Premium	When the market can only be partially served, focus on both customer groups. When the market can be served completely,if $\frac{\theta_P}{\theta_E}$ is low, only serve economy customers. Otherwise serve both customer groups.
	Economy	When the demand can be partially served, focus on both groups. When the market can be served completely,if $\frac{\theta_P}{\theta_E}$ is low, only serve economy customers. Otherwise serve both customer groups.
	Both	When the market can only be partially served, always focus on both customer groups. When the entire market can be served, if the proportion of economy customers is significant, exclusively focus on premium customers. Otherwise, focus on both customer groups.

serving only premium customers is never the equilibrium strategy for any retailer.

In summary, when considering a non-uniform distribution scenario, in cases where the whole demand market can be served, it always most profitable for the leader to cater only on premium customers. The follower would better off serving only economy customers in such situations.

4.7.3 Comparison of the uniform and non-uniform distribution of customers along the demand line

Comparing the equilibrium decisions in uniform and non-uniform customer distribution scenarios highlights the significant variations in strategies required for each situation. In heterogeneous markets featuring non-uniform customer distributions, it's vital to avoid oversimplification by assuming uniformity. Instead, accurate modeling and in-depth study of non-uniformity are essential, as they better reflect real-world conditions. Neglecting to consider non-uniform distribution can result in sub-equilibrium choices and inaccurate conclusions in market analysis and strategy formulation.

4.8 Conclusion

This paper examines the competition between two retailers in a heterogeneous market, focusing on factors such as location, pricing strategy, and quality of services. The market comprises two distinct customer segments: premium and economy customers. The proximity of a retailer to customers, along with the pricing and service quality they provide, significantly influences their market share and, consequently, their profitability. In this competitive landscape, the two retailers vie for a larger market share and increased profits. We investigate this problem in two distinct situations.

In the initial scenario, customers are uniformly distributed throughout the market. Our analysis reveals that when customers enter the market simultaneously, they opt for different locations but establish identical pricing and service quality levels. Conversely, when one retailer is already present in the market, the second retailer selects a distinct location, maintains the same pricing strategy, but enhances the service quality level offered.

In the second scenario, customers are distributed non-uniformly across the market. Premium customers tend to be more densely concentrated on one side, while economy customers are concentrated on the opposite side. Our analysis indicates that retailers consistently opt for the same location and provide identical pricing and service quality levels in this situation.

In conclusion, our analysis demonstrates significant variations in equilibrium decisions between scenarios where customers are uniformly dispersed and situations where customers are non-uniformly distributed within the market. Consequently, it is imperative to model market heterogeneity in a manner that accurately reflects real-world conditions, avoiding the oversimplification of assuming uniformity.

Chapter 5

Competition for location, price and number of services

5.1 Introduction

In highly competitive retail environments, firms often differentiate themselves not only through pricing and product assortment but also by offering complementary services that enhance the overall customer experience. These ancillary services —such as in-store coffee shops, restaurants, optometry clinics, free parking, or Wi-Fi—have become strategic tools for attracting and retaining customers. Such services may not directly generate revenue but can increase store traffic, lengthen customer dwell time, and improve perceived value, thereby influencing customer choice and willingness to pay.

Retailers like *Walmart* have implemented this approach by integrating pharmacies, financial services, and fast food outlets within their stores, effectively transforming their locations into one-stop destinations Ellickson and Grieco (2016). Similarly, *IKEA* combines furniture retailing with restaurants and childcare areas, encouraging longer visits and higher spending. These strategies suggest that offering value-added services can serve as a form of non-price competition that complements traditional location-pricing decisions.

In this chapter, we study the competition between two retailers selling the same product, who compete through their location, pricing strategies, and the number of ancillary services they offer. In addition to selling the product, each retailer may provide complementary services such as a coffee shop, restaurant, or children's playground.

The market consists of two customer segments: premium and economy. Premium customers have higher reservation prices and are willing to pay more for both the product and the associated services, whereas economy customers have lower reservation prices and are more price-sensitive.

Customers derive utility from the ancillary services offered by a retailer. The greater the number of ancillary services provided, the higher the utility customers obtain from purchasing from that retailer, and consequently, the more willing they are to buy from it.

It is assumed that customers derive the same level of utility from different types of ancillary services . Hence, only the number of ancillary services offered affects the utility a customer gains from purchasing from a retailer, not the specific nature of those services. For instance, whether a retailer provides free parking, Wi-Fi, or a food court and a children's play area, the utility customers derive from purchasing from that retailer remains unchanged. Moreover, it is the variety of services that matters, not their magnitude. For instance, customers' utility is unaffected by whether the retailer operates a large or small coffee shop.

In this setting, we assume that one retailer is already established in the market, having determined its location, pricing strategy, and service offerings. A second retailer is considering entering the market and must make its own decisions regarding location, pricing, and number of ancillary services in response to the incumbent's strategy.

The market is represented as a line of length λ, with the locations of the existing retailer (leader) and a new retailer (follower) denoted by X_1 and X_2, respectively. Without loss of generality, we assume that the leader is positioned to the left of the follower. Figure 5.1 illustrates the market setup.

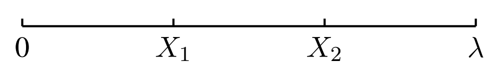

Figure 5.1: Depiction of the market

We need to define parameters and decision variables to model the problem.

5.1.1 Parameters

R_P: Reservation price of premium customers.

R_E: Reservation price of economy customers.

It should be noted that the reservation price of the premium customers is strictly greater than that of the economy customers: $R_P > R_E$.

λ: Length of the demand line.

t: Traveling time contribution to the utility.

γ: The percentage of the market composed of premium customers.

C: Fixed cost for establishing an ancillary service.

Π_i: Profit function of retailer i, $i \in \{1, 2\}$.

d_i: Demand covered by retailer i, $i \in \{1, 2\}$.

θ: Contribution of services to the utility .

u_i: Utility a customer drives from purchasing from retailer i.

5.1.2 Decision variables

P_i: price set by retailer i, $i \in \{1, 2\}$.

X_i: location of retailer i along the X axis, $i \in \{1, 2\}$.

N_i: Number of ancillary services offered by retailer i, $i \in \{1, 2\}$.

Although N should ideally be an integer variable, for the sake of analytical tractability, we assume that it can take real (non-integer) values. This assumption further ensures that the objective functions are differentiable.

5.1.3 Utility function

To model the utility that customers derive from ancillary services , this chapter adopts a concave utility function of the form $\sqrt{N}$, where N represents the number of additional services provided by the retailer. This square-root formulation captures the intuitive idea that while offering more services enhances the customer experience, the incremental benefit of each additional service decreases.

Concave utility functions, particularly square-root or logarithmic forms, are commonly used in economic modeling to represent preferences with diminishing marginal returns (Mas-Colell, Whinston, & Green, 1995). This approach ensures that utility increases with service offerings, but at a decreasing rate, which aligns with consumer behavior observed in multi-service retail environments.

On the cost side, it is assumed that the total cost of providing ancillary services increases linearly with the number of services offered. This simplification enhances analytical tractability while preserving the essential trade-off between improving customer utility and incurring operational expenses. Although real-world cost structures may be more complex—potentially involving fixed costs, capacity constraints, or economies of scale—the linear cost assumption serves as a reasonable first-order approximation in the absence of detailed cost data.

Consider an arbitrary customer of type j, where $j \in \{P, E\}$, located at point y along the market line. The utility that this customer obtains from purchasing from retailer i equals their reservation price for the product, minus the price charged by retailer i, plus the utility derived from the ancillary services offered by retailer i, and minus the disutility of traveling to retailer i:

$$u_{ij}(y) = (R_j - P_i) + \theta\sqrt{N_i} - t|(y - X_i)|, forj \in \{P, E\} \tag{192}$$

Without loss of generality, let's assume $\theta = 1$.

There are customers located both to the right and to the left of retailer i who may wish to purchase from them. Some customers are indifferent between buying the product from retailer i and not buying at all, meaning that their utility from purchasing from retailer i is equal to zero. This condition can be expressed as setting $u_i(y) = 0$:

$$|y - X_i| = \frac{R_j + \sqrt{N_i} - P_i}{t}, forj \in \{P, E\} \tag{193}$$

According to (193), the farthest premium customer that can be served by retailer i is located $\frac{R_P+\sqrt{N_i}-P_i}{t}$ units away in either direction. Similarly, the farthest economy customer who may choose to purchase from retailer i is located $\frac{R_E+\sqrt{N_i}-P_i}{t}$ units away in either direction.

Figure 5.2 illustrates this concept. The orange line represents the range of premium customers who may choose retailer 1, while the blue line indicates the range of economy customers who are willing to purchase from retailer 1.

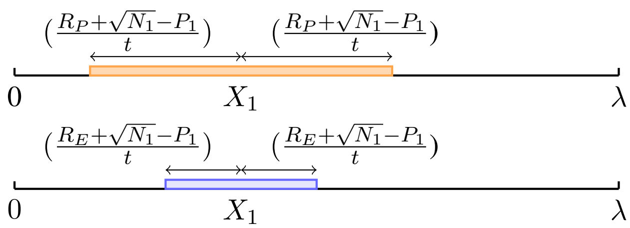

Figure 5.2: Maximum distance a retailer can cover

We assume that a customer will make a purchase from a retailer only if their utility from buying the product from that retailer is positive. It is important to note that if the utility of purchasing from both retailers is positive, the customer will choose the retailer that provides the higher utility.

The problem is analyzed under two different situations: (1) when the two customer groups are uniformly distributed along the demand line, and (2) when they are non-uniformly distributed along the demand line.

5.2 Case one: Customers are uniformly distributed along the demand line

When customers are uniformly distributed along the demand line, three situations may arise: (1) both premium and economy customers are partially served, (2) all premium customers and a portion of economy customers are served, or (3) both groups are fully served. Without loss of generality, we assume that retailer 1 is located on the left side of the demand line, while retailer 2 is located on the right.

The equilibrium location of the retailers in all three situations is:

$$X_1 = \frac{R_P + \sqrt{N_1} - P_1}{t} \tag{194}$$

$$X_2 = \lambda - \frac{R_P + \sqrt{N_2} - P_2}{t} \tag{195}$$

Figure 5.3 shows the relative locations of the two retailers and their market shares.

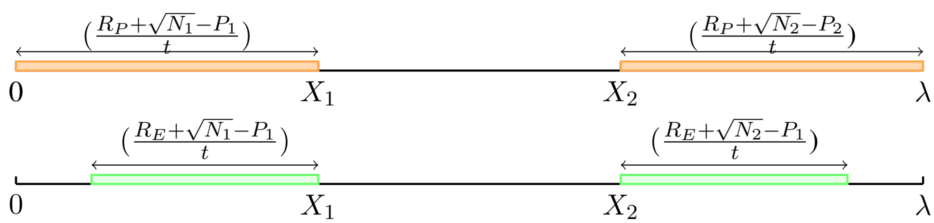

Figure 5.3: The equilibrium location of the two retailers based on the problem parameters

5.2.1 Partial coverage of both customer groups

When transportation costs are high and reservation prices of both customer groups are low, neither customer group can be fully served. Each retailer serves only a portion of customers to maximize profit. The profit function for each retailer is calculated as the demand they can cover multiplied by the price they offer, minus the cost of providing services. That is:

$$\pi_i = \sum_{j=E}^{P} d_{ij} \times P_i - C \times N_i \tag{196}$$

Each retailer can serve premium customers located up to $\frac{R_P+\sqrt{N_i}-P_i}{t}$ units to its left or right. Consequently, the total premium demand that a retailer can serve spans $2 \times \frac{R_P+\sqrt{N_i}-P_i}{t}$ distance units. Similarly, the total economy demand that a retailer can serve spans $2 \times \frac{R_E+\sqrt{N_i}-P_i}{t}$ distance units.

Figure 5.4 demonstrates this situation.

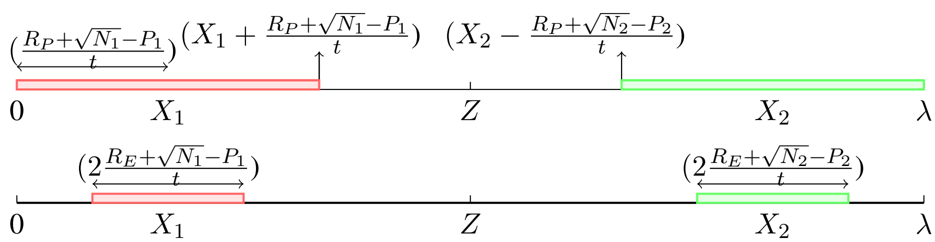

Figure 5.4: Partial coverage of premium and economy customers

Considering the fact that γ percent of customers are premium and $(1-\gamma)$ percent are economy customers, the profit function of each retailer would be as follows:

$$\pi_i = \frac{2P_i}{\lambda}(\frac{\gamma R_P + (1-\gamma)R_E + \sqrt{N_i} - P_i}{t}) - CN_i \tag{197}$$

Solving to equilibriumly:

$$P_i^* = \frac{C\lambda t\big(\gamma R_P + (1-\gamma)R_E\big)}{2C\lambda t - 1} \tag{198}$$

$$N_i^* = \frac{\big(\gamma R_P + (1-\gamma)R_E\big)^2}{(2C\lambda t - 1)^2} \tag{199}$$

$$\pi_i^* = \frac{C\big(\gamma R_P + (1-\gamma)R_E\big)^2}{2C\lambda t - 1} \tag{200}$$

The equilibrium price, number of services, and profit for both retailers are identical due to the symmetric situation.

Equations (198) and (199) indicate that $N_i = (\frac{P_i}{\lambda C t})^2$. There exists a positive relationship between price and the number of services : as the number of services increases, the price also rises. This occurs because offering additional services incurs higher costs. Consequently, when the price is low, the retailer cannot afford to provide many ancillary services . Conversely, if the retailer aims to increase the number of ancillary services , they must charge a higher price to cover the additional costs.

The equilibrium price and the equilibrium number of services depend on the reservation prices of both customer groups, R_E and R_P. This relationship arises because both premium and economy customers are partially served. Consequently, the number of potential customers in each group is determined by the reservation price associated with that group.

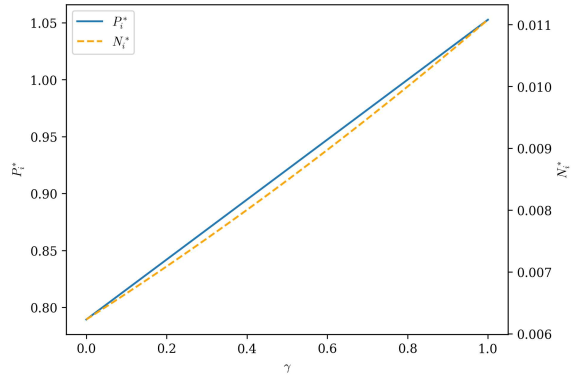

Figure 5.5: Effect of γ on equilibrium decisions in 5.2.1

Figure 5.5 shows the effect of γ on equilibrium price and number of services. As the share of premium customers (γ) increases, both the equilibrium price and the equilibrium number of services rise. However, the number of services grows at a slower rate. This pattern can be explained by the mathematical structure of the model: while the equilibrium price increases linearly with the weighted average reservation price $(\gamma R_P + (1 - \gamma)R_E)$, the equilibrium number of services increases quadratically with the same term. As a result, the growth in number of ancillary services becomes less pronounced at higher values of γ.

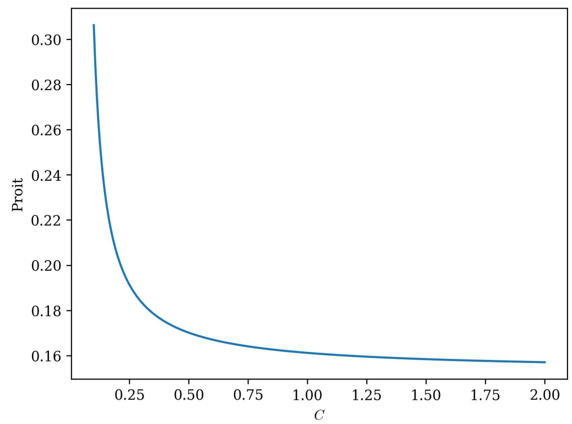

Figure 5.6: Effect of service establishment cost on profit in 5.2.1

Figure 5.6 illustrates the effect of the service establishment cost (C) on the retailer's equilibrium profit. As the cost of providing services increases, the equilibrium profit decreases. This occurs because both the equilibrium price and the equilibrium number of services decrease with higher C, but the decline in equilibrium price is more steep than the number of ancillary services.

5.2.2 Full coverage of premium customers and partial coverage of economy customers

This situation arises when transportation costs are low relative to the reservation price for premium customers but high for economy customers.

Figure 5.7 illustrates this situation. Premium customers located within the gray-shaded segment of the demand line can be served by both retailers, indicating potential market overlap in the premium segment. In contrast, only a portion of economy customers can be served. The orange lines represent the segments of the economy demand that are covered, while the red and blue lines correspond to the potential premium markets of Retailer 1 and Retailer 2, respectively.

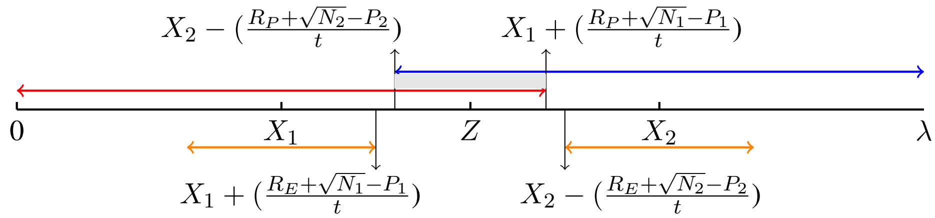

Figure 5.7: Full coverage of premium and partial coverage of economy customers

We need to find point Z where a customer is indifferent between buying from either retailer.

$$z = \frac{X_1 + X_2}{2} + \frac{P_2 - P_1}{2t} + \frac{\sqrt{N_1} - \sqrt{N_2}}{2t} = \frac{\lambda}{2} + \frac{P_2 - P_1}{2t} + \frac{\sqrt{N_1} - \sqrt{N_2}}{2t} \tag{201}$$

Equation (201) shows that when both retailers offer the same price and number of services, they share the market equally. However, if one retailer sets a lower price or provides a higher number of services, its market share exceeds half of the total customer base.

The profit functions are:

$$\pi_1 = \frac{P_1}{\lambda}\Big(\gamma(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t} + \frac{\sqrt{N_1} - \sqrt{N_2}}{2t}) + 2(1-\gamma)(\frac{R_E + \sqrt{N_1} - P_1}{t})\Big) - CN_1 \tag{202}$$

$$\pi_2 = \frac{P_2}{\lambda}\Big(\gamma(\frac{\lambda}{2} + \frac{P_1 - P_2}{2t} + \frac{\sqrt{N_2} - \sqrt{N_1}}{2t}) + 2(1-\gamma)(\frac{R_E + \sqrt{N_2} - P_2}{t})\Big) - CN_2 \tag{203}$$

Solving the Stakelberg game:

$$P_1^* = -\frac{8C\lambda t\big(4(1-\gamma)R_E + \gamma\lambda t\big)\big(C\lambda t(10\gamma - 16) + 3\gamma^2 - 10\gamma + 8\big)}{A} \tag{204}$$

$$P_2^* = \frac{\big(4C\gamma\lambda t + \gamma(3\gamma - 4)\big)\, P_1^* + 4C\gamma\lambda^2 t^2 + 16C\lambda t(1-\gamma)R_E}{(4-3\gamma)\big(8C\lambda t + 3\gamma - 4\big)} \tag{205}$$

$$N_i^* = (P_i^* \frac{(4-3\gamma)}{4tC\lambda})^2 \tag{206}$$

Where

$$A = (544\gamma^2 - 1536\gamma + 1024)C^2\lambda^2 t^2 + (384\gamma^3 - 1664\gamma^2 + 2304\gamma - 1024)C\lambda t + (63\gamma^4 - 384\gamma^3 + 832\gamma^2 - 768\gamma + 256).$$

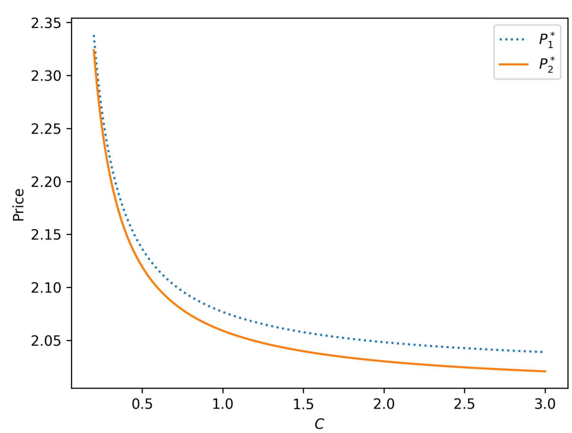

Figure 5.8: Effect of service establishment costs on price in 5.2.2

Figure 5.8 compares the prices set by the leader and the follower. It can be observed that the follower sets a strictly lower price than the leader. This outcome arises because the follower observes the leader's decision and acts subsequently. As a result, the follower can equilibriumly adjust its strategy by offering a lower price in order to attract a larger share of customers.

Equation (206) shows that the number of services offered by each retailer is proportional to the price it sets. Since the follower sets a lower price, it consequently offers fewer ancillary services.

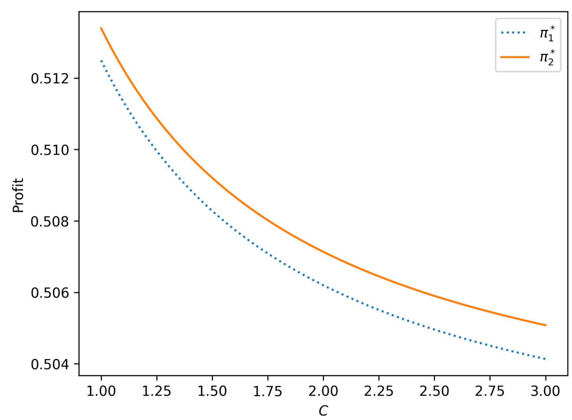

Figure 5.9: Effect of service establishment costs on profit in 5.2.2

Figure 5.9 compares the equilibrium profits of the leader and the follower. It is evident that the follower achieves a higher profit. Although the leader sets a higher price, it attracts fewer customers. Moreover, the leader offers a higher number of services, which increases its costs and results in a lower profit compared to the follower

5.2.3 Full coverage of both customer groups

Here, the reservation price for both premium and economy customers is high enough to cause the potential markets of the two retailers to overlap for both groups of customers.

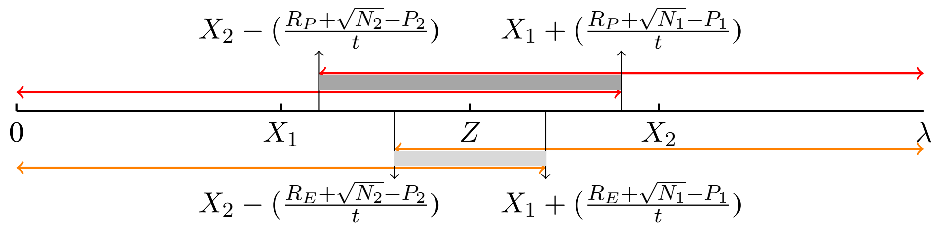

Figure 5.10: Full coverage of both premium and economy customers

Figure 5.10 illustrates this situation. The orange arrows indicate the potential market of each retailer for economy customers. It can be observed that there is an overlap between the potential economy markets of the two retailers, represented by the light gray shaded segment. The red arrows show the potential premium markets of the two retailers, and their overlap is depicted by the dark gray shaded segment. It is worth noting that the overlap in the premium markets is greater than that in the economy markets, as premium customers have higher reservation prices than economy customers.

The objective functions are:

$$\pi_1 = \frac{P_1}{\lambda}(\frac{\lambda}{2} + \frac{P_2 - P_1}{2t} + \frac{\sqrt{N_1} - \sqrt{N_2}}{2t}) - CN_1 \quad (207)$$

$$\pi_2 = \frac{P_2}{\lambda}(\frac{\lambda}{2} + \frac{P_1 - P_2}{2t} + \frac{\sqrt{N_2} - \sqrt{N_1}}{2t}) - CN_2 \tag{208}$$

Solving the Stakelberg game:

$$P_1^* = \frac{48C^2\lambda^2t^2 - 14C\lambda t + 1}{2C(16C\lambda t - 3)} \tag{209}$$

$$P_2^* = \frac{4\lambda t\,(5C\lambda t - 1)}{16C\lambda t - 3} \tag{210}$$

$$N_1^* = \left(\frac{6C\lambda t - 1}{2C(16C\lambda t - 3)}\right)^2 \tag{211}$$

$$N_2^* = \left(\frac{5C\lambda t - 1}{C(16C\lambda t - 3)}\right)^2 \tag{212}$$

$$\pi_1^* = \frac{36C^2\lambda^2t^2 - 12C\lambda t + 1}{4C(16C\lambda t - 3)} \tag{213}$$

$$\pi_2^* = \frac{200C^3\lambda^3t^3 - 105C^2\lambda^2t^2 + 18C\lambda t - 1}{C(16C\lambda t - 3)^2} \tag{214}$$

$$C\lambda t > \frac{1}{5}. \tag{215}$$

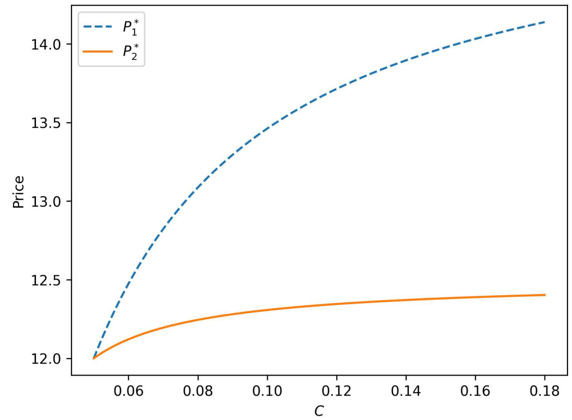

Figure 5.11: Comparison of equilibrium prices in full coverage of both customer groups

Figure 5.11 compares the prices set by the leader and the follower when both the economy and premium markets are fully covered. The results show that the follower's price is consistently lower than the leader's. This enables the follower to attract more customers.

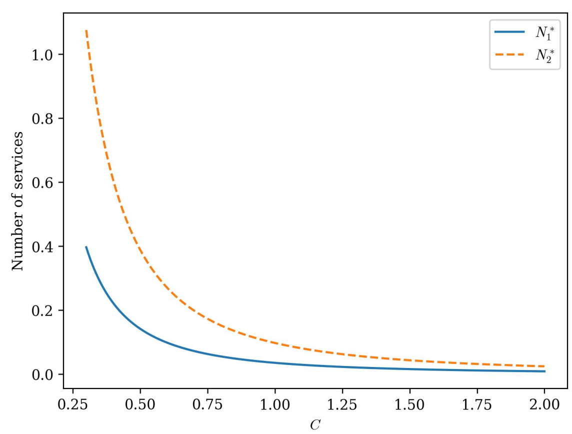

Figure 5.12: Comparison of equilibrium number of services in full coverage of both customer groups

Figure 5.12 compares the number of services offered by the leader and the follower. The follower provides a higher number of services than the leader. Although the follower sets a lower price, they attract a larger customer base, which generates higher total revenue and allows for greater investment in ancillary services . Consequently, the follower ultimately offers a higher number of ancillary service.

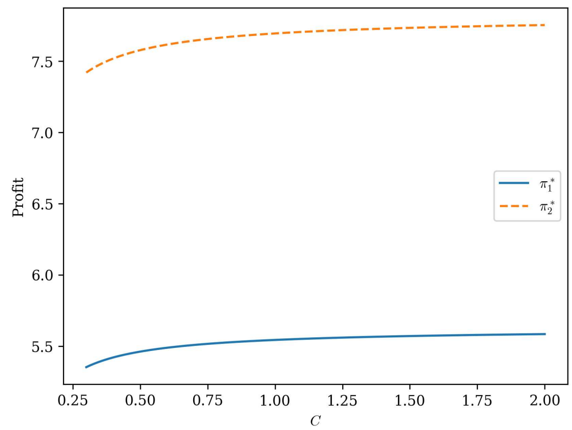

Figure 5.13: Comparison of equilibrium profit in full coverage of both customer groups

Figure 5.13 compares the profits of the leader and the follower. The follower earns a strictly higher profit. This outcome arises because, in a symmetric setting, the follower benefits from observing the leader's decisions and can equilibriumly adjust their own strategy in response. By offering a greater number of services and setting a lower price, the follower attracts more customers and enhances overall customer utility, which ultimately leads to higher profitability.

In the full-coverage case (section 5.2.3), where both premium and economy customers are fully served, the follower sets a lower price but offers a higher number of services, and yet achieves a higher profit than the leader. This outcome may appear counterintuitive, as one might expect a lower price and higher number of services to reduce profitability. However, under full coverage, the total number of customers in the market is fixed, and competition is driven primarily by differences in perceived utility rather than by price alone. By offering more services, the follower increases customer utility, attracts a larger share of the market, and compensates for the lower price through higher demand and overall revenue.

This outcome contrasts with the partial-coverage case (section 5.2.2), where not all customers are served and the leader benefits from setting a lower price to expand market coverage. Under full coverage, the follower cannot rely on price to expand demand, and therefore competes more effectively through service differentiation. Consequently, the follower's strategy of combining lower prices with increased number of services allows it to capture a larger share of the market and achieve higher overall profit.

Proposition 5.1. *When customers are uniformly distributed along the demand line, the equilibrium outcomes* (P_i^*, N_i^*, π_i^*) *belong to different feasibility regions depending on the premium and economy market coverage conditions:*

(i) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{4} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{4},$$

then

$$(P_i^*,\, N_i^*,\, \pi_i^*) \in \mathcal{F}(R_P,\, R_E).$$

(ii) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{4} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{4},$$

then

$$(P_i^*,\, N_i^*,\, \pi_i^*) \in \mathcal{F}(R_E).$$

(iii) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{4} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{4},$$

then

$$(P_i^*,\, N_i^*,\, \pi_i^*) \notin \mathcal{F}(R_P,\, R_E).$$

Proof. The proof proceeds by examining each case separately:

(i) When $\left(\frac{R_P+\sqrt{N_i}-P_i}{t} < \frac{\lambda}{4}\right)$ and $\left(\frac{R_E+\sqrt{N_i}-P_i}{t} < \frac{\lambda}{4}\right)$, both premium and economy markets are only partially covered. Therefore, the retailer's market share depends R_P and R_E. Hence, the equilibrium decisions and profit are also functions of R_P and R_E (section 5.2.1).

(ii) When $\left(\frac{R_P+\sqrt{N_i}-P_i}{t} > \frac{\lambda}{4}\right)$ and $\left(\frac{R_E+\sqrt{N_i}-P_i}{t} < \frac{\lambda}{4}\right)$, all premium customers are served, while the economy group is only partially served. Consequently, the market share depends solely on R_E. Since the equilibrium decisions and profit depend on the market share, they are functions of R_E (section 5.2.2).

(iii) When $\left(\frac{R_P+\sqrt{N_i}-P_i}{t} > \frac{\lambda}{4}\right)$ and $\left(\frac{R_E+\sqrt{N_i}-P_i}{t} > \frac{\lambda}{4}\right)$, both groups are fully served. Hence, the entire market is covered, and the market share is independent of R_P and R_E. Consequently, the equilibrium decisions are not a function of R_P and R_E (section 5.2.3).

$\square$

Proposition 5.2. *When customers are uniformly distributed along the demand line:*

(i) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{4} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{4},$$

then

$$P_1^* = P_2^*, \qquad N_1^* = N_2^*, \qquad \pi_1^* = \pi_2^*.$$

(ii) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{4} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{4},$$

then

$$P_1^* < P_2^*, \qquad N_1^* < N_2^*, \qquad \pi_1^* < \pi_2^*.$$

(iii) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{4} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{4},$$

then

$$P_1^* > P_2^*, \qquad N_1^* < N_2^*, \qquad \pi_1^* < \pi_2^*.$$

5.3 Case two: Customers are non-uniformly distributed along the demand line

The second case explored in this paper involves the non-uniform distribution of two customer groups: premium customers and economy customers, along the demand line. This situation is considered more realistic. In this context, it is assumed that the number of premium customers is relatively low on the left side of the demand line and increases gradually as you move towards the right side. Conversely, the number of economy customers is higher on the left side of the demand line and gradually decreases as you progress towards the right side. This case is often encountered in densely populated metropolitan areas, where wealthier individuals tend to reside in specific neighborhoods, while less affluent residents live in other neighborhoods.

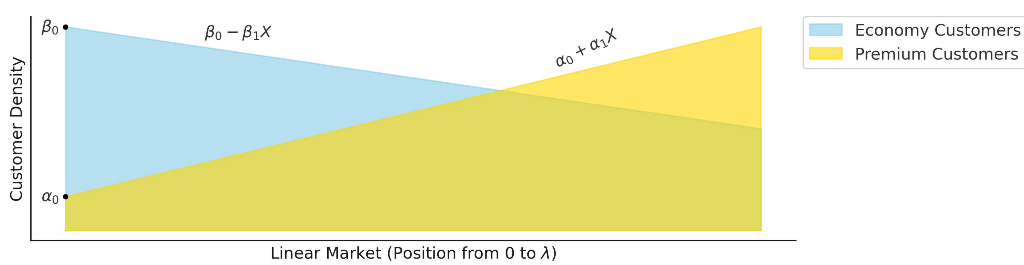

Figure 5.14: Depiction of the non-uniform market

Figure 5.14 serves as an illustration of the situation described.

Some new parameters need to be defined here:

α_0: the number of premium customers on the left side of the demand line (point 0).

α_1: the rate with which the number of premium customers increases when moving toward the right side of the demand line.

β_0: the number of economy customers on the left side of the demand line (point 0).

β_1: the rate with which the number of economy customers decreases along the demand line.

At any given point x on the demand line, where ($0 < x < \lambda$), the premium demand is ($f_P(x) = \alpha_0 + \alpha_1 x$) and the economy demand is ($f_E(x) = \beta_0 - \beta_1 x$).

N.B. Throughout section 5.3, we assume $\alpha_1 \neq \beta_1$. The number of premium and economy customers at any given point are $f_P(x) = \alpha_0 + \alpha_1 x$, and $f_E(x) = \beta_0 - \beta_1 x$, respectively. Hence, the total number of customers along the market is $f_T(x) = f_P(x) + f_E(x) = \alpha_0 + \beta_0 + (\alpha_1 - \beta_1)\, x$. When $\alpha_1 = \beta_1$, the total density becomes uniform (constant in x). Consequently, $\alpha_1 = \beta_1$ falls outside the scope of our non-uniform analysis and is excluded. All results below are stated under the maintained assumption $\alpha_1 \neq \beta_1$.

N.B. It was previously proved in chapter 3 that the two retailers will always co-locate when the distribution of customers is non-uniform. This result is used in the following analysis.

Similar to the uniform case, depending on the magnitude of the travel costs relative to customers' reservation prices, three distinct market coverage cases may arise. In the first case, both premium and economy segments are only partially served. In the second case, the premium segment is fully covered, whereas the economy segment is served only partially. In the third case, both premium and economy customers are fully served across the entire market.

In the subsequent sub-sections, each of these three distinct cases is modeled and analyzed in detail.

5.3.1 Both premium and economy customers can only be partially served

This situation arises when the travel cost is relatively high compared to the reservation prices of both premium and economy customers. Consequently, the maximum distance that each retailer can cover for either customer segment becomes less than half of the market length ($\frac{R_P+\sqrt{N_i}-P_i}{t} < \frac{\lambda}{2}$, $\frac{R_E+\sqrt{N_i}-P_i}{t} < \frac{\lambda}{2}$). In other words, once the maximum service distance that a retailer can cover on either side is less than half of the total market length, λ, the overall coverage is indeed less than λ. This implies that each retailer can serve only a segment of the market rather than its entirety. Therefore, both customer groups are partially served because the retailer's effective range does not extend across the full market length.

Each retailer is assumed to locate at a point X_i^*, whose equilibrium value will be determined later. The maximum distance that a retailer can serve premium customers is given by $\frac{R_P+\sqrt{N_i}-P_i}{t}$. Hence, the retailer can attract premium customers located within the interval $[X_i^* - \frac{R_P+\sqrt{N_i}-P_i}{t}, X_i^* + \frac{R_P+\sqrt{N_i}-P_i}{t}]$. Similarly, the maximum distance that the retailer can serve economy customers is $\frac{R_E+\sqrt{N_i}-P_i}{t}$, so the retailer covers economy customers within the range $[X_i^* - \frac{R_E+\sqrt{N_i}-P_i}{t},\ X_i^* + \frac{R_E+\sqrt{N_i}-P_i}{t}]$.

$$\pi_i = \frac{P_i}{2}\left(\int_{X_i-\frac{R_P+\sqrt{N_i}-P_i}{t}}^{X_i+\frac{R_P+\sqrt{N_i}-P_i}{t}} (\alpha_0 + \alpha_1 x)\, dx + \int_{X_i-\frac{R_E+\sqrt{N_i}-P_i}{t}}^{X_i+\frac{R_E+\sqrt{N_i}-P_i}{t}} (\beta_0 - \beta_1 x)\, dx\right) - CN_i \tag{216}$$

Subject to:

$$0 < \frac{R_E + \sqrt{N_i} - P_i}{t} \tag{217}$$

$$\frac{R_P + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{2} \tag{218}$$

The objective function (216) represents the sum of each retailer's profit from serving premium and economy customers, minus the cost of establishing services. Since the two retailers are co-located, customers choose between them at random. As a result, each retailer earns half the revenue from premium customers (i.e., half the number of premium customers multiplied by the price) and half the revenue from economy customers (i.e., half the number of economy customers multiplied by the price), while bearing the full cost of providing the services.

Constraint (217) ensures that the maximum distance each retailer can serve for economy customers remains positive. Since $R_P > R_E$, this constraint also ensures that the maximum distance each retailer can serve for premium customers remains positive.

Constraint (218) implies that premium customers are only partially served. In other words, the maximum distance that a retailer can serve for premium customers is less than half of the total market length. Since $R_P > R_E$, this also implies that economy customers are only partially served.

Solving to first-order-conditions results:

$$P_i^* = \frac{2Ct\,(R_P\alpha_1 - R_E\beta_1) + (R_E - R_P)(\alpha_0\beta_1 + \alpha_1\beta_0)}{2Ct\,(\alpha_1 - \beta_1)} \tag{219}$$

$$N_i^* = \left(\frac{(R_P - R_E)(\alpha_0\beta_1 + \alpha_1\beta_0)}{2Ct\,(\alpha_1 - \beta_1)}\right)^2 \tag{220}$$

$$X_i^* = \frac{S^* - (\alpha_0 + \beta_0)}{\alpha_1 - \beta_1} \tag{221}$$

Where:

$$S^* = \frac{2Ct\,(R_P - R_E)(\alpha_0\beta_1 + \alpha_1\beta_0)}{2Ct\,(R_E\beta_1 - R_P\alpha_1) + (R_P - R_E)(\alpha_0\beta_1 + \alpha_1\beta_0)} \tag{222}$$

The equilibrium location X_i^* represents the equilibrium position of retailer i along the market line, determined by the spatial distribution and relative importance of premium and economy customers. The term S^* in the numerator captures the overall market potential, which depends on the reservation prices of both customer groups (R_P and R_E), the travel cost t, and the heterogeneity parameters α and β. Subtracting $(\alpha_0 + \beta_0)$ adjusts this potential for the baseline density of customers at the market boundaries. The denominator $(\alpha_1 - \beta_1)$ reflects the difference in the spatial slopes of the two customer segments and therefore drives the direction and magnitude of the location shift.

When $\alpha_1 > \beta_1$, premium customers become more dominant, encouraging retailers to locate closer to the premium side to benefit from higher willingness to pay. Conversely, when $\beta_1 > \alpha_1$, economy customers dominate the spatial distribution, leading the retailers to position themselves nearer to that group to capture higher demand volume, even at lower prices.

The equilibrium price P_i^* reflects the balance between cost factors, represented by C and t, and market characteristics, represented by the reservation prices of premium and economy customers (R_P and R_E) and their spatial distribution parameters α and β.

The numerator combines two opposing effects. The term $2Ct(R_P\alpha_1 - R_E\beta_1)$ captures how differences in reservation prices and customer densities influence retailers' ability to charge higher prices when customers with higher willingness to pay are more concentrated in specific regions. The second term $(R_E - R_P)(\alpha_0\beta_1 + \alpha_1\beta_0)$ adjusts for the baseline asymmetry in customer distribution, accounting for how relative densities at the two ends of the market affect equilibrium pricing.

Economically, P_i^* increases when premium customers have significantly higher reservation prices than economy customers, when premium customers are more spatially concentrated ($\alpha_1 > \beta_1$), and when travel and service costs are moderate, allowing retailers to differentiate without losing demand. Conversely, when the economy segment dominates ($\beta_1 > \alpha_1$), the equilibrium price decreases due to higher price sensitivity.

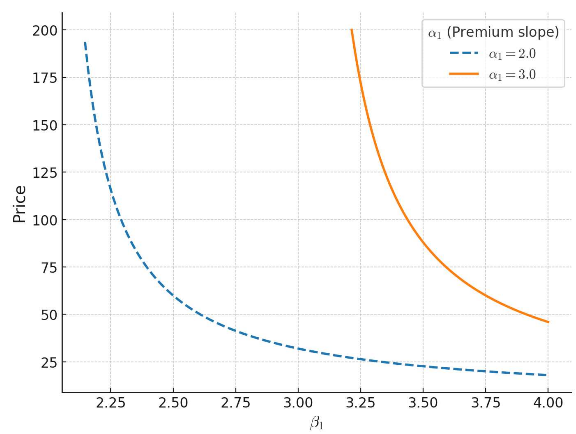

Figure 5.15: Effect of α_1 and β_1 on equilibrium price in 5.3.1

Figure 5.15 illustrates the effect of α_1 and β_1 on the equilibrium price P_i^*. For fixed values of α_1, the equilibrium price decreases as β_1 increases. This occurs because a higher slope of the economy customer density (β_1) implies a larger concentration of economy customers, who are typically more price-sensitive and have lower reservation prices. Consequently, retailers must reduce prices to attract and retain these customers. Conversely, an increase in α_1 leads to higher equilibrium prices, as a steeper premium density indicates a larger share of premium customers with higher willingness to pay. In this case, retailers can charge higher prices while still maintaining demand from the premium segment.

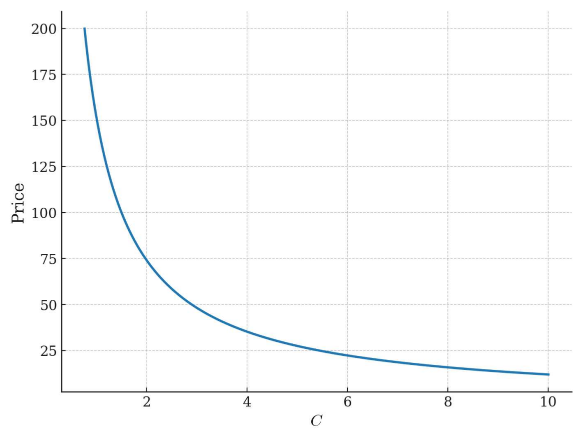

Figure 5.16: Effect of service establishment cost on equilibrium price in 5.3.1

Figure 5.16 represents the effect of service establishment cost on equilibrium price. The equilibrium price decreases steadily as the cost of providing services increases.

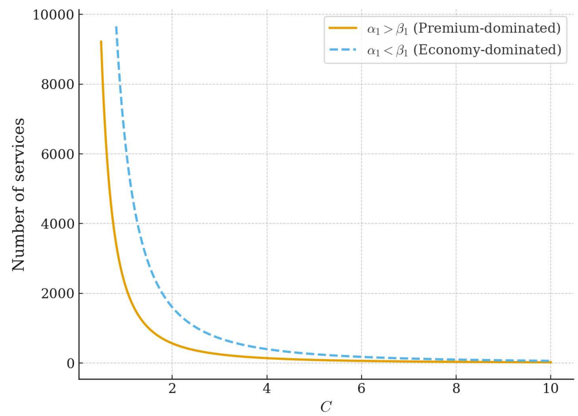

Figure 5.17: Effect of service establishment cost on equilibrium number of services in 5.3.1

Figure 5.17 represents the effect of service establishment costs on the number of services. The equilibrium number of services decreases steeply when cost of establishing services increases.

5.3.2 Full coverage of premium customers and partial coverage of economy customers

This situation occurs when the travel cost is relatively low for premium customers compared to their reservation price, but relatively high for economy customers.

Since all premium customers are served, it is the distribution of economy customers that determines the retailers' locations. Given that economy customers are more concentrated on the left side of the demand line, the retailers' equilibrium locations tend to shift toward that side as well.

To maximize the number of economy customers covered—and consequently maximize their profit—the two retailers choose the following locations:

$$X_1^* = X_2^* = \frac{R_E + \sqrt{N_i} - P_i}{t} \tag{223}$$

The profit function of each retailer would be:

$$\pi_i = \frac{P_i}{2}(\int_0^{\lambda} (\alpha_0 + \alpha_1 x)\,dx + \int_0^{\lambda - 2(\frac{R_E + \sqrt{N_i} - P_i}{t})} (\beta_0 - \beta_1 x)\,dx) - CN_i \tag{224}$$

Subject to:

$$\frac{\lambda}{2} < \frac{R_P + \sqrt{N_i} - P_i}{t} \tag{225}$$

$$0 < \frac{R_E + \sqrt{N_i} - P_i}{t} \tag{226}$$

$$\frac{R_E + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{2} \tag{227}$$

Equation (224) represents the sum of each retailer's profit from serving premium and economy customers, minus the cost of establishing services. Since the two retailers are co-located, customers choose between them at random. As a result, each retailer earns half the revenue from premium customers (i.e., half the number of premium customers multiplied by the price) and half the revenue from economy customers (i.e., half the number of economy customers multiplied by the price), while bearing the full cost of providing the services.

Constraint (225) implies that the premium market is fully covered.

Constraint (226) ensures that the maximum distance each retailer can serve for economy customers remains positive.

Constraint (227) implies that the economy market is only partially covered.

Solving to first-order-conditions results:

$$P_i^* = R_P - \frac{t\lambda}{2} + \frac{1}{4C}\left(\alpha_0\lambda + \frac{\alpha_1}{2}\lambda^2 + \frac{2\beta_0}{t}(R_P - R_E) - \frac{2\beta_1}{t^2}(R_P - R_E)^2\right) \tag{228}$$

$$N_i^* = \left[\frac{1}{4C}\left(\alpha_0\lambda + \frac{\alpha_1}{2}\lambda^2 + \frac{2\beta_0}{t}(R_P - R_E) - \frac{2\beta_1}{t^2}(R_P - R_E)^2\right)\right]^2 \tag{229}$$

$$\pi_i^* = \frac{1}{2}\left[\alpha_0\lambda + \frac{\alpha_1}{2}\lambda^2 + \frac{2\beta_0}{t}(R_P - R_E) - \frac{2\beta_1}{t^2}(R_P - R_E)^2\right]\left(R_P - \frac{t\lambda}{2}\right) \tag{230}$$

$$+\frac{1}{16C}\left[\alpha_0\lambda + \frac{\alpha_1}{2}\lambda^2 + \frac{2\beta_0}{t}(R_P - R_E) - \frac{2\beta_1}{t^2}(R_P - R_E)^2\right]^2 \tag{231}$$

The closed-form expression for the equilibrium price (228), provides valuable insights into how demand, and cost parameters jointly determine retailers' pricing decisions. The formula reveals two main components: a baseline term $\left(R_P - \frac{t\lambda}{2}\right)$, representing the effective price cap when service costs are high, and an adjustment term proportional to $\frac{1}{4C}$, which amplifies or dampens price depending on demand and heterogeneity conditions. An increase in the service cost C monotonically reduces the equilibrium price, as higher operating costs compress the feasible profit margin (figure 5.18).

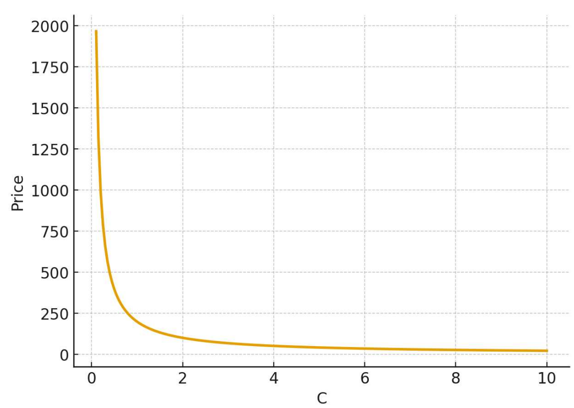

Figure 5.18: Effect of service establishment cost on equilibrium price in 5.3.2

Conversely, stronger market demand—captured by higher values of α_0, α_1, or β_0—raises the equilibrium price through greater potential market coverage and higher willingness to pay. The difference in reservation prices between premium and economy segments $(R_P - R_E)$ also plays a central role: a wider gap encourages higher pricing, but this effect diminishes as customer heterogeneity increases, due to the negative quadratic term involving β_1.

Travel cost t and market length λ enter with opposite effects: while larger t reduces the effective reach and thus the price, a moderate increase in λ can expand the accessible market and justify higher prices when demand density (α_0, α_1) is sufficiently strong. Overall, this closed-form solution captures the balance between cost efficiency, spatial differentiation, and market heterogeneity in determining equilibrium price.

The closed-form solution for the equilibrium number of services (229), illustrates how cost and demand parameters jointly influence the retailer's equilibrium number of services. The expression shows that N_i^* is inversely proportional to the square of the service establishment cost C, indicating that as service provision becomes more expensive, the retailer sharply reduces the number of offered services (figure 5.19).

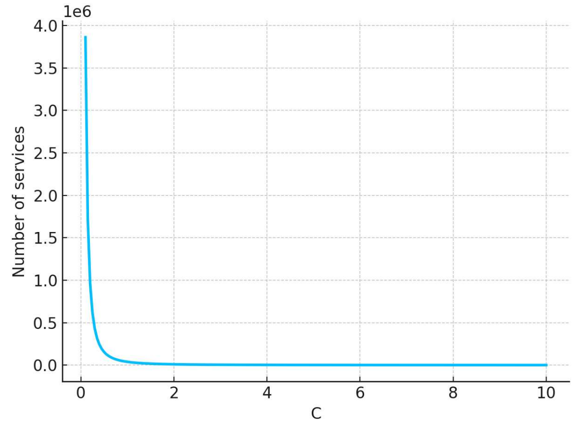

Figure 5.19: Effect of service establishment cost on equilibrium number of services in 5.3.2

Conversely, higher demand potential—reflected by larger values of α_0, α_1, or β_0—significantly increases N_i^*, as these parameters expand the size and profitability of the market. The difference between the premium and economy reservation prices $(R_P - R_E)$ also has a nonlinear effect: while a wider gap initially increases N_i^* by improving overall willingness to pay, excessive heterogeneity reduces it due to the negative quadratic term weighted by β_1.

Additionally, higher travel cost t lowers the effective service reach and thus diminishes the equilibrium service level, whereas a larger market length λ boosts the equilibrium number of services when demand intensity is strong enough. Overall, this formulation emphasizes that the retailer's service decision is highly sensitive to operational costs and market structure, balancing cost.

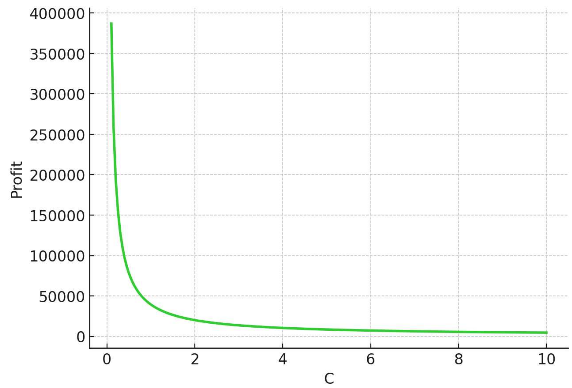

Figure 5.20: Effect of service establishment cost on equilibrium profit in 5.3.2

Figure 5.20 represents the effect of service provision cost on the profit. Since both price and number of services decrease sharply with an increase in service provision cost, the equilibrium profit also follows the same behaviour.

5.3.3 Full coverage of both customer groups

This situation arises when the travel costs are relatively low compared to the reservation prices for both economy and premium customers.

The two retailers co-locate at the center of the demand line.

Their profit function is:

$$\pi_i = \frac{P_i}{2}(\int_0^{\lambda} (\alpha_0 + \alpha_1 x)\, dx + \int_0^{\lambda} (\beta_0 - \beta_1 x)\, dx) - CN_i \tag{232}$$

Subject to:

$$\frac{\lambda}{2} < \frac{R_E + \sqrt{N_i} - P_i}{t} \tag{233}$$

Equation (232) represents the objective function that each retailer seeks to maximize, consisting of total profit minus the cost of providing services. Since both retailers are co-located, customers choose between them randomly. Consequently, each retailer earns half the revenue from the total number of potential customers, calculated as half the customer base multiplied by the price.

constraint (233), implies that all of the economy customers can be served, which means all of the premium customers can also be served.

The equilibrium number of services and equilibrium price are:

$$N_i^* = 0 \tag{234}$$

$$P_i^* = R_E - \frac{\lambda t}{2} \tag{235}$$

Since the entire market can be fully served for both premium and economy customers, the coverage—and thus the number of customers—remains constant and is not affected by price. Given that the profit function is linear and increasing in P_i , the equilibrium price corresponds to the highest feasible price. The maximum feasible price is the one that makes the utility of the farthest economy customer to purchase from each retailer exactly zero.

Moreover, the profit function is linearly decreasing with respect to N_i , indicating that the number of services should be set at its minimum value, which is zero. This implies that, in this situation, no additional service is offered.

Proposition 5.3. *When customers are non-uniformly distributed along the demand line:*

(i) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{2} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{2},$$

then

$$(P_i^*,\, N_i^*,\, \pi_i^*) \in \mathcal{F}(R_P,\, R_E).$$

(ii) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{2} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} < \frac{\lambda}{2},$$

then

$$(P_i^*,\, N_i^*,\, \pi_i^*) \in \mathcal{F}(R_E).$$

(iii) If

$$\frac{R_P + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{2} \quad \textit{and} \quad \frac{R_E + \sqrt{N_i} - P_i}{t} > \frac{\lambda}{2},$$

then

$$(P_i^*,\, \pi_i^*) \in \mathcal{F}(R_E) \quad \textit{and} \quad N_i^* = 0.$$

Proof. The proof proceeds by examining each case separately:

(i) When $\left(\frac{R_P+\sqrt{N_i}-P_i}{t} < \frac{\lambda}{2}\right)$ and $\left(\frac{R_E+\sqrt{N_i}-P_i}{t} < \frac{\lambda}{2}\right)$, both premium and economy markets are only partially covered. Therefore, the retailer's market share depends R_P and R_E. Hence, the equilibrium decisions and profit are also functions of R_P and R_E (section 5.3.1).

(ii) When $\left(\frac{R_P+\sqrt{N_i}-P_i}{t} > \frac{\lambda}{2}\right)$ and $\left(\frac{R_E+\sqrt{N_i}-P_i}{t} < \frac{\lambda}{2}\right)$, all premium customers are served, while the economy group is only partially served. Consequently, the market share depends solely on R_E. The equilibrium decisions are functions of both R_E and R_P (section 5.3.2).

(iii) When $\left(\frac{R_P+\sqrt{N_i}-P_i}{t} > \frac{\lambda}{2}\right)$ and $\left(\frac{R_E+\sqrt{N_i}-P_i}{t} > \frac{\lambda}{2}\right)$, both groups are fully served. Hence, the entire market is covered, and the market share is independent of R_P and R_E. Consequently, the price and profit are only functions of R_E and not R_P, while the equilibrium number of services is constant and not dependent to either reservation price (section 5.3.3).

□

Proposition 5.4. *When customers are non-uniformly distributed along the demand line the equilibrium of the game is symmetric. In particular,*

$$P_1^* = P_2^*, \qquad N_1^* = N_2^*, \qquad \pi_1^* = \pi_2^*.$$

5.4 Analysis of the results

In this section, the results obtained in the previous analyses are examined and compared. The discussion is organized into three parts. First, the outcomes corresponding to the case in which customers are uniformly distributed along the market line are presented and analyzed. Next, the results for the case of a non-uniform customer distribution are discussed to highlight the effects of spatial concentration. Finally, the two cases are compared to identify both similarities and differences, with particular attention to how the spatial distribution of customers influences the retailers' equilibrium decisions regarding price, location, and the number of ancillary services offered.

5.4.1 Results of the uniform case

When customers are uniformly dispersed along the demand line, the two retailers are better off choosing different locations, contrary to the traditional Hotelling problem in which retailers co-locate.

Considering the equilibrium price and number of services, the follower always sets a higher price and offers a higher number of services to attract more customers, gain market share, and achieve higher profit compared to the leader.

Table 5.1 compares the parameters affecting the equilibrium price across different customer coverage scenarios.

Table 5.1: Parameters affecting the equilibrium price

Coverage	Price Parameters
Premium: partial, Economy: partial	R_E, R_P, λ,t,C,γ
Premium: full, Economy: partial	R_E, λ,t,C,γ.
Premium: full, Economy: full	R_E, λ,t,C.

It is evident that the equilibrium price and profit depend on the length of the demand line (which indicates the number of potential customers), travel cost, service establishment cost, and the distribution of customers along the demand line (i.e., the percentage of each customer group).

Moreover, when both customer groups are partially served, the equilibrium price and profit depend on the reservation prices of both economy and premium customers. However, when premium customers are fully served—regardless of whether economy customers are partially or fully served—the equilibrium price and profit depend only on the reservation price of economy customers and not on that of premium customers.

This is because, when both customer groups are partially served, the reservation price of each group defines the farthest economy and premium customers that can be reached. In other words, each group's reservation price determines the retailer's market share for that customer type. However, when premium customers are fully served, and given that their reservation price is higher than that of economy customers, only the reservation price of economy customers influences the equilibrium price and profit.

Table 5.2 shows the equilibrium price set by the leader for different values of R_P and R_E, when $\lambda = 10$, $C = 3$, $\gamma = 0.4$ and $t = 1$; The table clearly shows that the equilibrium price increases with broader demand coverage. Specifically, the leader sets a higher price when both customer groups are fully served, compared to when premium customers are fully served and

Table 5.2: equilibrium Price set by the leader for the three different cases in the uniform case

R_P	R_E	Price		
		I	II	III
1.0	0.5	0.175		
1.5	0.5	0.225		
1.5	1.0	0.300		
2.0	0.5	0.275		
2.0	1.0	0.350		
2.0	1.5	0.425		
2.5	0.5	0.325		
2.5	1.0	0.400		
2.5	1.5	0.475		
2.5	2.0	0.550		
3.0	1.0		0.473	
3.0	1.5		0.562	
3.0	2.0		0.651	
3.0	2.5		0.739	
4.0	1.0		0.473	
4.0	1.5		0.562	
4.0	2.0		0.651	
4.0	2.5		0.739	
5.0	1.0		0.473	
5.0	1.5		0.562	
5.0	2.0		0.651	
5.0	2.5		0.739	
6.0	1.0		0.473	
6.0	1.5		0.562	
6.0	2.0		0.651	
6.0	2.5		0.739	
3.0	2.5			0.625
4.0	2.5			0.625
4.0	3.5			0.625
5.0	2.5			0.625
5.0	3.5			0.625
5.0	4.5			0.625
6.0	2.5			0.625
6.0	3.5			0.625
6.0	4.5			0.625
6.0	5.5			0.625

economy customers are only partially served. Likewise, the price is higher in the latter case than when both customer groups are served partially.

Table 5.3 shows the equilibrium price set by the follower for different

Table 5.3: equilibrium Price set by the follower for the three different cases in the uniform case

R_P	R_E	Price		
		I	II	III
1.0	0.5	0.175		
1.5	0.5	0.225		
1.5	1.0	0.300		
2.0	0.5	0.275		
2.0	1.0	0.350		
2.0	1.5	0.425		
2.5	0.5	0.325		
2.5	1.0	0.400		
2.5	1.5	0.475		
2.5	2.0	0.550		
3.0	0.5		0.739	
3.0	1.0		0.910	
3.0	1.5		1.081	
3.0	2.0		1.251	
3.5	0.5		0.739	
3.5	1.0		0.910	
3.5	1.5		1.081	
3.5	2.0		1.251	
4.0	0.5		0.739	
4.0	1.0		0.910	
4.0	1.5		1.081	
4.0	2.0		1.251	
4.5	0.5		0.739	
4.5	1.0		0.910	
4.5	1.5		1.081	
4.5	2.0		1.251	
5.0	4.5			2.745
5.5	4.5			2.745
5.5	5.0			2.745
6.0	4.5			2.745
6.0	5.0			2.745
6.0	5.5			2.745
6.5	4.5			2.745
6.5	5.0			2.745
6.5	5.5			2.745
6.5	6.0			2.745

values of R_P and R_E, when $\lambda = 10$, $C = 3$, $\gamma = 0.4$ and $t = 1$; The table clearly shows that the equilibrium price increases with broader demand coverage. Specifically, the leader sets a higher price when both customer groups are fully served, compared to when premium customers are fully served and economy customers are only partially served. Likewise, the price is higher in the latter case than when both customer groups are served partially.

Comparing Table 5.2 with Table 5.3 shows that the follower chooses the same price as the leader when both customer groups are only partially served. However, when at least one customer group is fully served, the follower sets a higher price than the leader. Moreover, it can be observed that in the case where the full market for both customer groups is served, the equilibrium price becomes independent of the reservation prices of either customer group. This observation was also explained in proposition 5.1.

Considering the equilibrium number of services provided in each case, and given that it depends on the equilibrium price, it can be inferred that as customer coverage increases, retailers tend to offer a greater number of services. Furthermore, the follower typically provides more services in order to capture a larger market share and increase profit.

5.4.2 Results of the non-uniform case

When customers are non-uniformly distributed along the demand line, the two retailers ultimately choose the same location, provide the same number of services, and offer identical prices. Consequently, both retailers earn the same profit.

Table 5.4 compares the parameters affecting the equilibrium price in non-uniform case across different customer coverage case.

Table 5.4: Parameters affecting the equilibrium price

Coverage	Price Parameters
Premium: partial, Economy: partial	R_E, R_P, λ,t,C,α_0, α_1, β_0, β_1.
Premium: full, Economy: partial	R_PR$_E$, λ,t,C, α_0, α_1, β_0, β_1.
Premium: full, Economy: full	R_E, λ,t,C, α_0, α_1, β_0, β_1.

It goes without saying that the equilibrium price, number of services, and consequently the equilibrium profit depend not only on the total number of premium and economy customers, but also on how these two customer groups are distributed along the demand line.

Moreover, the length of the demand line, travel cost, and service establishment cost also influence the equilibrium variables under each customer coverage case.

When both premium and economy customers are partially served—typically when transportation costs are relatively high compared to the reservation prices of both groups—the equilibrium price depends on the reservation prices of both premium and economy customers. However, when at least the premium customers are fully served, the equilibrium price depends solely on the reservation price of the economy customers, and not on that of the premium customers.

This is because, when both customer groups are only partially served, the reservation price of each group determines how many customers from that group can be covered. However, when premium customers have a relatively high reservation price and are fully served, their reservation price no longer influences the total number of customers served. In this case, it is the reservation price of the economy customers that acts as an upper bound on the equilibrium price.

5.4.3 Comparison of the uniform and non-uniform cases

Comparing the results of the two cases—where customers are either uniformly or non-uniformly distributed along the demand line—reveals that customer distribution plays a critical role in determining the retailers' equilibrium strategies. The spatial structure of demand significantly affects how competitors position themselves, set prices, and determine the level of service differentiation.

In the uniform distribution case, across all three possible market coverage scenarios, the two retailers select distinct locations, reflecting spatial differentiation as a profitable competitive strategy. The entrant strategically locates farther from the incumbent, sets a higher price, and offers more ancillary services . This configuration allows both retailers to serve different segments of the market while avoiding direct price competition.

In contrast, under the non-uniform distribution case, the retailers ultimately choose to co-locate in all three coverage scenarios. The concentration of customers on each side of the market reduces the incentive for spatial differentiation, leading to an equilibrium where both firms occupy the same location, charge identical prices, and provide the same number of services. In this setting, the competition becomes primarily price- and service-based rather than location-based.

Furthermore, both the equilibrium price and the equilibrium number of services differ systematically between the two distribution settings, indicating that customer heterogeneity influences not only spatial choices but also pricing and service decisions. Consequently, in real-world applications, assuming a uniform customer distribution when the actual distribution is non-uniform can lead to inaccurate strategic conclusions and sub equilibrium market entry or pricing decisions.

5.5 Conclusion

In this chapter, we examined a market-entry problem in which a new retailer considers entering a heterogeneous market composed of two distinct customer segments, while an incumbent retailer is already established in the same market. The entrant aims to determine the equilibrium pricing strategy, the number of ancillary services to offer, and the best location to maximize profitability. The analysis integrates both demand-side heterogeneity and spatial competition, capturing how differences in customers' reservation prices and travel sensitivities affect the entrant's equilibrium decisions. This framework provides insights into how offering ancillary services and strategic positioning influence the success of new market entrants in competitive environments.

The analysis considers two alternative cases for customer distribution along the market line. In the first case, both customer groups are assumed to be uniformly distributed, representing a balanced market where customers are evenly spread across locations. In the second case, each group is more densely concentrated on one side of the line, capturing a non-uniform distribution that reflects differentiated spatial preferences. This distinction allows for a comparative analysis of how customer concentration and market asymmetry influence the entrant's equilibrium pricing, service, and location decisions.

Under the uniform distribution case, the two retailers differentiate themselves spatially by selecting distinct locations along the market line. The entrant adopts a strategy characterized by a higher price and a greater number of ancillary services compared to the incumbent. This outcome reflects the entrant's attempt to attract premium customers and compensate for its later market entry through ancillary services . The combination of spatial separation and service enhancement enables the entrant to establish a profitable niche within a competitive yet evenly distributed market.

In contrast, under the non-uniform distribution case, both retailers choose to co-locate at the same position along the market line. In equilibrium, they adopt identical pricing strategies and offer the same number of ancillary services . This outcome arises because customer concentration on each side of the market reduces the incentive for spatial differentiation, leading to direct competition at the central location. The symmetry in pricing and service levels reflects a more homogeneous competitive environment, where neither retailer can profitably deviate without losing customers.

The analytical results demonstrate that offering ancillary services constitutes an effective strategy for enhancing profitability. Although establishing such services involves additional costs, the presence of ancillary services increases customer attraction and broadens market coverage. By improving the perceived value and convenience for consumers, these services raise overall demand and enable the retailer to charge higher equilibrium prices. Consequently, even after accounting for the associated costs, the net effect of side-service provision remains positive, confirming that offering ancillary services is a profitable and sustainable competitive strategy.

The comparison between the two cases highlights that the spatial distribution of customers has a substantial impact on the retailers' equilibrium strategic decisions. Differences in how customers are dispersed across the market influence not only pricing and service levels but also the degree of spatial differentiation between competitors. Therefore, understanding and accurately modeling customer distribution is essential for effective market entry analysis and for designing competitive strategies that align with the underlying structure of demand.

Chapter 6

Conclusion

6.1 Findings and Contributions

This thesis investigated the role of customer heterogeneity in shaping strategic decisions in location-pricing problems within a competitive retail environment. By modeling a duopoly market on a linear Hotelling line, and segmenting customers into premium and economy types, the study captured varying reservation prices, service expectations, and spatial sensitivities. The goal was to understand how differences in customer characteristics affect equilibrium decisions regarding price, location, and service provision.

A series of game-theoretical models were developed to study both simultaneous and sequential market entry. In the simultaneous entry setting, modeled as a Bertrand game, retailers made decisions concurrently. In the sequential case, a Stackelberg framework was adopted to reflect the leader-follower dynamic. Across both settings, models were analyzed under different assumptions about customer distribution—uniform and non-uniform—to uncover their effect on equilibrium outcomes.

The findings consistently show that customer heterogeneity significantly impacts retailers' strategies. In uniform markets, firms differentiate in location but converge on pricing and service levels. In contrast, non-uniform markets drive convergence across all strategic dimensions. The mode of entry also plays a critical role: in uniform settings, the follower may benefit by offering superior service, while in non-uniform cases, this advantage tends to diminish.

This research contributes to the literature by extending classical location-pricing models to account for heterogeneous customer preferences and asymmetric entry. The results offer both theoretical insight into spatial competition and practical guidance for firms operating in differentiated markets. The results of this thesis have practical relevance across a range of industries, including restaurants, stores, hotels, retail chains, and online shopping platforms, where firms must make strategic decisions regarding location, pricing, and service differentiation in the presence of diverse customer segments.

6.2 Future research

There are several promising directions for future research that could build on the models and findings developed in this thesis.

First, while this study incorporated a basic form of non-uniform customer distribution, future work could explore more complex and realistic spatial distributions. For instance, instead of linear gradients, researchers could examine non-linear or segmented distributions that better reflect real-world market conditions, such as urban–suburban population patterns or clusters of high-income customers in specific zones. Such extensions could reveal new strategic behaviors and equilibrium outcomes.

Another avenue involves relaxing the assumption of similar transportation costs across customer types. It would be interesting to see when the transportation cost of premium customers is strictly higher than that of economy customers, would the results vary or not. And in case the results would be different form those obtained in this thesis, in which way and how much they would vary.

Additionally, future research could explore scenarios in which retailers compete based on sustainable development criteria. In the present study, customer utility is modeled primarily as a function of travel distance. However, future work could incorporate the environmental impact of transportation by considering total travel distance as a proxy for pollution, and potentially including its minimization as part of the objective function.

Furthermore, increasing customer awareness of social sustainability issues—such as working conditions, employee health, and fair compensation—opens new dimensions of competition among retailers. Firms may differentiate themselves by improving labor practices and signaling these efforts to consumers. Finally, different customer segments may assign varying levels of importance to product sustainability attributes, suggesting the need for models that account for heterogeneous preferences regarding environmental and social responsibility

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